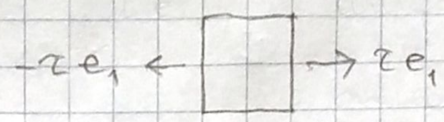
Moment tensor

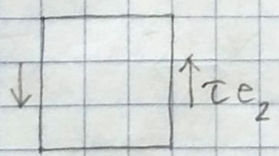


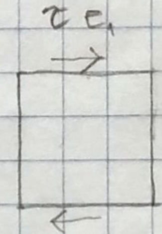
uniform traction

$$M_{\mathcal{F}_1} = \int_{\mathcal{F}_1} \tau e_1 \otimes \left(\frac{l_1}{2} e_1 + s_2 e_2 + s_3 e_3\right) dA$$

$$= \int_{\mathcal{F}_1} \tau e_1 \otimes \frac{l_1}{2} e_1 \, dA + \tau e_1 \otimes \underbrace{\int_{\mathcal{F}_1} (s_2 e_1 + s_3 e_3) \, dA}_{0}$$

$$M_{11} = \tau l_1 \int_{\mathcal{F}_1} e_1 \otimes e_1 \, dA = \tau l_1 A_{\mathcal{F}_1} \, e_1 \otimes e_1 = \tau V_{\mathcal{R}} \, e_1 \otimes e_1$$





$$M_{21} = \tau l_1 \int_{\mathcal{F}_1} e_2 \otimes e_1 \, dA = \tau \underbrace{l_1 A_{\mathcal{F}_1}}_{V_{\mathcal{R}}} e_2 \otimes e_1$$

$$M_{12} = \tau l_2 \int_{\mathcal{F}_2} e_1 \otimes e_2 \, dA = \tau \underbrace{l_2 A_{\mathcal{F}_2}}_{V_{\mathcal{R}}} e_1 \otimes e_2$$

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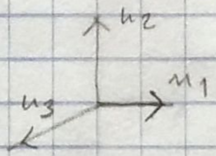
$$M_{21} + M_{12} = \tau V_{\mathcal{R}} \left(e_2 \otimes e_1 + e_1 \otimes e_2 \right)$$

$$\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

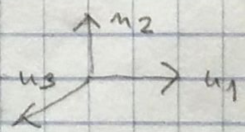
$$M_{21} - M_{12} = \tau V_{\mathcal{R}} \left(e_2 \otimes e_1 - e_1 \otimes e_2 \right)$$

$$\begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$A_{\mathcal{F}_1} = \mathrm{vol}(n_1, u_2, u_3)$$



$$A_{\mathcal{F}_2} = \mathrm{vol}(u_1, n_2, u_3)$$



$$A_{\mathcal{F}_3} = \mathrm{vol}(u_1, u_2, n_3)$$

outward unit normal vectors

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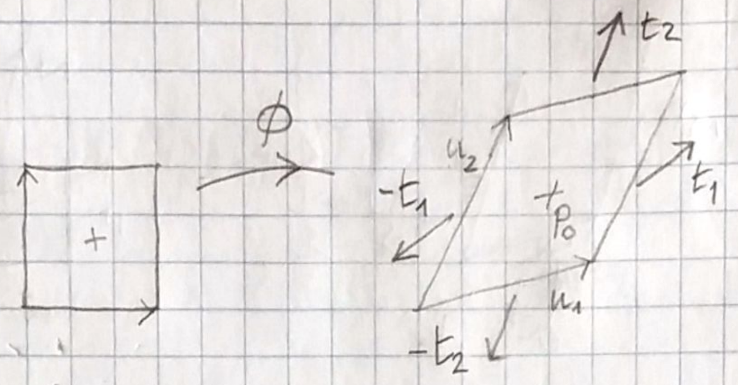

$$t_1 \otimes (x - p_0) = t_1 \otimes \left(x - p_0 - \frac{1}{2} u_1 + \frac{1}{2} u_1\right)$$
$$= t_1 \otimes \left(x - \left(p_0 + \frac{1}{2} u_1\right)\right) + t_1 \otimes \left(\frac{1}{2} u_1\right)$$

$$\int_{\mathcal{F}_1} t_1 \otimes (x - p_0)\, dA = \frac{1}{2} t_1 \otimes u_1 \int_{\mathcal{F}_1} dA = \frac{1}{2} A_{\mathcal{F}_1} t_1 \otimes u_1$$

$$\left(g_{\mathcal{F}_1} - \left(p_0 + \frac{1}{2} u_1\right)\right) = \frac{1}{A_{\mathcal{F}_1}} \int_{\mathcal{F}_1} \left(x - \left(p_0 + \frac{1}{2} u_1\right)\right) dA$$

defines the CENTROID $g_{\mathcal{F}_1}$ of the face $\mathcal{F}_1$. Because the face is a parallelogram its center is exactly $\left(p_0 + \frac{1}{2} u_1\right)$.

Hence

$$g_{\mathcal{F}_1} - \left(p_0 + \frac{1}{2} u_1\right) = 0$$

$$\frac{1}{2} t_1 \otimes u_1 A_{\mathcal{F}_1} - \frac{1}{2} t_1 \otimes u_1 A_{\mathcal{F}_1} = t_1 \otimes u_1 A_{\mathcal{F}_1}$$

$$M = (t_1 \otimes u_1) A_{\mathcal{F}_1} + (t_2 \otimes u_2) A_{\mathcal{F}_2} + (t_3 \otimes u_3) A_{\mathcal{F}_3}$$

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$$M n_1 = A_{\mathcal{F}_1} t_1 (u_1 \cdot n_1) + A_{\mathcal{F}_2} t_2 (u_2 \cdot n_1) + A_{\mathcal{F}_3} t_3 (u_3 \cdot n_1)$$

$$n_1 \cdot u_2 = 0 \qquad n_1 \cdot u_3 = 0 \qquad n_1 \in \mathrm{span}\{u_2, u_3\}^{\perp}$$

$$w_1 := u_1 - \underbrace{(u_1 \cdot n_1)}_{h_1} n_1 \;\Rightarrow\; w_1 \cdot n_1 = 0$$

$$\Rightarrow\; w_1 \in \mathrm{span}\{u_2, u_3\}$$

$$u_1 = w_1 + h_1 n_1$$

$$V_{\mathcal{R}} = \mathrm{vol}(u_1, u_2, u_3) = \underbrace{\mathrm{vol}(w_1, u_2, u_3)}_{\downarrow\, 0} + h_1 \underbrace{\mathrm{vol}(n_1, u_2, u_3)}_{A_{\mathcal{F}_1}}$$

$$M n_1 = A_{\mathcal{F}_1} t_1 h_1 = V_{\mathcal{R}} t_1$$

$$n_2 \cdot u_3 = 0 \qquad n_2 \cdot u_1 = 0 \qquad n_2 \in \mathrm{span}\{u_3, u_1\}^{\perp}$$

$$w_2 = u_2 - \underbrace{(u_2 \cdot n_2)}_{h_2} n_2 \;\Rightarrow\; w_2 \cdot n_2 = 0$$

$$\Rightarrow\; w_2 \in \mathrm{span}\{u_3, u_1\}$$

$$u_2 = w_2 + h_2 n_2$$

$$V_{\mathcal{R}} = \mathrm{vol}(u_1, u_2, u_3) = \mathrm{vol}(u_1, w_2, u_3) + h_2 \underbrace{\mathrm{vol}(u_1, n_2, u_3)}_{A_{\mathcal{F}_2}}$$

$$M n_2 = A_{\mathcal{F}_2} t_2 (u_2 \cdot n_2) = V_{\mathcal{R}} t_2$$

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