Energy imbalance and dissipation

$$S \cdot \nabla \bar{v} - \frac{d}{dt}\varphi(F) \geq 0 \qquad \nabla \bar{v} = \dot{F}$$

$$\hat{S}(F) \cdot \dot{F} = \frac{d}{dt}\varphi(F)$$

$$\underbrace{\left(S - \hat{S}(F)\right)}_{\text{dissipative stress } S^+} \cdot \dot{F} \geq 0$$

Replacing the definition of Piola stress

$$(\det F)\left(T - \hat{T}(F)\right)F^{-T} \cdot \dot{F} \geq 0$$

>0

$$\underbrace{\left(T - \hat{T}(F)\right)}_{\text{dissipative stress } T^+} \cdot \dot{F}F^{-1} \geq 0$$

We get also

$$(\det F)\hat{T}(F)F^{-T} \cdot \dot{F} = \frac{d}{dt}\varphi(F)$$

$$\hat{T}(F) \cdot \dot{F}F^{-1} = (\det F)^{-1}\frac{d}{dt}\varphi(F)$$

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Viscous dissipation

We can fulfil the imbalance principle in either form

$$S^+ \cdot \nabla \bar{v} \geq 0$$

or

$$T^+ \cdot \nabla v \geq 0$$

by choosing

$$T^+ = 2\mu \,\mathrm{sym}\,\nabla v$$

Since the inequality

$$\mu \,\mathrm{sym}\,\nabla v \cdot \nabla v \geq 0$$

can be written as

$$\mu \underbrace{(\mathrm{sym}\,\nabla v) \cdot (\mathrm{sym}\,\nabla v)}_{>0} \geq 0$$

then

$$\mu > 0$$

is the only condition to get the dissipation principle (energy imbalance) fulfilled

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