Incompressibility and inner pressure

An incompressible material is characterized by

$\det F = 1 \Rightarrow \operatorname{div} v = 0$ (isochoric motion)

Since $\operatorname{div} v = \operatorname{tr} \nabla v = 0$ then

$$\operatorname{sph}(\nabla v) = \frac{1}{3} \operatorname{tr} \nabla v = 0 \quad \Rightarrow \nabla v = \operatorname{dev} \nabla v$$

Hence

$$T \cdot \nabla v = (\operatorname{dev} T + \operatorname{sph} T) \cdot (\operatorname{dev} \nabla v + \operatorname{sph} \nabla v)$$
$$= \operatorname{dev} T \cdot \operatorname{dev} \nabla v = \operatorname{dev} T \cdot \nabla v$$

Response function from the strain energy

$$\hat{S}(F) \cdot \nabla \bar{v} = \frac{d}{dt} \varphi(F)$$

$$\underbrace{(\det F)}_{1} \hat{T}(F) F^{-T} \cdot \nabla \bar{v} = \frac{d}{dt} \varphi(F)$$

$$\check{T}(F) \cdot \nabla v = \frac{d}{dt} \varphi(F)$$

Because of incompressibility the spherical part of the stress is filtered out from the stress power above and the response function turns out to be a deviatoric tensor valued function.

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This is why the spherical part of the stress, denoted by $-pI$, enters explicitly the general characterization of the stress

$$T = \hat{T}(F) - pI + T^+$$

The dissipative stress T^+ is subject to the dissipation inequality

$$T^+ \cdot \nabla v \geq 0$$

which can possibly be fulfilled by choosing

$$T^+ = 2\mu \operatorname{sym} \nabla v$$

Should the material be compressible even the spherical part of the stress will be delivered by the rate of change of the strain energy as a response function.

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