[2018-05-03]

Incompressible hyperelastic material

$$\hat{S}(F)\cdot\dot{F} = \frac{d}{dt}\varphi(F) \qquad \forall \text{ isochoric motion}$$

isochoric

$$\underbrace{(\det F)}_{1}\hat{T}(F)\cdot\overbrace{\dot{F}F^{-1}}^{\text{isochoric}} = \frac{d}{dt}\varphi(F)$$

$$\hat{T}(F) = \operatorname{dev}\hat{T}(F) + \operatorname{sph}\hat{T}(F)$$

$$\operatorname{dev}\hat{T}(F)\cdot\dot{F}F^{-1} = \frac{d}{dt}\varphi(F)$$

$$T = \hat{T}_e(F) - pI \qquad \operatorname{tr}\hat{T}_e(F) = 0$$

originated from energy

neo-Hookean material

$$\varphi_H(F) = c_1(\iota_1 - 3)$$

$$\hat{S}(F) = 2c_1F \; ; \; \hat{T}(F) = 2c_1FF^T$$

$$\operatorname{dev}\hat{T}(F) = 2c_1\left(FF^T - \frac{1}{3}\operatorname{tr}(FF^T)I\right) \leftarrow \hat{T}_e(F)$$

$$\operatorname{dev}\hat{S}(F) = 2c_1\left(F - \frac{1}{3}\operatorname{tr}(FF^T)F^{-T}\right) \leftarrow \hat{S}_e(F)$$

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Uniaxial deformation (for incompressible materials)

$$-\tau e_1 \leftarrow \square \rightarrow \tau e_1$$

$$[F] = \begin{pmatrix} \lambda & & \\ & 1/\sqrt{\lambda} & \\ & & 1/\sqrt{\lambda} \end{pmatrix}$$

$$M^{ext} = A_{\mathcal{F}_1} \tau e_1 \otimes (\lambda \ell_1 e_1)$$

$$M^{ext} = V_{\mathcal{R}} \tau e_1 \otimes e_1$$

balance equations

$$f^{ext} = 0$$

$$\mathrm{skw}\, M^{ext} = 0$$

$$\mathrm{sym}\, M^{ext} = T V_{\mathcal{R}}$$

stress characterization

$$T = \hat{T}_e(F) - p I$$

strain energy $\varphi(F) = c_1(\iota_1 - 3)$ neo-Hookean

$$[F^T F] = \begin{pmatrix} \lambda^2 & & \\ & 1/\lambda & \\ & & 1/\lambda \end{pmatrix}$$

$$\iota_1 = \mathrm{tr}\, C = \lambda^2 + \frac{2}{\lambda}$$

$$\iota_3 = 1$$

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$$[C] = [F^T F] = \begin{pmatrix} \lambda^2 & & \\ & 1/\lambda & \\ & & 1/\lambda \end{pmatrix}$$

$$\iota_1 = \operatorname{tr} C = \lambda^2 + \frac{2}{\lambda} \qquad \iota_3 = 1$$

$$\frac{d}{dt}\iota_1 = \left(2\lambda - \frac{2}{\lambda^2}\right)\dot\lambda = 2\left(\lambda^2 - \frac{1}{\lambda}\right)\frac{\dot\lambda}{\lambda}$$

$$[\dot F F^{-1}] = \begin{pmatrix} 1 & & \\ & -\frac{1}{2}\lambda^{-\frac{3}{2}} & \\ & & -\frac{1}{2}\lambda^{-\frac{3}{2}} \end{pmatrix} \begin{pmatrix} \lambda^{-1} & & \\ & \lambda^{\frac{1}{2}} & \\ & & \lambda^{\frac{1}{2}} \end{pmatrix} \dot\lambda$$

$$= \begin{pmatrix} \lambda^{-1} & & \\ & -\frac{1}{2}\lambda^{-1} & \\ & & -\frac{1}{2}\lambda^{-1} \end{pmatrix} \dot\lambda = \begin{pmatrix} 1 & & \\ & -\frac{1}{2} & \\ & & -\frac{1}{2} \end{pmatrix} \frac{\dot\lambda}{\lambda}$$

$$[T] = \begin{pmatrix} \sigma_1 & & \\ & \sigma_2 & \\ & & \sigma_3 \end{pmatrix} \qquad \hat T_e(F) = \begin{pmatrix} \hat\sigma_1(\lambda) & & \\ & \hat\sigma_2(\lambda) & \\ & & \hat\sigma_3(\lambda) \end{pmatrix}$$

$$\operatorname{tr} \hat T_e(F) = 0$$

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$$\hat{T}_e(F) = 2c_1\left(FF^T - \frac{1}{3}\operatorname{tr}(FF^T)\,I\right) \qquad \text{neo-Hookean}$$

Since

$$[FF^T] = \begin{pmatrix} \lambda^2 & & \\ & 1/\lambda & \\ & & 1/\lambda \end{pmatrix}$$

we get

$$\operatorname{tr}(FF^T) = \lambda^2 + \frac{2}{\lambda} \qquad \operatorname{sph} FF^T$$

$$(1,1)\quad \lambda^2 - \frac{1}{3}\left(\lambda^2 + \frac{2}{\lambda}\right) = \frac{2}{3}\left(\lambda^2 - \frac{1}{\lambda}\right)$$

$$FF^T - \frac{1}{3}\operatorname{tr}(FF^T)\,I$$

$$(2,2)\quad \frac{1}{\lambda} - \frac{1}{3}\left(\lambda^2 + \frac{2}{\lambda}\right) = -\frac{1}{3}\left(\lambda^2 - \frac{1}{\lambda}\right)$$

$$[\hat{T}_e(F)] = 2c_1\begin{pmatrix} \frac{2}{3} & & \\ & -\frac{1}{3} & \\ & & -\frac{1}{3} \end{pmatrix}\left(\lambda^2 - \frac{1}{\lambda}\right) \qquad \text{deviatoric response}$$

$$\hat{T}_e(F) - pI = M/V_R \qquad \text{balance}$$

$$\begin{cases} \operatorname{dev}(\hat{T}_e(F) - pI) = \operatorname{dev}(M/V_R) \\ \operatorname{sph}(\hat{T}_e(F) - pI) = \operatorname{sph}(M/V_R) \end{cases}$$

$$\frac{M}{V_R} = \tau\, e_1 \otimes e_1 = \tau\left(e_1 \otimes e_1 - \frac{1}{3}I + \frac{1}{3}I\right)$$

$$\left.\begin{aligned} &\text{dev} && \hat{T}_e(F) = \tau\left(e_1 \otimes e_1 - \frac{1}{3}I\right) \\ &\text{sph} && -p = \tau\,\frac{1}{3} \end{aligned}\right\}\ \text{balance}$$

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$$
\begin{array}{ll}
(1,1) & \left\{\begin{array}{l}
\frac{4}{3}\, c \left(\lambda^2 - \frac{1}{\lambda}\right) = \tau - \frac{1}{3}\tau \\[2ex]
-\frac{2}{3}\, c \left(\lambda^2 - \frac{1}{\lambda}\right) = -\frac{1}{3}\tau \\[2ex]
-\frac{2}{3}\, c \left(\lambda^2 - \frac{1}{\lambda}\right) = -\frac{1}{3}\tau
\end{array}\right. \\
(2,2) & \\
(3,3) &
\end{array}
$$

dev balance

$$
\boxed{2c\left(\lambda^2 - \frac{1}{\lambda}\right) = \tau}
$$

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