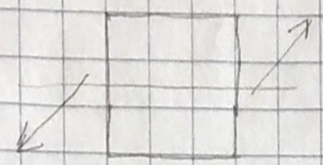
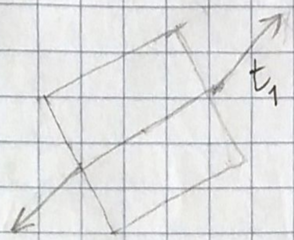
Uniaxial stretch and rotation



$$[F] = \begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \lambda & & \\ & \frac{1}{\sqrt{\lambda}} & \\ & & \frac{1}{\sqrt{\lambda}} \end{pmatrix}$$




$$t_1 = \tau \frac{\sqrt{2}}{2}(e_1 + e_2)$$

$$n_1 = F\bar{n}_1 = F(\ell_1 e_1)$$

$$M = A_{\mathcal{F}_1} \tau \frac{\sqrt{2}}{2}(e_1 + e_2) \otimes F(\ell_1 e_1)$$

$$= A_{\mathcal{F}_1} \tau \frac{\sqrt{2}}{2} \ell_1 (e_1 + e_2) \otimes R(\lambda e_1)$$

$$= A_{\mathcal{F}_1} \tau \frac{\sqrt{2}}{2} \lambda \ell_1 (e_1 + e_2) \otimes (\cos\theta\, e_1 + \sin\theta\, e_2)$$

$$= \frac{\sqrt{2}}{2} V_{\mathcal{B}} \tau \Big(\cos\theta (e_1 \otimes e_1 + e_2 \otimes e_1) + \sin\theta (e_1 \otimes e_2 + e_2 \otimes e_2)\Big)$$

$$\operatorname{skw} M = \frac{\sqrt{2}}{2} V_{\mathcal{B}} \tau \frac{1}{2}\Big(\cos\theta (e_2 \otimes e_1 - e_1 \otimes e_2) + \sin\theta (e_1 \otimes e_2 - e_2 \otimes e_1)\Big)$$

$$= \frac{\sqrt{2}}{2} V_{\mathcal{B}} \tau \frac{1}{2}(\cos\theta - \sin\theta)(e_2 \otimes e_1 - e_1 \otimes e_2)$$

[Notebook page scanned on 2018/09/19]

$$\mathrm{skw}\, M = 0 \quad \Rightarrow \quad \cos\vartheta = \sin\vartheta$$

$$\pm\frac{\sqrt{2}}{2} = \pm\frac{\sqrt{2}}{2}$$

$$\cos\vartheta = \frac{\sqrt{2}}{2}, \quad \sin\vartheta = \frac{\sqrt{2}}{2}$$

$$[R] = \frac{\sqrt{2}}{2}\begin{pmatrix} 1 & -1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & \sqrt{2} \end{pmatrix} \qquad [F] = \frac{\sqrt{2}}{2}\begin{pmatrix} \lambda & -1/\sqrt{\lambda} & 0 \\ \lambda & 1/\sqrt{\lambda} & 0 \\ 0 & 0 & \sqrt{2}/\sqrt{\lambda} \end{pmatrix}$$

$$\cos\vartheta = -\frac{\sqrt{2}}{2}, \quad \sin\vartheta = -\frac{\sqrt{2}}{2}$$

$$[R] = -\frac{\sqrt{2}}{2}\begin{pmatrix} 1 & -1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & -\sqrt{2} \end{pmatrix} \qquad [F] = -\frac{\sqrt{2}}{2}\begin{pmatrix} \lambda & -1/\sqrt{\lambda} & 0 \\ \lambda & 1/\sqrt{\lambda} & 0 \\ 0 & 0 & -\sqrt{2}/\sqrt{\lambda} \end{pmatrix}$$

$$\varphi(F) = \varphi(U) \qquad [U] = \begin{pmatrix} \lambda & & \\ & 1/\sqrt{\lambda} & \\ & & 1/\sqrt{\lambda} \end{pmatrix}$$

$$\hat{T}(F) \cdot \dot{F}F^{-1} = \dot{\varphi}(U)$$

It is convenient to resort to the general expression for the response function

[Notebook page scanned on 2018/09/19]

(uniaxial rotated)

$$\theta = \frac{\pi}{4}$$

$$[B] = \begin{pmatrix} \frac{1+\lambda^3}{2\lambda} & \frac{-1+\lambda^3}{2\lambda} & 0 \\ \frac{-1+\lambda^3}{2\lambda} & \frac{1+\lambda^3}{2\lambda} & 0 \\ 0 & 0 & \frac{1}{\lambda} \end{pmatrix}$$

$$\hat{T}_e(F) = 2c\left(B - \frac{1}{3}(\operatorname{tr} B) I\right) \qquad \text{neo-Hookean}$$

$$T = \hat{T}_e(F) - pI$$

$$[\hat{T}_e(F)] = 2c\,\frac{\lambda^3 - 1}{2\lambda} \begin{pmatrix} \frac{1}{3} & 1 & 0 \\ 1 & \frac{1}{3} & 0 \\ 0 & 0 & -\frac{2}{3} \end{pmatrix}$$

$$[M] = \frac{1}{2}\tau \begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} V_R$$

[Notebook page scanned on 2018/09/19]

$$\mathrm{tr}\,(T - M/V_R) = -3p - \tau$$

$$\mathrm{dev}\,(T - M/V_R) = \left(2c\left(\lambda^2 - \frac{1}{\lambda}\right) - \tau\right) \begin{pmatrix} \frac{1}{6} & \frac{1}{2} & 0 \\ \frac{1}{2} & \frac{1}{6} & 0 \\ 0 & 0 & -\frac{1}{3} \end{pmatrix}$$

$$\boxed{\begin{aligned} \tau + 3p &= 0 \\ 2c\left(\lambda^2 - \frac{1}{\lambda}\right) &= \tau \end{aligned}}$$

[Notebook page scanned on 2018/09/19]