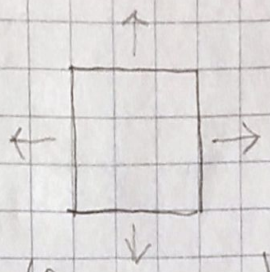
Biaxial deformation



$$t_1 = \tau e_1$$

$$t_2 = \tau e_2$$

$$[F] = \begin{pmatrix} \lambda & & \\ & \lambda & \\ & & \frac{1}{\lambda^2} \end{pmatrix}$$

$$[C] = \begin{pmatrix} \lambda^2 & & \\ & \lambda^2 & \\ & & \frac{1}{\lambda^4} \end{pmatrix} = [U^2] \qquad B = C$$

$$\hat{T}_e(F) = 2c\left(FF^T - \frac{1}{3}\operatorname{tr}(FF^T)I\right) \quad \text{neo-Hookean}$$

$$\lambda^2 - \frac{1}{3}\left(2\lambda^2 + \frac{1}{\lambda^4}\right) = \frac{1}{3}\left(\lambda^2 - \frac{1}{\lambda^4}\right)$$

$$\frac{1}{\lambda^4} - \frac{1}{3}\left(2\lambda^2 + \frac{1}{\lambda^4}\right) = -\frac{2}{3}\left(\lambda^2 - \frac{1}{\lambda^4}\right)$$

$$[\hat{T}_e(F)] = 2c\,\frac{1}{3}\left(\lambda^2 - \frac{1}{\lambda^4}\right)\begin{pmatrix} 1 & & \\ & 1 & \\ & & -2 \end{pmatrix}$$

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$$\frac{M}{V_R} = \hat{T}_e(F) - pI$$

balance & material response

$u_1 = \lambda \bar{u}_1$

$u_2 = \lambda \bar{u}_2$

$$M = A_{\mathcal{F}_1}\, \tau\, e_1 \otimes u_1 + A_{\mathcal{F}_2}\, \tau\, e_2 \otimes u_2$$

$$[\mathrm{cof}\,F] = \begin{pmatrix} \frac{1}{\lambda} & & \\ & \frac{1}{\lambda} & \\ & & \lambda^2 \end{pmatrix}$$

$$A_{\mathcal{F}_1} = A_{\bar{\mathcal{F}}_1} \|(\mathrm{cof}\,F)e_1\| = \ell_2 \ell_3 \frac{1}{\lambda}$$

$$A_{\mathcal{F}_2} = A_{\bar{\mathcal{F}}_2} \|(\mathrm{cof}\,F)e_2\| = \ell_3 \ell_1 \frac{1}{\lambda}$$

$$M = \tau\left(\ell_2 \ell_3 \frac{1}{\lambda}\, e_1 \otimes (\lambda \ell_1 e_1) + \ell_3 \ell_1 \frac{1}{\lambda}\, e_2 \otimes (\lambda \ell_2 e_2)\right)$$

$$M = \tau(\ell_1 \ell_2 \ell_3)(e_1 \otimes e_1 + e_2 \otimes e_2)$$

$$\frac{M}{V_R} = \tau(e_1 \otimes e_1 + e_2 \otimes e_2)$$

balance & response

$$\begin{cases} \mathrm{tr}\,\dfrac{M}{V_R} = -3p \quad \Rightarrow \quad 2\tau = -3p \quad \Rightarrow \quad p = -\dfrac{2}{3}\tau \\[2ex] \mathrm{dev}\,\dfrac{M}{V_R} = \hat{T}_e(F) \quad \Rightarrow \quad \tau\left(1 - \dfrac{2}{3}\right) = \dfrac{2}{3} c \left(\lambda^2 - \dfrac{1}{\lambda^4}\right) \end{cases}$$

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(biaxial deformation)

$$\boxed{\begin{aligned} p &= -\frac{2}{3}\tau \\ \tau &= 2c\left(\lambda^2 - \frac{1}{\lambda^4}\right) \end{aligned}}$$

Notice that if we add a traction on $\mathcal{F}_3$

$$t_3 = \tau\, e_3$$

we get

$$\frac{M}{V_{\mathcal{R}}} = \tau\left(e_1 \otimes e_1 + e_2 \otimes e_2 + e_3 \otimes e_3\right) = \tau I$$

As a consequence

$$p = -\frac{3}{3}\tau \qquad \Rightarrow \qquad p = -\tau$$

$$\tau\left(1 - \frac{3}{3}\right) = 2c\left(\lambda^2 - \frac{1}{\lambda^4}\right) \quad \Rightarrow \quad \lambda = 1$$

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