Power balance laws

$$\underbrace{\int_{\mathcal{R}_o} b_o \cdot v_o \, dV + \int_{\partial\mathcal{R}_o} t_o \cdot v_o \, dA}_{\text{external power expenditure}} = \int_{\mathcal{R}_o} S_o \cdot \nabla v_o \, dV \qquad \forall v_o$$

[→ G. (27.20)]

$$\int_{\mathcal{R}_o} \mu_o \dot{c} \rho_o \, dV = \underbrace{-\int_{\partial\mathcal{R}_o} \mu_o h_o \cdot n_o \, dA}_{\text{external power expenditure}} + \int_{\mathcal{R}_o} h_o \cdot \nabla \mu_o \, dV \qquad \forall \mu_o$$

Free energy imbalance principle

for any constitutive process (for any realizable evolution)

$$S_o \cdot \dot{F}_o + \mu_o \dot{c} \rho_o - h_o \cdot \nabla \mu_o - \frac{d}{dt} \psi \geq 0$$

[very general expression]

If we assume $\psi = \hat{\psi}(F_o, c)$

and $\dfrac{d}{dt}\psi = \hat{S}_o(F_o, c) \cdot \dot{F} + \rho_o \hat{\mu}_o(F_o, c)$

we get

$$\left(S_o - \hat{S}_o(F_o, c)\right) \cdot \dot{F} + \rho_o \left(\mu_o - \hat{\mu}_o(F_o, c)\right) \cdot \dot{c} - h_o \cdot \nabla \mu_o \geq 0$$

[Notebook page scanned on 2018/09/19]

## Fick's law

If we further assume

$$S_0 = \hat{S}_0(F_0, c) \quad \text{and} \quad \mu_0 = \hat{\mu}_0(F_0, c)$$

the free energy imbalance reduces to

$$-h_0 \cdot \nabla\mu_0 \geq 0$$

The simplest way to get this requirement fulfilled is to characterize the molar flux by

$$h_0 = -M_0 \nabla\mu_0 \qquad \text{Fick's law}$$

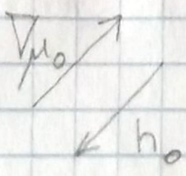
with $M_0$ a positive semidefinite tensor (mobility)

$$M_0 \nabla\mu_0 \cdot \nabla\mu_0 \geq 0$$

As a special case we get

$$h_0 = -\mathcal{M}_0 \nabla\mu_0$$

with $\mathcal{M}_0$ a non negative scalar (mobility coefficient)




characterizing $h_0$ as a vector opposite to $\nabla\mu_0$

[Notebook page scanned on 2018/09/19]