Displacement field u

$$u(x) = \phi(x) - x$$

$$\nabla u \, e = \lim_{h \to 0} \frac{u(\bar{c}(h)) - u(\bar{c}(0))}{h} \qquad \bar{c}'(0) = e$$

$$= \lim_{h \to 0} \frac{(\phi(\bar{c}(h)) - \bar{c}(h)) - (\phi(\bar{c}(0)) - \bar{c}(0))}{h}$$

$$= \lim_{h \to 0} \frac{\phi(\bar{c}(h)) - \phi(\bar{c}(0))}{h} - \lim_{h \to 0} \frac{\bar{c}(h) - \bar{c}(0)}{h}$$

$$= \lim_{h \to 0} \frac{c(h) - c(0)}{h} - \lim_{h \to 0} \frac{\bar{c}(h) - \bar{c}(0)}{h}$$

$$= c'(0) - \bar{c}'(0) = F\bar{c}'(0) - \bar{c}'(0) = (F - I)e$$

$$\Rightarrow \quad \nabla u = F - I \quad , \quad F = I + \nabla u$$

$$[\nabla u] = \begin{pmatrix} u_{1,1} & u_{1,2} & u_{1,3} \\ u_{2,1} & u_{2,2} & u_{2,3} \\ u_{3,1} & u_{3,3} & u_{3,3} \end{pmatrix}$$

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$$\int S \cdot \nabla v \, dV$$

$$\frac{1}{2}$$

$$S = \hat{S}_e(F) + S^+ \qquad\qquad S^+ = (\det F)\, T^+ F^{-T}$$

$$T = \hat{T}_e(F) + T^+ \qquad\qquad T^+ = 2\mu\, \mathrm{sym}\left(\dot{F} F^{-1}\right)$$

$$\hat{T}_e(F) = 2k_I\, J^{-\frac{5}{3}}\left(B - \frac{1}{3}\iota_1 I\right) + 2k_v\,(J-1)$$

$$\hat{S}_e(F) = 2k_I\, J^{-\frac{2}{3}}\left(F - \frac{1}{3}\iota_1 F^{-T}\right) + 2k_v\,(J-1)\, J F^{-T}$$

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$$\int_{\mathcal{F}_1} \bar{p}\, e_1\, dA = \int_{\bar{\mathcal{F}}_1} p\, a_1\, dA$$

$$\bar{p} = p\, \frac{A_{\bar{\mathcal{F}}_1}}{A_{\mathcal{F}_1}} \qquad \int_{\mathcal{F}_1} dA = \int_{\bar{\mathcal{F}}_1} a\, dA$$

$$a_1 = \frac{A_{\bar{\mathcal{F}}_1}}{A_{\mathcal{F}_1}} = \|(\operatorname{cof} F) e_1\| = \left(\operatorname{cof} F_{11}^2 + \operatorname{cof} F_{21}^2 + \operatorname{cof} F_{31}^2 \right)^{1/2}$$

$$\underline{\bar{p} = p\, a_1}$$

uniaxial deformation

$$\begin{cases} \dfrac{p}{2k_{I}} = J\left(\lambda^2 - \dfrac{1}{\lambda} \right) \\[4mm] J = 1 + \dfrac{1}{3}\dfrac{p}{2k_V} \end{cases}$$

p traction per unit current area

$2k_I$ shear modulus

$2k_V$ bulk modulus

COMSOL:

$\bar{p}$: traction to be applied (as a "weak contribution")

compare $u1$ at the centre of face $\mathcal{F}_1$

to λl_1 (as a solution to the uniaxial problem)

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	001	001a	001b (fine)	001c (coarse)
k_I	5 GPa	5 GPa		
k_V	50 GPa	50 GPa		
L	0.005	0.005		
R	$L/4$	$L/4$		
u_1/L	-0.25	-0.3159	-0.3148	-0.3158
λ	0.75	0.6839	0.6853	0.6845

$$\lambda = 0.67257$$

$$(L + u_1) = \lambda L$$

$$u_1 = (\lambda - 1) L$$

$$u_1/L = (\lambda - 1)$$

p force per unit current area

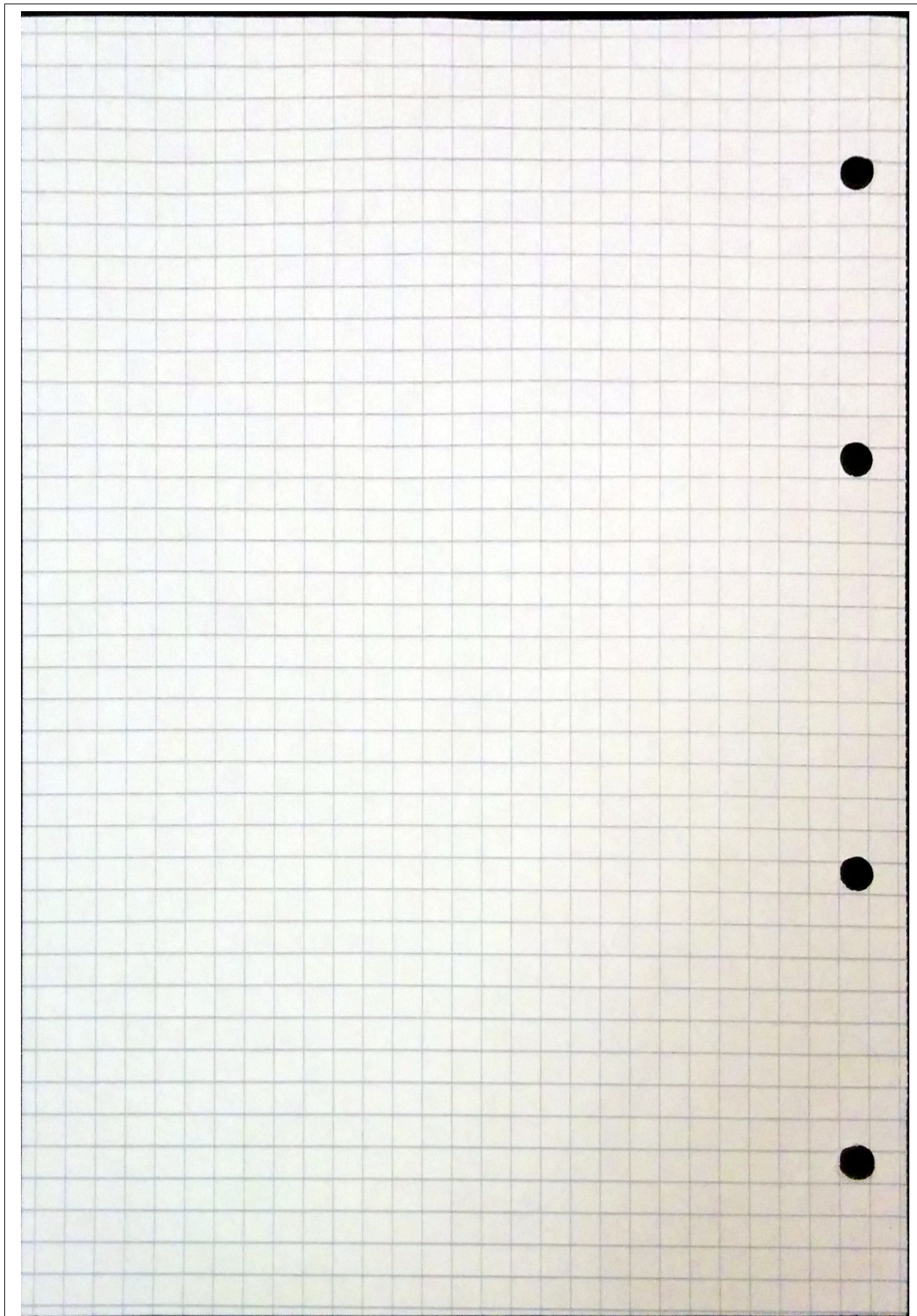$\bar{p}$ force per unit reference area

$$\bar{p} = p \, \frac{A_F}{A_{\bar{F}}}$$

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consol

field　　　　　　　　　　　　test field

u　　$u1$　　　　　　　　　　　　test ($u1$)
　　$u2$　　　　　　　　　v　　test ($u2$)
　　$u3$　　　　　　　　　　　　test ($u3$)

F
$1+u1x$	$u1y$	$u1z$
$u2x$	$1+u2y$	$u2z$
$u3x$	$u3y$	$1+u3z$

$\dot F$
$u1xt$	$u1yt$	$u1zt$
$u2xt$	$u2yt$	$u2zt$
$u3xt$	$u3yt$	$u3zt$

- The space of test velocity fields is isomorphic to the space of displacement fields.
- The space of test velocity gradients is, as a consequence, isomorphic to the space of displacement field gradients.

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```
class_comsol-model-(15-01).txt          Mon Jun 08 15:24:40 2015          1

 1: Dp11      1+u1x
 2: Dp21      u2x
 3: Dp31      u3x
 4: Dp12      u1y
 5: Dp22      1+u2y
 6: Dp32      u3y
 7: Dp13      u1z
 8: Dp23      u2z
 9: Dp33      1+u3z
10: f11       Dp11
11: f21       Dp21
12: f31       Dp31
13: f12       Dp12
14: f22       Dp22
15: f32       Dp32
16: f13       Dp13
17: f23       Dp23
18: f33       Dp33
19: B11       f11*f11+f12*f12+f13*f13
20: B12       f11*f21+f12*f22+f13*f23
21: B13       f11*f31+f12*f32+f13*f33
22: B21       f21*f11+f22*f12+f23*f13
23: B22       f21*f21+f22*f22+f23*f23
24: B23       f21*f31+f22*f32+f23*f33
25: B31       f31*f11+f32*f12+f33*f13
26: B32       f31*f21+f32*f22+f33*f23
27: B33       f31*f31+f32*f32+f33*f33
28: detF      -(f13*f22*f31)+f12*f23*f31+f13*f21*f32-f11*f23*f32-f12*f21*f33+f11*f22*f33
29: J         detF
30: i1        B11+B22+B33
31: i2        (i1^2-(B11^2+B12^2+B13^2+B21^2+B22^2+B23^2+B31^2+B32^2+B33^2))/2
32: cfF11     -(f23*f32)+f22*f33
33: cfF12     f23*f31-f21*f33
34: cfF13     -(f22*f31)+f21*f32
35: cfF21     f13*f32-f12*f33
36: cfF22     -(f13*f31)+f11*f33
37: cfF23     f12*f31-f11*f32
38: cfF31     -(f13*f22)+f12*f23
39: cfF32     f13*f21-f11*f23
40: cfF33     -(f12*f21)+f11*f22
41: cfDp11    cfF11
42: cfDp21    cfF21
43: cfDp31    cfF31
44: cfDp12    cfF12
45: cfDp22    cfF22
46: cfDp32    cfF32
47: cfDp13    cfF13
48: cfDp23    cfF23
49. cfDp33    cfF33
50: finv11    cfF11/detF
51: finv21    cfF12/detF
52: finv31    cfF13/detF
53: finv12    cfF21/detF
54: finv22    cfF22/detF
55: finv32    cfF23/detF
56: finv13    cfF31/detF
57: finv23    cfF32/detF
58: finv33    cfF33/detF
59: f11t      u1xt
60: f21t      u2xt
61: f31t      u3xt
62: f12t      u1yt
63: f22t      u2yt
64: f32t      u3yt
65: f13t      u1zt
66: f23t      u2zt
67: f33t      u3zt
68: Dv11      f11t*finv11+f12t*finv21+f13t*finv31
69: Dv21      f21t*finv11+f22t*finv21+f23t*finv31
70: Dv31      f31t*finv11+f32t*finv21+f33t*finv31
```

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```
class_comsol-model-(15-01).txt        Mon Jun 08 15:24:40 2015          2

 71: Dv12     f11t*finv12+f12t*finv22+f13t*finv32
 72: Dv22     f21t*finv12+f22t*finv22+f23t*finv32
 73: Dv32     f31t*finv12+f32t*finv22+f33t*finv32
 74: Dv13     f11t*finv13+f12t*finv23+f13t*finv33
 75: Dv23     f21t*finv13+f22t*finv23+f23t*finv33
 76: Dv33     f31t*finv13+f32t*finv23+f33t*finv33
 77: S11      (Se11+Sdiss11)
 78: S21      (Se21+Sdiss21)
 79: S31      (Se31+Sdiss31)
 80: S12      (Se12+Sdiss12)
 81: S22      (Se22+Sdiss22)
 82: S32      (Se32+Sdiss32)
 83: S13      (Se13+Sdiss13)
 84: S23      (Se23+Sdiss23)
 85: S33      (Se33+Sdiss33)
 86: Se11     2*kI*J^(-2/3)*(f11-(i1/3)*finv11)+2*kV*(J-1)*cfF11
 87: Se12     2*kI*J^(-2/3)*(f12-(i1/3)*finv21)+2*kV*(J-1)*cfF12
 88: Se13     2*kI*J^(-2/3)*(f13-(i1/3)*finv31)+2*kV*(J-1)*cfF13
 89: Se21     2*kI*J^(-2/3)*(f21-(i1/3)*finv12)+2*kV*(J-1)*cfF21
 90: Se22     2*kI*J^(-2/3)*(f22-(i1/3)*finv22)+2*kV*(J-1)*cfF22
 91: Se23     2*kI*J^(-2/3)*(f23-(i1/3)*finv32)+2*kV*(J-1)*cfF23
 92: Se31     2*kI*J^(-2/3)*(f31-(i1/3)*finv13)+2*kV*(J-1)*cfF31
 93: Se32     2*kI*J^(-2/3)*(f32-(i1/3)*finv23)+2*kV*(J-1)*cfF32
 94: Se33     2*kI*J^(-2/3)*(f33-(i1/3)*finv33)+2*kV*(J-1)*cfF33
 95: Sdiss11  cfF11*Tdiss11+cfF21*Tdiss12+cfF31*Tdiss13
 96: Sdiss12  cfF12*Tdiss11+cfF22*Tdiss12+cfF32*Tdiss13
 97: Sdiss13  cfF13*Tdiss11+cfF23*Tdiss12+cfF33*Tdiss13
 98: Sdiss21  cfF11*Tdiss21+cfF21*Tdiss22+cfF31*Tdiss23
 99: Sdiss22  cfF12*Tdiss21+cfF22*Tdiss22+cfF32*Tdiss23
100: Sdiss23  cfF13*Tdiss21+cfF23*Tdiss22+cfF33*Tdiss23
101: Sdiss31  cfF11*Tdiss31+cfF21*Tdiss32+cfF31*Tdiss33
102: Sdiss32  cfF12*Tdiss31+cfF22*Tdiss32+cfF32*Tdiss33
103: Sdiss33  cfF13*Tdiss31+cfF23*Tdiss32+cfF33*Tdiss33
104: T11      (Te11+Tdiss11)
105: T21      (Te21+Tdiss21)
106: T31      (Te31+Tdiss31)
107: T12      (Te12+Tdiss12)
108: T22      (Te22+Tdiss22)
109: T32      (Te32+Tdiss32)
110: T13      (Te13+Tdiss13)
111: T23      (Te23+Tdiss23)
112: T33      (Te33+Tdiss33)
113: Te11     2*kI*J^(-5/3)*(B11-i1/3)+2*kV*(J-1)
114: Te12     2*kI*J^(-5/3)*B12
115: Te13     2*kI*J^(-5/3)*B13
116: Te21   . 2*kI*J^(-5/3)*B21
117: Te22     2*kI*J^(-5/3)*(B22-i1/3)+2*kV*(J-1)
118: Te23     2*kI*J^(-5/3)*B23
119: Te31     2*kI*J^(-5/3)*B31
120: Te32     2*kI*J^(-5/3)*B32
121: Te33     2*kI*J^(-5/3)*(B33-i1/3)+2*kV*(J-1)
122: Tdiss11 mu*(Dv11+Dv11)
123: Tdiss12 mu*(Dv12+Dv21)
124: Tdiss13 mu*(Dv13+Dv31)
125: Tdiss21 mu*(Dv21+Dv12)
126: Tdiss22 mu*(Dv22+Dv22)
127: Tdiss23 mu*(Dv23+Dv32)
128: Tdiss31 mu*(Dv31+Dv13)
129: Tdiss32 mu*(Dv32+Dv23)
130: Tdiss33 mu*(Dv33+Dv33)
131: TSph     (T11+T22+T33)/3
132: phi      kI*(J^(-2/3)*i1-3)+kV*(J-1)^2
```

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