Course on Formal Methods 2010-2011

Logic & Theorem Proving

27 June 2011

1. Given the formula $((x \vee y) \wedge z) \Rightarrow ((x \vee \neg y) \Rightarrow \neg z)$, give (*if they exist*) two assignments to variables x, y, z that make the formula true and two other assignments that make it false.

2. Transform the formula $(\neg x \vee y) \Rightarrow (\neg x \Rightarrow (x \wedge y))$ into CNF.

3. Prove $(A \Rightarrow \neg B) \Rightarrow \neg(A \wedge B)$ using natural deduction by indicating the rule applied at each step.

4. Prove $((\exists x.(P\,x)) \Rightarrow (\forall y.(Q\,y))) \Rightarrow \forall x.\forall y.((P\,x) \Rightarrow (Q\,y))$ using natural deduction by indicating the rule applied at each step.

5. Given the λ-expression $t = (\lambda x.x(\lambda w.xw))(\lambda y.yw)$, mark each variable occurrence in t as binding, bound or free. For each bound occurrence, indicate the corresponding binding occurrence. Then, apply β-reduction to t by underlining the redex at each step.

6. Let $\Sigma = \{f : \sigma \rightarrow \tau \rightarrow \rho,\ g : \tau \rightarrow \tau \rightarrow \sigma\}$ and $\Gamma = \{x : \tau, y : \tau\}$. Derive a type judgement for the term $\lambda y^{\tau}.\, f\,(g\,x\,y)\,x$.

7. An *algebraic expression* over any type T of elements is either a variable denoted by an element in T or the addition of two algebraic expressions over T or the difference of two algebraic expressions over T or the multiplication of two algebraic expressions over T. Give a definition of the type `AlgExp` using the derived rule for data type definition, discuss its characteristics and finally give an instance of a term of the defined data type.