Course on Formal Methods 2010-2011

1 March 2012

Rewriting

1. Let R be the following trs on a signature $\Sigma = \{a, b, f, g\}$:

$$\begin{array}{rcl} f(x,a) & \rightarrow & x \\ f(a,x) & \rightarrow & x \\ f(x,x) & \rightarrow & x \\ g(x, g(b,y)) & \rightarrow & g(x,y) \\ g(a, f(g(b,x),x)) & \rightarrow & g(a,x) \end{array}$$

i) Give an *ordering on terms* such that R be terminating with respect to such a term ordering.
ii) Check that R is locally confluent.

2. Let R be the following trs describing an equational theory E on the signature $\Sigma = \{a, f, g, h\}$:

$$\begin{array}{rcl} g(x, f(y)) & \rightarrow & h(g(x,y), x) \\ g(x,a) & \rightarrow & a \\ h(x, f(a)) & \rightarrow & f(x) \end{array}$$

i) Give an *ordering on terms* such that R be terminating with respect to such a term ordering.
ii) Check that R is confluent.
iii) Solve modulo E the equation $g(x,y) = h(x,y)$ by applying the E-unification algorithm based on normal and basic narrowing. Give the derivation tree with all the narrowing steps of the first level of the tree and half of the second level, plus all possible normalization steps.

Logic & Theorem Proving

1. Given the formula $f = \neg((x \Rightarrow \neg y) \wedge z)$, give (*if they exist*) two assignments to variables x, y, z that make the formula true and two other assignments that make it false.

2. Transform the formula f given in Exercise 1 into CNF.

3. Prove $(A \vee B) \Rightarrow \neg((\neg A) \wedge (\neg B))$ using natural deduction *by indicating the rule applied at each step.*

4. Prove $(\forall x.\ \forall y.\ (P\,x\,y) \Rightarrow \neg(P\,y\,x)) \Rightarrow (\forall x. \neg(P\,x\,x))$ using natural deduction *by indicating the rule applied at each step.*

5. Given the λ-expression $t = (\lambda x.x(\lambda y.x(y\,a)))(\lambda z.zy)$, mark each variable occurrence in t as binding, bound or free. For each bound occurrence, indicate the corresponding binding occurrence. Then, reduce t to β-normal form by underlining the redex at each step.

6. Let $\Sigma = \{f : \sigma \to \rho,\ g : \rho \to \sigma \to \tau,\ h : \sigma \to \tau \to \rho\}$ and $\Gamma = \{x : \sigma,\ y : \sigma\}$. Derive a type judgement for the term $\lambda x^{\sigma}.\, h\,x\,(g\,(f\,y)\,x)$.

7. A *list of pairs* over any types T_1 and T_2 of elements can be either an *empty* list or a *pair* (e_1, e_2), where $e_1 \in T_1$ and $e_2 \in T_2$, followed by a list of pairs over T_1 and T_2. Give a definition of the type `PairList` using the derived rule for data type definition, discuss its characteristics and finally give an instance of a term of the defined data type.