Contents of the course

Functional analysis in applied mathematics and engineering

2008-2009

Corrado Lattanzio & Olga Rozanova

Basic Functional Analysis

Normed linear spaces. Normed linear spaces. Continuity, convergence, compactness. Banach spaces. Denseness, separability. Contractions and applications.

Integration theory of functions. Riemann integral. Lebesgue measure and Lebesgue integral. L_P spaces.

Linear operators. Bounded operators. Strong convergence and convergence in norm. Uniform and strong continuity. Banach-Steinhaus Theorem. Open Mapping Theorem. Closed Graph Theorem. Linear functionals. Hahn-Banach Theorem. Dual and bidula spaces. Relfexive spaces. Weak and weak-* convergence. Weak-* compactness. Semicontinuity. Compact operators.

Hilbert spaces. Inner product. Orthogonality. Orthogonal Projection Theorem. Riesz Representation Theorem. Lax-Milgram Theorem. Adjont operator. Self-adjoint operators. Positive operators.

Calculus in Banach spaces. Bochner integral. Uniform and strong measurability of operators. Gateaux and Fréchet derivatives. Taylor's formula.

Abstract equations

Differential equations. Dynamical systems. Linear and nonlinear differential equations. Partial differential equations. Sobolev spaces. Distribution theory. Weak derivatives.

Semigroup theory. Semigroups. Infinitesimal generator. Hille-Yosida Theorem. Non homogeneous abstract evolution equations. Analytic semigroups.

Spectral theory and applications. Spectral theory in finite dimension: Alternative Theorem. Spectrum of closed linear operators in linear normed spaces. Spectral theory for compact and normal operators. Fredholm Alternative Theorem. Spectral decomposition for unbounded operators. Applications to partial differential equations.

Applications

Linear system theory. Controllability. Stability. Osservability. Linear quadratic cost control problems. Time optimal control. Controllability and osservability for infinite-dimensional linear systems.

Textbook

R. F. Curtain, A. J. Pritchard, *Functional Analysis in Modern Applied Mathematics,* Academic Press, 1977.