Dynamical systems and bifurcation theory
Metodi analitici per problemi differenziali

Test of 10 November 2008

Duration: 120 min.

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Exercise 1

Find and classify equilibrium points of the nonlinear system

$$\dot{X} = f(X), \quad X = \begin{pmatrix} x \\ y \\ z \end{pmatrix}, \quad f(X) = \begin{pmatrix} y^2 - y \\ z + y(x-1) \\ y - x \end{pmatrix}.$$

Exercise 2

Consider the system

$$\dot{X} = f(X), \quad X = \begin{pmatrix} x \\ y \end{pmatrix}, \quad f(X) = \begin{pmatrix} -3y - 2x^5 \\ 3x - y^3 \end{pmatrix}.$$

Justifying all answers:

1. Write the linearization of that system about the origin; classify the origin and draw the phase portrait for the linearized system. Using only the linearization, what can we say about the nature of the origin for the nonlinear system?
2. Find an appropriate Liapunov function and study the stability of the origin.
3. Using polar coordinates, study the nature of the origin (be as accurate as possible).
4. Draw the phase portrait for the nonlinear system.

Exercise 3

Using the potential energy

$$U(x) = -\int_{-1}^{x} f(y)dy,$$

study the Newtonian system defined by the equation

$$\ddot{x} = f(x), \quad f(x) = 4x - 4x^3.$$