Dynamical systems and bifurcation theory
Metodi analitici per problemi differenziali

Test of 12 January 2009

Duration (total): 120 min.

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Exercise 1

Consider the linear system

$$\dot{x} = Ax, \quad A = \begin{pmatrix} -2 & 0 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}.$$

1. Using the exponential of the matrix A, find the solution of the linear system with initial condition $x(0) = x_0$.
2. Determine stable, unstable and center subspaces for that system and deduce stability properties for $x = 0$.

Exercise 2

Consider the system

$$\dot{X} = f(X), \quad X = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}, \quad f(X) = \begin{pmatrix} x_1 + 3x_2 + \frac{1}{2}(x_1 - x_2)^2 \\ 3x_1 + x_2 + \frac{1}{2}(x_1 - x_2)^2 \end{pmatrix}.$$

Justifying all answers:

1. Write the linearization of that system about the origin, determine stable, unstable and center subspaces, classify the origin and draw the phase portrait for the linear system. Deduce the classification for the nonlinear system.
2. Write the nonlinear system as follows:
   $$\dot{Y} = BY + G(Y),$$
   where B is diagonal and $G(Y)$ is quadratic in y_1 and y_2. Without solving the nonlinear system explicitly, find and draw the stable and unstable manifolds, first in Y and then in X.
3. Solve the nonlinear system explicitly, first in Y and then in X, and check the results of point 2.