Dynamical systems and bifurcation theory
Metodi analitici per problemi differenziali

Test of 27 January 2009

Duration (total): 120 min.

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Exercise 1

Consider the linear system

$$\dot{x} = Ax, \quad A = \begin{pmatrix} 0 & 1 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix}.$$

1. Using the exponential of the matrix A, find the solution of the linear system with initial condition $x(0) = x_0$.
2. Determine stable, unstable and center subspaces for that system and deduce stability properties for $x = 0$.

Exercise 2

Consider the system

$$\dot{X} = f(X), \quad X = \begin{pmatrix} x \\ y \end{pmatrix}, \quad f(X) = \begin{pmatrix} 2y + 3x\left(x^2+y^2\right)^{3/2} \cos\left(\frac{1}{\sqrt{x^2+y^2}}\right) \\ -2x + 3y\left(x^2+y^2\right)^{3/2} \cos\left(\frac{1}{\sqrt{x^2+y^2}}\right) \end{pmatrix}; \quad f(0) = 0.$$

Justifying all answers:

1. Write the linearization of that system about the origin; classify the origin and draw the phase portrait for the linearized system. Using only the linearization, what can we say about the nature of the origin for the nonlinear system?
2. Study the nature of the origin for the nonlinear system (be as accurate as possible, that is, deduce stability properties and classify it).
3. Draw the phase portrait for the nonlinear system.