

# FUNCTIONAL ANALYSIS IN APPLIED MATHEMATICS AND ENGINEERING — FIRST PART

Test of 10 February 2009

Duration: 60 min.

Family and first name: \_\_\_\_\_

Matricola: \_\_\_\_\_

## Exercise 1

1. Using contraction mapping theorem, prove existence and uniqueness of solutions for the equation  $x - \frac{1}{3}e^x = 0$  in the interval  $[0, 1]$ .
2. Find the three successive approximations of this solution starting from  $x = 0$ .

## Exercise 2

Prove that any countable subset of  $\mathbb{R}$  is measurable and, more precisely, it has zero measure.

## Exercise 3

Consider the operator  $A : X \rightarrow C[a, b]$  defined by  $(Au)(t) = u(t)$ , where  $X = C^1[a, b]$  with the norm  $\|u\|_X = \|u'\|_\infty + |u(a)|$ . Prove that the operator is linear, bounded, continuous and evaluate its norm.

## Exercise 4

1. Give the definition of the dual space of a normed linear space  $X$ .
2. Give the definition of reflexive spaces and show that weak and  $*$ -weak convergences are equivalent for reflexive spaces.