Functional analysis in applied mathematics and engineering — Second part

Test of 10 February 2009

Duration: 60 min.

Family and first name: ______________________________

Matricola: ______________________________

Exercise 1

Evaluate $D^3 H(x)$ in the sense of distributions, where $H(x)$ denotes the Heaviside function

$$H(x) = \begin{cases} 1 & x > 0 \\ 0 & x \leq 0. \end{cases}$$

Exercise 2

In the space $L_2([0, 2\pi]; \mathbb{R})$, Find the projection of the vector $x(t) = t - 1$ into the linear subspaces $\mathcal{S}$ and $\mathcal{S}^\perp$, where

$$\mathcal{S} = Span\left\{ \frac{1}{\sqrt{\pi}} \sin(t), \frac{1}{\sqrt{\pi}} \sin(2t) \right\}.$$

Exercise 3

Define the spectrum of a bounded operator and then describe the eigenvalues and eigenvector spaces of a self–adjoint, compact operator on an Hilbert space.

Exercise 4

Define the notion of adjoint operator for a linear and bounded operator $A \in \mathcal{L}(H)$, H being an Hilbert space (explain why the definition is well–posed, using in an appropriate way the Riesz Representation Theorem).

How the above definition must be modified if A is an *unbounded* operator defined in $D(A) \subset H$?