DYNAMICAL SYSTEMS AND BIFURCATION THEORY

METODI ANALITICI PER PROBLEMI DIFFERENZIALI

Test of 22 May 2009

Duration (total): 120 min.

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Exercise 1

Consider the linear system

$$\dot{x} = Ax, \quad A = \begin{pmatrix} 1 & 2 \\ 2 & 2 \end{pmatrix}.$$

1. Reduce the matrix A in Jordan canonical form.
2. Find the solution of the linear system with initial condition $x(0) = x_0$.
3. Draw the phase portrait both in the coordinates for which A is reduced in Jordan canonical form and in the original coordinates (be as specific as possible).
4. Classify the origin $x = 0$ according to the previous discussion (be as specific as possible).

Exercise 2

Consider the system

$$\dot{X} = f(X), \quad X = \begin{pmatrix} x \\ y \end{pmatrix}, \quad f(X) = \begin{pmatrix} -4y + 3x^5 \\ 4x + 3yx^4 \end{pmatrix}.$$

Justifying all answers:

1. Write the linearization of that system about the origin; classify the origin and draw the phase portrait for the linearized system. Using only the linearization, what can we say about the nature of the origin for the nonlinear system?
2. Study the nature of the origin for the nonlinear system in polar coordintates (be as specific as possible, that is, deduce stability properties and classify it).
3. Draw the phase portrait for the nonlinear system.