

FUNCTIONAL ANALYSIS IN APPLIED MATHEMATICS
AND ENGINEERING

Test of 30 June 2009

Duration: approx. 60 min.

Family and first name: _____

Matricola: _____

Exercise 1

Prove that $L^4([0, 2]) \subset L^1([0, 2])$.

Hint: use in an appropriate way the Hölder inequality.

Exercise 2

Evaluate the limit in the sense of distributions of the following sequence:

$$u_n(x) = \begin{cases} 2n, & \text{if } x \in \left[-\frac{1}{n}, \frac{1}{n}\right], \\ 0, & \text{otherwise.} \end{cases}$$

Exercise 3

State and make comments (for instance, provide examples) for Orthogonal Projection Theorem in Hilbert spaces.

Exercise 4

In the space $H = L_2(a, b)$ with the usual inner product, consider the operator $T : H \rightarrow H$ defined by:

$$(Tu)(t) = \int_a^b k(t, s)u(s)ds,$$

where $k(t, s) \in L_2((a, b) \times (a, b))$. Prove T is linear and bounded and afterwards evaluate the adjoint T^* of T .