FUNCTIONAL ANALYSIS IN APPLIED MATHEMATICS AND ENGINEERING

Test of 25 September 2009

Duration: approx. 60 min.

Family and first name: _____

Matricola: _____

Exercise 1

Prove that if $f, g \in C([a,b])$, then

$$f = g \text{ almost everywhere in } [a,b] \quad \Leftrightarrow \quad f(x) = g(x) \text{ for any } x \in [a,b].$$

Exercise 2

Let f be the linear functional defined by

$$f(x) = \sum_{i=1}^{+\infty} x_i,$$

for any sequence of real numbers $x = (x_1, x_2, \ldots)$. Prove that f is bounded in ℓ_1 and evaluate its norm.
Is f bounded in ℓ_∞? Justify your answer.

Exercise 3

State and make comments (for instance, provide examples and conterexamples) on different notions of convergences (strong, weak, *–weak) in Banach spaces.

Exercise 4

In $L_2(0, \pi)$, find the distance between $x(t) = t$ and the subspace

$$S = Span \left\{ \sqrt{\frac{2}{\pi}} \sin(t), \sqrt{\frac{2}{\pi}} \sin(2t) \right\}.$$