Family and first name: _____

Matricola: _____

CFU: _____

Exercise 1 [only for 9 CFU]

Consider the function $f : [0,1] \to \mathbb{R}$ defined by:

$$f(x) = \begin{cases} 3 & \text{if } x = \dfrac{2n}{n^2+1} \\ 1 & \text{otherwise,} \end{cases}$$

where $n \in \mathbb{N}$.

1. Prove that f is measurable (explain your answer).

2. Evaluate $\displaystyle\int_0^1 f(x)dx$ (explain your answer).

Exercise 2 [6 and 9 CFU]

For any $1 < p < +\infty$, let $\{x^k\} \subset \ell_p$ be the sequence defined by:

$$x^k = (0, \ldots, 0, \underbrace{1}_{k\text{-th place}}, 0, 0, \ldots).$$

Prove that $x_k \rightharpoonup 0$ (weakly) in ℓ_p, but not strongly in ℓ_p.

Exercise 3 [6 and 9 CFU]

Describe the notions of Gateaux and Fréchet derivatives in Banach spaces.

Exercise 4 [only for 9 CFU]

In $L_2(-1,1)$, find the distance between $x(t) = t^2$ and the subspace

$$S = Span\{1, t\}.$$