Functional analysis in applied mathematics and engineering

and engineering

Test of 1 February 2010

Duration: approx. 60 min.

Family and first name: _____

Matricola: _____

CFU: _____

Exercise 1 [6 and 9 CFU]

Prove that any countable set has zero measure.

Exercise 2 [only for 9 CFU]

Evaluate in the sense of distribution $D(x^2\delta(x))$, where $\delta(x)$ denotes the Dirac's delta.

Exercise 3 [6 and 9 CFU]

Describe the notions of linear, bounded operators, norm of an operator and give examples of (linear) bounded and unbounded operators.

Exercise 4 [only for 9 CFU]

In the Hilber space of bilateral sequences $\{x_n\}_{n\in\mathbb{Z}}$ with complex values with inner product

$$\langle x, y \rangle = \sum_{-\infty}^{+\infty} x_n \overline{y_n}$$

and norm

$$\|x\|_2 = \sum_{-\infty}^{+\infty} |x_n|^2,$$

define the linear and bounded operator

$$(Ax)_n = x_{n-1}.$$

Prove that A is invertible and $A^* = A^{-1}$.