

FUNCTIONAL ANALYSIS IN APPLIED MATHEMATICS
AND ENGINEERING

Test of 24 February 2010

Duration: approx. 60 min.

Family and first name: _____

Matricola: _____

CFU: _____

Exercise 1 [only for 9 CFU]

Using contraction mapping theorem, prove existence and uniqueness of solutions $f \in C([a, b])$ for the following Fredholm integral equation:

$$f(x) = \lambda \int_a^b k(x, y)f(y)dy,$$

where $k \in C([a, b] \times [a, b])$ and $\lambda \in \mathbb{R}$ is *sufficiently small*.

Exercise 2 [6 and 9 CFU]

Prove that $L^2([-1, 1]) \subset L^1([-1, 1])$.

Hint: use in an appropriate way the Schwarz inequality.

Exercise 3 [only for 9 CFU]

Evaluate $D\chi_{[-1, 1]}(x)$ in the sense of distributions, where $\chi_{[-1, 1]}(x)$ denotes the characteristic function of the interval $[-1, 1]$:

$$\chi_{[-1, 1]}(x) = \begin{cases} 1, & \text{if } x \in [-1, 1], \\ 0, & \text{otherwise.} \end{cases}$$

Exercise 4 [6 and 9 CFU]

Give the definition of Hilbert spaces and introduce the notion of Fourier series in Hilbert spaces.