Complex Analysis

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Duration 120 min.

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Exercise 1 [8 points]

Consider the real valued function $u(x, y) = x(ae^x \cos y + bx) - ye^x \sin y$. Find the values $a, b \in \mathbb{R}$ such that u is the real part of an analytic function. Then for these values find the corresponding imaginary part and write down the complete function in the complex variable $z = x + iy$.
Justify all answers.

Exercise 2 [6 points]

Let $f : \mathbb{C} \to \mathbb{C}$ be an analytic function such that $\lim_{|z| \to +\infty} f(z)$ exists and it is finite. Prove that f is a constant function.
Justify all answers.

Exercise 3 [9 points]

Using complex analysis techniques, compute the following integral (principal value):

$$P.V. \int_{-\infty}^{+\infty} \frac{\cos x}{(x^2 + 2)(x - 1)} \, dx.$$

Justify all answers.

Exercise 4 [9 points]

Find a conformal transformation such that the imagine of the disk $A = \{z \in \mathbb{C} : |z - 2| < 2\}$ is the half–plane $B = \{z \in \mathbb{C} : \operatorname{Im} z > \operatorname{Re} z\}$. Do not use known formulas/transformations, but compute them explicitly and draw the sets A and B.
Justify all answers.