

COMPLEX ANALYSIS

July 7, 2016

Duration 120 min.

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Exercise 1 [8 points]

Determine the number of zeros of the polynomial $P(z) = 4z^2 - z^5 + 1$ in the region $C = \{z \in \mathbb{C} : 1 < |z| < 2\}$. Justify all answers.

Exercise 2 [6 points]

Let $f : \mathbb{C} \rightarrow \mathbb{C}$ be an analytic function such that $\operatorname{Re} f(z) > 2$. Prove that f is a constant function. Justify all answers.

Exercise 3 [9 points]

Compute the following integral

$$\int_{\gamma_R} (z-2)^2 \left(e^{\frac{2}{z-2}} + \frac{\sin(z)}{(z(z-1))^2} \right) dz,$$

where $\gamma_R = \{z \in \mathbb{C} : |z| = R; R > 0, R \neq 1, 2\}$ oriented in the positive sense. Justify all answers.

Exercise 4 [9 points]

Using Laplace transform, solve the following Cauchy problem

$$y''' + 4y' = 1; \quad y(0) = 1, \quad y'(0) = 0, \quad y''(0) = -1.$$

Justify all answers.