Complex analysis

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Duration 120 min.

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## Exercise 1 [6 points]

Given an integer $N \geq 0$ find the number of solutions (counted with multiplicity) of the equation $3z^N = z^2 + 1$ contained in the disk $|z| \leq 1$.
Justify all answers.

## Exercise 2 [8 points]

Classify the isolated singularities of the following functions and compute the corresponding residues:

1) $f(z) = \dfrac{1}{\mathrm{e}^z - 1} - \dfrac{1}{z}$;

2) $f(z) = \exp\left(\frac{z}{1-z}\right)$.

Justify all answers.

## Exercise 3 [9 points]

Compute the Fourier transform of

$$f(x) = \frac{x}{2 + x^4}.$$

Do not use "known formulas", but compute everything explicitly and justify all answers.

## Exercise 4 [9 points]

1) Find the image of the set $E = \{z \in \mathbb{C} : \operatorname{Im} z > 0\}$ by the function $f(z) = \log z$.

2) Find a conformal transformation which maps the strip $A = \{z \in \mathbb{C} : 0 < \operatorname{Im} z < \pi\}$ in the disk $B = \{z \in \mathbb{C} : |z| < 1\}$.

Do not use "known formulas", but compute everything explicitly, draw the needed sets and justify all answers.