Complex analysis

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Duration 120 min.

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Exercise 1 [6 points]

Consider the family of functions $u : \mathbb{R}^2 \to \mathbb{R}$ given by $u(x, y) = ax^3 + 3xy^2 - 6bxy$, $(a, b \in \mathbb{R})$. Find $a, b \in \mathbb{R}$ such that u is the real part of an analytic function f and determine $f(z)$.
Justify all answers.

Exercise 2 [8 points]

Let consider

$$f(z) = \frac{1}{(z-1)(z-3i)}.$$

Find singularities and residues of f, and, after writing it in "sum of simple fractions" (for this, the residues just evaluated could be very useful ...), find its Laurent expansion centered at $z_0 = 0$ and which converges for $z = 2$.
Justify all answers.

Exercise 3 [9 points]

Compute

$$P.V. \int_{-\infty}^{+\infty} \frac{(x^2+2)\mathrm{e}^{2ix}}{x(x^2+9)}.$$

Exercise 4 [9 points]

Find a (conformal) fractional linear transformation mapping the circle $\{|x| = 1\}$ and the real axis $\{\operatorname{Im} z = 0\}$ in the two bisector lines $\{\mathrm{Re\,z} = \mathrm{Im\,z}\}$ e $\{\mathrm{Re\,z} = -\mathrm{Im\,z}\}$, respectively.
Justify all answers.