Complex analysis

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Duration 120 min.

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Exercise 1 [8 points]

Let consider $P(z) = z^4 - 6z + 3$. Find the number of zeros (counted with their multiplicity) of $P(z)$ contained in the annulus $1 \leq |z| < 2$.
Justify all answers.

Exercise 2 [8 points]

Given $\gamma \in \mathbb{R}$, compute

$$\int_{-\infty}^{+\infty} \frac{e^{i\gamma x}}{\cosh x}\, dx.$$

We suggest to use the rectangular path joining the points $(R, 0), (R, \pi), (-R, \pi), (-R, 0)$.
Justify all answers.

Exercise 3 [8 points]

Find a conformal transformation which maps the disk $A = \{z \in \mathbb{C} : |z - 2i| < 1\}$ into the halfspace $B = \{z \in \mathbb{C} : \operatorname{Im} z < \operatorname{Re} z\}$. Do not use known formulas/transformations, but compute them explicitly and draw the sets A and B.
Justify all answers.

Exercise 4 [8 points]

Using Laplace transform, solve the following Cauchy problem

$$\begin{cases} x' = 3x - 4y \\ y' = 2x - 3y \end{cases} \quad ; \quad x(0) = 1,\ y(0) = -1.$$

Justify all answers.