COMPLEX ANALYSIS

January 24, 2017
Duration 120 min.

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Exercise 1 [8 points]

Classify the isolated singularities of the following functions and compute the corresponding residues:

1) $f(z) = \dfrac{1}{\cos(z) - 1} - \dfrac{2}{z^2}$;

2) $f(z) = \sin\left(\dfrac{z+2}{z}\right)$.

Justify all answers.

Exercise 2 [6 points]

Let $f : \mathbb{C} \to \mathbb{C}$ be an analytic function such that $\operatorname{Im} f(z) > 1$. Prove that f is a constant function.
Justify all answers.

Exercise 3 [8 points]

Find a conformal transformation which maps the set $A = \{z \in \mathbb{C} \ : \ \operatorname{Re} z < 0 , \operatorname{Im} z > 0\}$ into the set $B = \{z \in \mathbb{C} \ : \ |z - 1| < 2\}$.
Do not use known formulas/transformations, but compute them explicitly and draw all needed sets.
Justify all answers.

Exercise 4 [10 points]

Let
$$f(x) = \frac{1}{1 + x^2} \quad \text{and} \quad g(x) = \frac{\sin x}{x}.$$

Compute their Fourier transform $\hat{f}(k)$, $\hat{g}(k)$ and then compute

$$\int_{-\infty}^{+\infty} \frac{\sin x}{x(1 + x^2)}.$$

Do not use known formulas/transformations, but compute them explicitly.
Justify all answers.