COMPLEX ANALYSIS

February 10, 2017
Duration 120 min.

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Exercise 1 [8 points]

Find the number of solutions (counted with multiplicity) of the equation $e^z = 4z^2 - 1$ such that $|z| < 1$.
Justify all answers.

Exercise 2 [8 points]

Using complex analysis techniques, compute the following integral:

$$\int_{-\infty}^{+\infty} \frac{\sin x}{x^2 + 2x + 2}\, dx.$$

Justify all answers.

Exercise 3 [8 points]

Consider the real valued function $u(x,y) = x \cos x (e^y + e^{-y}) + y \sin x (e^y - e^{-y})$. Verify that u is the real part of an analytic function and then find the corresponding imaginary part $v(x,y)$. Finally, write down the complete function $u(x,y) + iv(x,y)$ as a function $f(z)$ of the complex variable $z = x + iy$.
Justify all answers.

Exercise 4 [8 points]

Using Laplace transform, solve the following Cauchy problem:

$$y''' + y = 2e^{-t}, \quad y(0) = 1,\ y'(0) = 2,\ y''(0) = -3.$$

Justify all answers.