Cognome e nome: _____

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Exercise 1 [6 points]

Let $f : \mathbb{C} \to \mathbb{C}$ be an analytic function such that $\operatorname{Re} f(z) > 0$. Prove that f is a constant function.
Justify all answers.

Exercise 2 [8 points]

Using complex analysis techniques, compute

$$P.V. \int_0^{+\infty} \frac{x \sin x}{2 + x^2} \, dx.$$

Justify all answers.

Exercise 3 [10 points]

Let consider

$$f(z) = \frac{z}{(z+1)^2(z^2-1)}.$$

Find singularities and residues of f and find all Laurent expansions of f centered at $z_0 = -1$.
Justify all answers.

Exercise 4 [8 points]

Using Laplace transform, solve the following Cauchy problem:

$$\begin{cases} x' = x - 4y \\ y' = 3x - y \end{cases} \quad ; \quad x(0) = 0, \ y(0) = 2.$$

Justify all answers.