Master Course in Mathematical Engineering — 2013/14
Advanced Analysis I– DT0012 – 6 CFU
Lecturer: C. Lattanzio

Distributions. Locally integrable functions. The space of test function $\mathcal{D}(\Omega)$. Distributions. Distributions associated to Locally integrable functions. Singular distributions. Examples. Operations on distributions: sum, products times functions, change of variables, restrictions, tensor product. Differentiation and his properties; comparison with classical derivatives. Differentiation of jump functions. Partition of unity. Support of a distribution; compactly supported distributions.

Convolution. Convolution in L^p spaces. Regularity of the convolution. Regularizing sequences and smoothing by means of convolutions. Convolution between distributions and regularization of distributions. Denseness of $\mathcal{D}(\Omega)$ in $\mathcal{D}'(\Omega)$.

Sobolev spaces. Definition of weak derivatives and his motivation. Sobolev spaces $W^{k,p}(\Omega)$ and their properties. Interior and global approximation by smooth functions. Extensions. Traces. Embeddings theorems: Gagliardo–Nirenberg–Sobolev inequality and Embedding theorem for $p < n$. Embedding theorem for $p = n$. Hölder spaces. Morrey inequality. Embedding theorem for $p > n$. Sobolev inequalities in the general case. Compact embeddings: Rellich–Kondrachov theorem, Poincaré inequalities. Characterization of the dual space H^{-1}.

Second order parabolic equations. Definition of parabolici operator. Weak solutions for linear parabolic equations. existence of weak solutions: Galerkin approximation, construction of approximating solutions, energy estimates, existence and uniqueness of solutions.
Existence of solutions of viscous scalar conservation laws.

First order nonlinear hyperbolic equations. Scalar conservation laws: derivation, examples. Weak solutions, Rankine-Hugoniot conditions, entropy conditions. L^1 stability, uniqueness and comparison for weak entropy solutions. Convergence of the vanishing viscosity and existence of the weak, entropy solution. Riemann problem.
Definition of hyperbolic system. Quasilinear hyperbolic systems, symmetric and symmetrizable systems. Existence of solutions: approximations, a priori estimate, local existence of classical solutions.

Textbooks:

- H. Brezis, *Functional Analysis, Sobolev Spaces and Partial Differential Equations.* Universitext, Springer.
- C.M. Dafermos., *Hyperbolic Conservation Laws in Continuum Physics*, Springer.
- L.C. Evans, *Partial Differential Equations.* Graduate Studies in Mathematics, Vol. 19, AMS.
- G. Gilardi, *Analisi 3.* McGraw–Hill.
- M.E. Taylor, *Partial Differential Equations, Nonlinear equations.* Vol. 3, Springer.
- V.S. Vladimirov, *Equations of Mathematical Physics.* Marcel Dekker, Inc.