Master Course in Mathematical Engineering — 2019/20
Complex Analysis – 6 CFU
Lecturer: C. Lattanzio

Analytic functions. Complex numbers; limits. Elementary functions: exponential, trigonometric functions, logarithm and complex powers. Continuous and analytic functions; Cauchy-Riemann equations and inverse function theorem. Harmonic conjugates.

Cauchy's theorem. Contour integrals; fundamental theorem of calculus; path independence theorem. Cauchy's theorem for a rectangle (Cauchy-Goursat Theorem) and Cauchy's theorem for a disk. Deformation theorem and homotopy form of Cauchy's theorem. Index of a closed path, Cauchy's integral formulas. Liouville Theorem; Morera's Theorem; maximum module's principle.

Series representation of analytic functions. Convergent series of analytic functions; power series and Taylor's theorem. Zeroes of analytic functions and analytic continuation. Laurent expansion; singularities.

Residues. Definition and calculus of residues. Residues' Theorem; residues and behavior at infinity. Root-pole counting theorem; principle of argument; Rouche's Theorem.

Conformal mappings. Definition of conformal mappings; conformal mapping theorem; examples. Riemann mapping theorem. Fractional linear transformations; cross ratios.

Laplace transform. Definition of Laplace transform; basic properties; convergence and uniqueness theorems; complex inversion formula.

Fourier transform. Fourier transform in L^1; basic properties. Fourier transform in L^2; Plancherel's Theorem; inverse Fourier transform.

Textbooks:
- J.E. Marsden, M.J. Hoffman. *Basic complex analysis.* Freeman
- W. Rudin. *Real and complex analysis.* Mc Graw Hill
- L.V. Ahlfors. *Complex Analysis.* McGraw-Hill