Master Course in Mathematical Engineering — 2014/15
Applied Partial Differential Equations and Fluid Dynamics– DT0049 – 9 CFU
Lecturer: C. Lattanzio

First order partial differential equations. Definition of characteristic vectors and characteristic surfaces. Characteristics for (semi)linear partial differential equations of first order in two independent variables. Existence and uniqueness to initial value problems for first order semilinear partial differential equations. Duhamel's principle for non homogeneous first order partial differential equations.

Second order partial differential equations. Classification of second order semilinear partial differential equations in two independent variables. Canonical form for second order semilinear partial differential equations in two independent variables. Classification for second order semilinear partial differential equations in many independent variables.

Heat equation. Derivation of heat equation and well–posed problems in one space dimension. Solution of Cauchy–Dirichlet problem for one dimensional heat equation by means of Fourier method of separation of variables. Energy method and uniqueness. Maximum principle. Fundamental solution. Solution of global Cauchy problem. Non homogeneous problem: Duhamel's principle.

Laplace equation. Laplace and Poisson equation: well-posed problems; uniqueness by means of energy method. Mean value property and maximum principles. Laplace equation in a disk by means of separation of variables. Poisson's formula. Harnack's inequality and Liouville's Theorem. Fundamental solution of Laplace operator. Solution of Poisson's equation in the whole space. Green's functions and Green's representation formula.

Wave equation. Transversal vibrations of a string. Well–posed problems in one space dimension. D'Alembert formula. Characteristic parallelogram. Domain of dependence and range of influence. Fundamental solution for one dimensional wave equation. Duhamel's principle for non homogeneous one dimensional wave equations. Special solutions of multi–d wave equation: planar and spherical waves. Well–posedness for initial, boundary value problems: uniqueness by means of energy estimates. Separation of variables. Domain of dependence and range of influence in several space variables. Fundamental solution for multi–dimensional wave equation. Solution of 3–d wave equation: Kirchhoff's formula and strong Huygens' principle. Wave equation in two dimensions: method of descent. Fundamental solution in 2–d. Duhamel's principle for non homogeneous wave equation in 3–d: delayed potentials.

Fluid dynamics. Continuum approach for the study of fluid motion. The notion of stress tensor. Statics of fluids and the stress tensor in a fluid at rest. Kinematics of fluids: spatial and material description of the flow, material derivative, Reynolds transport theorem, local analysis of flow. Dynamics of fluids: the equations of motion. Constitutive equations for viscous fluids. The Navier–Stokes equations. Scaling and dimensional analysis. Unidirectional flows. Vorticity equation. Flow at large values of the Reynolds number. Flow at low values of the Reynolds number.

Textbooks:

- L.C. Evans, *Partial Differential Equations.* Graduate Studies in Mathematics, Vol. 19, AMS, 2010.

- S. Salsa, *Partial Differential Equations in Actions: from Modelling to Theory.* Springer–Verlag Italia, 2008.

- S. Salsa, G. Verzini, *Equazioni a derivate parziali: complementi ed esercizi.* Springer–Verlag Italia. 2005.

- E. C. Zachmanoglou, Dale W. Thoe, *lntroduction to Partial Differential Equations with Applications.* Dover Publications, Inc., 1986.