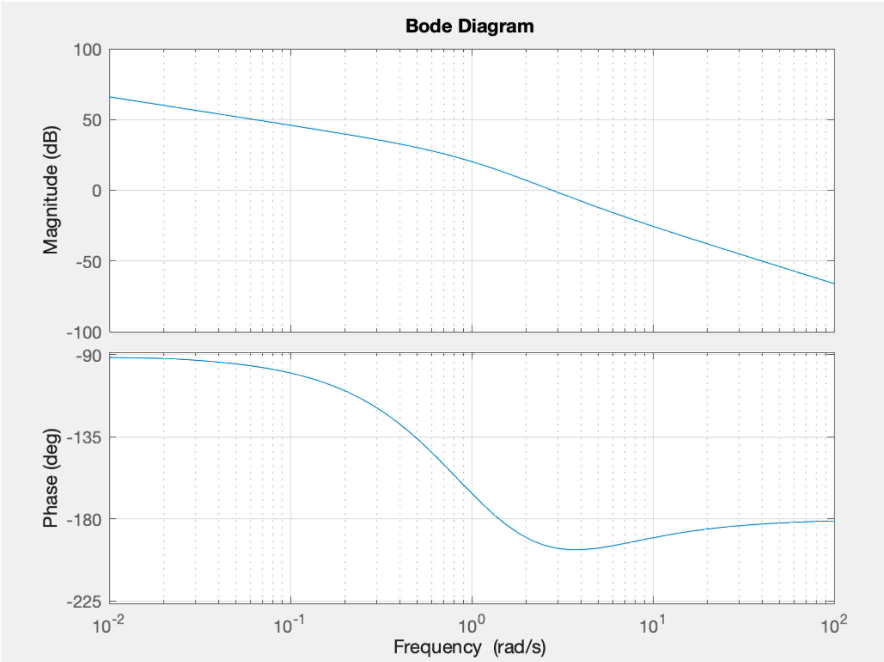
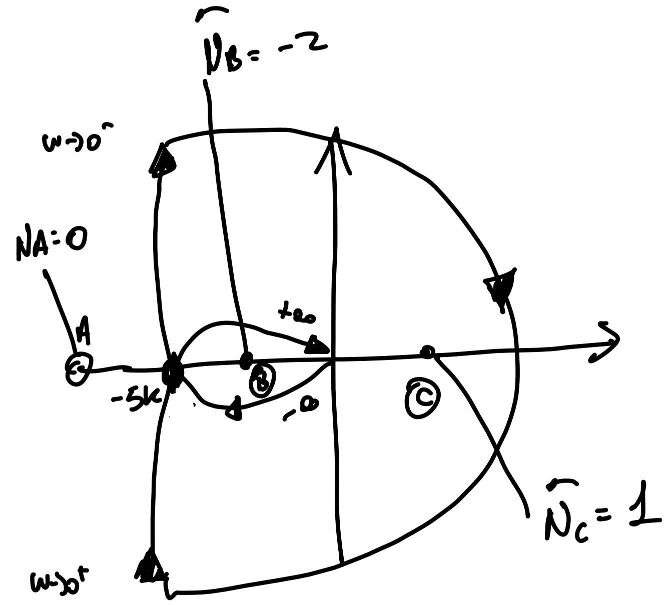
Pb 1 $\quad W(s) = K\,\dfrac{5(s+4)}{s(s+1)^2}$





$D_{CH}(s) = s^3 + 2s^2 + (5K+1)s + 20K$

3	1	$5K+1$
2	2	$20K$
1	$1-5K$	0
0	$20K$	

	(< 0)	0 … $\frac{1}{5}$	(> $\frac{1}{5}$)
1	+	+	+
2	+	+	+
$1-5K$	+	+	−
K	−	+	+
	1V	0V	2V

$1-5K > 0 \iff K < \frac{1}{5}$

stabilità asintotica per $K \in \left(0, \frac{1}{5}\right)$

instabilità per $K \in (-\infty, 0)$ $(P_{CH} = 1)$ e $K \in \left(\frac{1}{5}, +\infty\right)$ $(P_{CH} = 2)$

Pb 2

$$e^{At} = \left[\begin{array}{c|c} \frac{1}{3}e^{-2t} + \frac{2}{3}e^{4t} & \frac{2}{3}e^{-2t} - \frac{2}{3}e^{4t} \\ \hline \frac{1}{3}e^{-2t} - \frac{1}{3}e^{4t} & \frac{2}{3}e^{-2t} + \frac{1}{3}e^{4t} \end{array}\right]$$

$w(t) = e^{-2t}$ (modo associato a $\lambda = 4$ non eccitabile)

$$W(s) = \frac{1}{s+2}$$

$$Y_g(s) = \frac{1}{s(s+2)} \rightarrow y_g(t) = \frac{1}{2}\delta_{-1}(t) - \frac{e^{-2t}}{2}$$

Pb 3 $\quad W(z) = \dfrac{2}{z - 1/5}$

$$Y_g(z) = \frac{z}{z-1} \cdot \frac{2}{z-1/5} = \frac{2z}{(z-1)(z-1/5)}$$

$$\frac{Y_g(z)}{z} = \frac{R_1}{z-1} + \frac{R_2}{z-1/5}$$

$$R_1 = \lim_{z\to 1} \frac{2}{\frac{4}{5}} = 5/2$$

$$R_2 = \lim_{z\to \frac{1}{5}} \frac{2}{-4/5} = -5/2$$

$$Y_g(z) = 5/2 \cdot \frac{z}{z-1} - 5/2 \frac{z}{z-1/5} \leftrightarrow y_g(t) = \frac{5}{2}\delta_{-1}(t) - \frac{5}{2}\left(\frac{1}{5}\right)^t$$

$$y_{om}(t) = \frac{1}{10}\left|W(e^{j\frac{\pi}{2}})\right| \sin\left(\frac{\pi}{2}t + \angle W(e^{j\frac{\pi}{2}})\right) = \frac{1}{10}\left|W(j)\right| \sin\left(\frac{\pi}{2}t + \angle W(j)\right)$$

$$W(j) = \frac{2}{j - \frac{1}{5}} \qquad |W(j)| = \frac{2}{\sqrt{\frac{1}{25}+1}} = \frac{10}{\sqrt{26}} \qquad \angle W(j) = \angle(1) - \angle\left(-\frac{1}{5}+j\right) = 1.7682$$

$$y_{forz}(t) = \frac{1}{\sqrt{26}} \sin\left(\frac{\pi}{2}t + 1.7682\right)$$

Pb.4

$$R = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ -1 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{bmatrix} \qquad \mathrm{Im}(R) = \mathrm{span}\left\{ \begin{pmatrix} 0 \\ 1 \\ -1 \\ -1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ -1 \\ 0 \end{pmatrix} \right\}$$

$$= \mathrm{span}\left\{ \begin{pmatrix} 0 \\ 0 \\ 0 \\ -1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ -1 \\ 0 \end{pmatrix} \right\}$$

$$Q = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 2 & 0 & 0 & 0 \\ -2 & 2 & 2 & 0 \end{bmatrix} \qquad \mathrm{Ker}(Q) = \mathrm{span}\left\{ \begin{pmatrix} 0 \\ 1 \\ -1 \\ 0 \end{pmatrix} \right\}$$

$$\mathcal{X}_1 = \mathcal{R} \cap \mathcal{I} = \mathcal{I}$$

$$\mathcal{X}_2 : \mathcal{X}_1 \oplus \mathcal{X}_2 = \mathcal{R} \implies \mathcal{X}_2 = \mathrm{span}\left\{ \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \right\}$$

$$\mathcal{X}_3 : \mathcal{X}_1 \oplus \mathcal{X}_3 = \mathcal{I} \implies \mathcal{X}_3 = \{0\}$$

$$\mathcal{X}_4 : \mathcal{X}_1 \oplus \dots \oplus \mathcal{X}_4 = \mathbb{R}^4 \implies \mathcal{X}_4 = \mathrm{span}\left\{ \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} \right\}$$

Pb 5

$$J(x) = \begin{bmatrix} -1 & -2(x_2+1) \\ 4(x_2+1) & 4x_1 - k \end{bmatrix} \Bigg|_{x_e = (0,-1)} = \begin{bmatrix} -1 & 0 \\ 0 & -k \end{bmatrix}$$

$$\begin{cases} \text{se } k > 0 & \implies x_e \text{ loc. A.S.} \\ \text{se } k = 0 & \implies \text{caso critico} \\ \text{se } k < 0 & \implies x_e \text{ instabile} \end{cases}$$

$$\boxed{\xi_1 = x_1 \qquad \xi_2 = x_2 + 1}$$

$\underline{k=0}$ $\Rightarrow$ $\begin{cases} \dot{x}_1 = -x_1 - (x_2+1)^2 \\ \dot{x}_2 = 4x_1(x_2+1) \end{cases}$ $\qquad$ $\dot{\xi}_1 = -\xi_1 - \xi_2^2$

$\dot{\xi}_2 = 4\xi_1\xi_2$

$$V(\xi) = \frac{1}{2}\xi_1^2 + \frac{\alpha}{2}\xi_2^2$$

$$\dot{V}(\xi) = [\xi_1 \mid \alpha\xi_2] \left[\frac{-\xi_1 - \xi_2^2}{4\xi_1\xi_2} \right] = -\xi_1^2 - \xi_1\xi_2^2 + 4\alpha\xi_1\xi_2^2$$

scelgo $\alpha = 1/4$ $\Rightarrow$ $V(\xi) = \frac{1}{2}\xi_1^2 + \frac{1}{8}\xi_2^2$

$\dot{V}(\xi) = -\xi_1^2 \leq 0$ $\Rightarrow$ per $k=0$ x_e è sempl. stabl.