

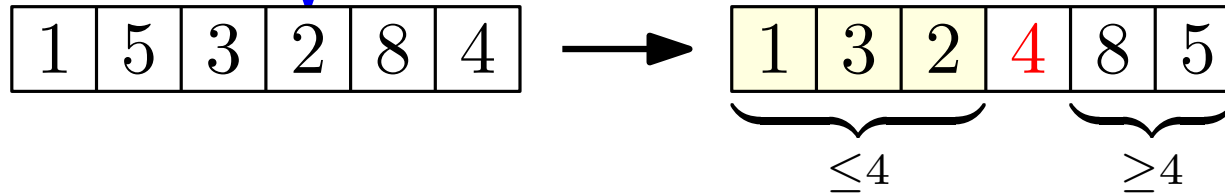
# Sorting

- $k$ -th smallest element

$O(n)$  comparisons

```
std::nth_element(vec.begin(), vec.begin()+k, vec.end());
```

$k = 3$



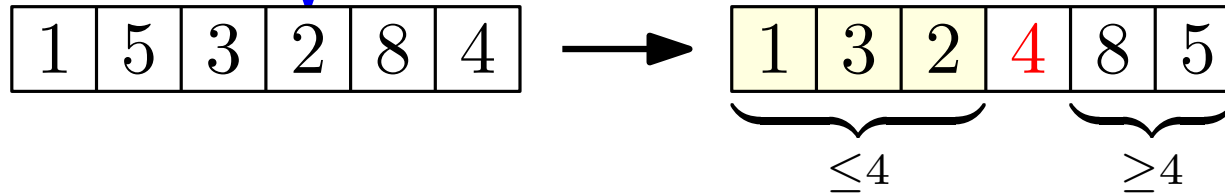
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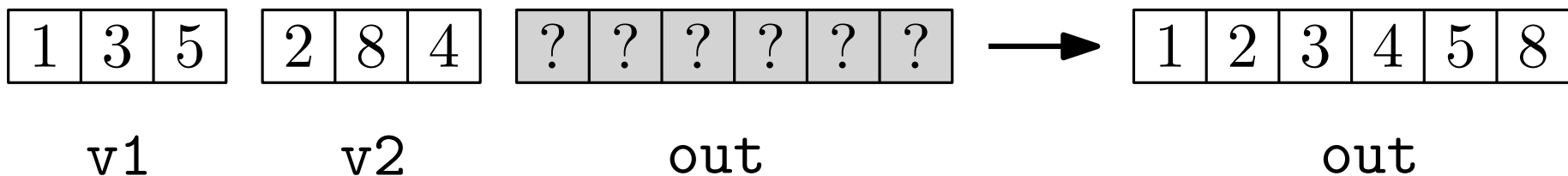
$k = 3$



- Merge

$O(n_1 + n_2)$  comparisons

```
std::merge(v1.begin(), v1.end(),  
          v2.begin(), v2.end(),  
          out.begin());
```



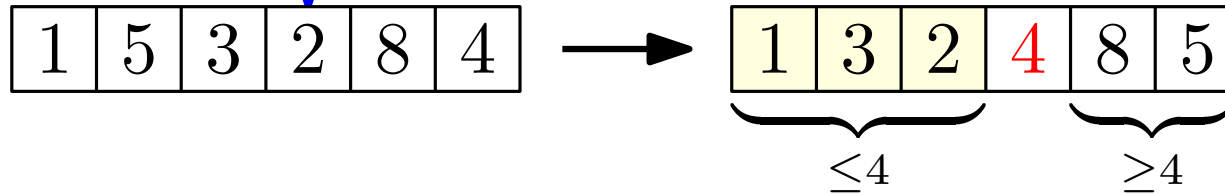
# Sorting

- $k$ -th smallest element

$O(n)$  comparisons

```
std::nth_element(vec.begin(), vec.begin()+k, vec.end());
```

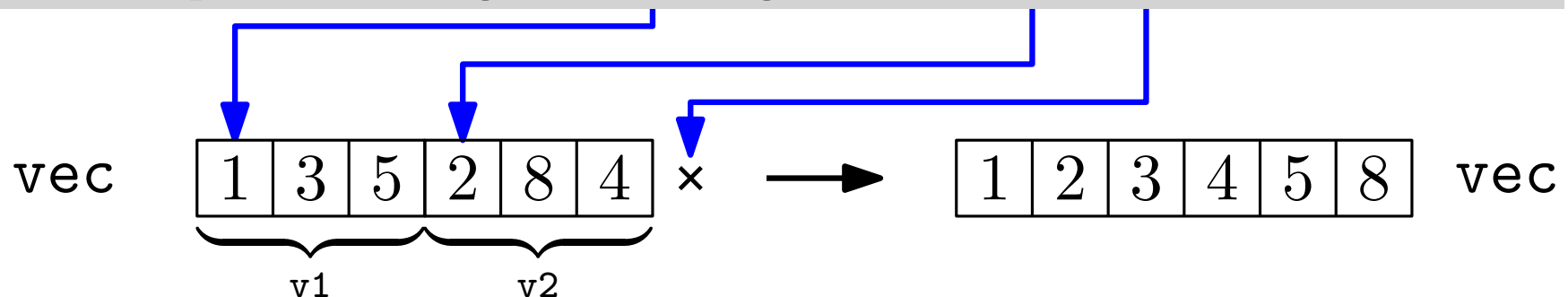
$k = 3$



- Merge

$O(n_1 + n_2)$  comparisons

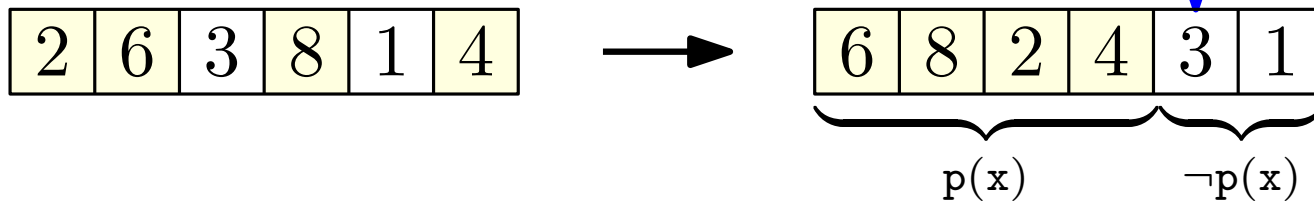
```
std::inplace_merge(vec.begin(), mid, vec.end());
```



# Partitioning

$O(n)$  calls to  $p$

```
std::partition(vec.begin(), vec.end(), p);
```



```
bool p(int i) { return i%2==0; }
```

```
std::partition(vec.begin(), vec.end(), [](int i){ return i%2==0; });
```

See also:

```
std::stable_partition(vec.begin(), vec.end(), p);
```

```
std::partition_point(vec.begin(), vec.end(), p);
```

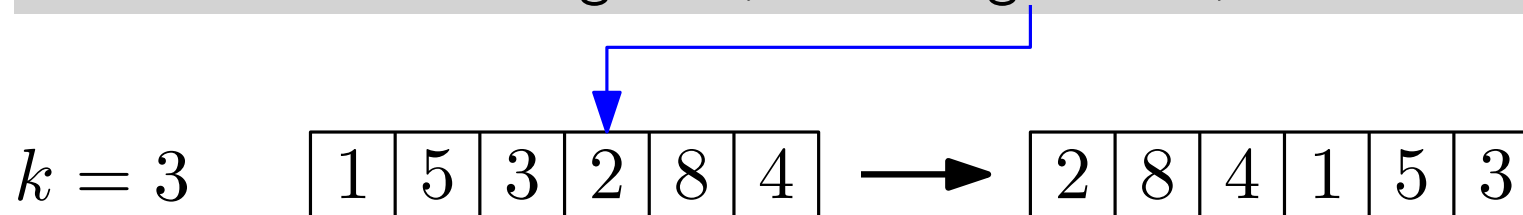
```
bool std::is_partitioned(vec.begin(), vec.end(), p);
```

# Permutations

- Rotations

$O(n)$

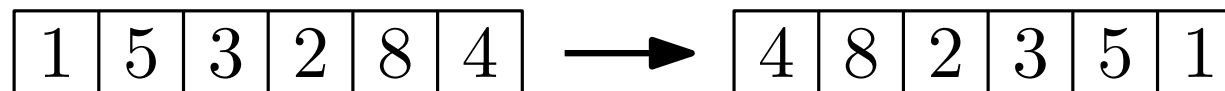
```
std::rotate(vec.begin(), vec.begin()+k, vec.end());
```



- Reversing a permutation

$O(n)$

```
std::reverse(vec.begin(), vec.end());
```



- Shuffling

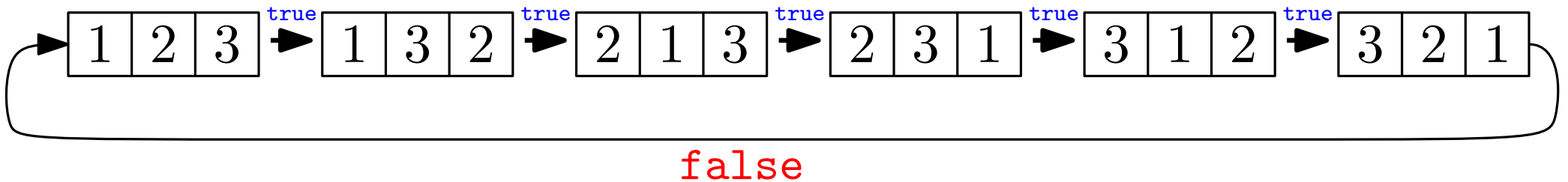
$O(n)$

```
std::shuffle(vec.begin(), vec.end(), rng);
```

# Permutations

- Next/Previous permutation in lexicographic order  $O(n)$

```
bool std::next_permutation(vec.begin(), vec.end());
```

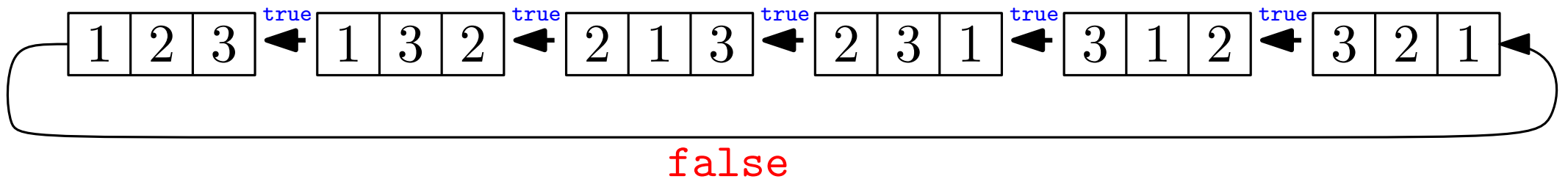


# Permutations

- Next/Previous permutation in lexicographic order  $O(n)$

```
bool std::next_permutation(vec.begin(), vec.end());
```

```
bool std::prev_permutation(vec.begin(), vec.end());
```



# Permutations

- Can  $v_2$  be obtained as a permutation of  $v_1$  ?

```
bool std::is_permutation(v1.begin(), v1.end(), v2.begin(), v2.end())
```

- Does  $v_1$  precede  $v_2$  (w.r.t. the lexicographic order)?

```
bool std::lexicographical_compare(v1.begin(), v1.end(),  
                                v2.begin(), v2.end());
```

- Are  $v_1$  and  $v_2$  the same permutation?

```
bool std::equal(v1.begin(), v1.end(), v2.begin(), v2.end());
```





# Value Queries & Transformations

- Counting

```
std::count(vec.begin(), vec.end(), 5);
```

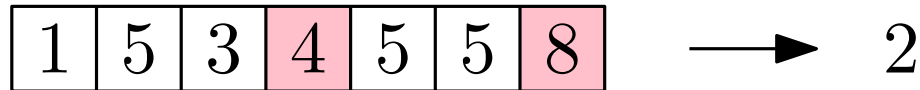


# Value Queries & Transformations

- Counting

```
std::count(vec.begin(), vec.end(), 5);
```

```
std::count_if(vec.begin(), vec.end(), [](int i){ return i%2==0; });
```

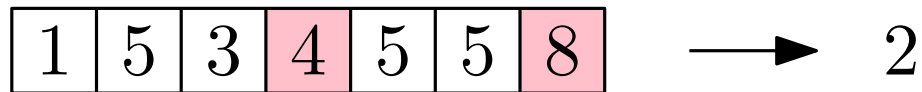


# Value Queries & Transformations

- Counting

```
std::count(vec.begin(), vec.end(), 5);
```

```
std::count_if(vec.begin(), vec.end(), [](int i){ return i%2==0; });
```

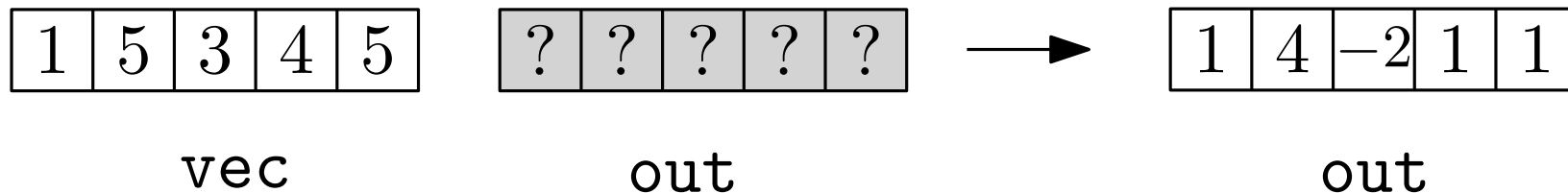


- Adjacent differences



```
std::adjacent_difference(vec.begin(), vec.end(), out.begin(), binary_op);
```

$$\text{out}[0] = \text{vec}[0] \qquad \text{out}[i] = \text{vec}[i] \oplus \text{vec}[i - 1]$$



# Value Queries & Transformations

- Reduce



```
std::reduce(vec.begin(), vec.end(), init, binary_op);
```

**Note:** `binary_op` needs to be commutative and associative.

**Returns:**  $init \oplus \left( \bigoplus_{i=0}^{n-1} \text{vec}[i] \right)$

Example:

```
std::reduce(vec.begin(), vec.end(), 2, [](int x, int y) { return x*y; });
```



# Value Queries & Transformations

- Reduce



```
std::reduce(vec.begin(), vec.end(), init, binary_op);
```

**Note:** `binary_op` needs to be commutative and associative.

**Returns:**  $init \oplus \left( \bigoplus_{i=0}^{n-1} \text{vec}[i] \right)$

Example:

```
std::reduce(vec.begin(), vec.end(), 2, [](int x, int y) { return x*y; });
```

$$\boxed{1 \mid 5 \mid 3} \longrightarrow 2 \prod_{i=0}^2 \text{vec}[i] = 2 \cdot 1 \cdot 5 \cdot 3 = 30$$

# Value Queries & Transformations

- Reduce



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std::reduce(vec.begin(), vec.end(), init, binary_op);
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Example:

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```

$$\boxed{1 \mid 5 \mid 3} \longrightarrow 2 \prod_{i=0}^2 \text{vec}[i] = 2 \cdot 1 \cdot 5 \cdot 3 = 30$$

```
std::reduce(vec.begin(), vec.end());
```

$$\boxed{1 \mid 5 \mid 3} \longrightarrow \sum_{i=0}^2 \text{vec}[i] = 1 + 5 + 3 = 9$$

# Value Queries & Transformations

- Transform (w. unary transform)

```
std::transform(vec.begin(), vec.end(), out.begin(), unary_op);
```

Applies `unary_op()` to every element of `vec`, stores the results in `out`.

**Note:** `unary_op` must not have side effect and must not modify the elements of `vec`.

Example:

```
int unary_op(const int x) { return x+1; }
```

vec 

|   |   |   |   |
|---|---|---|---|
| 2 | 5 | 3 | 1 |
|---|---|---|---|

 $\longrightarrow$ 

|   |   |   |   |
|---|---|---|---|
| 3 | 6 | 4 | 2 |
|---|---|---|---|

 out

# Value Queries & Transformations

- Transform (w. binary transform)

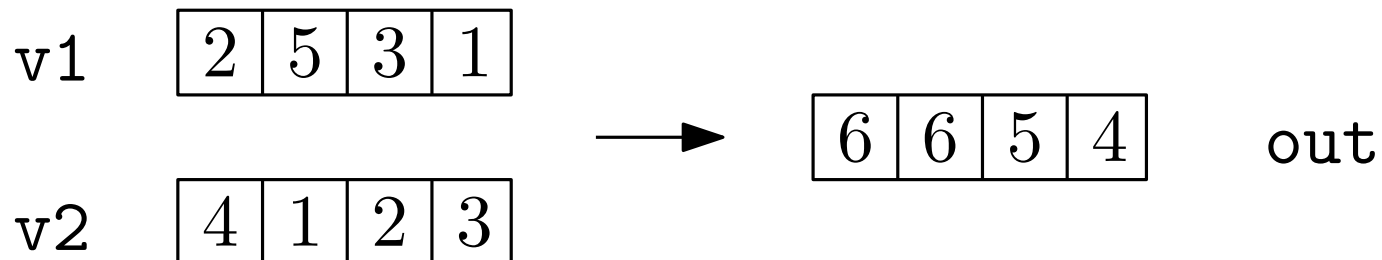
```
std::transform(v1.begin(), v1.end(), v2.begin(), out.begin(), binary_op);
```

Applies `binary_op()` to every pair of corresponding elements of `v1` and `v2`, stores the results in `out`.

**Note:** `binary_op` must not have side effect and must not modify the elements of `v1` or `v2`.

Example:

```
int binary_op(const int x, const int y) { return x+y; }
```





# Value Queries & Transformations

- Transform reduce (w. unary transform)



```
std::transform_reduce(vec.begin(), vec.end(), init, binary_op, unary_op);
```

**Note:** `binary_op` needs to be commutative and associative.

**Returns:**  $init \oplus \left( \bigoplus_{i=0}^{n-1} \text{unary\_op}(\text{vec}[i]) \right)$

Example:

```
std::transform_reduce(vec.begin(), vec.end(), 1, std::sum<int>,
                    [](int x) {return x*x});
```



# Value Queries & Transformations

- Transform reduce (w. unary transform)



```
std::transform_reduce(vec.begin(), vec.end(), init, binary_op, unary_op);
```

**Note:** `binary_op` needs to be commutative and associative.

**Returns:**  $init \oplus \left( \bigoplus_{i=0}^{n-1} \text{unary\_op}(\text{vec}[i]) \right)$

Example:

```
std::transform_reduce(vec.begin(), vec.end(), 1, std::sum<int>,
                    [](int x) {return x*x});
```

$$\boxed{2} \boxed{5} \boxed{3} \longrightarrow 1 + \sum_{i=0}^2 (\text{vec}[i])^2 = 1 + 4 + 25 + 9 = 39$$

# Value Queries & Transformations

- Transform reduce (w. binary transform)



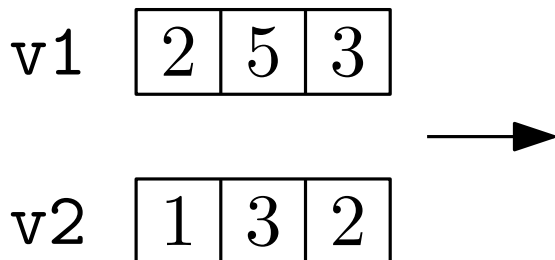
```
std::transform_reduce(v1.begin(), v1.end(), v2.begin(), init, op1, op2);
```

**Note:** op1 and op2 need to be commutative and associative.

**Returns:**  $init \oplus \left( \bigoplus_{i=0}^{n-1} v1[i] \otimes v2[i] \right)$

Example:

```
std::transform_reduce(v1.begin(), v1.end(), v2.begin(), 1,  
                    std::plus<int>, std::multiplies<int>);
```



# Value Queries & Transformations

- Transform reduce (w. binary transform)



```
std::transform_reduce(v1.begin(), v1.end(), v2.begin(), init, op1, op2);
```

**Note:** op1 and op2 need to be commutative and associative.

**Returns:**  $init \oplus \left( \bigoplus_{i=0}^{n-1} v1[i] \otimes v2[i] \right)$

Example:

```
std::transform_reduce(v1.begin(), v1.end(), v2.begin(), 1,  
                    std::plus<int>, std::multiplies<int>);
```

v1 

|   |   |   |
|---|---|---|
| 2 | 5 | 3 |
|---|---|---|

v2 

|   |   |   |
|---|---|---|
| 1 | 3 | 2 |
|---|---|---|

$$\longrightarrow 1 + \sum_{i=0}^2 (v1[i] \cdot v2[i]) = 1 + 2 + 15 + 6 = 24$$

# Value Queries & Transformations

- Inclusive scan



```
std::inclusive_scan(v.begin(), v.end(), out.begin(), binary_op, init);
```

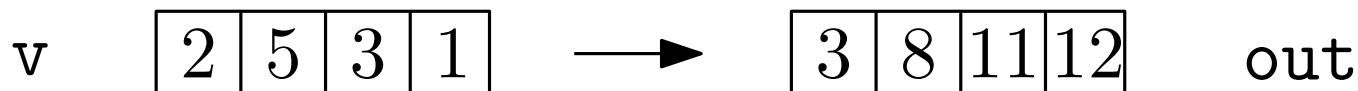
**Note:** `binary_op` needs to be commutative and associative.

For every  $j = 0, 1, \dots, v.size() - 1$ , sets:

$$\text{out}[j] = \text{init} \oplus \left( \bigoplus_{i=0}^j v[i] \right)$$

**Example:**

```
std::inclusive_scan(v.begin(), v.end(), out.begin(), std::plus<>, 1);
```



# Value Queries & Transformations

- **Exclusive** scan



```
std::exclusive_scan(v.begin(), v.end(), out.begin(), binary_op, init);
```

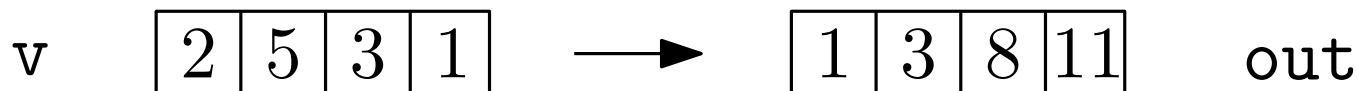
**Note:** `binary_op` needs to be commutative and associative.

For every  $j = 0, 1, \dots, v.size() - 1$ , sets:

$$\text{out}[j] = \text{init} \oplus \left( \bigoplus_{i=0}^{j-1} v[i] \right)$$

**Example:**

```
std::exclusive_scan(v.begin(), v.end(), out.begin(), std::plus<>, 1);
```



# Value Queries & Transformations

- All of:  $\forall x \in \text{vec}, p(x)$  ?

```
bool std::all_of(vec.begin(), vec.end(), p);
```

- Any of:  $\exists x \in \text{vec} : p(x)$  ?

```
bool std::any_of(vec.begin(), vec.end(), p);
```

- None of:  $\nexists x \in \text{vec} : p(x)$  ?     $\forall x \in \text{vec} : \neg p(x)$  ?

```
bool std::none_of(vec.begin(), vec.end(), p);
```

# Value Queries & Transformations

- All of:  $\forall x \in \text{vec}, p(x)$  ?

```
bool std::all_of(vec.begin(), vec.end(), p);
```

- Any of:  $\exists x \in \text{vec} : p(x)$  ?

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bool std::any_of(vec.begin(), vec.end(), p);
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- None of:  $\nexists x \in \text{vec} : p(x)$  ?     $\forall x \in \text{vec} : \neg p(x)$  ?

```
bool std::none_of(vec.begin(), vec.end(), p);
```

Example:

```
bool p(int x) { return x%3 == 0 };
```

|   |   |   |   |   |   |
|---|---|---|---|---|---|
| 1 | 5 | 3 | 2 | 8 | 4 |
|---|---|---|---|---|---|



# Value Queries & Transformations

- All of:  $\forall x \in \text{vec}, p(x)$  ?

```
bool std::all_of(vec.begin(), vec.end(), p);
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bool std::any_of(vec.begin(), vec.end(), p);
```

- None of:  $\nexists x \in \text{vec} : p(x)$  ?     $\forall x \in \text{vec} : \neg p(x)$  ?

```
bool std::none_of(vec.begin(), vec.end(), p);
```

Example:

```
bool p(int x) { return x%3 == 0 };
```

|   |   |   |   |   |   |
|---|---|---|---|---|---|
| 1 | 5 | 3 | 2 | 8 | 4 |
|---|---|---|---|---|---|

`all_of(..., p) = false`

`any_of(..., p) = true`

`none_of(..., p) = false`

# Value Queries & Transformations

- All of:  $\forall x \in \text{vec}, p(x)$  ?

```
bool std::all_of(vec.begin(), vec.end(), p);
```

- Any of:  $\exists x \in \text{vec} : p(x)$  ?

```
bool std::any_of(vec.begin(), vec.end(), p);
```

- None of:  $\nexists x \in \text{vec} : p(x)$  ?     $\forall x \in \text{vec} : \neg p(x)$  ?

```
bool std::none_of(vec.begin(), vec.end(), p);
```

Example:

```
bool p(int x) { return x%3 == 0 };
```

```
std::vector<int> vec();
```

# Value Queries & Transformations

- All of:  $\forall x \in \text{vec}, p(x)$  ?

```
bool std::all_of(vec.begin(), vec.end(), p);
```

- Any of:  $\exists x \in \text{vec} : p(x)$  ?

```
bool std::any_of(vec.begin(), vec.end(), p);
```

- None of:  $\nexists x \in \text{vec} : p(x)$  ?     $\forall x \in \text{vec} : \neg p(x)$  ?

```
bool std::none_of(vec.begin(), vec.end(), p);
```

## Example:

```
bool p(int x) { return x%3 == 0 };
```

```
std::vector<int> vec();
```

`all_of(..., p) = true`

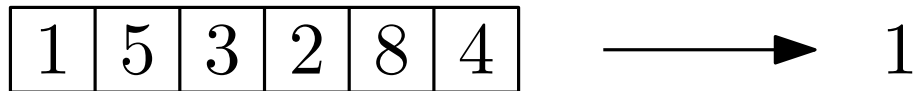
`any_of(..., p) = false`

`none_of(..., p) = true`

# Linear Search

- Minimum/Maximum of a collection

```
std::min_element(vec.begin(), vec.end());
```

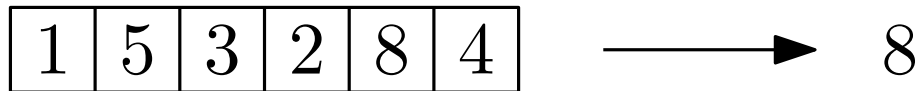


# Linear Search

- Minimum/Maximum of a collection

```
std::min_element(vec.begin(), vec.end());
```

```
std::max_element(vec.begin(), vec.end());
```



# Linear Search

- Minimum/Maximum of a collection

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std::min_element(vec.begin(), vec.end());
```

```
std::max_element(vec.begin(), vec.end());
```

```
std::minmax_element(vec.begin(), vec.end());
```

|   |   |   |   |   |   |
|---|---|---|---|---|---|
| 1 | 5 | 3 | 2 | 8 | 4 |
|---|---|---|---|---|---|

 $\longrightarrow$  `std::pair(1, 8)`

# Linear Search

- Minimum/Maximum of a collection

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std::min_element(vec.begin(), vec.end());
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std::max_element(vec.begin(), vec.end());
```

```
std::minmax_element(vec.begin(), vec.end());
```

|   |   |   |   |   |   |
|---|---|---|---|---|---|
| 1 | 5 | 3 | 2 | 8 | 4 |
|---|---|---|---|---|---|

 → std::pair(1, 8)

- Searching for an element

```
std::find(vec.begin(), vec.end(), 9);
```

|   |   |   |   |   |   |
|---|---|---|---|---|---|
| 6 | 9 | 3 | 5 | 9 | 4 |
|---|---|---|---|---|---|



# Linear Search

- Minimum/Maximum of a collection

```
std::min_element(vec.begin(), vec.end());
```

```
std::max_element(vec.begin(), vec.end());
```

```
std::minmax_element(vec.begin(), vec.end());
```

|   |   |   |   |   |   |
|---|---|---|---|---|---|
| 1 | 5 | 3 | 2 | 8 | 4 |
|---|---|---|---|---|---|

 → std::pair(1, 8)

- Searching for an element

```
std::find(vec.begin(), vec.end(), 2);
```

|   |   |   |   |   |   |
|---|---|---|---|---|---|
| 6 | 9 | 3 | 5 | 9 | 4 |
|---|---|---|---|---|---|

 ×





# Linear Search

- Minimum/Maximum of a collection

```
std::min_element(vec.begin(), vec.end());
```

```
std::max_element(vec.begin(), vec.end());
```

```
std::minmax_element(vec.begin(), vec.end());
```

|   |   |   |   |   |   |
|---|---|---|---|---|---|
| 1 | 5 | 3 | 2 | 8 | 4 |
|---|---|---|---|---|---|

 → std::pair(1, 8)

- Searching for an element

```
std::find(vec.begin(), vec.end(), 2);
```

```
std::find_if(vec.begin(), vec.end(), [](int x) { return x%3; });
```

|   |   |   |   |   |   |
|---|---|---|---|---|---|
| 6 | 9 | 3 | 5 | 9 | 4 |
|---|---|---|---|---|---|

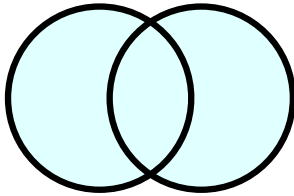


# Set Operations

**Note:** For the purpose of the following operations, sets are just sorted containers (not necessarily `std::set`).

# Set Operations

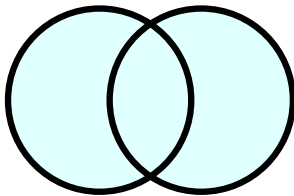
**Note:** For the purpose of the following operations, sets are just sorted containers (not necessarily `std::set`).



```
std::set_union(v1.begin(), v1.end(), v2.begin(), v2.end(), out_it);
```

# Set Operations

**Note:** For the purpose of the following operations, sets are just sorted containers (not necessarily `std::set`).



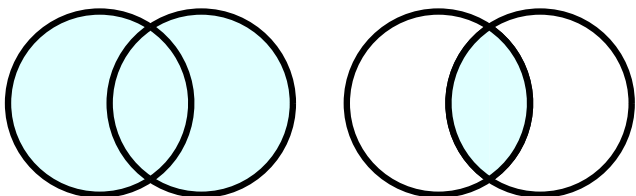
```
std::set_union(v1.begin(), v1.end(), v2.begin(), v2.end(), out_it);
```

Example:

```
std::vector<int> v1 { 1, 2, 3 };  
std::vector<int> v2 { 2, 4, 5 };  
std::vector<int> out();  
  
std::set_union(v1.begin(), v1.end(), v2.begin(), v2.end(),  
              std::back_inserter(out));  
  
// out contains 1, 2, 3, 4, 5
```

# Set Operations

**Note:** For the purpose of the following operations, sets are just sorted containers (not necessarily `std::set`).

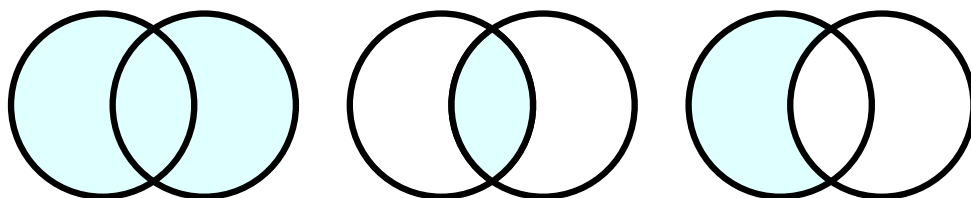


```
std::set_union(v1.begin(), v1.end(), v2.begin(), v2.end(), out_it);
```

```
std::set_intersection(v1.begin(), v1.end(), v2.begin(), v2.end(), out_it);
```

# Set Operations

**Note:** For the purpose of the following operations, sets are just sorted containers (not necessarily `std::set`).



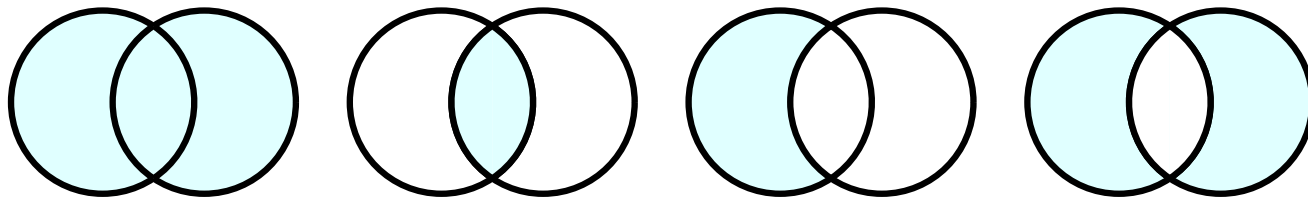
```
std::set_union(v1.begin(), v1.end(), v2.begin(), v2.end(), out_it);
```

```
std::set_intersection(v1.begin(), v1.end(), v2.begin(), v2.end(), out_it);
```

```
std::set_difference(v1.begin(), v1.end(), v2.begin(), v2.end(), out_it);
```

# Set Operations

**Note:** For the purpose of the following operations, sets are just sorted containers (not necessarily `std::set`).



```
std::set_union(v1.begin(), v1.end(), v2.begin(), v2.end(), out_it);
```

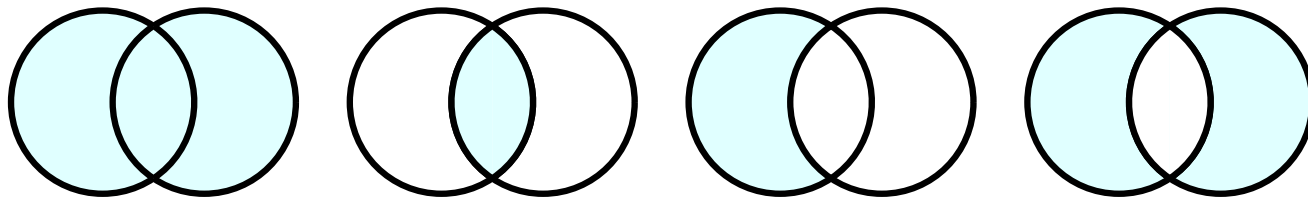
```
std::set_intersection(v1.begin(), v1.end(), v2.begin(), v2.end(), out_it);
```

```
std::set_difference(v1.begin(), v1.end(), v2.begin(), v2.end(), out_it);
```

```
std::set_symmetric_difference(v1.begin(), v1.end(), v2.begin(), v2.end(),  
                             out_it);
```

# Set Operations

**Note:** For the purpose of the following operations, sets are just sorted containers (not necessarily `std::set`).



```
std::set_union(v1.begin(), v1.end(), v2.begin(), v2.end(), out_it);
```

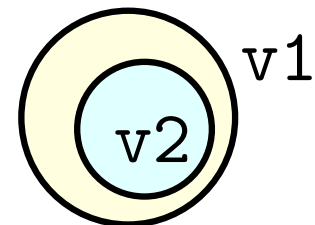
```
std::set_intersection(v1.begin(), v1.end(), v2.begin(), v2.end(), out_it);
```

```
std::set_difference(v1.begin(), v1.end(), v2.begin(), v2.end(), out_it);
```

```
std::set_symmetric_difference(v1.begin(), v1.end(), v2.begin(), v2.end(),  
                             out_it);
```

```
bool std::includes(v1.begin(), v1.end(), v2.begin(), v2.end());
```

**Returns:** true iff v2 is a subset of v1





# Binary Search

- Is 5 in the collection?

```
bool std::binary_search(vec.begin(), vec.end(), 6);
```



# Binary Search

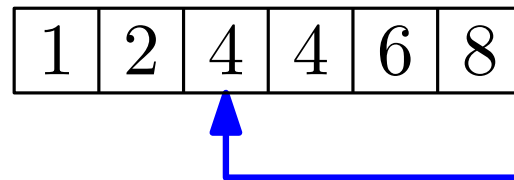
- Is 5 in the collection?

```
bool std::binary_search(vec.begin(), vec.end(), 6);
```



- First index  $i$  s.t.  $v[i] \geq 3$  (Where should 3 be inserted?)

```
std::lower_bound(vec.begin(), vec.end(), 3);
```



# Binary Search

- Is 5 in the collection?

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bool std::binary_search(vec.begin(), vec.end(), 6);
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- First index  $i$  s.t.  $v[i] \geq 3$  (Where should 3 be inserted?)

```
std::lower_bound(vec.begin(), vec.end(), 3);
```

- First index  $i$  s.t.  $v[i] > 4$

```
std::upper_bound(vec.begin(), vec.end(), 4);
```



# Binary Search

- Is 5 in the collection?

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bool std::binary_search(vec.begin(), vec.end(), 6);
```

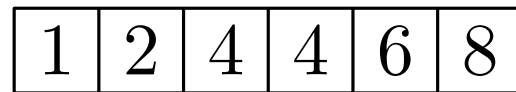


- First index  $i$  s.t.  $v[i] \geq 3$  (Where should 3 be inserted?)

```
std::lower_bound(vec.begin(), vec.end(), 3);
```

- First index  $i$  s.t.  $v[i] > 4$

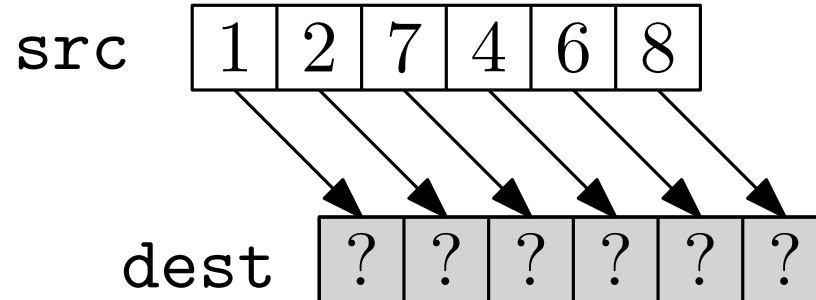
```
std::upper_bound(vec.begin(), vec.end(), 4);
```



```
std::equal_range(vec.begin(), vec.end(), 4); → std::pair(lb, ub);
```

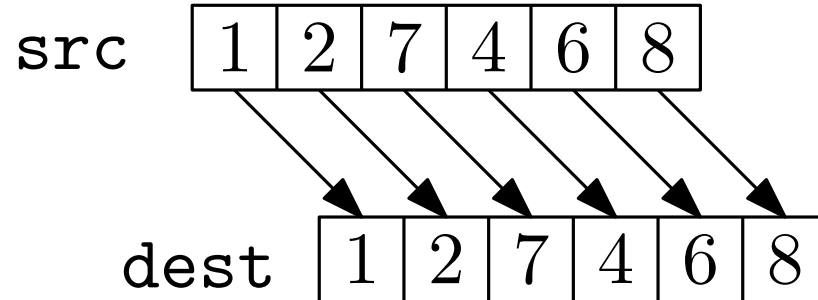
# Copying collections

```
std::copy(src.begin(), src.end(), dest.begin());
```



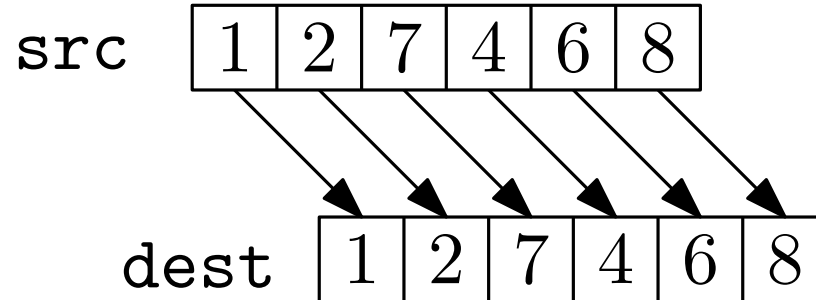
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```
std::copy(src.begin(), src.end(), dest.begin());
```

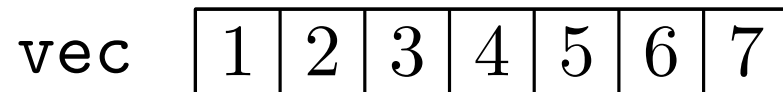


# Copying collections

```
std::copy(src.begin(), src.end(), dest.begin());
```

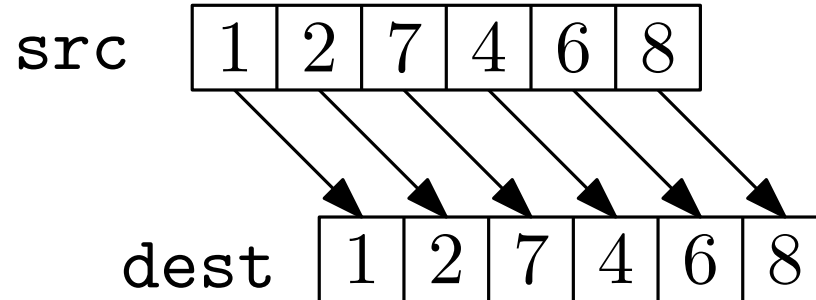


```
std::copy(vec.begin(), vec.begin()+4, vec.begin()+2);
```

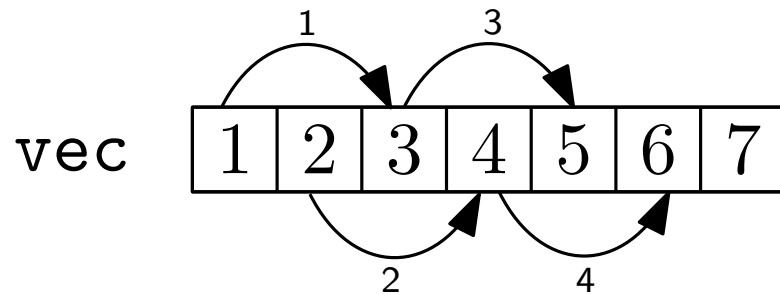


# Copying collections

```
std::copy(src.begin(), src.end(), dest.begin());
```



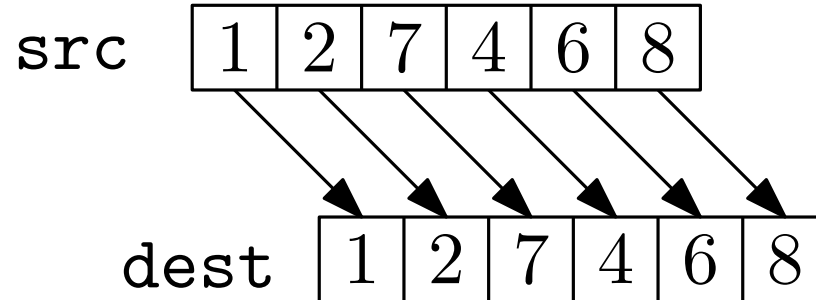
```
std::copy(vec.begin(), vec.begin()+4, vec.begin()+2);
```



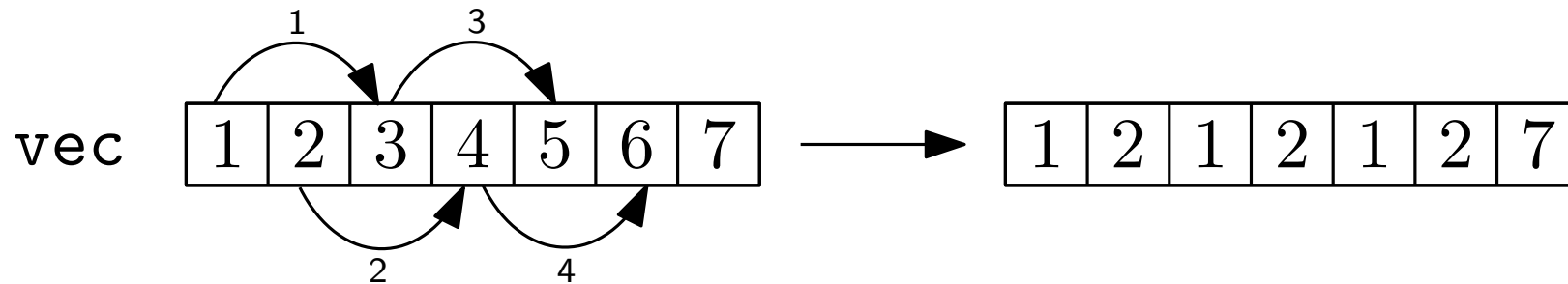


# Copying collections

```
std::copy(src.begin(), src.end(), dest.begin());
```

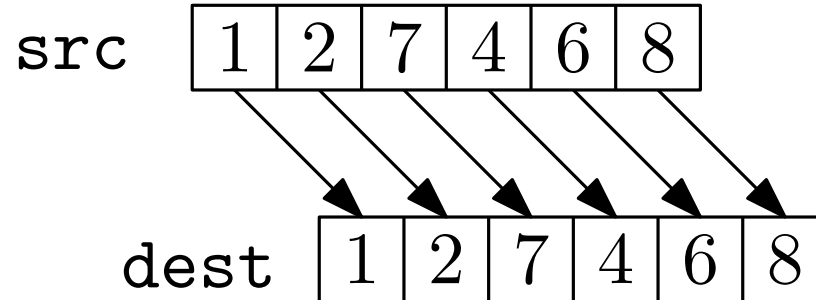


```
std::copy(vec.begin(), vec.begin()+4, vec.begin()+2);
```

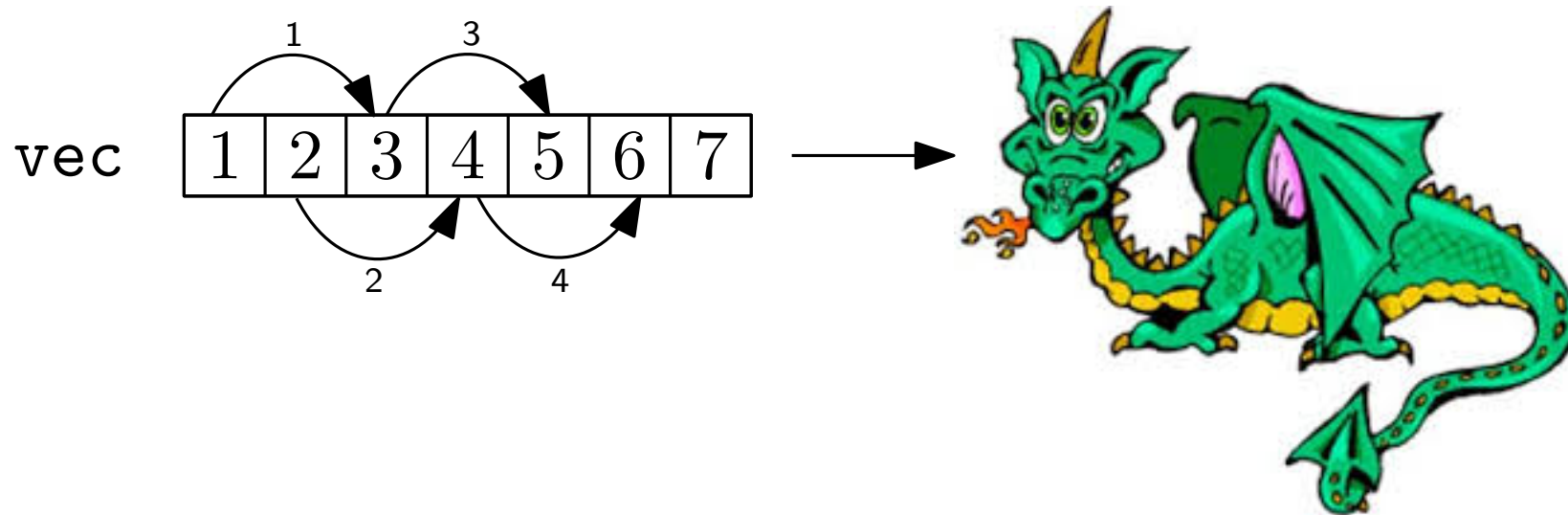


# Copying collections

```
std::copy(src.begin(), src.end(), dest.begin());
```

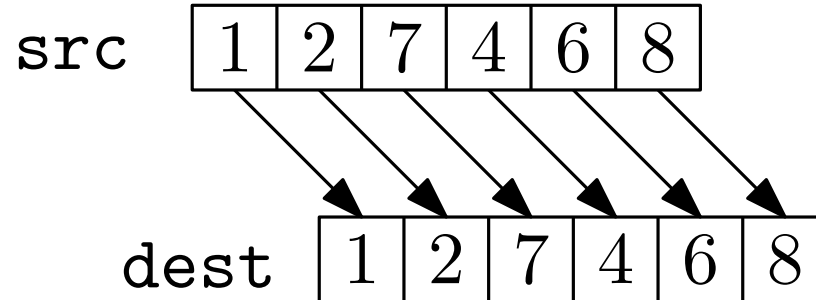


```
std::copy(vec.begin(), vec.begin()+4, vec.begin()+2);
```

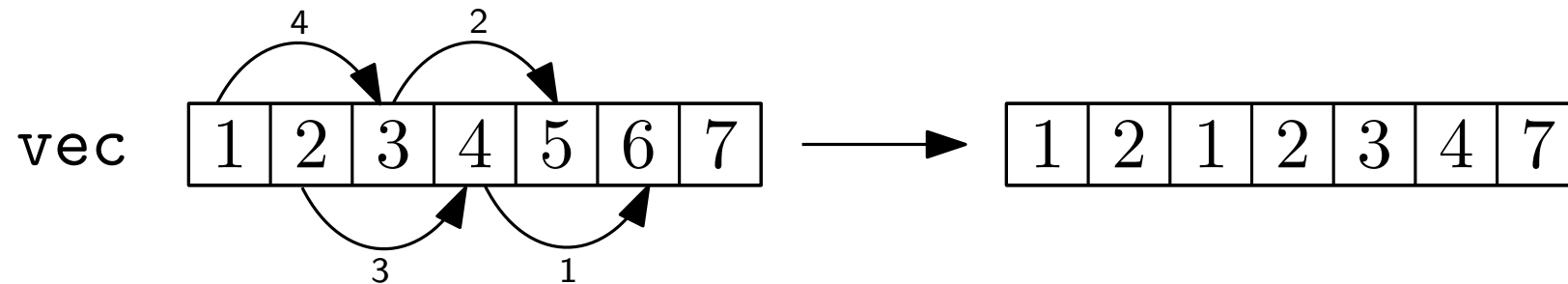


# Copying collections

```
std::copy(src.begin(), src.end(), dest.begin());
```

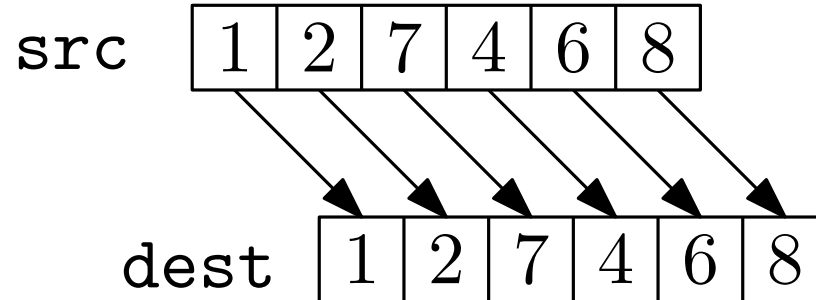


```
std::copy_backward(vec.begin(), vec.begin()+4, vec.begin()+5);
```

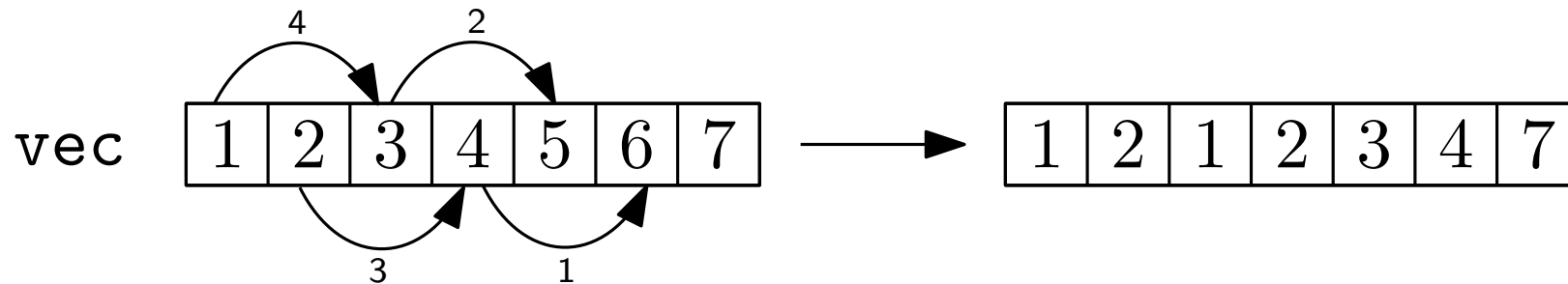


# Copying collections

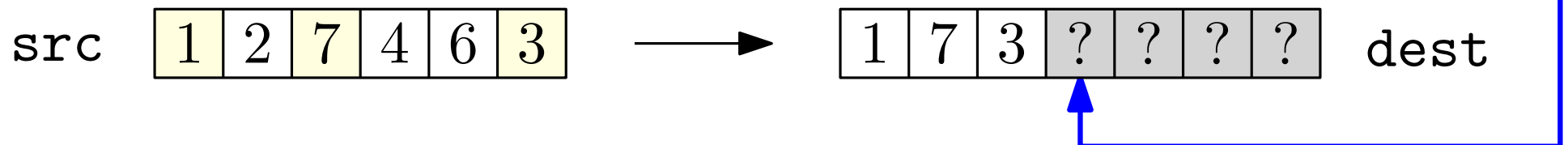
```
std::copy(src.begin(), src.end(), dest.begin());
```



```
std::copy_backward(vec.begin(), vec.begin()+4, vec.begin()+5);
```

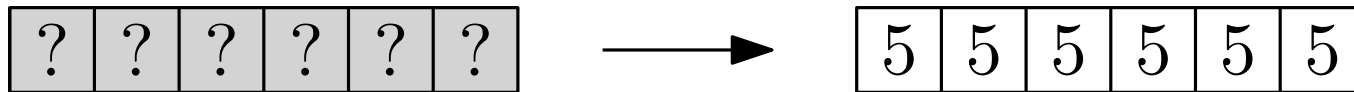


```
std::copy_if(src.begin(), src.end(), dest.begin(), [](int x){ return x%2; });
```

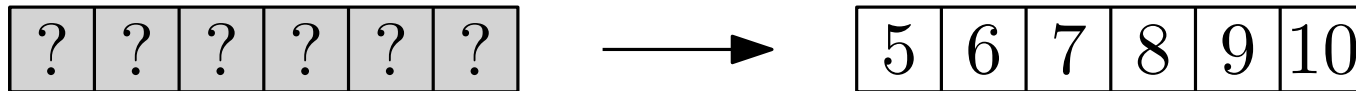


# Initializing collections

```
std::fill(vec.begin(), vec.end(), 5);
```



```
std::iota(vec.begin(), vec.end(), 5);
```



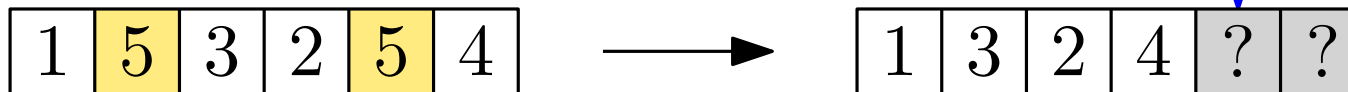
```
std::generate(vec.begin(), vec.end(), f);
```

Shorthand for:

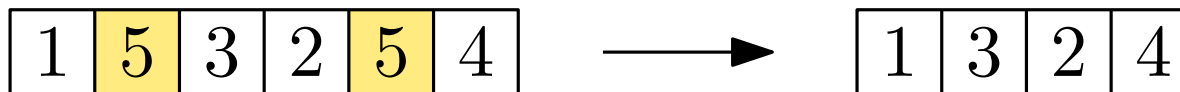
```
vec[0] = f();  
vec[1] = f();  
...
```

# Deleting elements

```
std::remove(vec.begin(), vec.end(), 5);
```

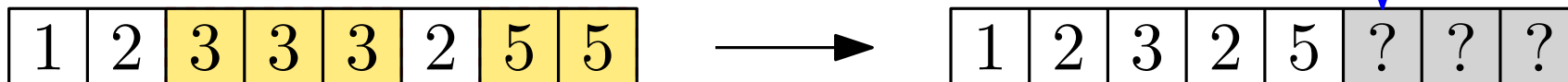


```
vec.erase(std::remove(vec.begin(), vec.end(), 5), vec.end());
```



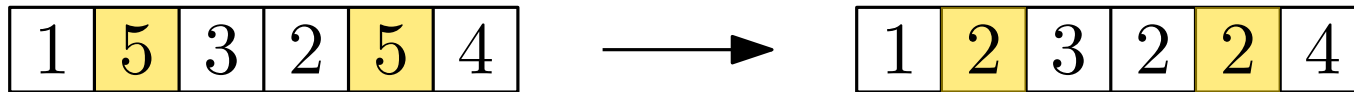
See also: `std::remove_if`.

```
std::unique(vec.begin(), vec.end());
```

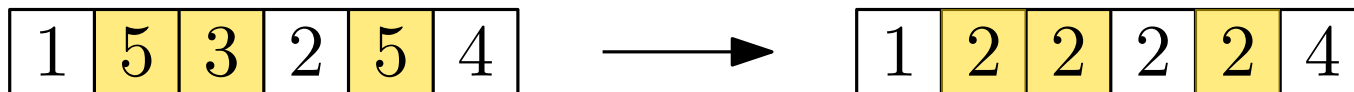


# Replacing elements

```
std::replace(vec.begin(), vec.end(), 5, 2);
```



```
std::replace_if(vec.begin(), vec.end(), [](int x) { return x%2; }, 2);
```



# Foreach

- Executes `f()` on every element between between `vec.begin()` (inclusive) and `vec.end()` (exclusive), in order.

```
std::foreach(vec.begin(), vec.end(), f);
```



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Example:

```
void f(int& x) { x = x*x; }
```



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```
std::foreach(vec.begin(), vec.end(), f);
```

Example:

```
void f(int& x) { x = x*x; }
```



Consider using range-based for loops, when appropriate:

```
for(int& x : vec) { x = x*x; }
```