### Information Systems and Network Security

Docente: Stefano Leucci



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### Basic Info

### **Course length:** 48 hours (6 CFU)

• 24 lectures

### When/where:

- Tuesday 9:30 11:30 **Room:** A1.2
- Thursday 11:30 13:30 **Room:** A1.2



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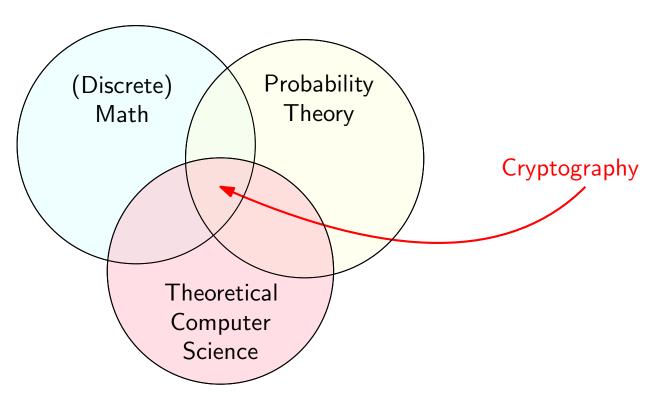
### Office hours:

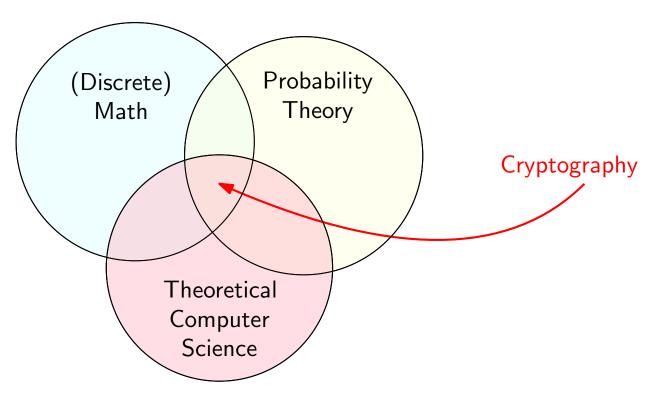
- Thursday 16:30 18:30
- Please send an email to stefano.leucci@gmail.com or ask before/after class

#### **Course material:**

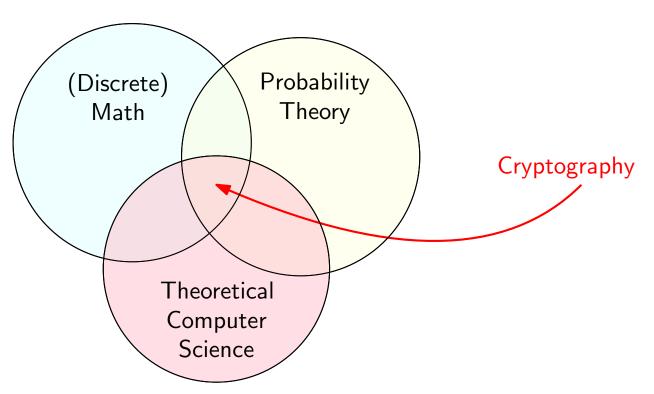
https://people.disim.univaq.it/~stefanoleucci/isns24/





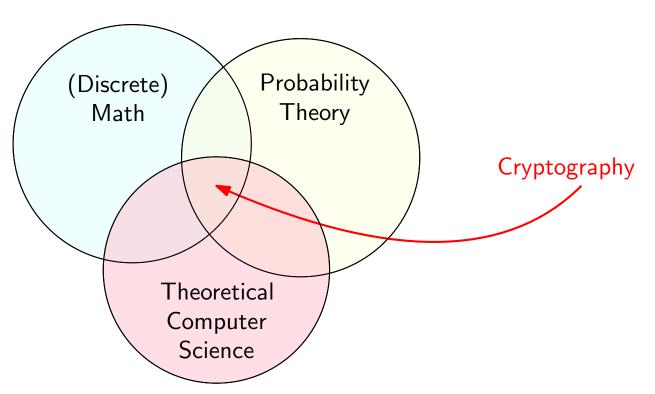


(Discrete) Math: Basic algebra, Modular arithmetic, some concepts from group theory and number theory



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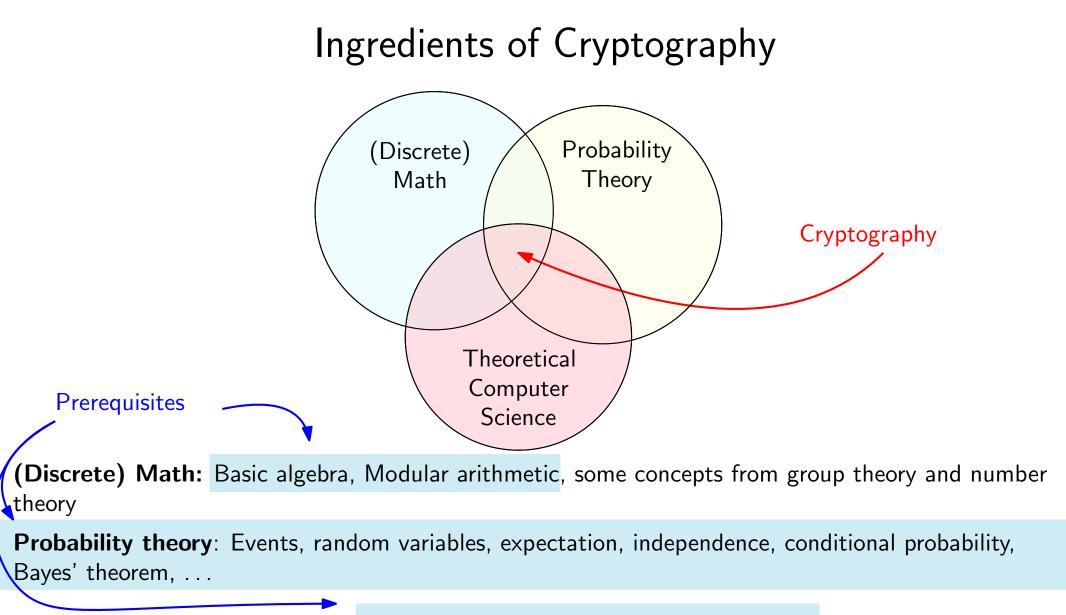
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Theoretical computer science: Algorithms, complexity, asymptotic notation, reductions

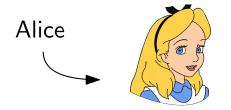


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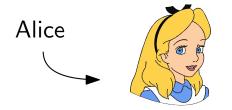


Two entities wish to communicate





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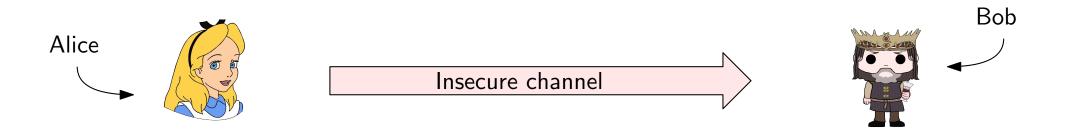


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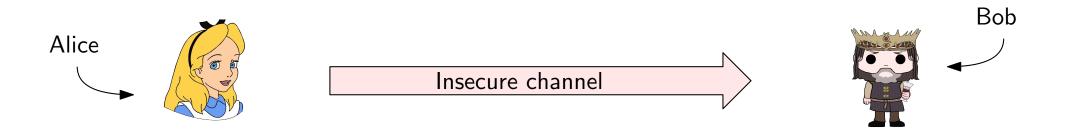
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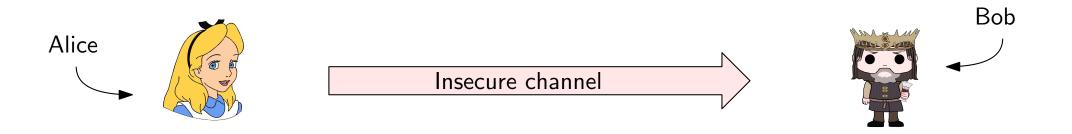


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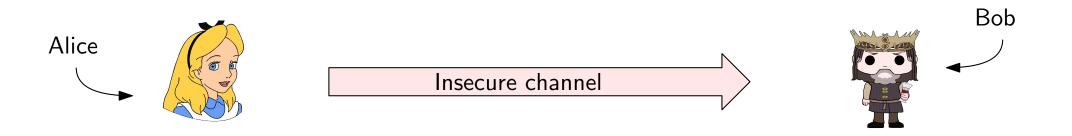


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What does "secure" mean?



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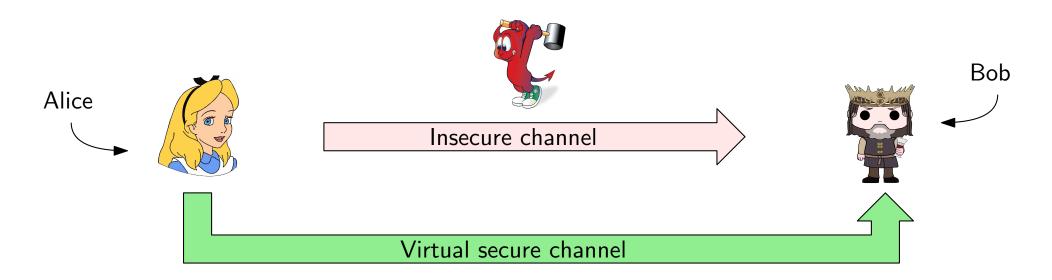


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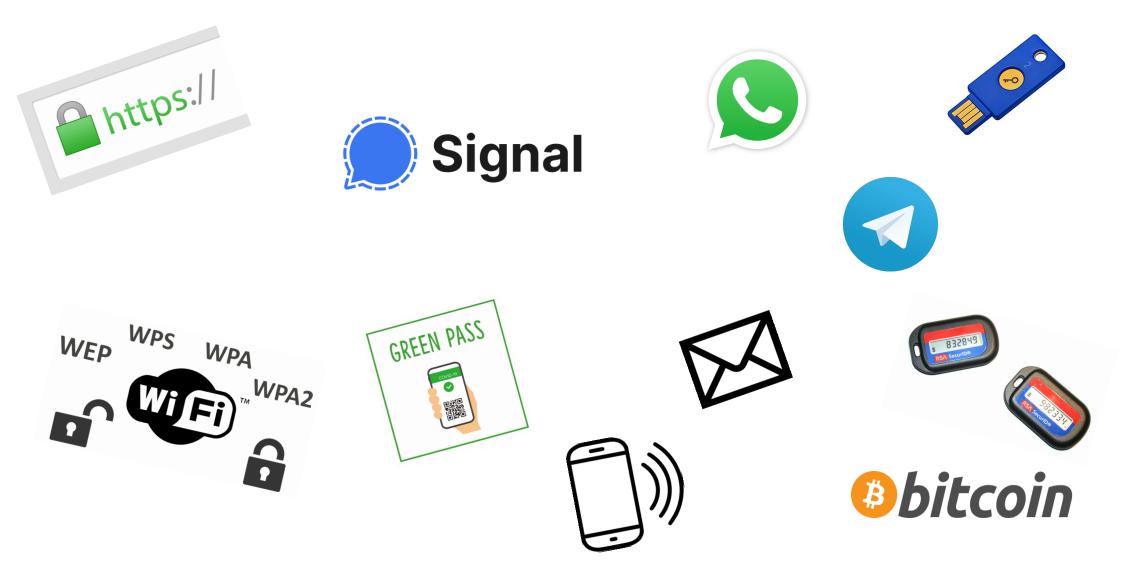
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- Authentication: any message received by Bob can be confirmed to have originated from Alice (we can detect any message injected by the adversary)
- Integrity: it is not possible to alter the contents of a message sent across the channel (without the tampering being detected)



Intuitively, we established a virtual "secure" communication channel on top of an underlying insecure channel



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YOU ARE Here

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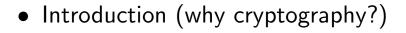
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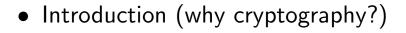
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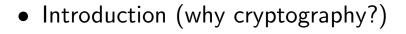


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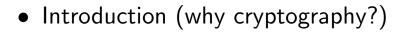
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# Private-key

cryptography

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Topics

- Hash functions
- Key distribution

At the "boundary" between private-key and public-key cryptography

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Public-key cryptography

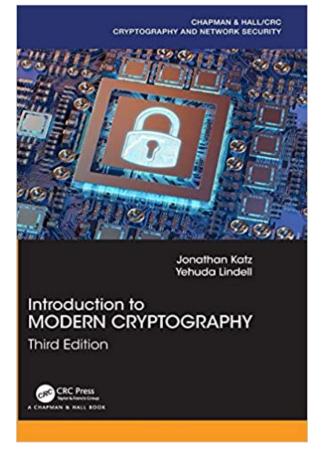
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At the "boundary" between private-key and public-key cryptography

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- Public-key cryptography, Hybrid cryptography
- Digital signatures
- Digital certificates, SSL/TLS
- Advanced applications: secret sharing, multiparty computation, zero-knowledge proofs

Public-key cryptography

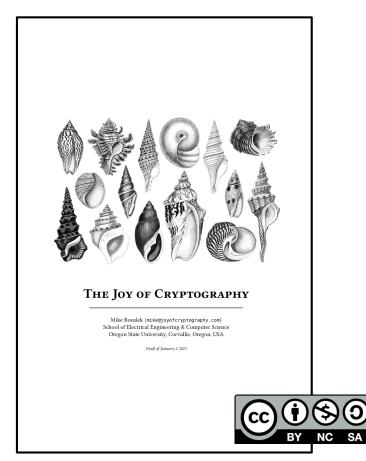
### Books



Introduction to Modern Cryptography

Jonathan Katz, Yehuda Lindell

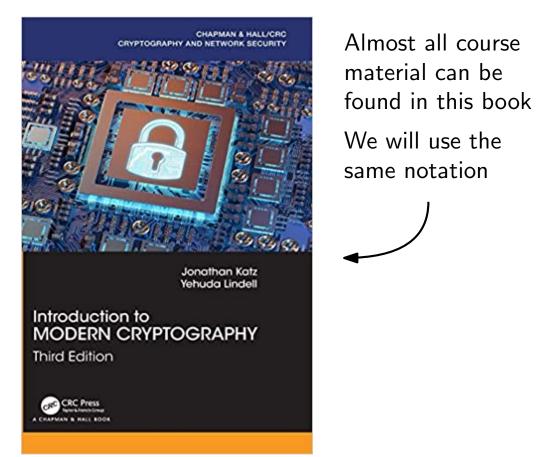
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The Joy Of Cryptography

Mike Rosulek https://joyofcryptography.com/

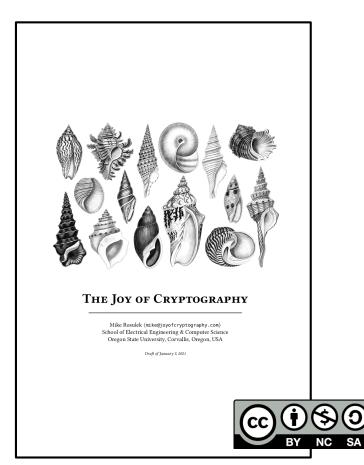
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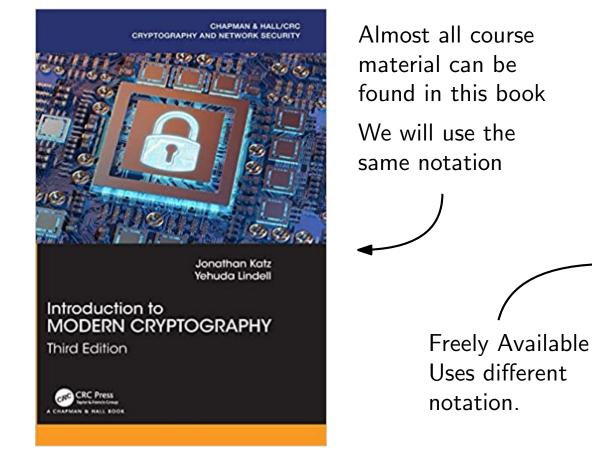
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### Books



THE JOY OF CRYPTOGRAPHY Mike Rosulek (mike@joyofcryptography.com School of Electrical Engineering & Computer Science Oregon State University, Corvallis, Oregon, USA Draft of January 3, 2021

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### Exams

Written exam:

- Questions on the theoretical concepts (e.g., security definitions)
- Exercises (e.g., prove security, carry out an attack, etc...)



### Some advanced applications

Imagine some sensitive information that is kept by a single agent

- A master encryption key
- Your bitcoin wallet
- Nuclear codes
- ...





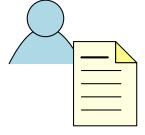
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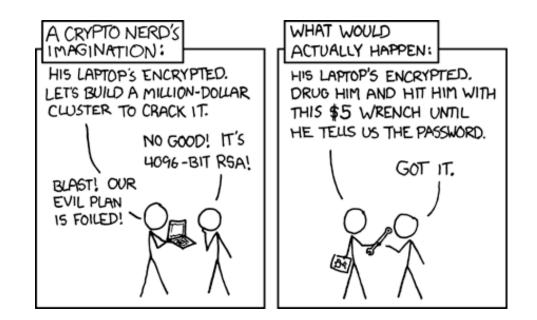
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Single point of failure!



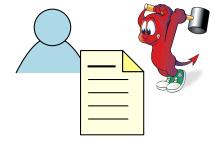
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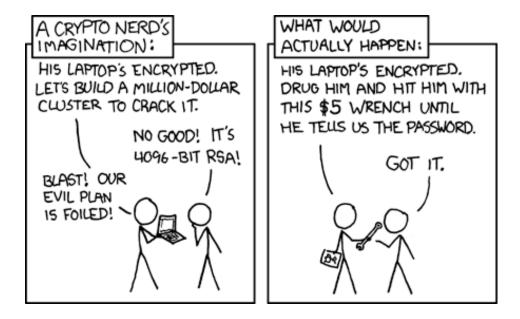






### Single point of failure!

An attacker can compromise one machine and steal the sensitive information



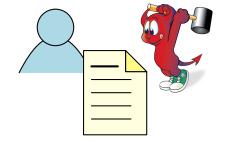
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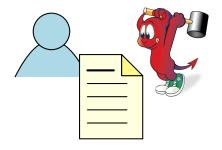
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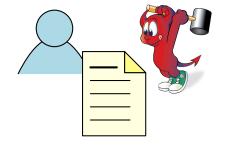


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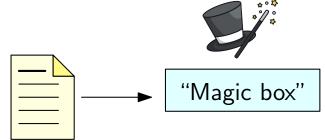
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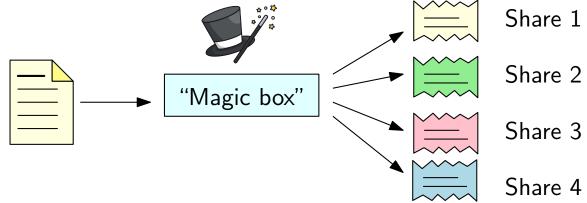
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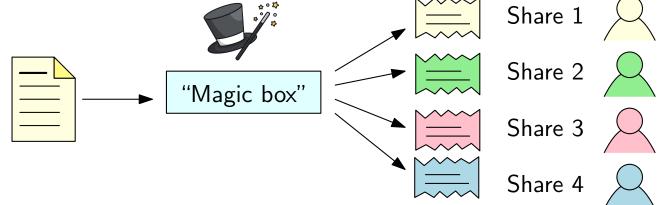
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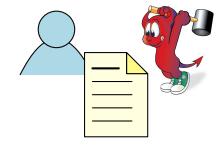
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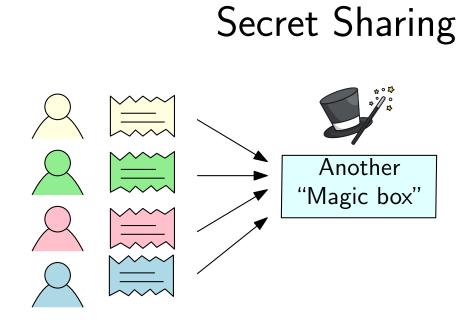
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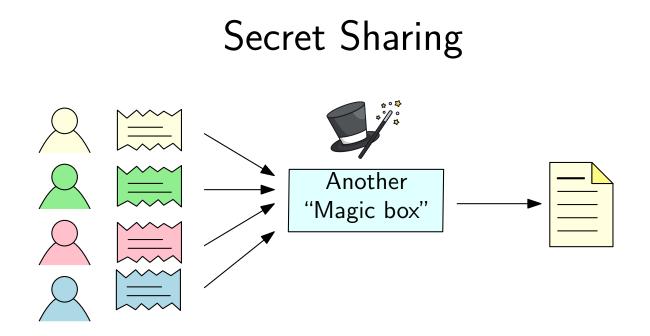
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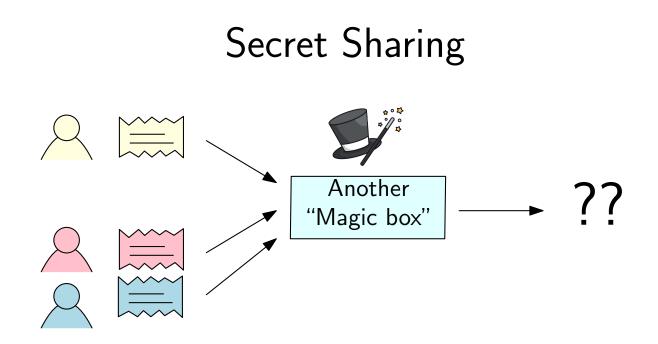




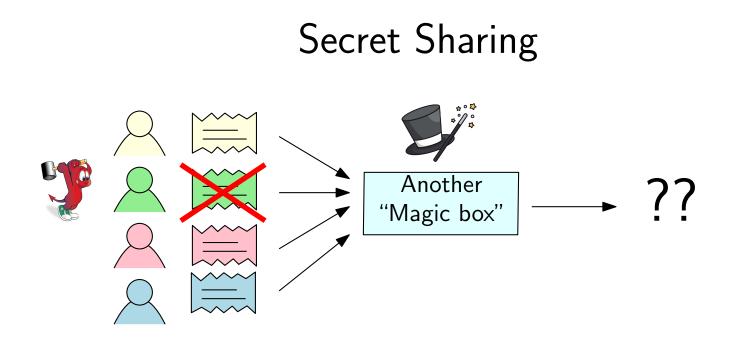
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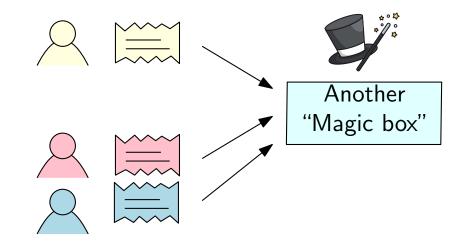
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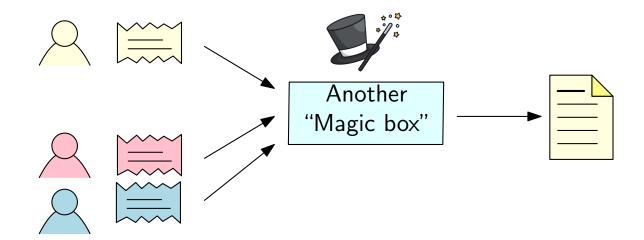


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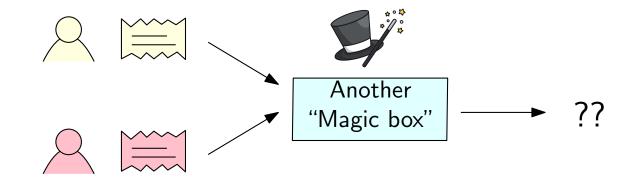


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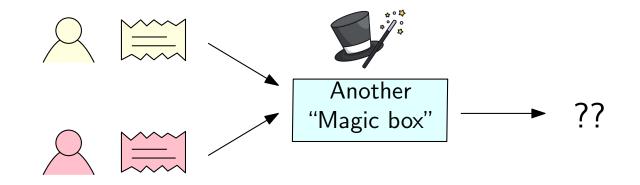


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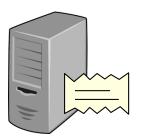
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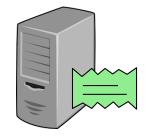
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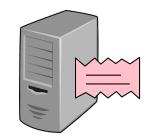


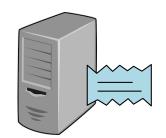
*t*-out-of-*n* threshold secret-sharing scheme

• Agents can be different servers connected over the Internet

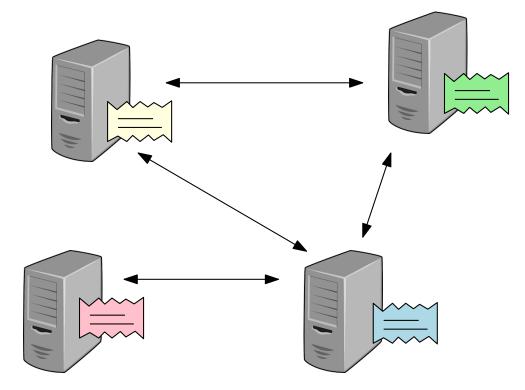




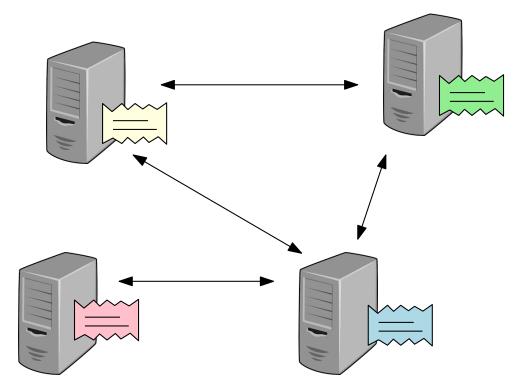




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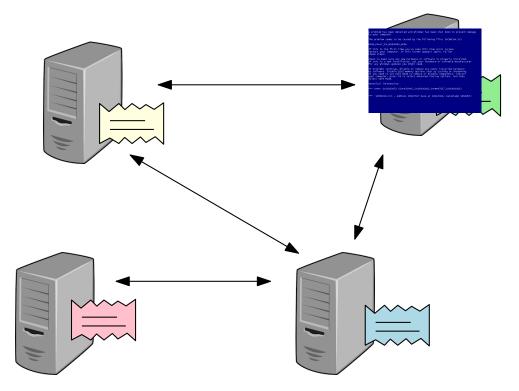
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## Secret Sharing

- Agents can be different servers connected over the Internet
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- The system remains secure if < t servers are compromised
- The system remains operational (the secret can be recovered) if  $\leq n t$  servers are unavailable

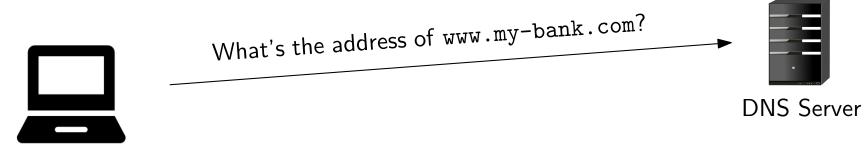
The Domain Name System is the system responsible for converting human-readable domain names into IP addresses



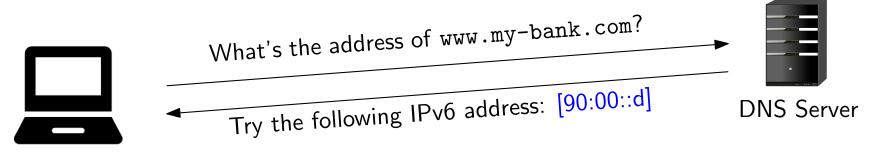


**DNS** Server

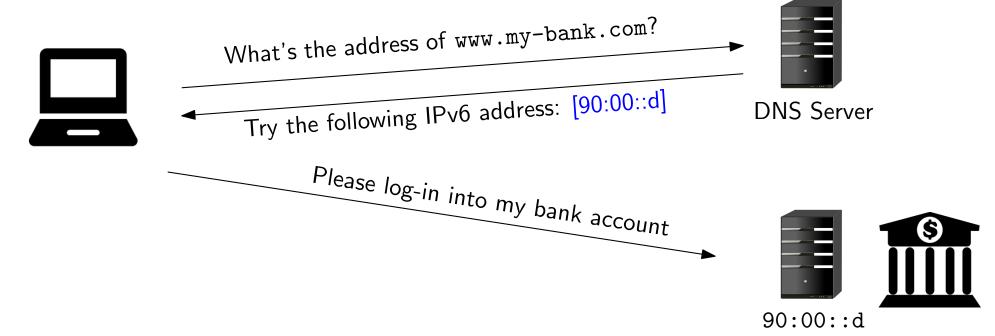
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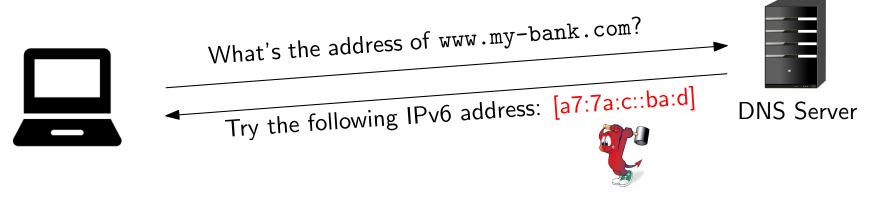
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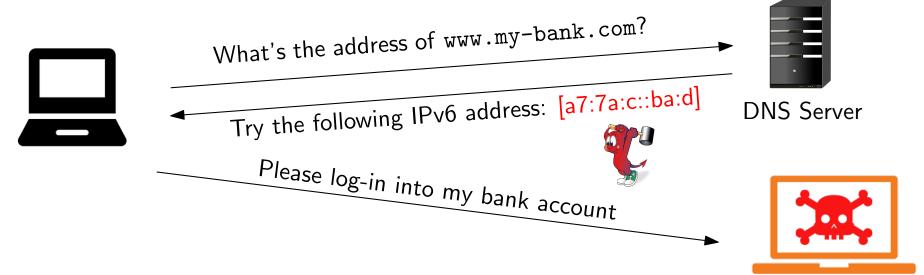


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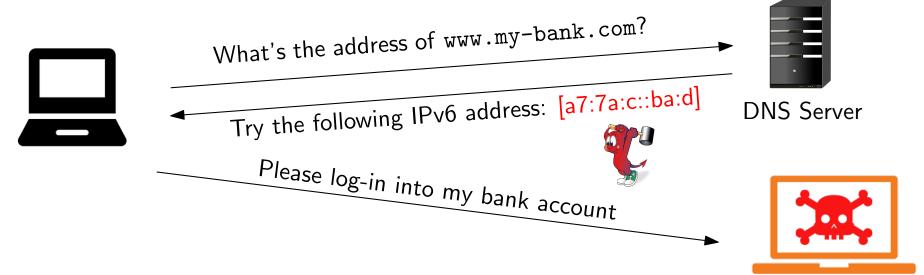
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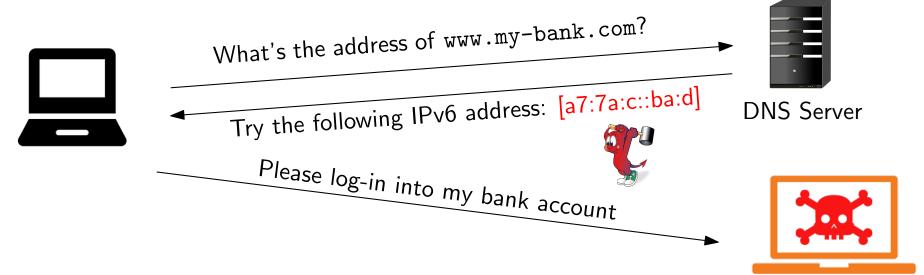
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Who can be trusted with the master cryptographic keys to the system?

DNSSEC is managed by the Internet Corporation for Assigned Names and Numbers (ICANN)

The master key is split into 7 pieces and distributed on smart cards to 7 geographically diverse people

At least five key-holding members of this fellowship would have to meet at a secure data center in the United States to reboot [DNSSEC] in case of a very unlikely system collapse.

"If you round up five of these guys, they can decrypt [the root key] should the West Coast fall in the water and the East Coast get hit by a nuclear bomb"

— Richard Lamb, program manager for DNSSEC at ICANN.



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"If you round up five of these guys, they can decrypt [the root key] should the West Coast fall in the water and the East Coast get hit by a nuclear bomb"

— Richard Lamb, program manager for DNSSEC at ICANN.

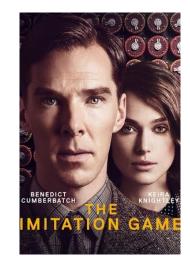
# 

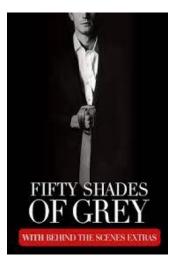
n = 7 t = 5

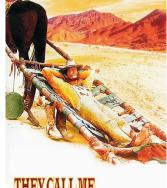
5-out-of-7 threshold secret-sharing scheme

Alice and Bob want to decide on a movie to watch





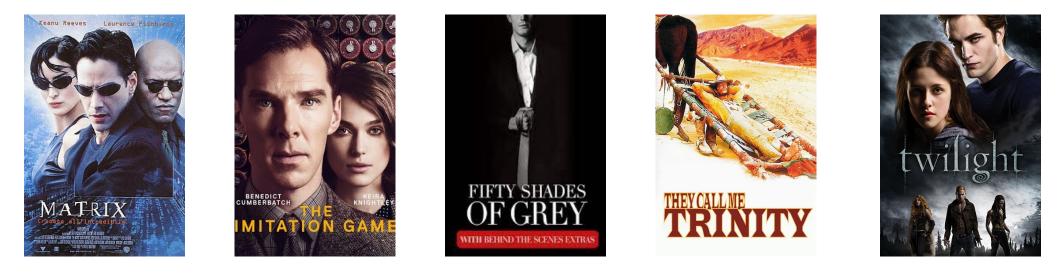








Alice and Bob want to decide on a movie to watch



Each of Alice and Bob has their own preferences...

However, they don't want to reveal that they like a movie unless the other person also likes it

Alice and Bob want to decide on a movie to watch

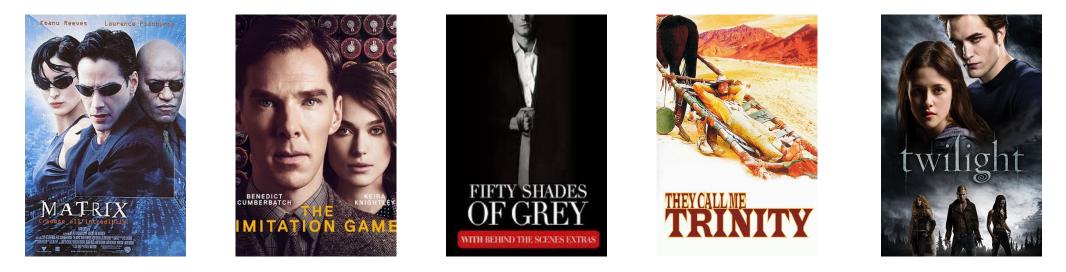


Each of Alice and Bob has their own preferences...

However, they don't want to reveal that they like a movie unless the other person also likes it

- They vote on one movie at a time until they find a movie that they both like
- Voting is done using a secure **2-party computation protocol**

Alice and Bob want to decide on a movie to watch



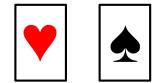
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Alice and Bob are **honest** (they follow the protocol) but **curious** 

They will use some cards with two different faces, say hearts and spades





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They will place 5 cards in a row on a table







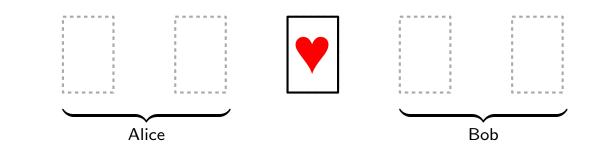
They will use some cards with two different faces, say hearts and spades

They will place 5 cards in a row on a table

The middle card is face up and is always a hearts card





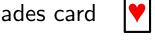


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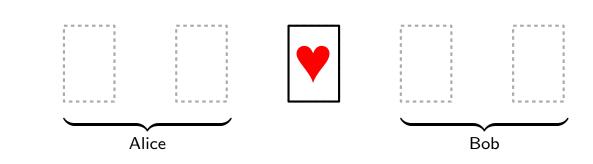
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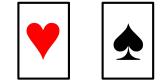
The middle card is face up and is always a hearts card

Each of Alice and Bob has a hearts card and a spades card





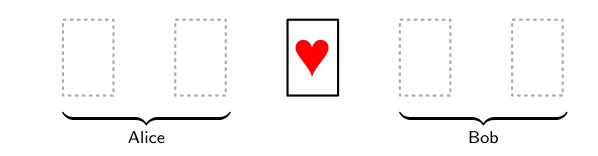




If Alice likes the movie, she will place her two cards face down in the order

Otherwise she will place her two cards **face down** in the order





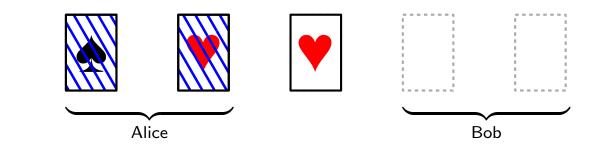




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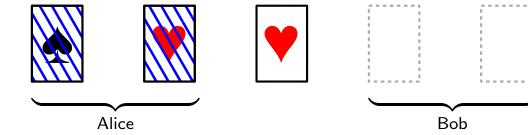


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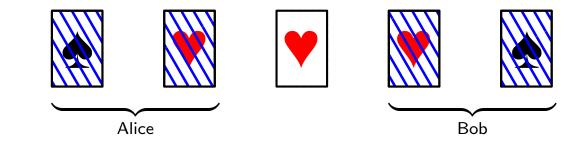


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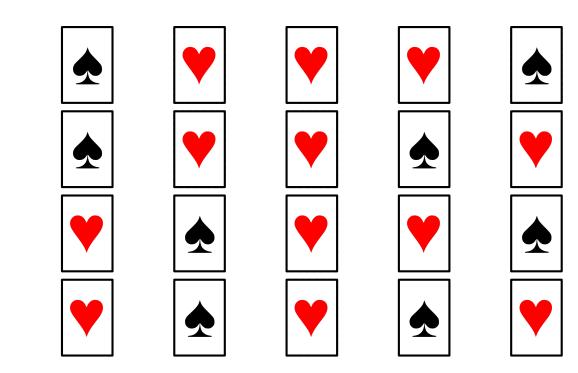






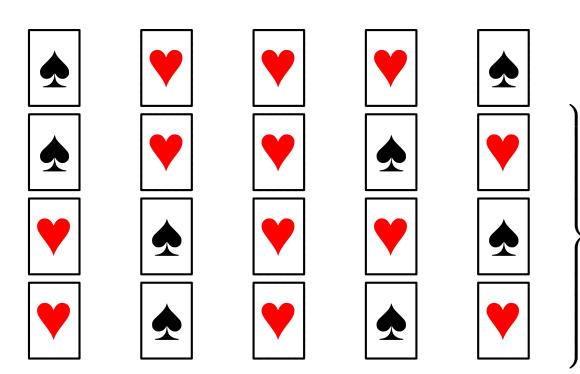
**Possible configurations:** 





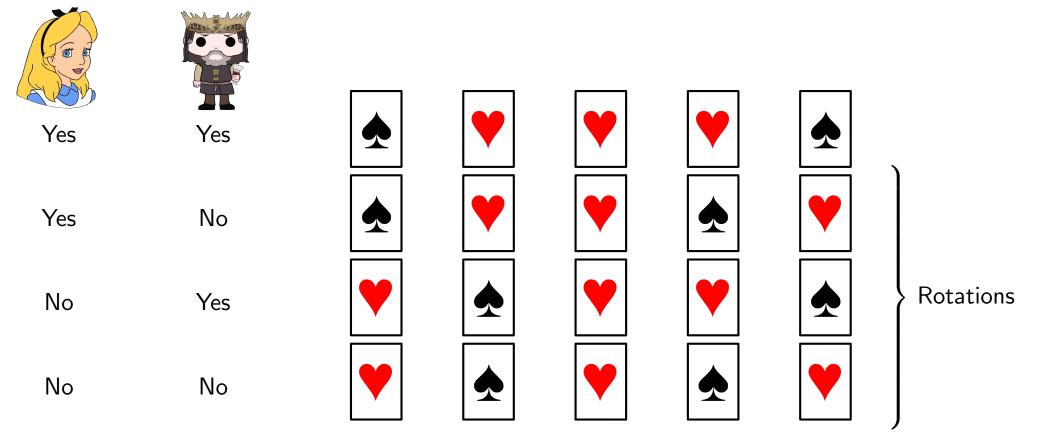
**Possible configurations:** 





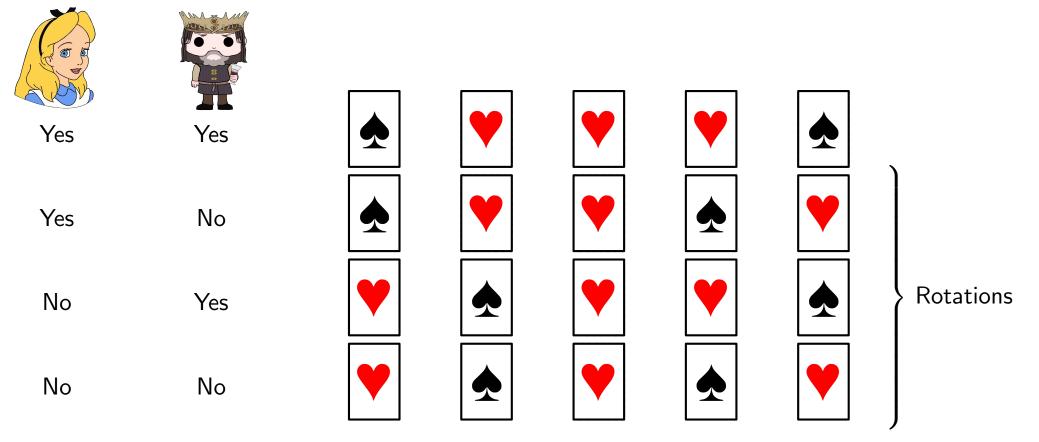
Rotations

**Possible configurations:** 



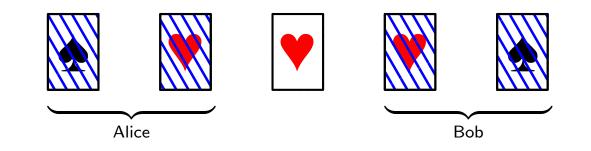
There are three hearts in a row if and only if both Alice and Bob like the movie

**Possible configurations:** 



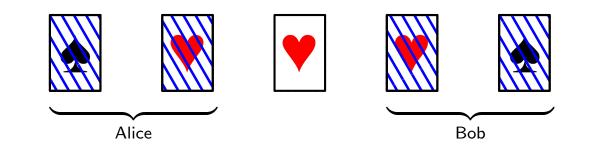
There are three hearts in a row if and only if both Alice and Bob like the movie This holds (in a modular sense) even if any rotations of the cards is considered!





Alice and Bob pile up the cards, all face down, ensuring that their order is preserved

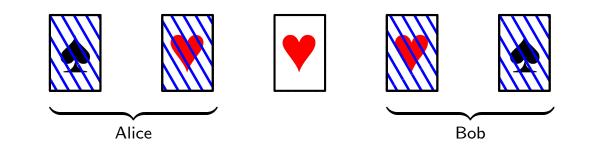




Alice and Bob pile up the cards, all face down, ensuring that their order is preserved

Alice and Bob take turns cutting the deck (i.e., they perform a rotation)



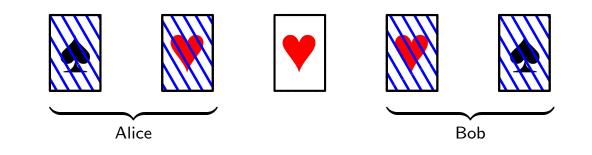


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Alice and Bob reveal the cards and watch the movie if and only if there are 3 consecutive hearts (in a modular fashion), otherwise they "pass"





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Alice and Bob take turns cutting the deck (i.e., they perform a rotation)

Alice and Bob reveal the cards and watch the movie if and only if there are 3 consecutive hearts (in a modular fashion), otherwise they "pass"

Observation: If they end up not watching the movie, all possible "pass" configurations are equiprobable

The previous scenario is a secure multiparty computation problem:

• There are n agents (parties)  $1, 2, \ldots, n$ 

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In the movie selection problem:

• 
$$n=2$$

•  $s_1, s_2 \in \{\mathsf{pass}, \mathsf{watch}\}$ 

• 
$$f(s_1, s_2) = \begin{cases} \text{watch} & \text{if } s_1 = s_2 = \text{watch} \\ \text{pass} & \text{otherwise} \end{cases}$$

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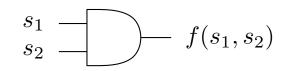
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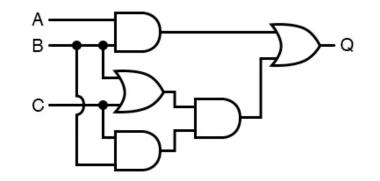
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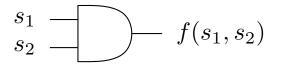
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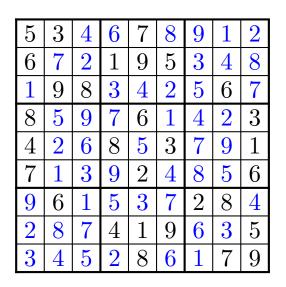
What about arbitrary circuits?

$\frac{5}{6}$	3			7				
6			1	9	5			
	9	8					6	
8				6				3
8 4			8		3			1
7				2				6
	6					2	8	
			4	1	9			5
				8			7	9

5	3	4	6	7	8	9	1	2
6	7	2	1	9	5	3	4	8
1	9	8	3	4	2	5	6	7
8	5	9	7	6	1	4	2	3
4	2	6	8	5	3	7	9	1
7	1	3	9	2	4	8	5	6
9	6	1	5	3	7	2	8	4
2	8	7	4	1	9	6	3	5
3	4	5	2	8	6	1	7	9

$\frac{5}{6}$	3			7				
6			1	9	5			
	9	8					6	
8				6				3
$\frac{8}{4}$			8		3			1
7				2				6
	6					2	8	
			4	1	9			5
				8			7	9



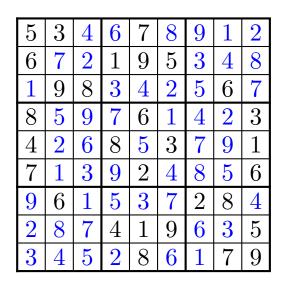


5 6	3			7				
6			1	9	5			
	9	8					6	
8				6				3
$\frac{8}{4}$			8		3			1
7				2				6
	6					2	8	
			4	1	9			5
				8			7	9

I know a solution to this Sudoku instance



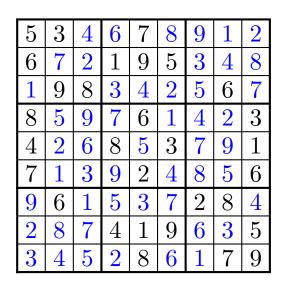




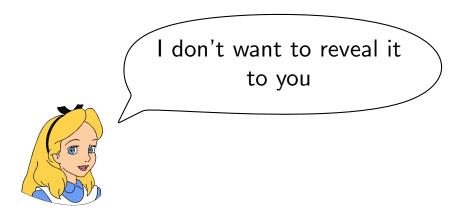
$\frac{5}{6}$	3			7				
6			1	9	5			
	9	8					6	
8				6				3
4			8		3			1
7				2				6
	6					2	8	
			4	1	9			5
				8			7	9







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6			1	9	5			
	9	8					6	
8				6				3
$\frac{8}{4}$			8		3			1
7				2				6
	6					2	8	
			4	1	9			5
				8			7	9





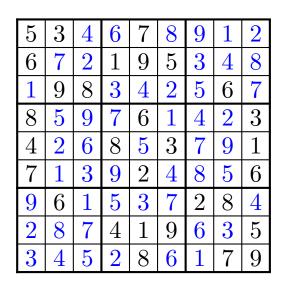
5	3	4	6	7	8	9	1	2
6	7	2	1	9	5	3	4	8
1	9	8	3	4	2	5	6	7
8	5	9	7	6	1	4	2	3
4	2	6	8	5	3	7	9	1
$\overline{7}$	1	3	9	2	4	8	5	6
9	6	1	5	3	7	2	8	4
2	8	7	4	1	9	6	3	5
3	4	5	2	8	6	1	7	9

,	5	3			7				
	6			1	9	5			
		9	8					6	
,	8				6				3
	4			8		3			1
	7				2				6
		6					2	8	
				4	1	9			5
					8			7	9

Then I don't believe you really have a solution





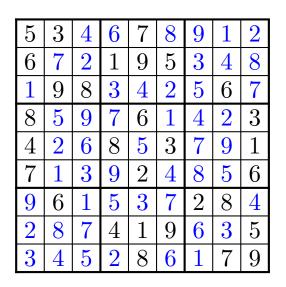


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			4	1	9			5
				8			7	9

I can prove to you that I have a solution without revealing anything about it





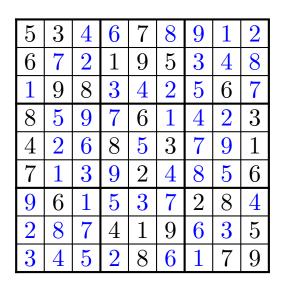


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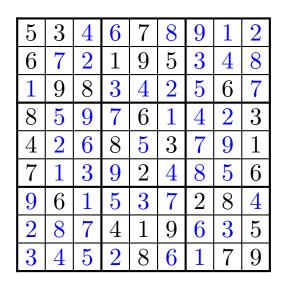
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I can prove to you that I have a solution without revealing anything about it

Zero Knowledge protocol







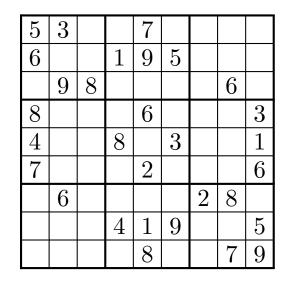
56	3			7				
6			1	9	5			
	9	8					6	
8				6				3
4			8		3			1
7				2				6
	6					2	8	
			4	1	9			5
				8			7	9

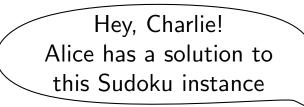
I am now convinced you have the solution





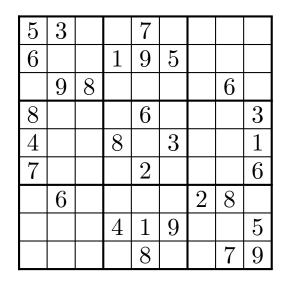


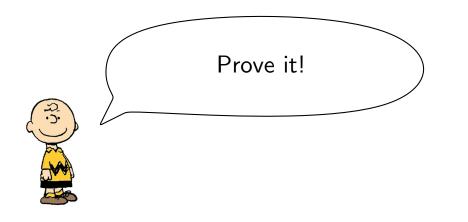












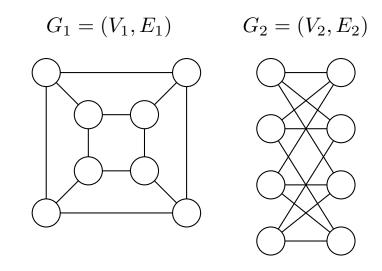


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$\frac{8}{4}$			8		3			1
7				2				6
	6					2	8	
			4	1	9			5
				8			7	9

Even if I definitely know she has a solution, somehow I have no way of proving that

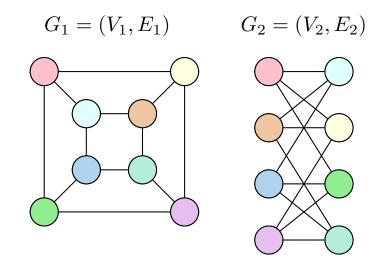






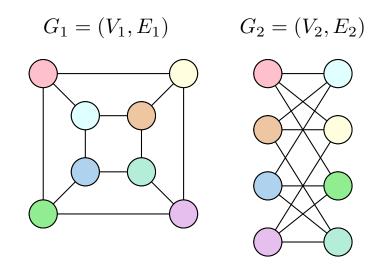
#### Graph isomorphism problem

 $G_1$  is isomorphic to  $G_2$  iff  $\exists$  bijection  $\pi: V_1 \to V_2$  s.t.  $(u, v) \in E_1 \iff (\pi(u), \pi(v)) \in E_2$ .



#### Graph isomorphism problem

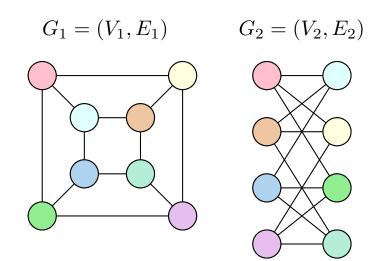
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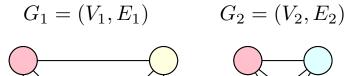
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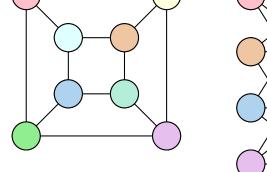
• Is in NP (the certificate is  $\pi$ )



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- Is in NP (the certificate is  $\pi$ )
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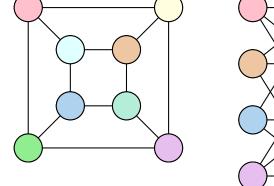
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- Is not known to be NP-Complete
- Is a candidate problem to be in the class NP-Intermediate

NP-Intermediate = NP  $\setminus$  P

$$G_1 = (V_1, E_1)$$
  $G_2 = (V_2, E_2)$ 



#### Graph isomorphism problem

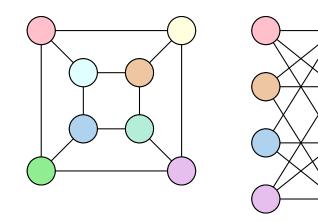
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• If NP-Intermediate  $\neq \emptyset$  then P $\neq$  NP.

$$G_1 = (V_1, E_1)$$
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Alice knows an isomorphism  $\pi$  between  $G_1$  and  $G_2$ 

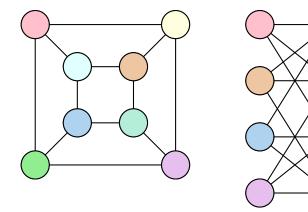
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  $G_2 = (V_2, E_2)$ 



Alice knows an isomorphism  $\pi$  between  $G_1$  and  $G_2$ 

Alice can use a Zero Knowledge protocol to convince bob that  $G_1$  and  $G_2$  are isomorphic without revealing  $\pi$ 

#### Graph isomorphism problem

- $G_1$  is isomorphic to  $G_2$  iff  $\exists$  bijection  $\pi: V_1 \to V_2$  s.t.  $(u, v) \in E_1 \iff (\pi(u), \pi(v)) \in E_2$ .
- Is in NP (the certificate is  $\pi$ )
- Is not known to be NP-Complete
- Is a candidate problem to be in the class NP-Intermediate
- If NP-Intermediate  $\neq \emptyset$  then P $\neq$  NP.

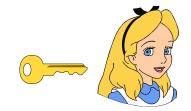
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## Basic definitions

There are two broad settings in which encryption is used:

The **private-key** or **symmetric** setting:

• Alice and Bob have a **shared**, **secret** key

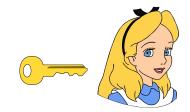




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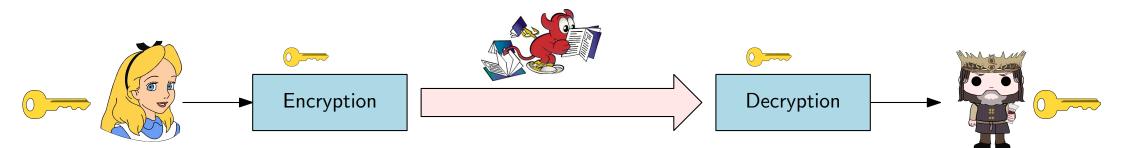




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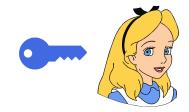
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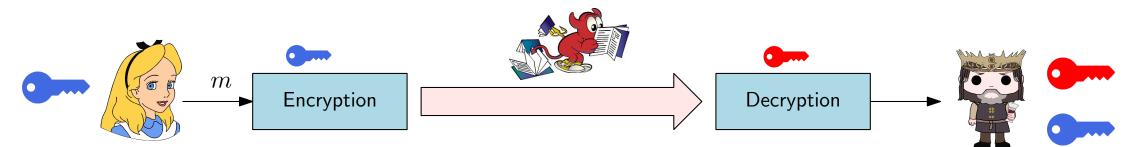
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For now, we will only be concerned with private-key cryptography

## Private-key cryptography

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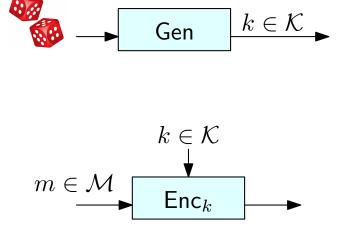
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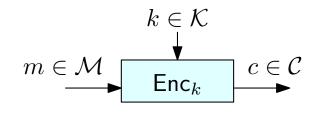


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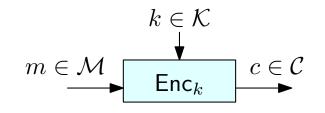
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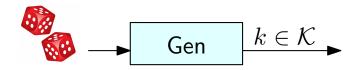
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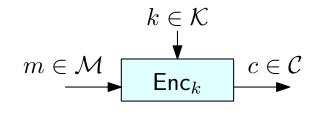
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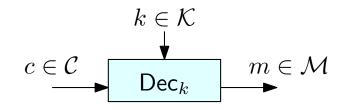
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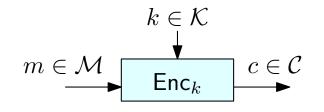
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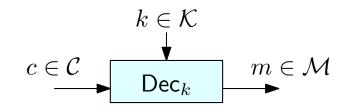
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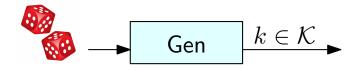
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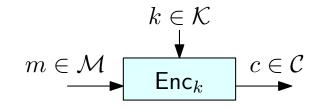
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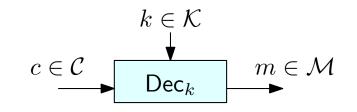
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**Perfect correctness:**  $\forall k \in \mathcal{K}, m \in \mathcal{M}$  if c can be output by  $Enc_k(m)$  then  $Dec_k(c) = m$ 







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If the key is leaked, it is easy to replace it

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- No need for Bob to implement Alice's weird scheme on his own (Bob can use public, vetted, implementations of well-known schemes)