## Information Systems and Network Security

Docente: Stefano Leucci


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## Basic Info

Course length: 48 hours (6 CFU)

- 24 lectures


## When/where:

- Tuesday 9:30-11:30

Room: A1.2

- Thursday 11:30-13:30

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- 24 lectures


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## Office hours:

- Thursday 16:30-18:30
- Please send an email to stefano.leucci@gmail.com or ask before/after class


## Course material:

https://people.disim.univaq.it/~stefanoleucci/isns24/

## Ingredients of Cryptography



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(Discrete) Math: Basic algebra, Modular arithmetic, some concepts from group theory and number theory

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Prerequisites

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Why cryptography?

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- Secrecy/Confidentiality: The contents messages sent over the channel will be known only to Alice and Bob
- Authentication: any message received by Bob can be confirmed to have originated from Alice (we can detect any message injected by the adversary)
- Integrity: it is not possible to alter the contents of a message sent across the channel (without the tampering being detected)


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D Signal


Bbitcoin

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## Private-key cryptography

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- Key distribution

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- Public-key cryptography, Hybrid cryptography
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Public-key cryptography

- Advanced applications: secret sharing, multiparty computation, zero-knowledge proofs


## Books



Introduction to
MODERN CRYPTOGRAPHY
Third Edition


Introduction to Modern Cryptography Jonathan Katz, Yehuda Lindell ISBN: 978-0815354369


The Joy Of Cryptography Mike Rosulek
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## Exams

Written exam:

- Questions on the theoretical concepts (e.g., security definitions)
- Exercises (e.g., prove security, carry out an attack, etc...)


Some advanced applications

## Secret Sharing

Imagine some sensitive information that is kept by a single agent

- A master encryption key

- Your bitcoin wallet
- Nuclear codes



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WHAT WOULD


ACTUALLY HAPPEN:
HIS LAPTOP'S ENCRYPTED. DRUG HIM AND HIT HIM WITH THIS \$5 WRENCH UNTL HE TEUS US THE PASSWORD.


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Single point of failure!
An attacker can compromise one machine and steal the sensitive information


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"Magic box"

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Share 1
Share 2


Share 3
Share 4

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## Idea:

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$t$-out-of- $n$ threshold secret-sharing scheme

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- The system remains secure if $<t$ servers are compromised
- The system remains operational (the secret can be recovered) if $\leq n-t$ servers are unavailable


## DNSSEC

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What's the address of www.my-bank.com?

DNS Server

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What's the address of www.my-bank.com?
Try the following IPv6 address: [90:00:: d]
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To prevent this, DNSSEC is used to authenticate DNS mappings
Who can be trusted with the master cryptographic keys to the system?

## DNSSEC

DNSSEC is managed by the Internet Corporation for Assigned Names and Numbers (ICANN)

The master key is split into 7 pieces and distributed on smart cards to 7 geographically diverse people

At least five key-holding members of this fellowship would have to meet at a secure data center in the United States to reboot [DNSSEC] in case of a very unlikely system collapse.
"If you round up five of these guys, they can decrypt [the root key] should the West Coast fall in the water and the East Coast get hit by a nuclear bomb"

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Alice and Bob are honest (they follow the protocol) but curious

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Each of Alice and Bob has a hearts card and a spades card
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If Alice likes the movie, she will place her two cards face down in the order
 Otherwise she will place her two cards face down in the order
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## Choosing a movie with secret preferences

Possible configurations:


Yes
Yes

No

## No




There are three hearts in a row if and only if both Alice and Bob like the movie This holds (in a modular sense) even if any rotations of the cards is considered!

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Observation: If they end up not watching the movie, all possible "pass" configurations are equiprobable

## Secure Multiparty Computation

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In the movie selection problem:

- $n=2$
- $s_{1}, s_{2} \in\{$ pass, watch $\}$
- $f\left(s_{1}, s_{2}\right)= \begin{cases}\text { watch } & \text { if } s_{1}=s_{2}=\text { watch } \\ \text { pass } & \text { otherwise }\end{cases}$


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What about arbitrary circuits?

## Zero Knowledge Proofs

| 5 | 3 |  |  | 7 |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 6 |  |  | 1 | 9 | 5 |  |  |  |
|  | 9 | 8 |  |  |  |  | 6 |  |
| 8 |  |  |  | 6 |  |  |  | 3 |
| 4 |  |  | 8 |  | 3 |  |  | 1 |
| 7 |  |  |  | 2 |  |  |  | 6 |
|  | 6 |  |  |  |  | 2 | 8 |  |
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| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 6 | 7 | 2 | 1 | 9 | 5 | 3 | 4 | 8 |
| 1 | 9 | 8 | 3 | 4 | 2 | 5 | 6 | 7 |
| 8 | 5 | 9 | 7 | 6 | 1 | 4 | 2 | 3 |
| 4 | 2 | 6 | 8 | 5 | 3 | 7 | 9 | 1 |
| 7 | 1 | 3 | 9 | 2 | 4 | 8 | 5 | 6 |
| 9 | 6 | 1 | 5 | 3 | 7 | 2 | 8 | 4 |
| 2 | 8 | 7 | 4 | 1 | 9 | 6 | 3 | 5 |
| 3 | 4 | 5 | 2 | 8 | 6 | 1 | 7 | 9 |


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| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 6 |  |  | 1 | 9 | 5 |  |  |  |
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| 5 | 3 | 4 | 6 | 7 | 8 | 9 | 1 | 2 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 6 | 7 | 2 | 1 | 9 | 5 | 3 | 4 | 8 |
| 1 | 9 | 8 | 3 | 4 | 2 | 5 | 6 | 7 |
| 8 | 5 | 9 | 7 | 6 | 1 | 4 | 2 | 3 |
| 4 | 2 | 6 | 8 | 5 | 3 | 7 | 9 | 1 |
| 7 | 1 | 3 | 9 | 2 | 4 | 8 | 5 | 6 |
| 9 | 6 | 1 | 5 | 3 | 7 | 2 | 8 | 4 |
| 2 | 8 | 7 | 4 | 1 | 9 | 6 | 3 | 5 |
| 3 | 4 | 5 | 2 | 8 | 6 | 1 | 7 | 9 |


| 5 | 3 |  |  | 7 |  |  |  |  |
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Really? Show it to me!

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Then I don't believe you really have a solution

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Hey, Charlie!
Alice has a solution to this Sudoku instance

## Zero Knowledge Proofs

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## Prove it!

## Zero Knowledge Proofs

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## Zero Knowledge Proofs



## Graph isomorphism problem

$G_{1}$ is isomorphic to $G_{2}$ iff $\exists$ bijection $\pi: V_{1} \rightarrow V_{2}$ s.t. $(u, v) \in E_{1} \Longleftrightarrow(\pi(u), \pi(v)) \in E_{2}$.

## Zero Knowledge Proofs


$G_{2}=\left(V_{2}, E_{2}\right)$


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$$
N P \text {-Intermediate }=N P \backslash P
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- If NP-Intermediate $\neq \emptyset$ then $\mathrm{P} \neq \mathrm{NP}$.


## Zero Knowledge Proofs



Alice knows an isomorphism $\pi$ between $G_{1}$ and $G_{2}$

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## Zero Knowledge Proofs



Alice knows an isomorphism $\pi$ between $G_{1}$ and $G_{2}$

Alice can use a Zero Knowledge protocol to convince bob that $G_{1}$ and $G_{2}$ are isomorphic without revealing $\pi$

## Graph isomorphism problem

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## Basic definitions

## Types of cryptography

There are two broad settings in which encryption is used:

The private-key or symmetric setting:

- Alice and Bob have a shared, secret key



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- Messages are encrypted and decrypted using the same key



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For now, we will only be concerned with private-key cryptography

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$\operatorname{Dec}_{k}(c)$ denotes an execution of Dec with inputs $k$ and $c$
Perfect correctness: $\forall k \in \mathcal{K}, m \in \mathcal{M}$ if $c$ can be output by $\operatorname{Enc}_{k}(m)$ then $\operatorname{Dec}_{k}(c)=m$


## Kerckhoffs' principle

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If the key is leaked, it is easy to replace it


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- Some of these mistakes are really subtle (we will see some of them in the course)


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- No need for Bob to implement Alice's weird scheme on his own (Bob can use public, vetted, implementations of well-known schemes)

