Caesar cipher

An example of a simple symmetric encryption scheme is the Caesar cipher

"If he had anything confidential to say, he wrote it in cipher, that is, by so changing the order of the letters of the alphabet, that not a word could be made out. If anyone wishes to decipher these, and get at their meaning, he must substitute the fourth letter of the alphabet, namely D, for A, and so with the others."

– Suetonius, Life of Julius Caesar

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$$\begin{array}{cccccccccccccccccccccccccc}
\mathtt{A} & \mathtt{B} & \mathtt{C} & \mathtt{D} & \mathtt{E} & \mathtt{F} & \mathtt{G} & \mathtt{H} & \mathtt{I} & \mathtt{J} & \mathtt{K} & \mathtt{L} & \mathtt{M} & \mathtt{N} & \mathtt{O} & \mathtt{P} & \mathtt{Q} & \mathtt{R} & \mathtt{S} & \mathtt{T} & \mathtt{U} & \mathtt{V} & \mathtt{W} & \mathtt{X} & \mathtt{Y} & \mathtt{Z} \\
\downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\
\mathtt{D} & \mathtt{E} & \mathtt{F} & \mathtt{G} & \mathtt{H} & \mathtt{I} & \mathtt{J} & \mathtt{K} & \mathtt{L} & \mathtt{M} & \mathtt{N} & \mathtt{O} & \mathtt{P} & \mathtt{Q} & \mathtt{R} & \mathtt{S} & \mathtt{T} & \mathtt{U} & \mathtt{V} & \mathtt{W} & \mathtt{X} & \mathtt{Y} & \mathtt{Z} & \mathtt{A} & \mathtt{B} & \mathtt{C}
\end{array}$$

Caesar cipher: example

$m =$ `A T T A C K A T D A W N`

$$\Big\downarrow \; \mathsf{Enc}(m)$$

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$$m = \texttt{A T T A C K A T D A W N}$$

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$$m =$$

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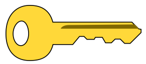
$m =$ R E T R E A T N O W

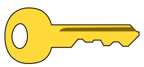
Shift ciphers

The Caesar cipher is a special type of *shift cipher*

In a shift cipher, each character is replaced with the character k positions down the alphabet (in a modular fashion)

The *key* of the cipher is the integer k 

(the key is also called the *shift* of the cipher)



$k = 5$

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓
F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E

Shift ciphers

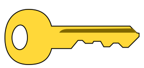
$m = \texttt{F L A N K T H E E N E M Y}$

$$\Big\downarrow\ \mathsf{Enc}_5(m)$$

$c = \texttt{K Q F S P Y M J J S J R D}$

$c = \texttt{X J S I M J Q U}$

$$\Big\downarrow\ \mathsf{Dec}_5(c)$$



$k = 5$

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓
F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E

Shift ciphers

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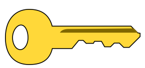
$$\Big\downarrow\ \mathsf{Dec}_5(c)$$

$$m = \texttt{S E N D H E L P}$$



$k = 5$

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓
F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E

Shift ciphers

Message space: $\mathcal{M} = \{A, \ldots, Z\}^*$

Shift ciphers

Message space: $\mathcal{M} = \{0, 1, \ldots, 25\}^*$

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Ciphertext space:

Shift ciphers

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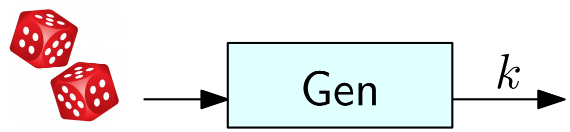
Key space: $\mathcal{K} = \{0, \ldots, 25\}$

Key generation:



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Message space: $\mathcal{M} = \{0, 1, \ldots, 25\}^*$

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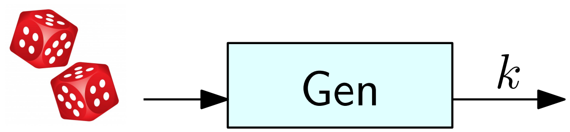
Key generation: return k chosen u.a.r. from $\mathcal{K}$



Shift ciphers

Message space: $\mathcal{M} = \{0, 1, \dots, 25\}^*$

$m = m_1 m_2 \dots m_\ell$

Ciphertext space: $\mathcal{C} = \{0, \dots, 25\}^*$

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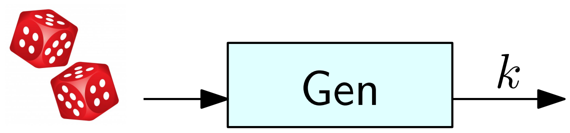
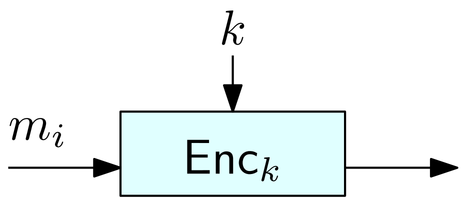


Encryption function:

$$\mathsf{Enc}_k(m) = \mathsf{Enc}_k(m_1) \| \mathsf{Enc}_k(m_2) \| \dots \| \mathsf{Enc}_k(m_\ell)$$



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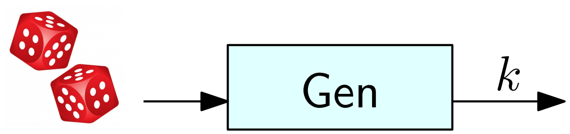
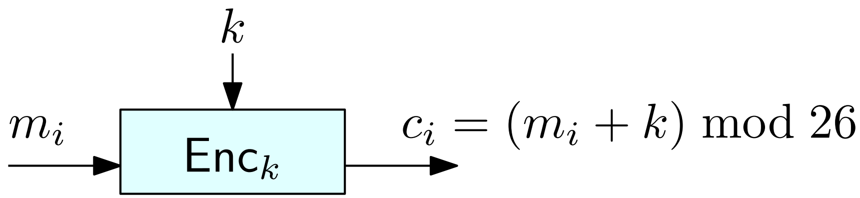


Encryption function:

$$\mathsf{Enc}_k(m) = \mathsf{Enc}_k(m_1) \| \mathsf{Enc}_k(m_2) \| \ldots \| \mathsf{Enc}_k(m_\ell)$$

$$\mathsf{Enc}_k(m_i) = (m_i + k) \bmod 26$$



Shift ciphers

Message space: $\mathcal{M} = \{0, 1, \ldots, 25\}^*$

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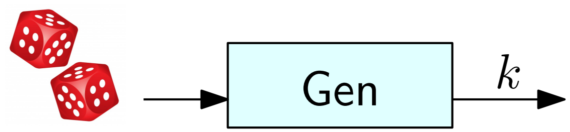

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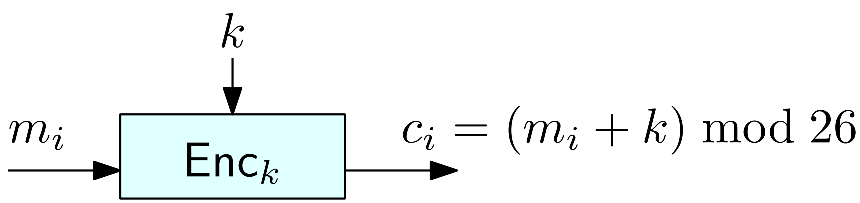
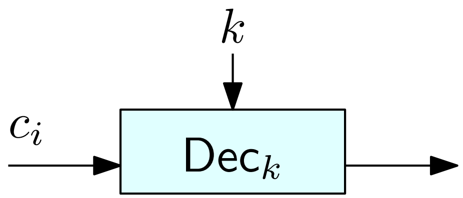


Decryption function:

$$\mathsf{Dec}_k(c) = \mathsf{Dec}_k(c_1) \| \mathsf{Dec}_k(c_2) \| \ldots \| \mathsf{Dec}_k(c_\ell)$$



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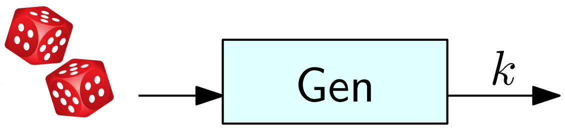

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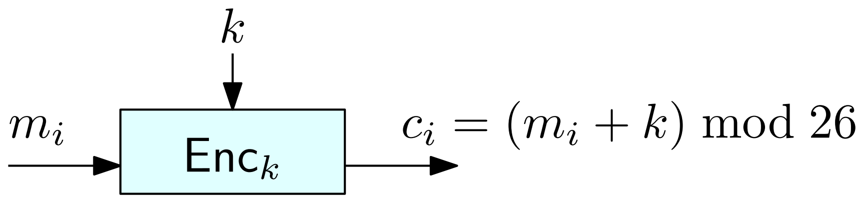

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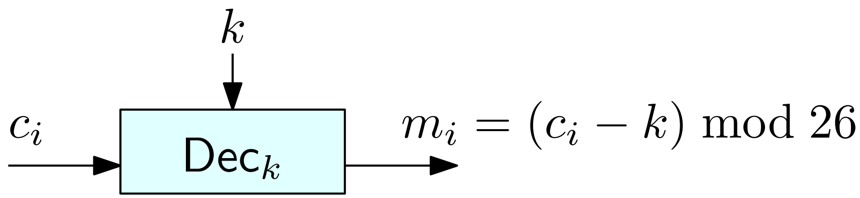

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Correctness:

We need to prove that $\mathsf{Dec}_k(\mathsf{Enc}_k(m)) = m$

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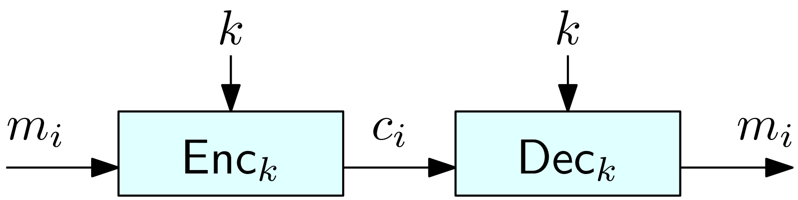

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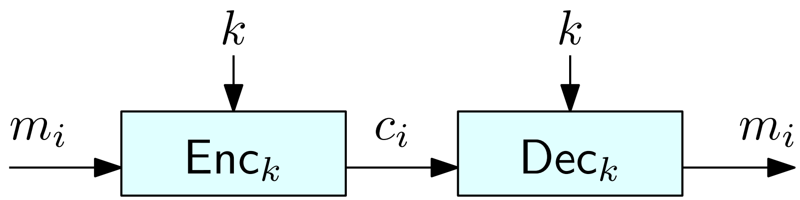

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$\mathsf{Dec}_k(\mathsf{Enc}_k(m_i)) \ = \mathsf{Dec}_k((m_i + k) \bmod 26)$ (definition of Enc_k)

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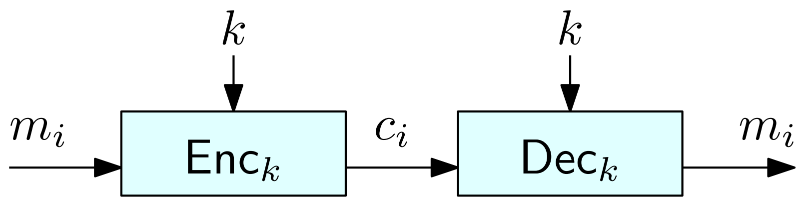

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$$\begin{aligned}
\mathsf{Dec}_k(\mathsf{Enc}_k(m_i)) &= \mathsf{Dec}_k((m_i + k) \bmod 26) && \text{(definition of Enc}_k\text{)} \\
&= (((m_i + k) \bmod 26) - k) \bmod 26 && \text{(definition of Dec}_k\text{)}
\end{aligned}$$

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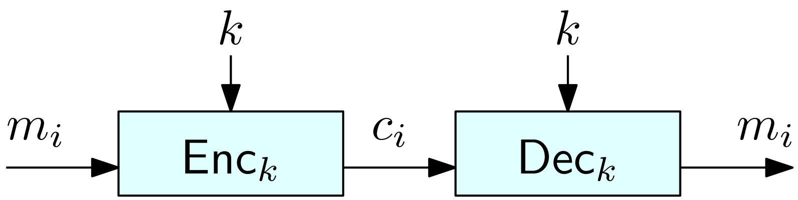

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&= (((m_i + k) \bmod 26) - k) \bmod 26 && \text{(definition of } \mathsf{Dec}_k\text{)} \\
&= (m_i + k - k) \bmod 26 && \text{(properties of } \mathrm{mod}\text{)}
\end{aligned}$$

Shift ciphers

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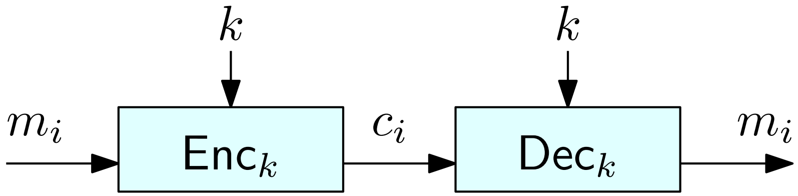

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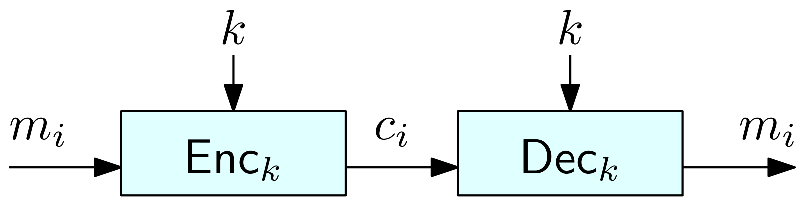

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\mathsf{Dec}_k(\mathsf{Enc}_k(m_i)) &= \mathsf{Dec}_k((m_i + k) \bmod 26) && \text{(definition of } \mathsf{Enc}_k\text{)} \\
&= (((m_i + k) \bmod 26) - k) \bmod 26 && \text{(definition of } \mathsf{Dec}_k\text{)} \\
&= (m_i + k - k) \bmod 26 && \text{(properties of } \mathrm{mod}\text{)} \\
&= m_i \bmod 26 \\
&= m_i && (m_i < 26)
\end{aligned}
$$

Shift ciphers

Are shift ciphers secure?

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We can use a **brute-force** (or **exhaustive search**) attack

In a brute-force attack, the adversary systematically tries all possible keys until the correct one is found.

Shift ciphers

Brute-force attack:

$$
\begin{aligned}
\mathsf{Dec}_0(c) &= \texttt{X J S I M J Q U} \\
\mathsf{Dec}_1(c) &= \texttt{W I R H L I P T} \\
\mathsf{Dec}_2(c) &= \texttt{V H Q G K H O S} \\
\mathsf{Dec}_3(c) &= \texttt{U G P F J G N R} \\
\mathsf{Dec}_4(c) &= \texttt{T F O E I F M Q} \\
\mathsf{Dec}_5(c) &= \texttt{S E N D H E L P} \\
\mathsf{Dec}_6(c) &= \texttt{R D M C G D K O} \\
&\vdots \\
\mathsf{Dec}_{24}(c) &= \texttt{Z L U K O L S W} \\
\mathsf{Dec}_{25}(c) &= \texttt{Y K T J N K R V}
\end{aligned}
$$

Shift ciphers

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$$\vdots$$

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$$\vdots$$

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Sufficient key-space principle: Any cipher should use a “large enough” key space to prevent brute-force attacks

(Monoalphabetic) Substitution ciphers

The key is now a permutation π of the alphabet $\Sigma = \{\mathtt{A}, \mathtt{B}, \ldots, \mathtt{Z}\}$

$\mathcal{K} = \{\pi : \Sigma \to \Sigma \mid \pi \text{ is a pemutation}\}$

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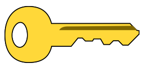
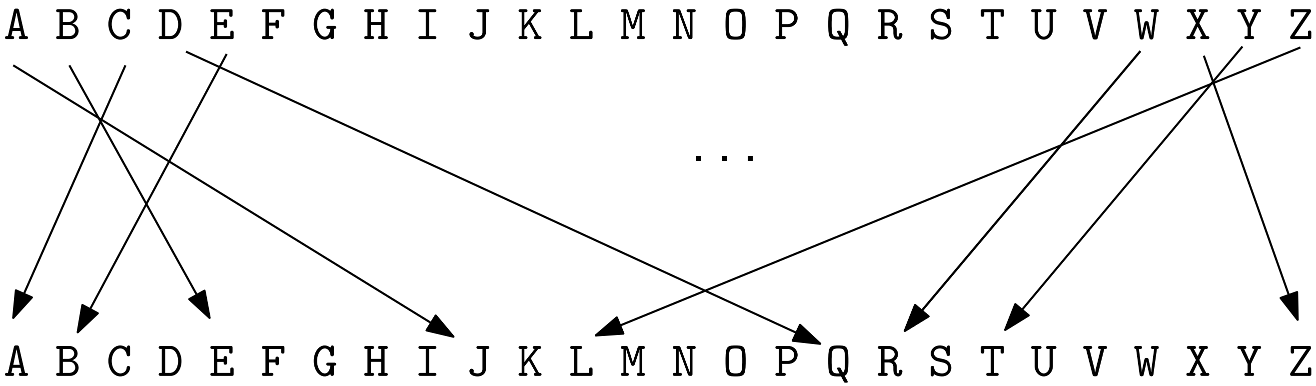


$k = \pi$



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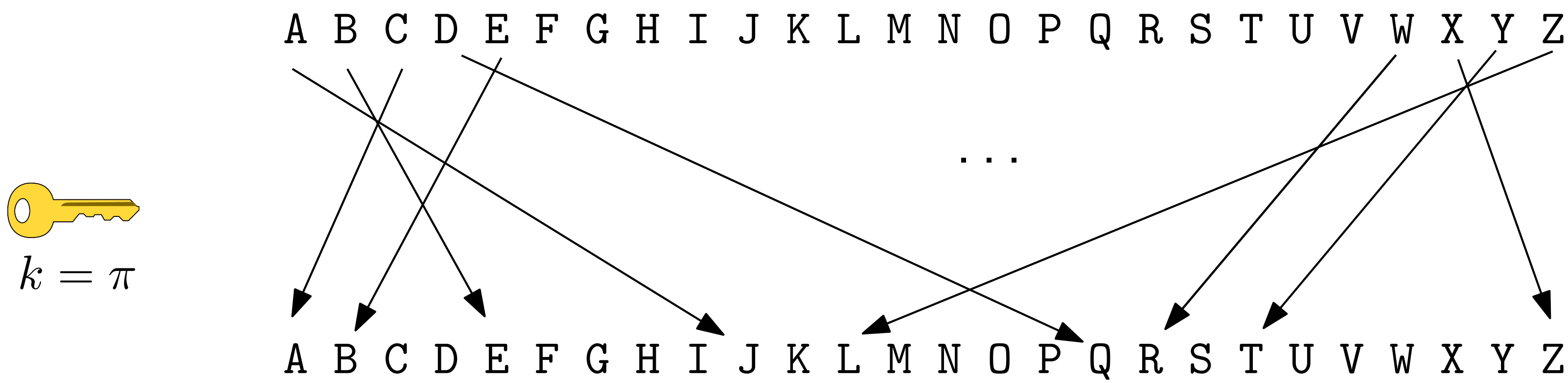


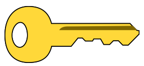
To encrypt a message, replace each character m_i in the plaintext with $k(m_i) = \pi(m_i)$

$\mathsf{Enc}_k(m) = k(m_1) \| k(m_2) \| \ldots \| k(m_\ell)$

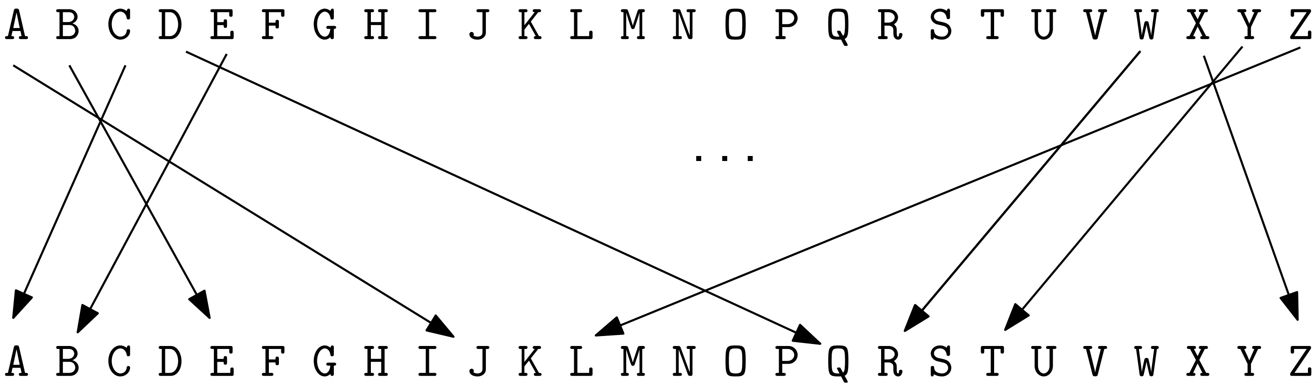
(Monoalphabetic) Substitution ciphers

The key is now a permutation π of the alphabet $\Sigma = \{\mathtt{A}, \mathtt{B}, \ldots, \mathtt{Z}\}$

$\mathcal{K} = \{\pi : \Sigma \to \Sigma \mid \pi \text{ is a pemutation}\}$



$k = \pi$



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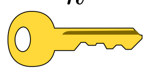
$\mathsf{Enc}_k(m) = k(m_1) \| k(m_2) \| \ldots \| k(m_\ell)$

To decrypt a message, replace each character c_i of the ciphertext with $k^{-1}(c_i) = \pi^{-1}(c_i)$

$\mathsf{Dec}_k(m) = k^{-1}(c_1) \| k^{-1}(c_2) \| \ldots \| k^{-1}(c_\ell)$

(Monoalphabetic) Substitution ciphers

$$\underbrace{\begin{array}{cccccccccccccccccccccccccc}
\mathtt{A} & \mathtt{B} & \mathtt{C} & \mathtt{D} & \mathtt{E} & \mathtt{F} & \mathtt{G} & \mathtt{H} & \mathtt{I} & \mathtt{J} & \mathtt{K} & \mathtt{L} & \mathtt{M} & \mathtt{N} & \mathtt{O} & \mathtt{P} & \mathtt{Q} & \mathtt{R} & \mathtt{S} & \mathtt{T} & \mathtt{U} & \mathtt{V} & \mathtt{W} & \mathtt{X} & \mathtt{Y} & \mathtt{Z} \\
\mathtt{J} & \mathtt{E} & \mathtt{A} & \mathtt{Q} & \mathtt{B} & \mathtt{Y} & \mathtt{D} & \mathtt{P} & \mathtt{V} & \mathtt{F} & \mathtt{K} & \mathtt{I} & \mathtt{N} & \mathtt{H} & \mathtt{M} & \mathtt{X} & \mathtt{U} & \mathtt{S} & \mathtt{W} & \mathtt{C} & \mathtt{O} & \mathtt{G} & \mathtt{R} & \mathtt{Z} & \mathtt{T} & \mathtt{L}
\end{array}}_{k}$$



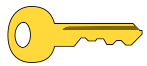
$$m = \mathtt{A\ W\ A\ I\ T\ O\ R\ D\ E\ R\ S}$$

$$\Big\downarrow \; \mathsf{Enc}_k(m)$$

(Monoalphabetic) Substitution ciphers

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
J	E	A	Q	B	Y	D	P	V	F	K	I	N	H	M	X	U	S	W	C	O	G	R	Z	T	L

k



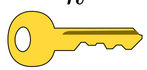
$$m = \texttt{A W A I T O R D E R S}$$

$$\big\downarrow \ \mathsf{Enc}_k(m)$$

$$c = \texttt{J R J V C M S Q B S W}$$

(Monoalphabetic) Substitution ciphers

$$\underbrace{\begin{array}{cccccccccccccccccccccccccc} \mathtt{A} & \mathtt{B} & \mathtt{C} & \mathtt{D} & \mathtt{E} & \mathtt{F} & \mathtt{G} & \mathtt{H} & \mathtt{I} & \mathtt{J} & \mathtt{K} & \mathtt{L} & \mathtt{M} & \mathtt{N} & \mathtt{O} & \mathtt{P} & \mathtt{Q} & \mathtt{R} & \mathtt{S} & \mathtt{T} & \mathtt{U} & \mathtt{V} & \mathtt{W} & \mathtt{X} & \mathtt{Y} & \mathtt{Z} \\ \mathtt{J} & \mathtt{E} & \mathtt{A} & \mathtt{Q} & \mathtt{B} & \mathtt{Y} & \mathtt{D} & \mathtt{P} & \mathtt{V} & \mathtt{F} & \mathtt{K} & \mathtt{I} & \mathtt{N} & \mathtt{H} & \mathtt{M} & \mathtt{X} & \mathtt{U} & \mathtt{S} & \mathtt{W} & \mathtt{C} & \mathtt{O} & \mathtt{G} & \mathtt{R} & \mathtt{Z} & \mathtt{T} & \mathtt{L} \end{array}}_{k}$$



$m = \mathtt{A\ W\ A\ I\ T\ O\ R\ D\ E\ R\ S}$

$\big\downarrow\ \mathsf{Enc}_k(m)$

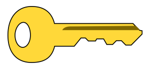
$c = \mathtt{J\ R\ J\ V\ C\ M\ S\ Q\ B\ S\ W}$

$c = \mathtt{B\ H\ B\ N\ T\ Q\ M\ R\ H}$

$\big\downarrow\ \mathsf{Dec}_k(c)$

(Monoalphabetic) Substitution ciphers

$$\underbrace{\begin{array}{cccccccccccccccccccccccccc}
\mathtt{A} & \mathtt{B} & \mathtt{C} & \mathtt{D} & \mathtt{E} & \mathtt{F} & \mathtt{G} & \mathtt{H} & \mathtt{I} & \mathtt{J} & \mathtt{K} & \mathtt{L} & \mathtt{M} & \mathtt{N} & \mathtt{O} & \mathtt{P} & \mathtt{Q} & \mathtt{R} & \mathtt{S} & \mathtt{T} & \mathtt{U} & \mathtt{V} & \mathtt{W} & \mathtt{X} & \mathtt{Y} & \mathtt{Z} \\
\mathtt{J} & \mathtt{E} & \mathtt{A} & \mathtt{Q} & \mathtt{B} & \mathtt{Y} & \mathtt{D} & \mathtt{P} & \mathtt{V} & \mathtt{F} & \mathtt{K} & \mathtt{I} & \mathtt{N} & \mathtt{H} & \mathtt{M} & \mathtt{X} & \mathtt{U} & \mathtt{S} & \mathtt{W} & \mathtt{C} & \mathtt{O} & \mathtt{G} & \mathtt{R} & \mathtt{Z} & \mathtt{T} & \mathtt{L}
\end{array}}_{k}$$



$$m = \mathtt{A\ W\ A\ I\ T\ O\ R\ D\ E\ R\ S}$$

$$\Big\downarrow \ \mathsf{Enc}_k(m)$$

$$c = \mathtt{J\ R\ J\ V\ C\ M\ S\ Q\ B\ S\ W}$$

$$c = \mathtt{B\ H\ B\ N\ T\ Q\ M\ R\ H}$$

$$\Big\downarrow \ \mathsf{Dec}_k(c)$$

$$m = \mathtt{E\ N\ E\ M\ Y\ D\ O\ W\ N}$$

(Monoalphabetic) Substitution ciphers

How many keys are there?

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If we tried 100 billion keys per second, we would need about 100 million years to find the right permutation k

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Observation (informal): A large keyspace is not a sufficient condition for a cipher to be secure

Substitution ciphers

Suppose that we somehow have deciphered a small portion of the ciphertext

We can replace each known ciphertext symbol x with its plaintext $k^{-1}(x)$ and then use the partially decrypted message to make further guesses about k

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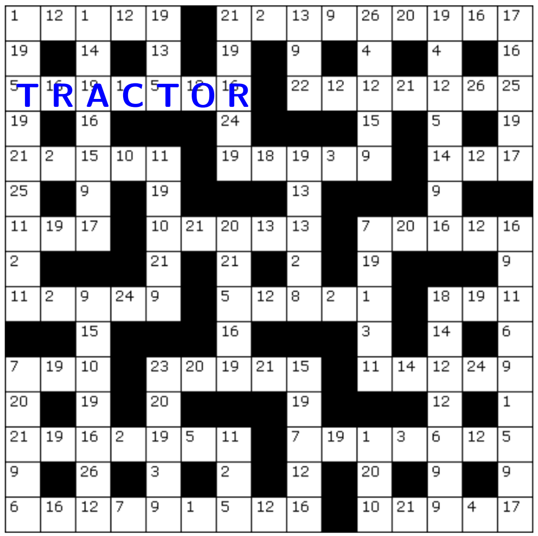
A similar example: codebreaker word puzzle



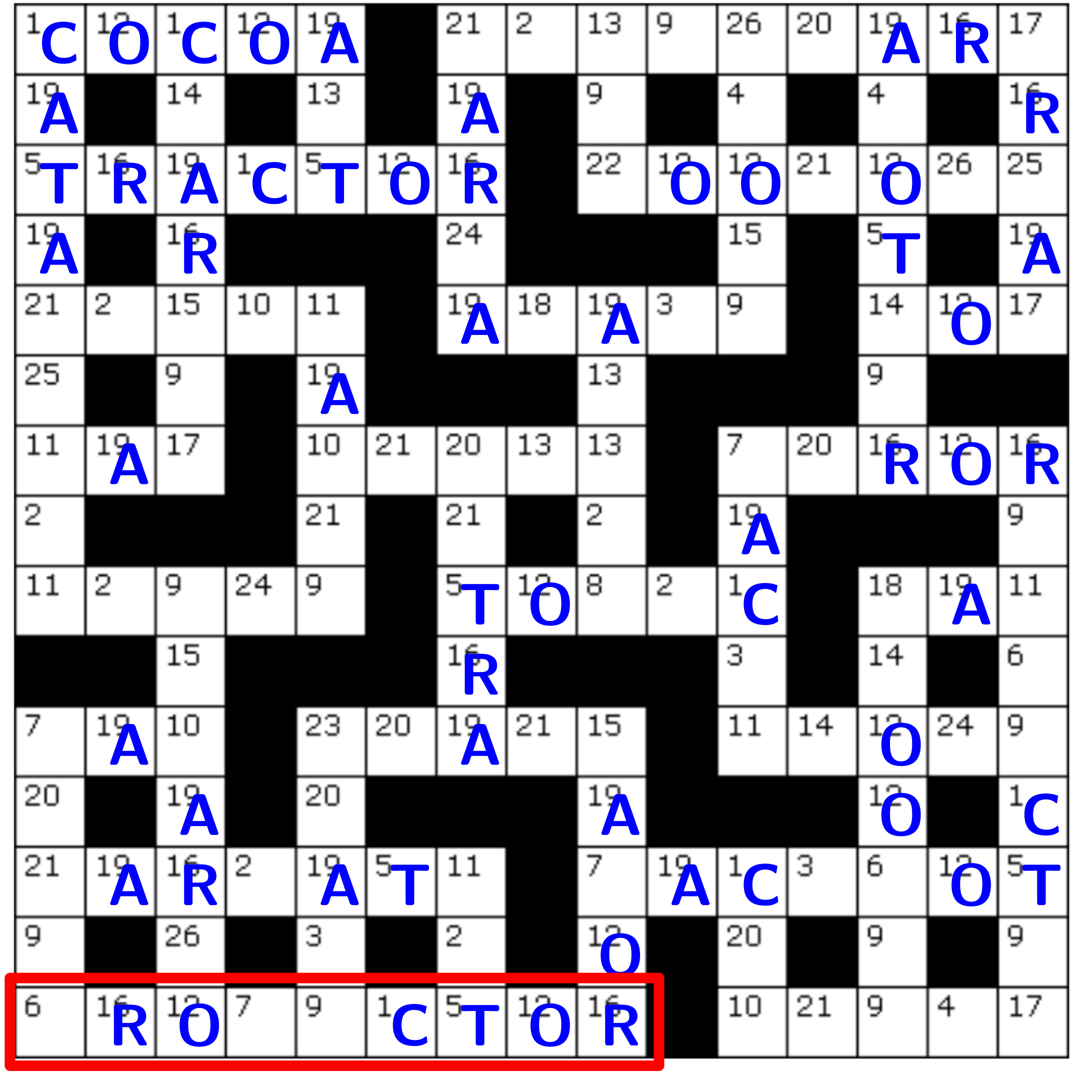
Substitution ciphers

1	12	1	12	19		21	2	13	9	26	20	19	16	17
19		14		13		19		9		4		4		16
5 T	16 R	19 A	1 C	5 T	12 O	16 R		22	12	12	21	12	26	25
19		16				24				15		5		19
21	2	15	10	11		19	18	19	3	9		14	12	17
25		9		19				13				9		
11	19	17		10	21	20	13	13		7	20	16	12	16
2				21		21		2		19				9
11	2	9	24	9		5	12	8	2	1		18	19	11
		15				16				3		14		6
7	19	10		23	20	19	21	15		11	14	12	24	9
20		19		20				19				12		1
21	19	16	2	19	5	11		7	19	1	3	6	12	5
9		26		3		2		12		20		9		9
6	16	12	7	9	1	5	12	16		10	21	9	4	17

Substitution ciphers

1 C	12 O	1 C	12 O	19 A		21	2	13	9	26	20	19 A	16 R	17
19 A		14		13		19 A		9		4		4		16 R
5 T	16 R	19 A	1 C	5 T	12 O	16 R		22	12 O	12 O	21	12 O	26	25
19 A		16 R				24				15		5 T		19 A
21	2	15	10	11		19 A	18	19 A	3	9		14	12 O	17
25		9		19 A				13				9		
11	19 A	17		10	21	20	13	13		7	20	16 R	12 O	16 R
2				21		21		2		19 A				9
11	2	9	24	9		5 T	12 O	8	2	1 C		18	19 A	11
		15				16 R				3		14		6
7	19 A	10		23	20	19 A	21	15		11	14	12 O	24	9
20		19 A		20				19 A				12 O		1 C
21	19 A	16 R	2	19 A	5 T	11		7	19 A	1 C	3	6	12 O	5 T
9		26		3		2		12 O		20		9		9
6	16 R	12 O	7	9	1 C	5 T	12 O	16 R		10	21	9	4	17

Substitution ciphers



Substitution ciphers

Substitution ciphers

Substitution ciphers

1 C	12 O	1 C	12 O	19 A	■	21	2	13	9 E	26	20	19 A	16 R	17 D
19 A	■	14	■	13	■	19 A	■	9 E	■	4	■	4	■	16 R
5 T	16 R	19 A	1 C	5 T	12 O	16 R	■	22	12 O	12 O	21	12 O	26	25
19 A	■	16 R	■	■	■	24	■	■	■	15	■	5 T	■	19 A
21	2	15	10	11 S	■	19 A	18	19 A	3	9 E	■	14	12 O	17 D
25	■	9 E	■	19 A	■	■	■	13	■	■	■	9 E	■	■
11 S	19 A	17	■	10	21	20	13	13	■	7 J	20	16 R	12 O	16 R
2	■	■	■	21	■	21	■	2	■	19 A	■	■	■	9 E
11 S	2	9 E	24	9 E	■	5 T	12 O	8	2	1 C	■	18	19 A	11 S
■	■	15	■	■	■	16 R	■	■	■	3	■	14	■	6 P
7 J	19 A	10	■	23	20	19 A	21	15	■	11 S	14	12 O	24	9 E
20	■	19 A	■	20	■	■	■	19 A	■	■	■	12 O	■	1 C
21	19 A	16 R	2	19 A	5 T	11 S	■	7 J	19 A	1 C	3	6 P	12 O	5 T
9 E	■	26	■	3	■	2	■	12 O	■	20	■	9 E	■	9 E
6 P	16 R	12 O	7 J	9 E	1 C	5 T	12 O	16 R	■	10	21	9 E	4	17 D

Substitution ciphers

Substitution ciphers

1 C	12 O	1 C	12 O	19 A		21	2	13	9 E	26	20 U	19 A	16 R	17 D
19 A		14		13		19 A		9 E		4		4		16 R
5 T	16 R	19 A	1 C	5 T	12 O	16 R		22	12 O	12 O	21	12 O	26	25
19 A		16 R				24				15		5 T		19 A
21	2	15	10	11 S		19 A	18	19 A	3	9 E		14	12 O	17 D
25		9 E		19 A				13				9 E		
11 S	19 A	17		10	21	20 U	13	13		7 J	20 U	16 R	12 O	16 R
2				21		21		2		19 A				9 E
11 S	2	9 E	24	9 E		5 T	12 O	8	2	1 C		18	19 A	11 S
		15				16 R				3		14		6 P
7 J	19 A	10		23	20 U	19 A	21	15		11 S	14	12 O	24	9 E
20 U		19 A		20 U				19 A				12 O		1 C
21	19 A	16 R	2	19 A	5 T	11 S		7 J	19 A	1 C	3	6 P	12 O	5 T
9 E		26		3		2		12 O		20 U		9 E		9 E
6 P	16 R	12 O	7 J	9 E	1 C	5 T	12 O	16 R		10	21	9 E	4	17 D

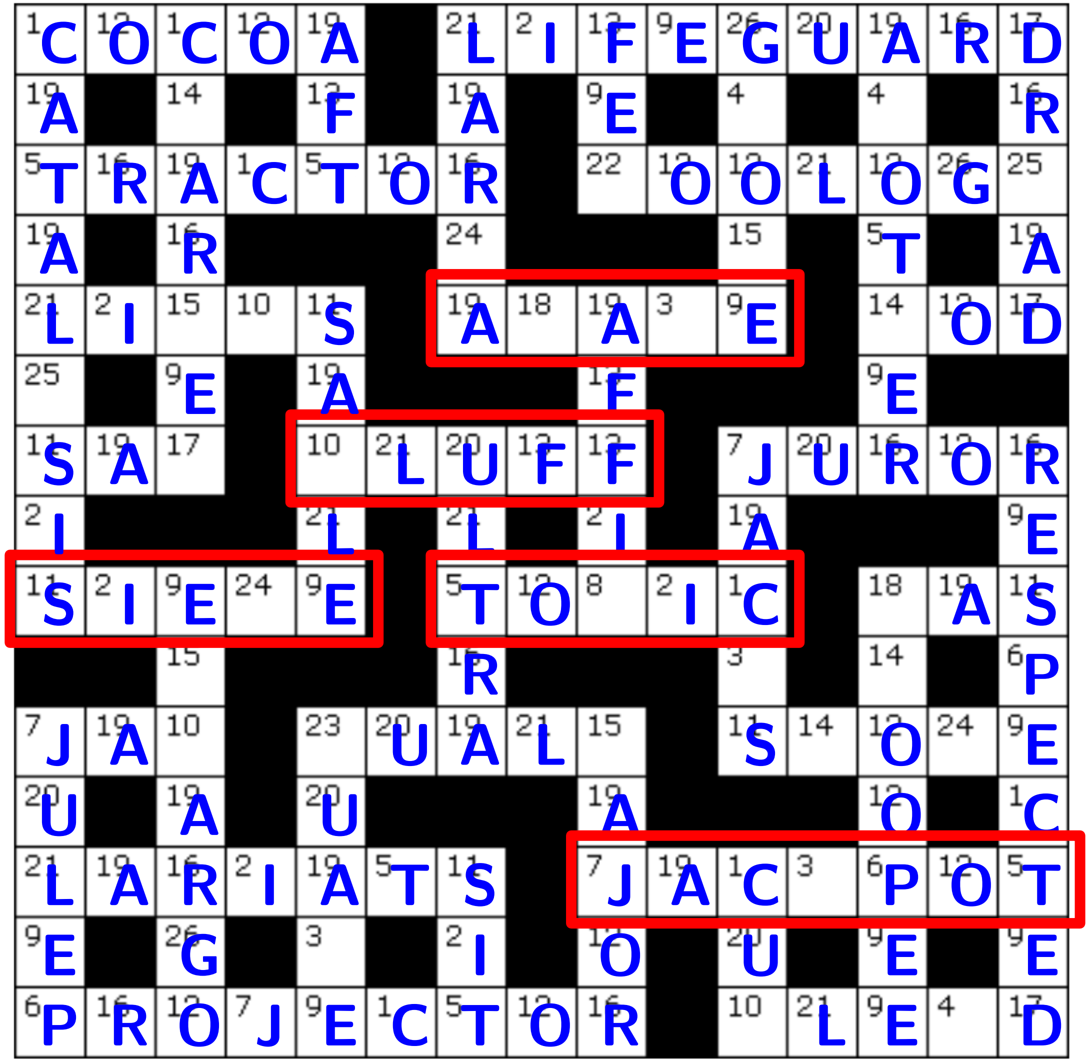
Substitution ciphers

1 C	12 O	1 C	12 O	19 A	■	21	2	13	9 E	26	20 U	19 A	16 R	17 D
19 A	■	14	■	13	■	19 A	■	9 E	■	4	■	4	■	16 R
5 T	16 R	19 A	1 C	5 T	12 O	16 R	■	22	12 O	12 O	21	12 O	26	25
19 A	■	16 R	■	■	■	24	■	■	■	15	■	5 T	■	19 A
21	2	15	10	11 S	■	19 A	18	19 A	3	9 E	■	14	12 O	17 D
25	■	9 E	■	19 A	■	■	■	13	■	■	■	9 E	■	■
11 S	19 A	17	■	10	21	20 U	13	13	■	7 J	20 U	16 R	12 O	16 R
2	■	■	■	21	■	21	■	2	■	19 A	■	■	■	9 E
11 S	2	9 E	24	9 E	■	5 T	12 O	8	2	1 C	■	18	19 A	11 S
■	■	15	■	■	■	16 R	■	■	■	3	■	14	■	6 P
7 J	19 A	10	■	23	20 U	19 A	21	15	■	11 S	14	12 O	24	9 E
20 U	■	19 A	■	20 U	■	■	■	19 A	■	■	■	12 O	■	1 C
21	19 A	16 R	2	19 A	5 T	11 S	■	7 J	19 A	1 C	3	6 P	12 O	5 T
9 E	■	26	■	3	■	2	■	12 O	■	20 U	■	9 E	■	9 E
6 P	16 R	12 O	7 J	9 E	1 C	5 T	12 O	16 R	■	10	21	9 E	4	17 D

Substitution ciphers

1 C	12 O	1 C	12 O	19 A	■	21 L	2 I	13 F	9 E	26 G	20 U	19 A	16 R	17 D
19 A	■	14	■	13 F	■	19 A	■	9 E	■	4	■	4	■	16 R
5 T	16 R	19 A	1 C	5 T	12 O	16 R	■	22	12 O	12 O	21 L	12 O	26 G	25
19 A	■	16 R	■	■	■	24	■	■	■	15	■	5 T	■	19 A
21 L	2 I	15	10	11 S	■	19 A	18	19 A	3	9 E	■	14	12 O	17 D
25	■	9 E	■	19 A	■	■	■	13 F	■	■	■	9 E	■	■
11 S	19 A	17	■	10	21 L	20 U	13 F	13 F	■	7 J	20 U	16 R	12 O	16 R
2 I	■	■	■	21 L	■	21 L	■	2 I	■	19 A	■	■	■	9 E
11 S	2 I	9 E	24	9 E	■	5 T	12 O	8	2 I	1 C	■	18	19 A	11 S
■	■	15	■	■	■	16 R	■	■	■	3	■	14	■	6 P
7 J	19 A	10	■	23	20 U	19 A	21 L	15	■	11 S	14	12 O	24	9 E
20 U	■	19 A	■	20 U	■	■	■	19 A	■	■	■	12 O	■	1 C
21 L	19 A	16 R	2 I	19 A	5 T	11 S	■	7 J	19 A	1 C	3	6 P	12 O	5 T
9 E	■	26 G	■	3	■	2 I	■	12 O	■	20 U	■	9 E	■	9 E
6 P	16 R	12 O	7 J	9 E	1 C	5 T	12 O	16 R	■	10	21 L	9 E	4	17 D

Substitution ciphers



Substitution ciphers

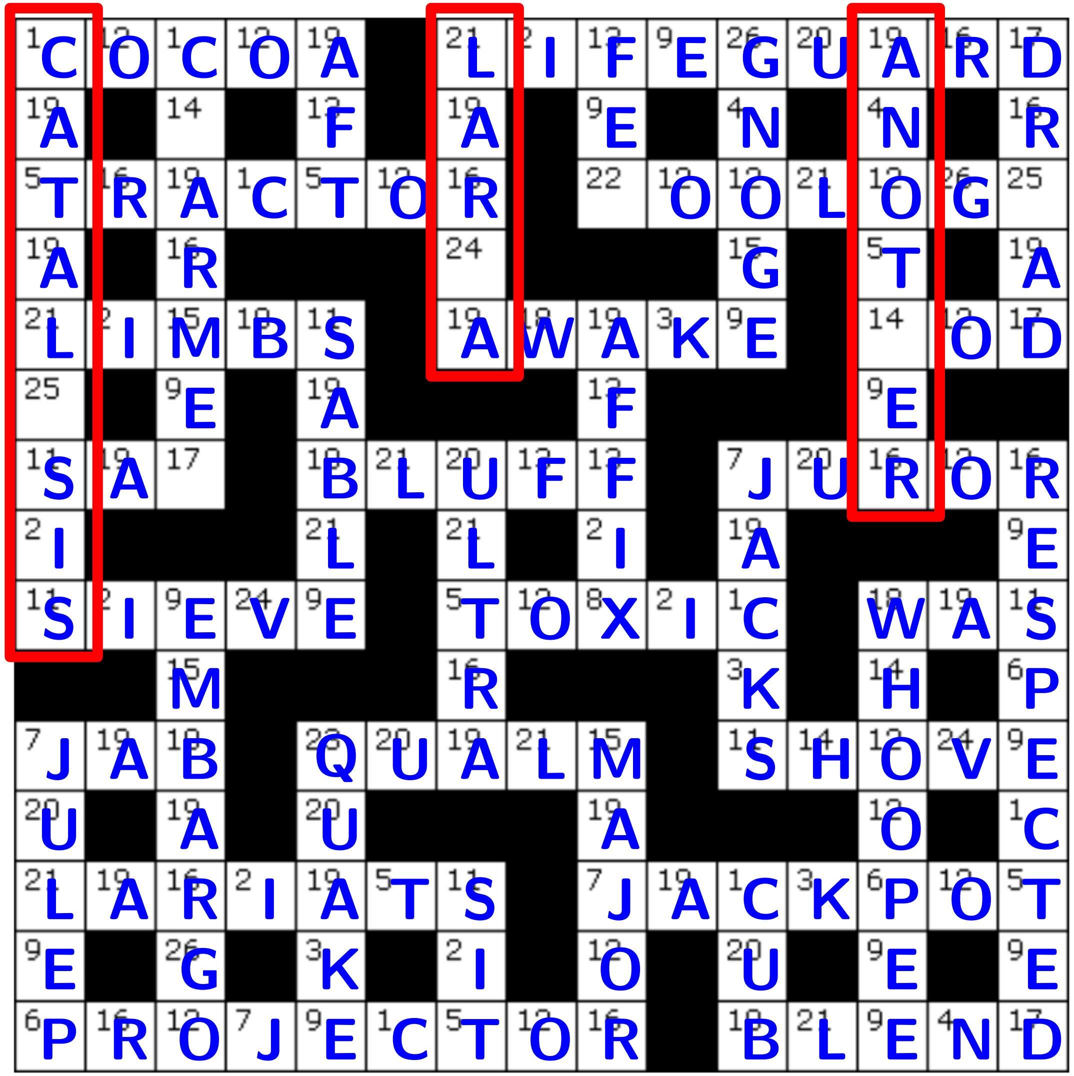
1 C	12 O	1 C	12 O	19 A	■	21 L	2 I	13 F	9 E	26 G	20 U	19 A	16 R	17 D
19 A	■	14	■	13 F	■	19 A	■	9 E	■	4	■	4	■	16 R
5 T	16 R	19 A	1 C	5 T	12 O	16 R	■	22	12 O	12 O	21 L	12 O	26 G	25
19 A	■	16 R	■	■	■	24	■	■	■	15	■	5 T	■	19 A
21 L	2 I	15	10 B	11 S	■	19 A	18 W	19 A	3 K	9 E	■	14	12 O	17 D
25	■	9 E	■	19 A	■	■	■	13 F	■	■	■	9 E	■	■
11 S	19 A	17	■	10 B	21 L	20 U	13 F	13 F	■	7 J	20 U	16 R	12 O	16 R
2 I	■	■	■	21 L	■	21 L	■	2 I	■	19 A	■	■	■	9 E
11 S	2 I	9 E	24 V	9 E	■	5 T	12 O	8 X	2 I	1 C	■	18 W	19 A	11 S
■	■	15	■	■	■	16 R	■	■	■	3 K	■	14	■	6 P
7 J	19 A	10 B	■	23	20 U	19 A	21 L	15	■	11 S	14	12 O	24 V	9 E
20 U	■	19 A	■	20 U	■	■	■	19 A	■	■	■	12 O	■	1 C
21 L	19 A	16 R	2 I	19 A	5 T	11 S	■	7 J	19 A	1 C	3 K	6 P	12 O	5 T
9 E	■	26 G	■	3 K	■	2 I	■	12 O	■	20 U	■	9 E	■	9 E
6 P	16 R	12 O	7 J	9 E	1 C	5 T	12 O	16 R	■	10 B	21 L	9 E	4	17 D

Substitution ciphers

Substitution ciphers

1 C	12 O	1 C	12 O	19 A	■	21 L	2 I	13 F	9 E	26 G	20 U	19 A	16 R	17 D
19 A	■	14	■	13 F	■	19 A	■	9 E	■	4 N	■	4 N	■	16 R
5 T	16 R	19 A	1 C	5 T	12 O	16 R	■	22	12 O	12 O	21 L	12 O	26 G	25
19 A	■	16 R	■	■	■	24	■	■	■	15 G	■	5 T	■	19 A
21 L	2 I	15 M	10 B	11 S	■	19 A	18 W	19 A	3 K	9 E	■	14	12 O	17 D
25	■	9 E	■	19 A	■	■	■	13 F	■	■	■	9 E	■	■
11 S	19 A	17	■	10 B	21 L	20 U	13 F	13 F	■	7 J	20 U	16 R	12 O	16 R
2 I	■	■	■	21 L	■	21 L	■	2 I	■	19 A	■	■	■	9 E
11 S	2 I	9 E	24 V	9 E	■	5 T	12 O	8 X	2 I	1 C	■	18 W	19 A	11 S
■	■	15 M	■	■	■	16 R	■	■	■	3 K	■	14 H	■	6 P
7 J	19 A	10 B	■	23 Q	20 U	19 A	21 L	15 M	■	11 S	14 H	12 O	24 V	9 E
20 U	■	19 A	■	20 U	■	■	■	19 A	■	■	■	12 O	■	1 C
21 L	19 A	16 R	2 I	19 A	5 T	11 S	■	7 J	19 A	1 C	3 K	6 P	12 O	5 T
9 E	■	26 G	■	3 K	■	2 I	■	12 O	■	20 U	■	9 E	■	9 E
6 P	16 R	12 O	7 J	9 E	1 C	5 T	12 O	16 R	■	10 B	21 L	9 E	4 N	17 D

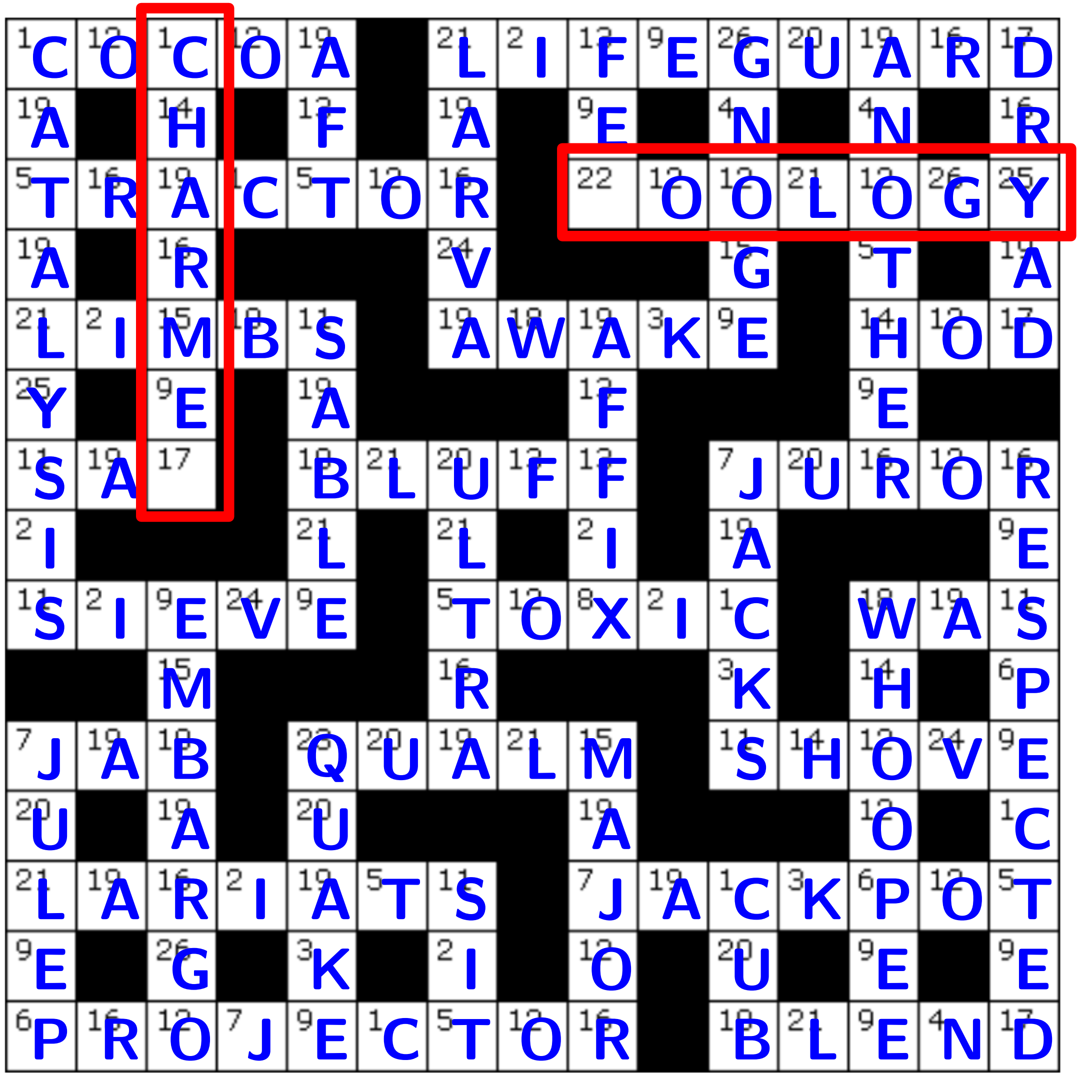
Substitution ciphers



Substitution ciphers

1 C	12 O	1 C	12 O	19 A	■	21 L	2 I	13 F	9 E	26 G	20 U	19 A	16 R	17 D
19 A	■	14 H	■	13 F	■	19 A	■	9 E	■	4 N	■	4 N	■	16 R
5 T	16 R	19 A	1 C	5 T	12 O	16 R	■	22	12 O	12 O	21 L	12 O	26 G	25 Y
19 A	■	16 R	■	■	■	24 V	■	■	■	15 G	■	5 T	■	19 A
21 L	2 I	15 M	10 B	11 S	■	19 A	18 W	19 A	3 K	9 E	■	14 H	12 O	17 D
25 Y	■	9 E	■	19 A	■	■	■	13 F	■	■	■	9 E	■	■
11 S	19 A	17	■	10 B	21 L	20 U	13 F	13 F	■	7 J	20 U	16 R	12 O	16 R
2 I	■	■	■	21 L	■	21 L	■	2 I	■	19 A	■	■	■	9 E
11 S	2 I	9 E	24 V	9 E	■	5 T	12 O	8 X	2 I	1 C	■	18 W	19 A	11 S
■	■	15 M	■	■	■	16 R	■	■	■	3 K	■	14 H	■	6 P
7 J	19 A	10 B	■	23 Q	20 U	19 A	21 L	15 M	■	11 S	14 H	12 O	24 V	9 E
20 U	■	19 A	■	20 U	■	■	■	19 A	■	■	■	12 O	■	1 C
21 L	19 A	16 R	2 I	19 A	5 T	11 S	■	7 J	19 A	1 C	3 K	6 P	12 O	5 T
9 E	■	26 G	■	3 K	■	2 I	■	12 O	■	20 U	■	9 E	■	9 E
6 P	16 R	12 O	7 J	9 E	1 C	5 T	12 O	16 R	■	10 B	21 L	9 E	4 N	17 D

Substitution ciphers



Substitution ciphers

1 C	12 O	1 C	12 O	19 A	■	21 L	2 I	13 F	9 E	26 G	20 U	19 A	16 R	17 D
19 A	■	14 H	■	13 F	■	19 A	■	9 E	■	4 N	■	4 N	■	16 R
5 T	16 R	19 A	1 C	5 T	12 O	16 R	■	22 Z	12 O	12 O	21 L	12 O	26 G	25 Y
19 A	■	16 R	■	■	■	24 V	■	■	■	15 G	■	5 T	■	19 A
21 L	2 I	15 M	10 B	11 S	■	19 A	18 W	19 A	3 K	9 E	■	14 H	12 O	17 D
25 Y	■	9 E	■	19 A	■	■	■	13 F	■	■	■	9 E	■	■
11 S	19 A	17 D	■	10 B	21 L	20 U	13 F	13 F	■	7 J	20 U	16 R	12 O	16 R
2 I	■	■	■	21 L	■	21 L	■	2 I	■	19 A	■	■	■	9 E
11 S	2 I	9 E	24 V	9 E	■	5 T	12 O	8 X	2 I	1 C	■	18 W	19 A	11 S
■	■	15 M	■	■	■	16 R	■	■	■	3 K	■	14 H	■	6 P
7 J	19 A	10 B	■	23 Q	20 U	19 A	21 L	15 M	■	11 S	14 H	12 O	24 V	9 E
20 U	■	19 A	■	20 U	■	■	■	19 A	■	■	■	12 O	■	1 C
21 L	19 A	16 R	2 I	19 A	5 T	11 S	■	7 J	19 A	1 C	3 K	6 P	12 O	5 T
9 E	■	26 G	■	3 K	■	2 I	■	12 O	■	20 U	■	9 E	■	9 E
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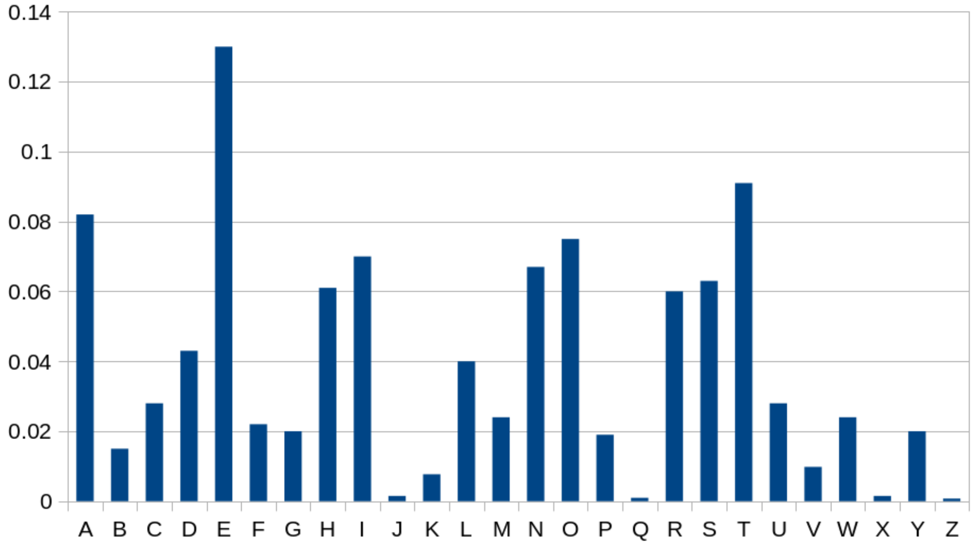
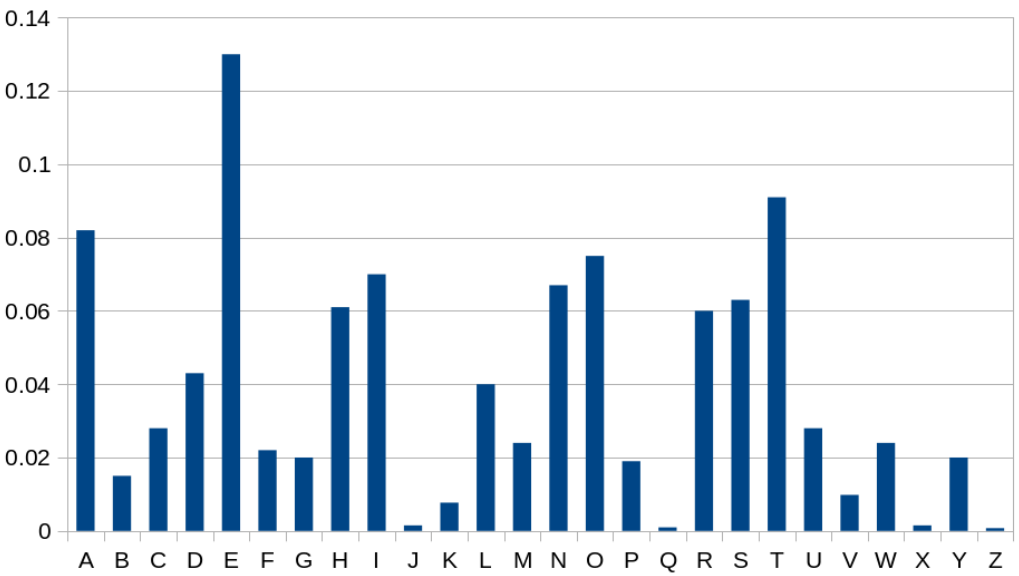


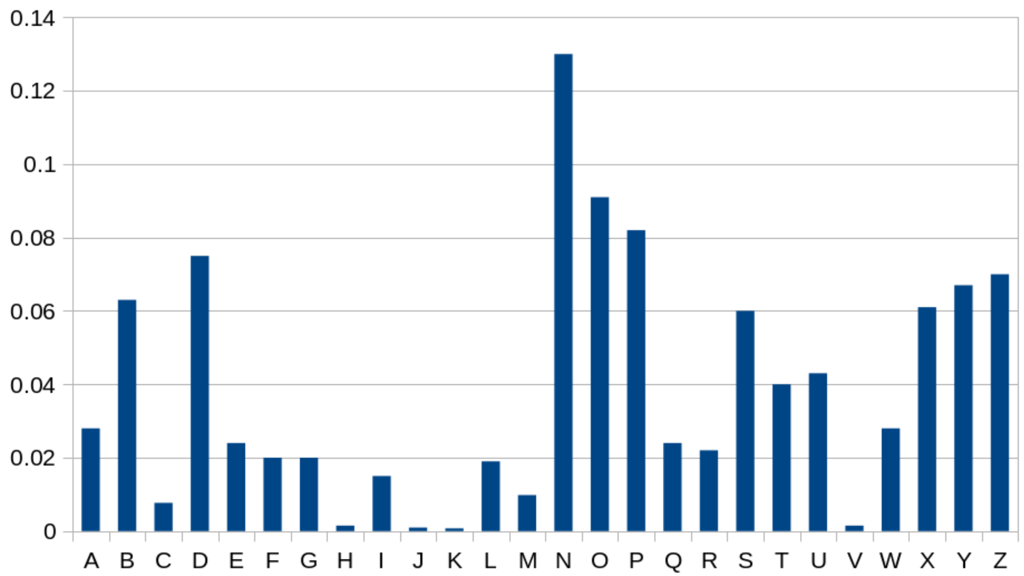
Substitution ciphers

Compare the expected frequencies in the message language with the observed frequencies in the ciphertext

Expected



Observed (in the ciphertext)



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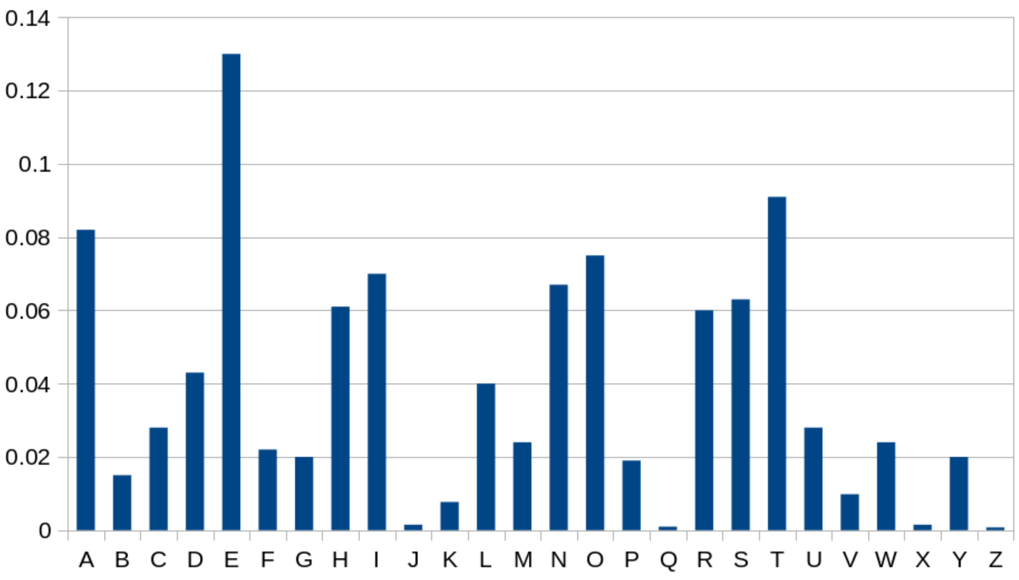

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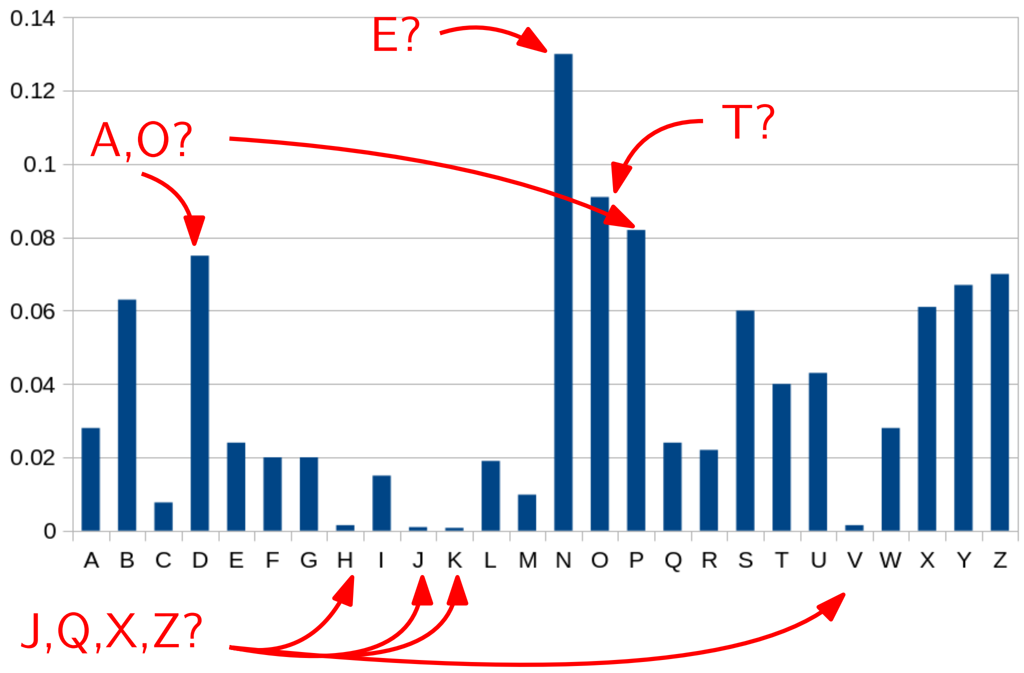
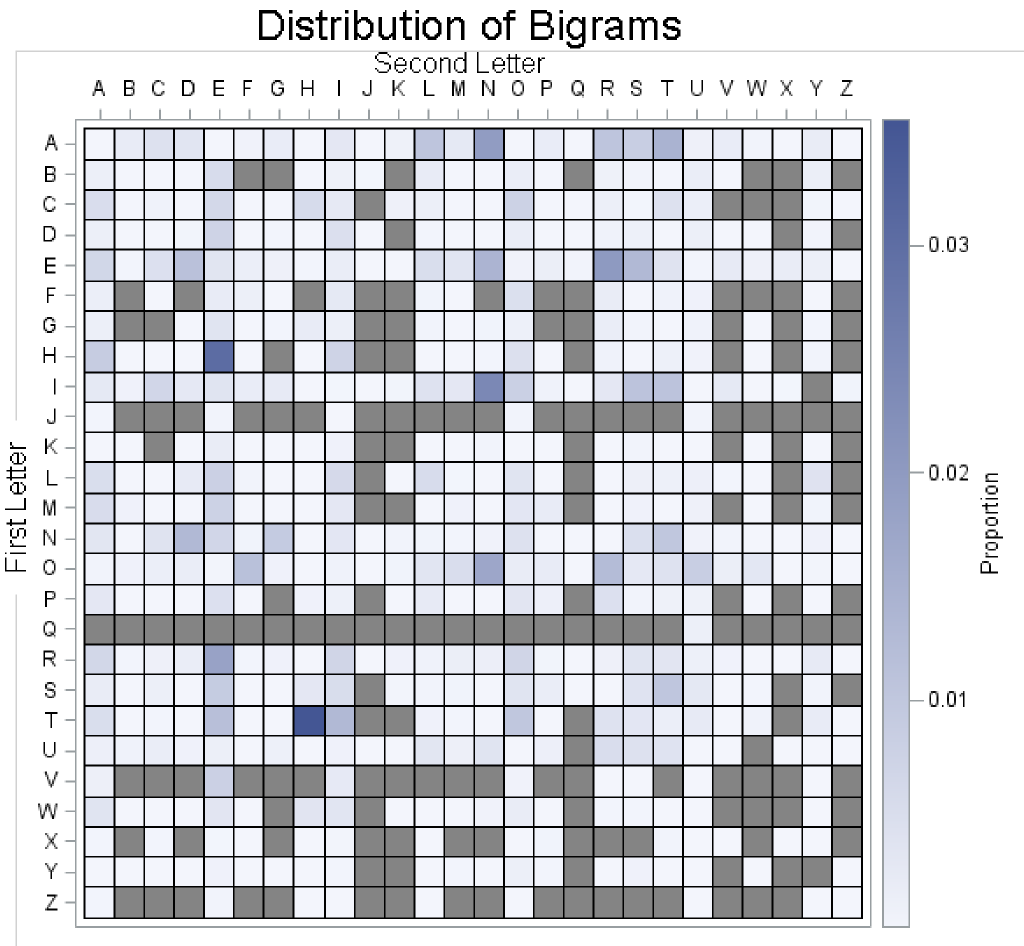


Guess part of the key and use the guesses to break the cipher (as shown before)

Substitution ciphers

The same analysis can be repeated for bigrams, trigrams, etc



Vigenère cipher

Monoalphabetic substitution ciphers are vulnerable to frequency analysis

Blaise de Vigenère
(1523 - 1596)

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$$s_i = \begin{cases} 0 & \text{if } k_i = \text{A} \\ 1 & \text{if } k_i = \text{B} \\ 2 & \text{if } k_i = \text{C} \\ \ldots & \\ 25 & \text{if } k_i = \text{Z} \end{cases}$$

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The generic i-th character m_i of the message $m = m_0 m_1 \ldots m_{\ell-1}$ is encrypted using a shift cipher with shift $s_{i \bmod t}$

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Vigenère cipher

$\mathcal{M} = \{A, \ldots, Z\}^*$

$\mathcal{K} = \{A, \ldots, Z\}^t$

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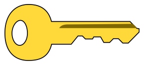 $k = $ `A C I D`

$m = $ `T H I S N I G H T` $\longrightarrow$ $\boxed{\mathsf{Enc}_k}$

Vigenère cipher

$\mathcal{M} = \{A, \ldots, Z\}^*$

$\mathcal{K} = \{A, \ldots, Z\}^t$

$\mathcal{C} = \{A, \ldots, Z\}^*$

k = `A C I D`

shifts = 0 2 8 3

m = `T H I S N I G H T`

Enc_k

Vigenère cipher

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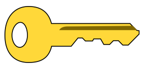 k = A C I D

shifts = 0 2 8 3

m = T H I S N I G H T

0 2 8 3 0 2 8 3 0

Enc_k

Vigenère cipher

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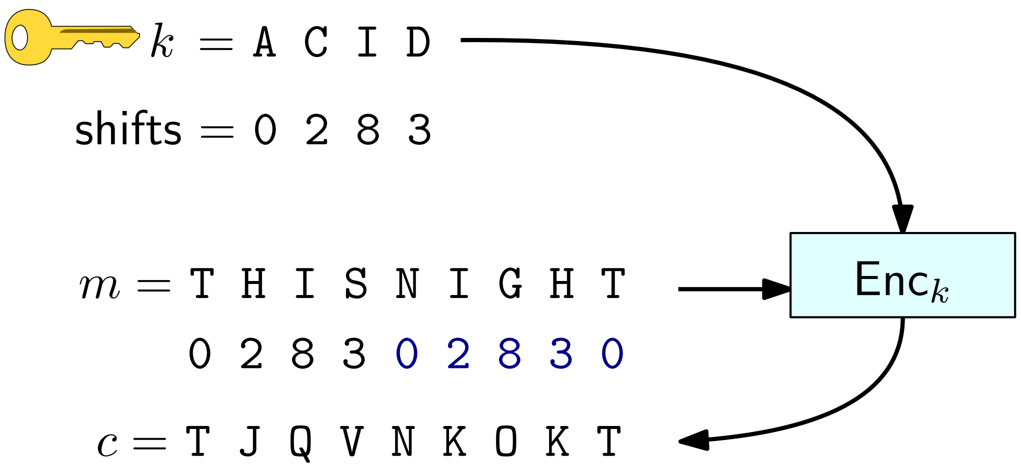

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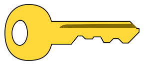
 k = A C I D

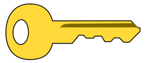
shifts = 0 2 8 3

m = T H I S N I G H T

0 2 8 3 0 2 8 3 0

Enc$_k$

c = T J Q V N K O K T

 k = A C I D

shifts = 0 2 8 3

c = A D Z U T R T D N

Dec$_k$

Vigenère cipher

$\mathcal{M} = \{A, \ldots, Z\}^*$

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$\mathcal{C} = \{A, \ldots, Z\}^*$

k = A C I D

shifts = 0 2 8 3

m = T H I S N I G H T

0 2 8 3 0 2 8 3 0

Enc_k

c = T J Q V N K O K T

k = A C I D

shifts = 0 2 8 3

c = A D Z U T R T D N

0 2 8 3 0 2 8 3 0

Dec_k

Vigenère cipher

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$\mathcal{K} = \{A, \ldots, Z\}^t$

$\mathcal{C} = \{A, \ldots, Z\}^*$

k = A C I D

shifts = 0 2 8 3

$m =$ T H I S N I G H T $\rightarrow$ $\boxed{\mathsf{Enc}_k}$

0 2 8 3 0 2 8 3 0

$c =$ T J Q V N K O K T

k = A C I D

shifts = 0 2 8 3

$c =$ A D Z U T R T D N $\rightarrow$ $\boxed{\mathsf{Dec}_k}$

0 2 8 3 0 2 8 3 0

$m =$ A B O R T P L A N

Vigenère cipher

A table called “tabula recta” can be used to aid encryption and decryption

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
A	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
B	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A
C	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B
D	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C
E	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D
F	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E
G	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F
H	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G
I	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H
J	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I
K	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J
L	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K
M	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L
N	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M
O	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N
P	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
Q	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
R	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q
S	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R
T	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S
U	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T
V	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U
W	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V
X	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W
Y	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X
Z	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y

Vigenère cipher

A table called "tabula recta" can be used to aid encryption and decryption

E.g., to encrypt the plaintext character `K` with the shift corresponding to the key character `F`, look up the letter at the intersection of the row labeled `K` and the column labeled `F` (or vice-versa)

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
A	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
B	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A
C	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B
D	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C
E	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D
F	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E
G	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F
H	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G
I	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H
J	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I
K	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J
L	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K
M	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L
N	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M
O	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N
P	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
Q	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
R	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q
S	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R
T	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S
U	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T
V	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U
W	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V
X	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W
Y	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X
Z	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y

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B	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A
C	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B
D	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C
E	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D
F	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E
G	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F
H	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G
I	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H
J	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I
K	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J
L	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K
M	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L
N	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M
O	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N
P	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
Q	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
R	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q
S	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R
T	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S
U	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T
V	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U
W	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V
X	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W
Y	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X
Z	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y

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	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
A	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
B	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A
C	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B
D	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C
E	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D
F	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E
G	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F
H	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G
I	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H
J	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I
K	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J
L	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K
M	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L
N	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M
O	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N
P	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
Q	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
R	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q
S	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R
T	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S
U	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T
V	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U
W	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V
X	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W
Y	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X
Z	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y

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To decrypt the ciphertext character `P` with the shift corresponding to the key character `F`, find `P` in the column corresponding to `F` and return the row label

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
A	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
B	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A
C	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B
D	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C
E	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D
F	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E
G	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F
H	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G
I	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H
J	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I
K	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J
L	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K
M	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L
N	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M
O	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N
P	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
Q	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
R	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q
S	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R
T	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S
U	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T
V	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U
W	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V
X	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W
Y	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X
Z	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y

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	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
A	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
B	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A
C	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B
D	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C
E	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D
F	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E
G	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F
H	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G
I	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H
J	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I
K	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J
L	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K
M	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L
N	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M
O	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N
P	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
Q	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
R	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q
S	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R
T	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S
U	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T
V	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U
W	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V
X	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W
Y	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X
Z	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y

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	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
A	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
B	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A
C	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B
D	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C
E	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D
F	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E
G	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F
H	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G
I	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H
J	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I
K	K	L	M	N	O	$\boxed{P}$	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J
L	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K
M	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L
N	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M
O	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N
P	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
Q	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
R	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q
S	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R
T	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S
U	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T
V	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U
W	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V
X	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W
Y	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X
Z	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y

Vigenère cipher

Is Vigenère cipher secure?

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Suppose that the adversary is somehow able to figure out what the length t of the key is

E.g.: $t = 4$

$c =$

```
A M A P A A U H K G O O T W F I O G G G T B T
Q I N N A V S M B T K Q O M O I W C P C T W T
U O I F A G O G T I M O U C F P B T W T B N P
W C P C Q B S J D G F A U O W B O E E K D A E
R K R E M L K B F P R O O T J C C S U O O F S
I Q I W U R B N F W M B T G A A U I E W D F L
Z L S F C Q Z O
```

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E.g.: $t = 4$

$$
c = \begin{array}{l}
\boxed{\mathtt{A}}\ \mathtt{M\ A\ P}\ \boxed{\mathtt{A}}\ \mathtt{A\ U\ H}\ \boxed{\mathtt{K}}\ \mathtt{G\ O\ O}\ \boxed{\mathtt{T}}\ \mathtt{W\ F\ I}\ \boxed{\mathtt{O}}\ \mathtt{G\ G\ G}\ \boxed{\mathtt{T}}\ \mathtt{B\ T} \\
\mathtt{Q}\ \boxed{\mathtt{I}}\ \mathtt{N\ N\ A}\ \boxed{\mathtt{V}}\ \mathtt{S\ M\ B}\ \boxed{\mathtt{T}}\ \mathtt{K\ Q\ O}\ \boxed{\mathtt{M}}\ \mathtt{O\ I\ W}\ \boxed{\mathtt{C}}\ \mathtt{P\ C\ T}\ \boxed{\mathtt{W}}\ \mathtt{T} \\
\mathtt{U\ O}\ \boxed{\mathtt{I}}\ \mathtt{F\ A\ G}\ \boxed{\mathtt{O}}\ \mathtt{G\ T\ I}\ \boxed{\mathtt{M}}\ \mathtt{O\ U\ C}\ \boxed{\mathtt{F}}\ \mathtt{P\ B\ T}\ \boxed{\mathtt{W}}\ \mathtt{T\ B\ N}\ \boxed{\mathtt{P}} \\
\mathtt{W\ C\ P}\ \boxed{\mathtt{C}}\ \mathtt{Q\ B\ S}\ \boxed{\mathtt{J}}\ \mathtt{D\ G\ F}\ \boxed{\mathtt{A}}\ \mathtt{U\ O\ W}\ \boxed{\mathtt{B}}\ \mathtt{O\ E\ E}\ \boxed{\mathtt{K}}\ \mathtt{D\ A\ E} \\
\boxed{\mathtt{R}}\ \mathtt{K\ R\ E}\ \boxed{\mathtt{M}}\ \mathtt{L\ K\ B}\ \boxed{\mathtt{F}}\ \mathtt{P\ R\ O}\ \boxed{\mathtt{O}}\ \mathtt{T\ J\ C}\ \boxed{\mathtt{C}}\ \mathtt{S\ U\ O}\ \boxed{\mathtt{O}}\ \mathtt{F\ S} \\
\mathtt{I}\ \boxed{\mathtt{Q}}\ \mathtt{I\ W\ U}\ \boxed{\mathtt{R}}\ \mathtt{B\ N\ F}\ \boxed{\mathtt{W}}\ \mathtt{M\ B\ T}\ \boxed{\mathtt{G}}\ \mathtt{A\ A\ U}\ \boxed{\mathtt{I}}\ \mathtt{E\ W\ D}\ \boxed{\mathtt{F}}\ \mathtt{L} \\
\mathtt{Z\ L}\ \boxed{\mathtt{S}}\ \mathtt{F\ C\ Q}\ \boxed{\mathtt{Z}}\ \mathtt{O}
\end{array}
$$

Vigenère cipher

Is Vigenère cipher secure? It has been considered secure for centuries...

Suppose that the adversary is somehow able to figure out what the length t of the key is

E.g.: $t = 4$

$c =$ A M A P A A U H K G O O T W F I O G G G T B T
Q I N N A V S M B T K Q O M O I W C P C T W T
U O I F A G O G T I M O U C F P B T W T B N P
W C P C Q B S J D G F A U O W B O E E K D A E
R K R E M L K B F P R O O T J C C S U O O F S
I Q I W U R B N F W M B T G A A U I E W D F L
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Q	I	N	N	A	V	S	M	B	T	K	Q	O	M	O	I	W	C	P	C	T	W	T
U	O	I	F	A	G	O	G	T	I	M	O	U	C	F	P	B	T	W	T	B	N	P
W	C	P	C	Q	B	S	J	D	G	F	A	U	O	W	B	O	E	E	K	D	A	E
R	K	R	E	M	L	K	B	F	P	R	O	O	T	J	C	C	S	U	O	O	F	S
I	Q	I	W	U	R	B	N	F	W	M	B	T	G	A	A	U	I	E	W	D	F	L
Z	L	S	F	C	Q	Z	O															

The ciphertext can be decomposed into n ciphertext $c^{(1)}, c^{(2)}, \ldots, c^{(t)}$.

Each $c^{(i)}$ is encrypted using the same shift

Each ciphertext can be attacked separately (but we cannot simply bruteforce them)

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- Consider some (unknown) sequence of characters that appears frequently in the plaintext (for example the word "the")

```
T H E M A N A N D T H E W O M A N R E T R I E V E D T H E L E T T E R F R O M T H E P O S T B O X
B E A D S B E A D S B E A D S B E A D S B E A D S B E A D S B E A D S B E A D S B E A D S B E A D
-------------------------------------------------------------------------------------------------
U L E P S O E N G L I I W R E B R R H L S M E Y W E X H H D F X T H J G V O P L I I P R K U F O A
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- Consider some (unknown) sequence of characters that appears frequently in the plaintext (for example the word "the")
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```
[T H E] M A N A N D [T H E] W O M A N R E T R I E V E D [T H E] L E T T E R F R O M [T H E] P O S T B O X
 B E A D S B E A D S B E A D S B E A D S B E A D S B E A D S B E A D S B E A D S B E A D S B E A D
-----------------------------------------------------------------------------------------------------
[U L E] P S O E N G L I I W R E B R R H L S M E Y W E [X H H] D F X T H J G V O P L I I P R K U F O A
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- When this happens, the corresponding portions of ciphertext will be equal

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B E A D S B E A D S B E A D S B E A D S B E A D S B E A D S B E A D S B E A D S B E A D S B E A D
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Obs: The distance between repeated patterns in the ciphertext is likely to be a multiple of the key length

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In the example the key length t is 5 and the distance between patterns is 30

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- **Option 2:** Kasiski's method
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Index of coincidence method

Let p_j be the expected frequency of the j-th letter $(j = 0. \dots, 25)$ in the language of the plaintext

Using the frequencies in the English language: $$\sum_{j=0}^{25} p_j^2 \approx 0.065$$

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Guess that the key length is τ and split the ciphertext into $c^{(1)}, \ldots, c^{(\tau)}$ sub-ciphertexts (as before).
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Compute $$S_\tau = \sum_{i=0}^{25} q_j^2$$

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$\implies S_\tau \approx \sum_{j=0}^{25} \left(\frac{1}{26}\right)^2 \approx 0.038$

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For a given i, let q_j be the observed frequency of the j-th letter of the alphabet in $c^{(i)}$.

Compute $$S_\tau = \sum_{i=0}^{25} q_j^2$$

The smallest value of τ such that $S_\tau \approx 0.065$ is probably the length of the key

This can be validated by repeating the check for other values of i

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- If the guess is correct then the i-th letter in the alphabet is mapped to the $(i + j)$-th letter $\pmod{26}$

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- If the guess is wrong, we expect $\sum_{i=0}^{25} p_i q_{(i+j) \bmod 26}$ to be "far enough" from 0.065

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Compute $I_j = \sum_{i=0}^{25} p_i q_{(i+j)} \bmod 26$ for all possible shifts j and choose the one for which I_j is closest to 0.065.

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Marian Adam Rejewski

Alan Mathison Turing

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What is the key of this cipher? The diameter of the rod!

To decrypt the ciphertext, simply wind it around a rod of the same diameter

Breaking the scytale cipher

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Wind the parchment around a cone

Look for the portion of the cone where letters start to line up and produce sensible words

The corresponding diameter is the key!

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```
      K I L L K I N G
m =   T O M O R R O W
      M I D N I G H T
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$$m = \begin{bmatrix} \mathtt{K} & \mathtt{I} & \mathtt{L} & \mathtt{L} & \mathtt{K} & \mathtt{I} & \mathtt{N} & \mathtt{G} \\ \mathtt{T} & \mathtt{O} & \mathtt{M} & \mathtt{O} & \mathtt{R} & \mathtt{R} & \mathtt{O} & \mathtt{W} \\ \mathtt{M} & \mathtt{I} & \mathtt{D} & \mathtt{N} & \mathtt{I} & \mathtt{G} & \mathtt{H} & \mathtt{T} \end{bmatrix}$$

Scytale cipher

What is the effect of winding the parchment around the scytale on the order of the characters in the plaintext?

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The scytale cipher is a (specific type of) transposition cipher!

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The plaintext is arranged in a matrix with n columns (and the appropriate number of rows)

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E.g., if the permutation is $4, 2, 1, 6, 5, 3$, then the ciphertext is:

$c =$ M G O T C H I O W D T T T O E E S R H X E I R T K E N M A O

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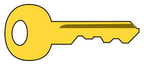
What is the key? 

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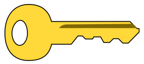
What is the key?  The pair (n, π)

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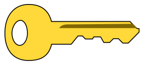
What is the key?  The pair (n, π)

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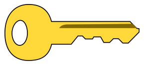
What is the key?  The pair (n, π)

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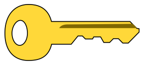
What is the key?  The pair (n, π)

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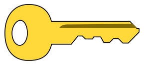
What is the key?  The pair (n, π)

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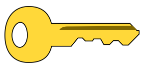
What is the key?  The pair (n, π)

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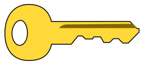
What is the key?  The pair (n, π)

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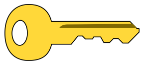
What is the key?  The pair (n, π)

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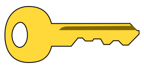
What is the key?  The pair (n, π)

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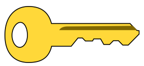
What is the key?  The pair (n, π)

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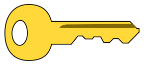
What is the key?  The pair (n, π)

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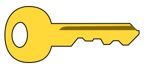
What is the key?  The pair (n, π)

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The plaintext can be found by reading the rows in order (left to right, top to bottom)

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Are columnar transposition ciphers secure?

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  (if the transposition cipher is regular, look at the divisors of the ciphertext's length)

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```
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- Look for anagrams (that simultaneously yield intelligible text on multiple rows)

Other transposition ciphers

To make cryptanalysis harder, a double (irregular) transposition cipher is often used:

- Pick two sub-keys $k_1 = (n_1, \pi_1)$ and $k_2 = (n_2, \pi_2)$ $\qquad k = (k_1, k_2)$

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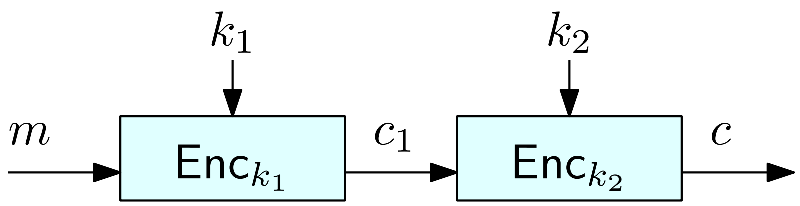

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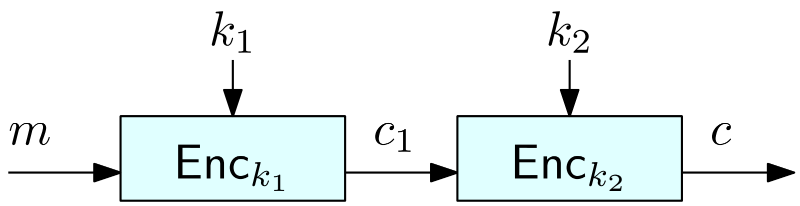

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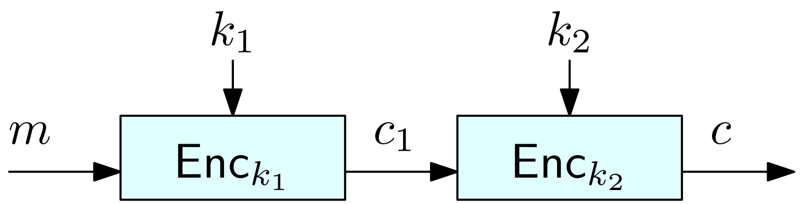

- Among the "manual" ciphers, the double transposition cipher is easy to carry out but hard to break
- Many other (more complex) transposition ciphers have been used

Other transposition ciphers

The Zodiac Z-340 cipher remained unsolved for 51 years!

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Ultimately, transposition ciphers are **not secure**:

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- If a key is close to the right one, a decryption of the ciphertext reveals parts of the plaintext

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What does secure mean?