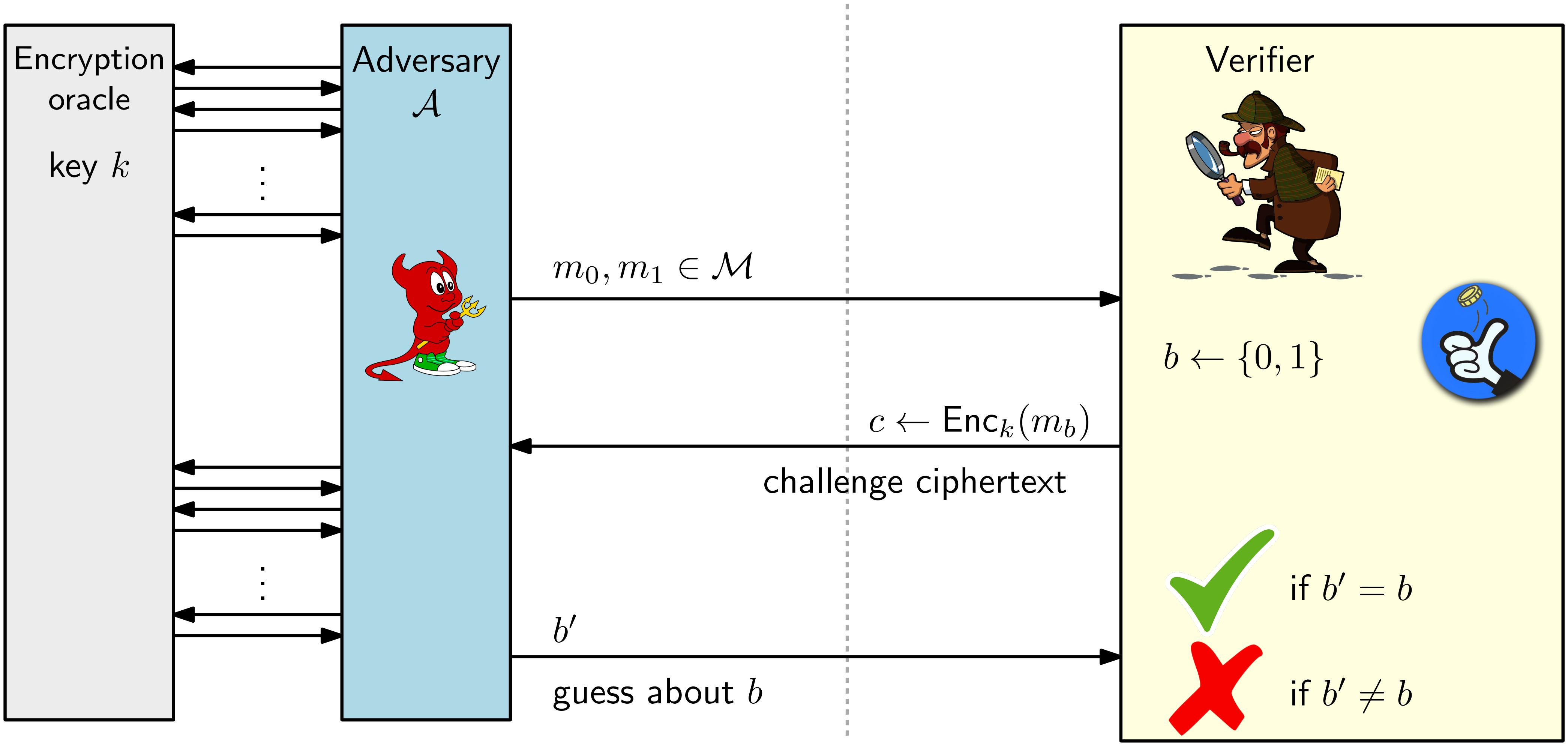
Recap: Modeling CPA security

A key $k \leftarrow \mathsf{Gen}(1^n)$ is generated



Definition of CPA-security

Definition: *A private key encryption scheme Π has indistinguishable encryptions under a chosen-plaintext attack (is* **CPA-secure***) if, for every probabilistic polynomial-time adversary $\mathcal{A}$, there is a negligible function ε such that:*

$$\Pr[\mathit{PrivK}^{cpa}_{\mathcal{A},\Pi}(n) = 1] \leq \frac{1}{2} + \varepsilon(n)$$

Any private-key encryption scheme that is CPA-secure is also CPA-secure for multiple encryptions

$\Downarrow$

If Π is CPA-secure then Π has indistinguishable multiple encryptions in the presence of an eavesdropper (and hence it is also EAV-secure)

Achieving CPA security (a first attempt)

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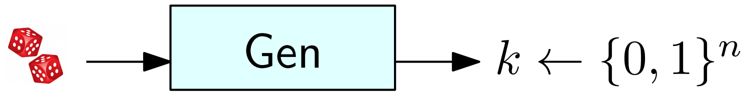
No stateless, deterministic encryption scheme can be CPA-secure

Actually achieving CPA security

Let $F : \{0,1\}^n \times \{0,1\}^n \to \{0,1\}^n$ be a (length-preserving) pseudorandom function

Consider the following encryption scheme $\Pi = (\mathsf{Gen}, \mathsf{Enc}, \mathsf{Dec})$:

- $\mathsf{Gen}(1^n)$: return a key k chosen u.a.r. from $\{0,1\}^n$



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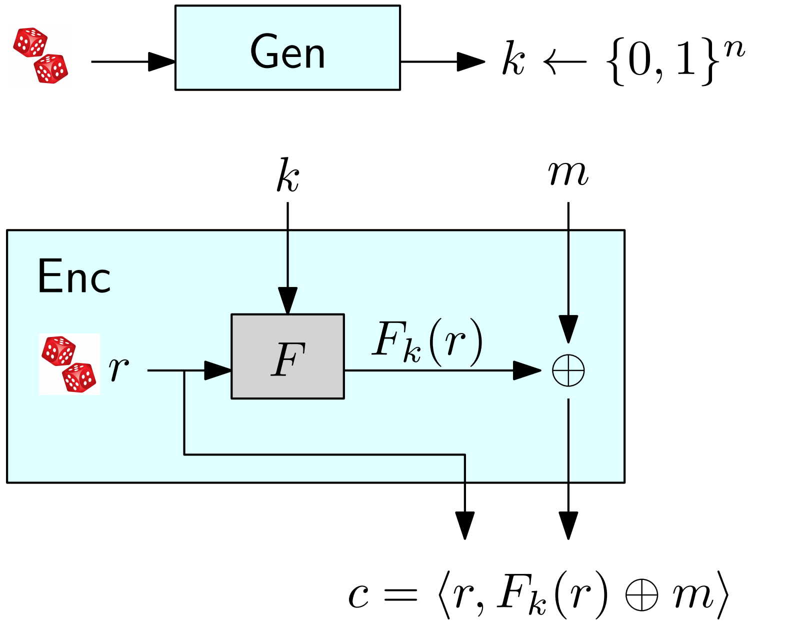

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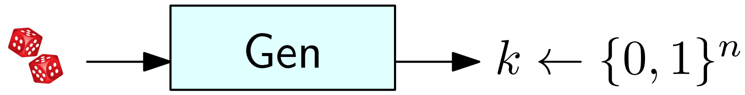

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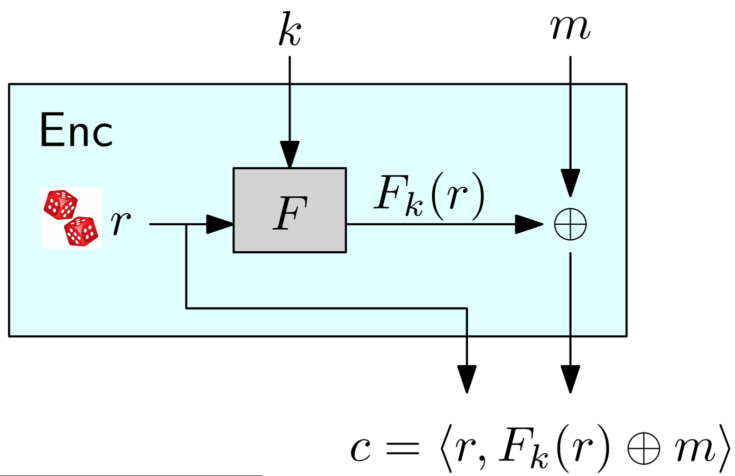

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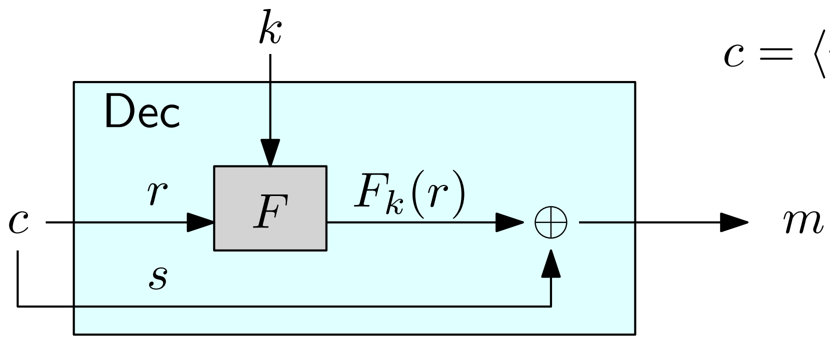

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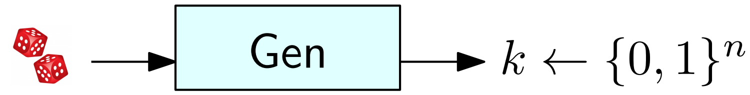

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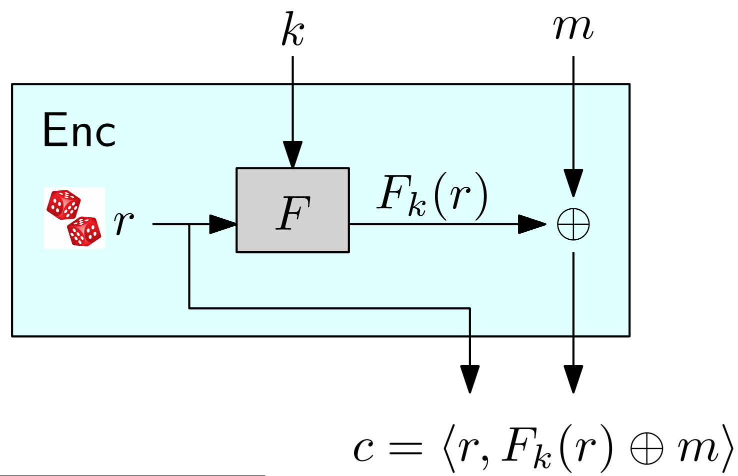

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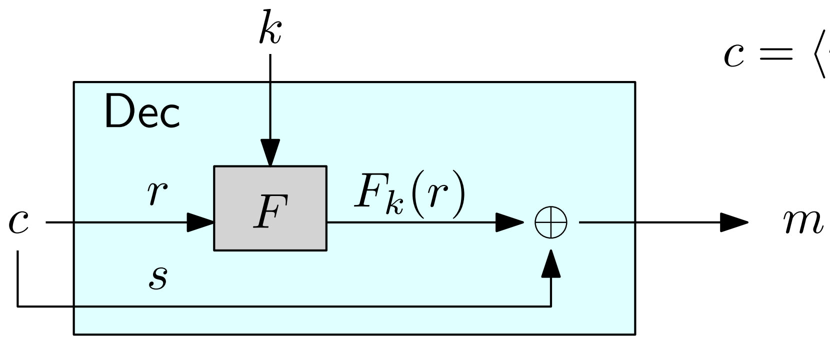

$\mathcal{M} = \{0,1\}^n \quad \mathcal{K} = \{0,1\}^n \quad \mathcal{C} = \{0,1\}^{2n}$

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- When a message needs to be encrypted, a **fresh** random string r is generated
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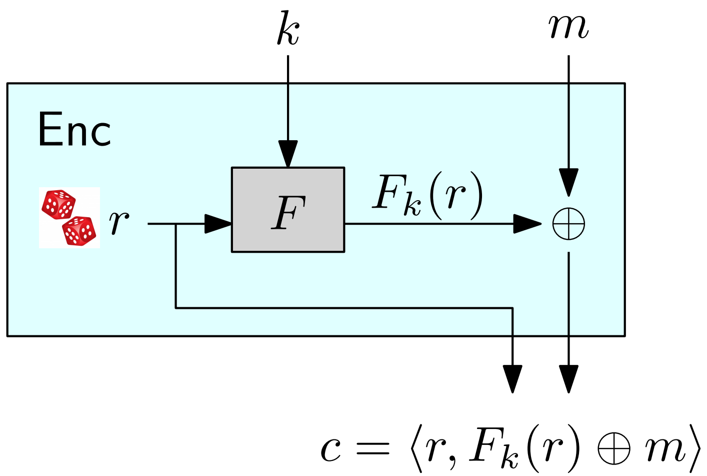

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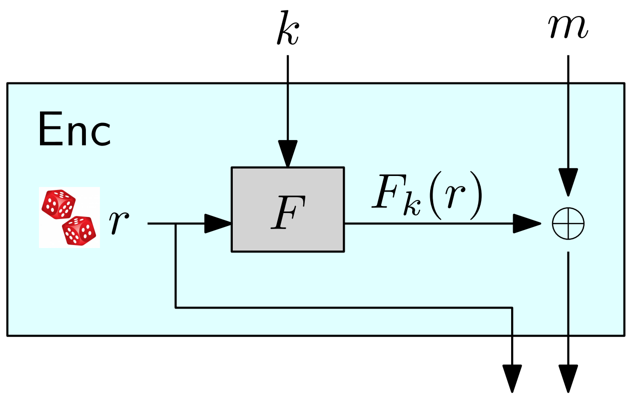

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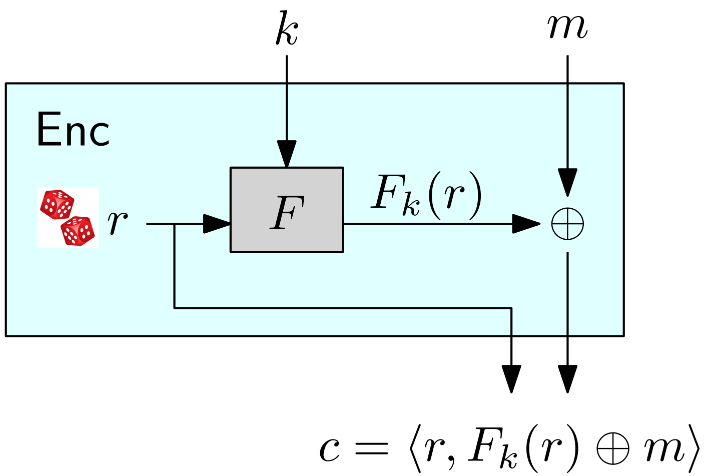

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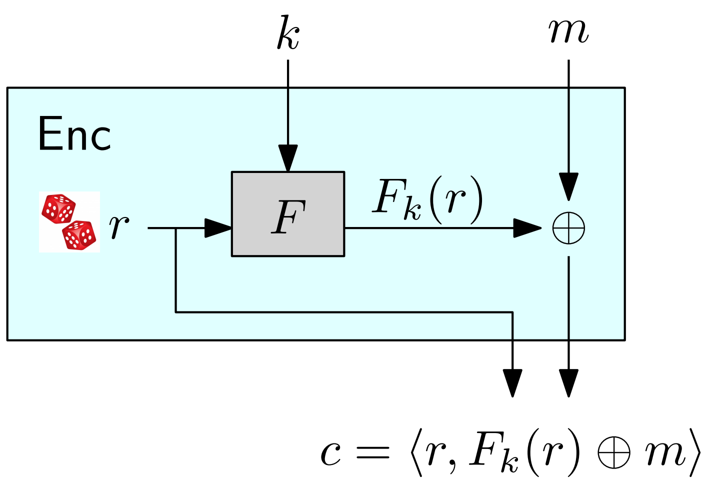

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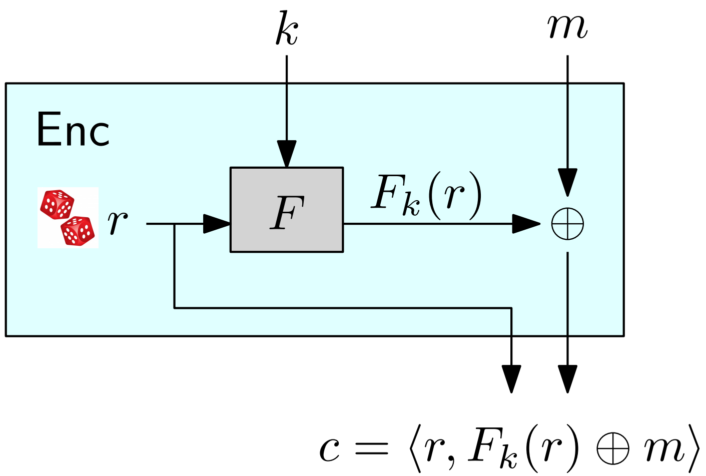
- The process behaves similarly to the "real" OTP if the parties were to"agree on a new key" after each message



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$$\Pr[D^{f(\cdot)}(1^n) = 1] = \Pr[\mathsf{PrivK}^{\mathsf{cpa}}_{\mathcal{A},\widetilde{\Pi}}(n) = 1]$$

$$\Big| \Pr[D^{F_k(\cdot)}(1^n) = 1] - \Pr[D^{f(\cdot)}(1^n) = 1] \Big| = \Big| \Pr[\mathsf{PrivK}^{\mathsf{cpa}}_{\mathcal{A},\Pi}(n) = 1] - \Pr[\mathsf{PrivK}^{\mathsf{cpa}}_{\mathcal{A},\widetilde{\Pi}}(n) = 1] \Big| = \varepsilon(n)$$

Not negligible!

Proof of security

Can we **prove** that this encryption scheme is secure?

High-level proof strategy:

- Consider a variant $\widetilde{\Pi}$ of Π in which a truly random function f is used instead of F
- Prove that if $\widetilde{\Pi}$ is CPA-secure then Π is CPA-secure ✔
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$\widetilde{\mathsf{Enc}}_k(m)$ (where $|m| = n$):

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- If n is too short, or it is not chosen from a uniform distribution then repeats might happen!

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- The last chunk needs special handling (padding, truncating the output, ...)

Encrypting long messages

What if we want to send a message that is longer than the security parameter (i.e., $|m| > n$) ?

Idea:

- We have shown that any CPA-secure encryption scheme is also secure for multiple encryptions
- Think of m as the concatenation of many messages of length n, i.e., $m = m_1 \,\|\, m_2 \,\|\, m_3 \,\|\, \ldots$
- Encrypt each m_i separately

Does it work? Yes, but...

- The ciphertext is (at least) twice as long as the plaintext
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Can we do better?

Stream ciphers

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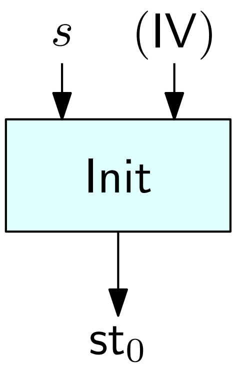
Warning: Sometimes the term "stream cipher" is used to refer to the encryption scheme built from the actual stream cipher (as defined here)

Stream ciphers

A stream cipher is a pair of deterministic polynomial-time algorithms

- **Init:** takes a n-bit seed s, and possibly a n-bit *initialization vector* (IV), and outputs a *state* st_0



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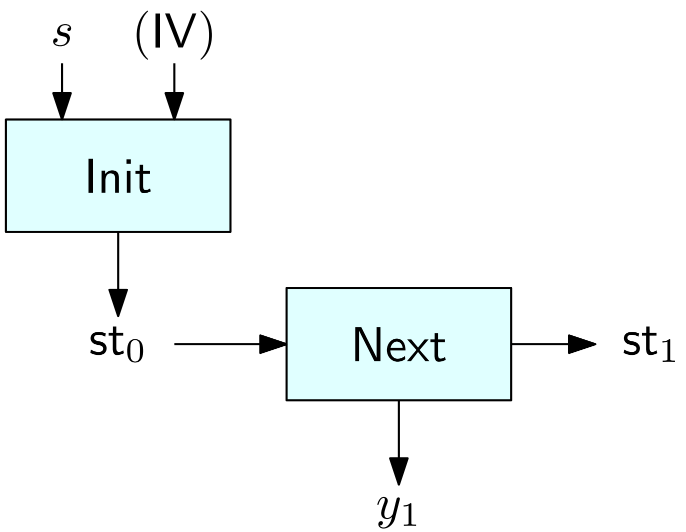

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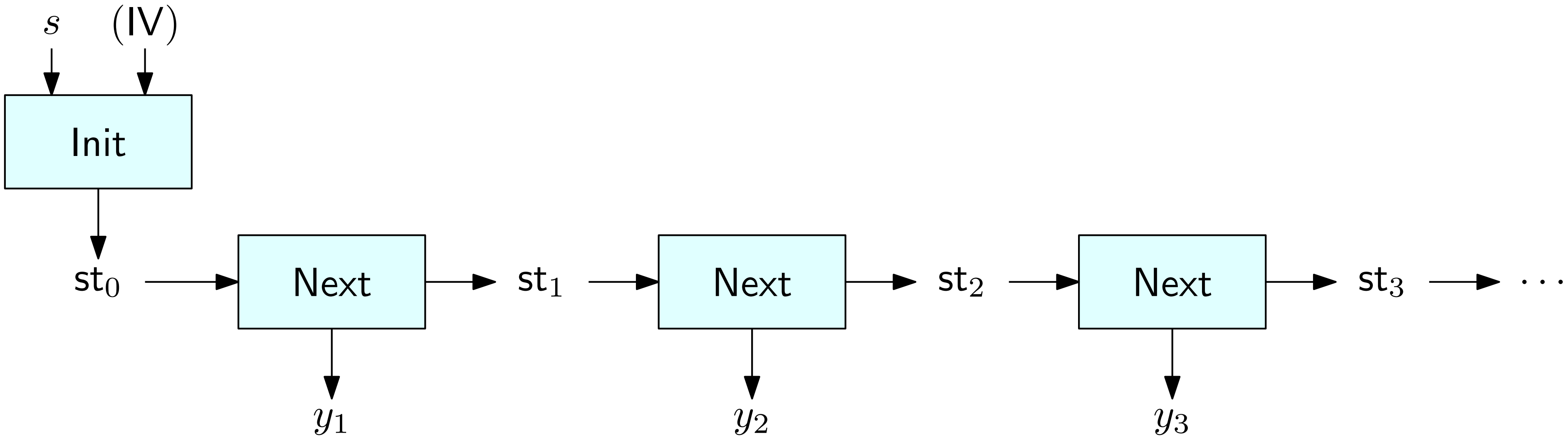

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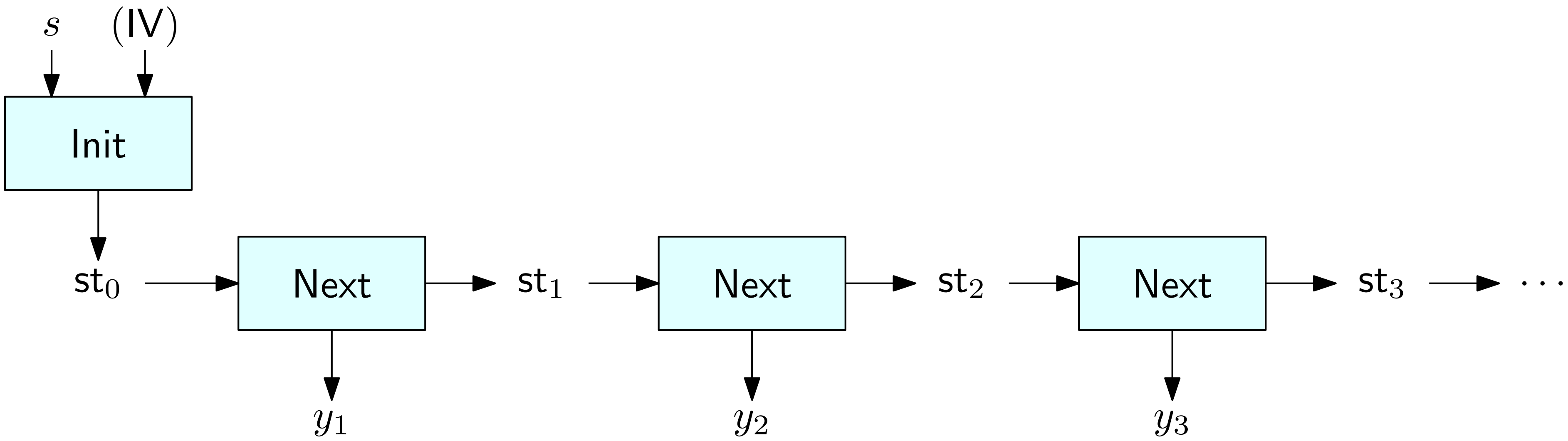

* In practice, **Next** can output multiple bits at once (e.g., a byte)

Secure Stream Ciphers

A stream cipher is secure if the output stream generated by starting from a seed chosen u.a.r. is pseudorandom

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If the stream cipher does not use IVs:

- Define the function $G^\ell(s)$ (from $\{0, 1\}^n$ to $\{0, 1\}^\ell$) as the string y of $\mathsf{GetBits}(\mathsf{Init}(s), 1^\ell)$
- The stream cipher is **secure** if $G^\ell(s)$ is a **pseudorandom generator** for any polynomial ℓ

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If the stream cipher uses IVs:

- Define the function $F_s^\ell(\mathsf{IV})$ (from $\{0, 1\}^n \times \{0, 1\}^n$ to $\{0, 1\}^\ell$) as the string y of $\mathsf{GetBits}(\mathsf{Init}(s, \mathsf{IV}), 1^\ell)$

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Secure Stream ciphers with IVs from Pseudorandom Functions

If we have a pseudorandom function $F : \{0,1\}^n \times \{0,1\}^n \to \{0,1\}^n$, we can use it to build a stream cipher that takes an initialization vector

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$\langle i \rangle$ = Binary encoding of i using $n/4$ bits

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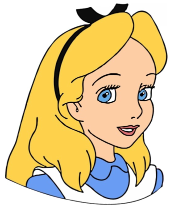
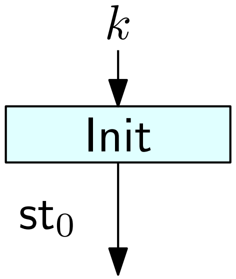
- **Unsynchronized mode**: The sender and receiver do not need to store any information during the communication session (i.e., they are stateless)
  - Useful for long messages, and communication over a long period of time. Does not require messages to be delivered in order
  - Each message uses its own IV
  - Needs IVs, Ciphertext length = message length + IV length ($\approx$ message length for long messages)

Synchronized mode





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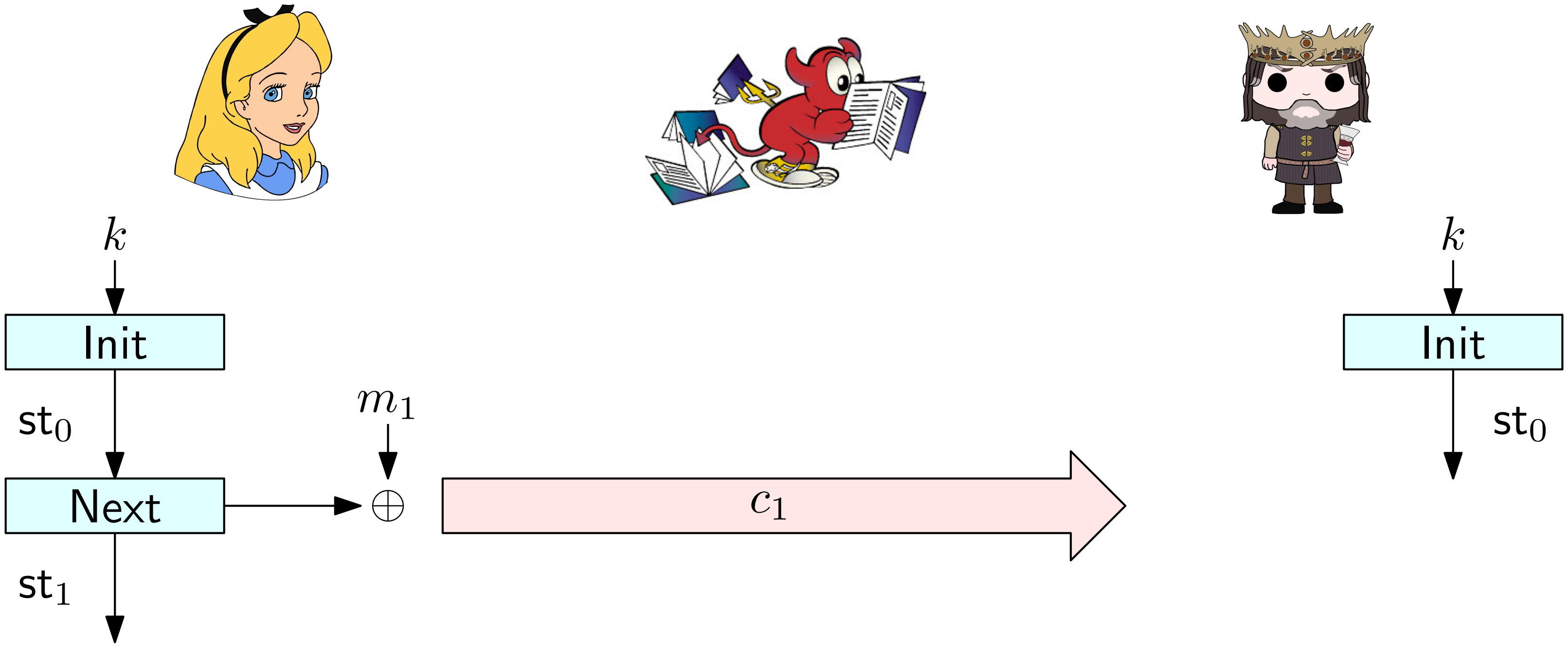

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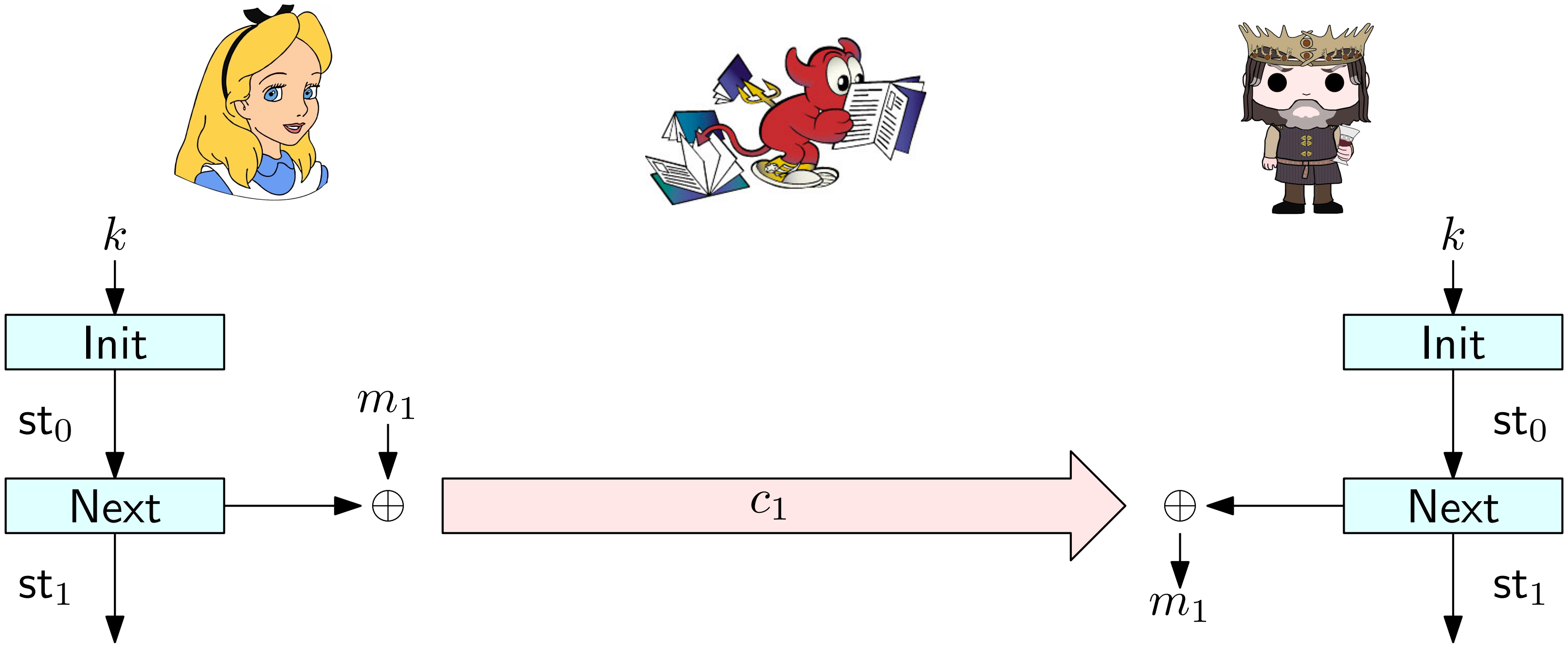

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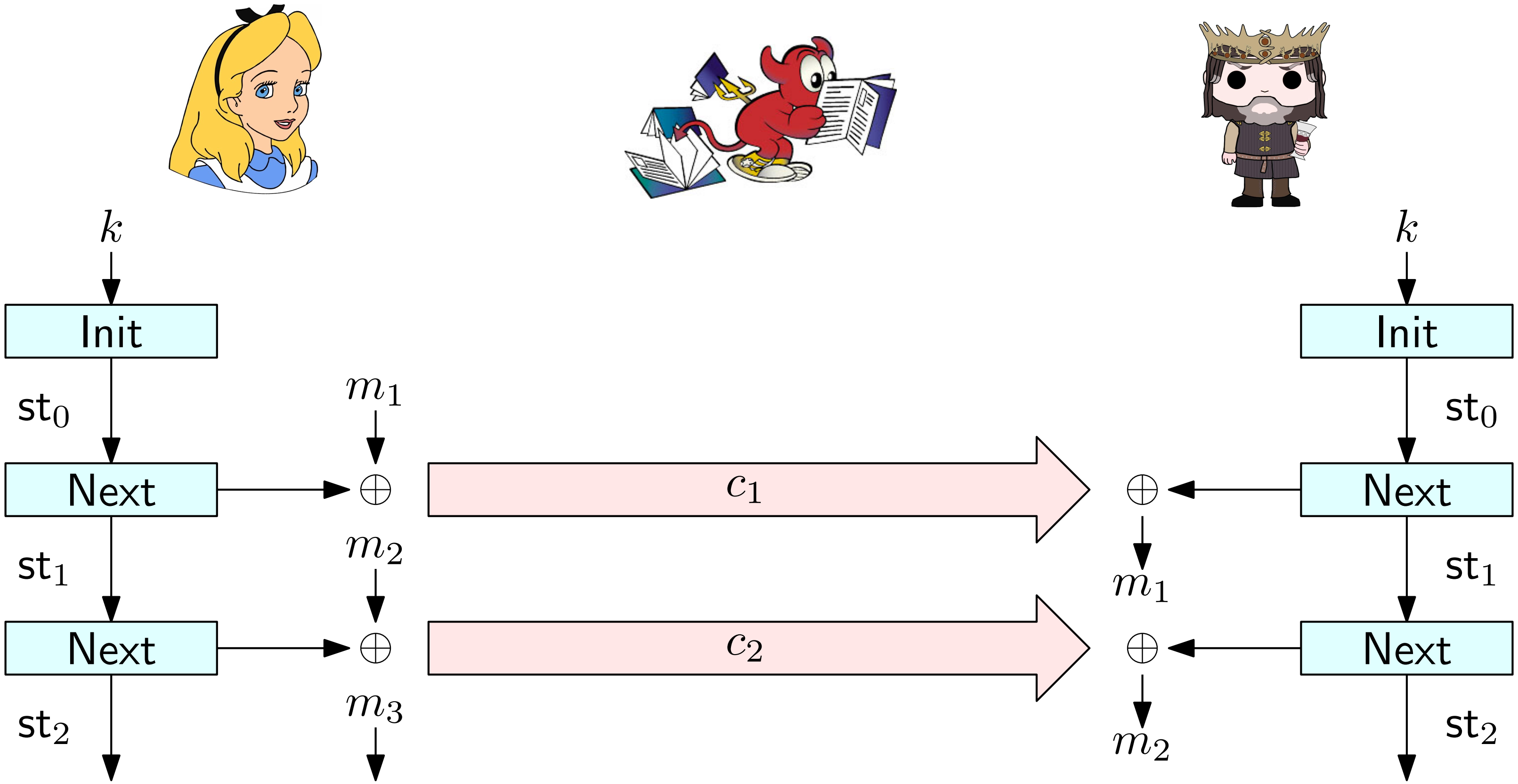

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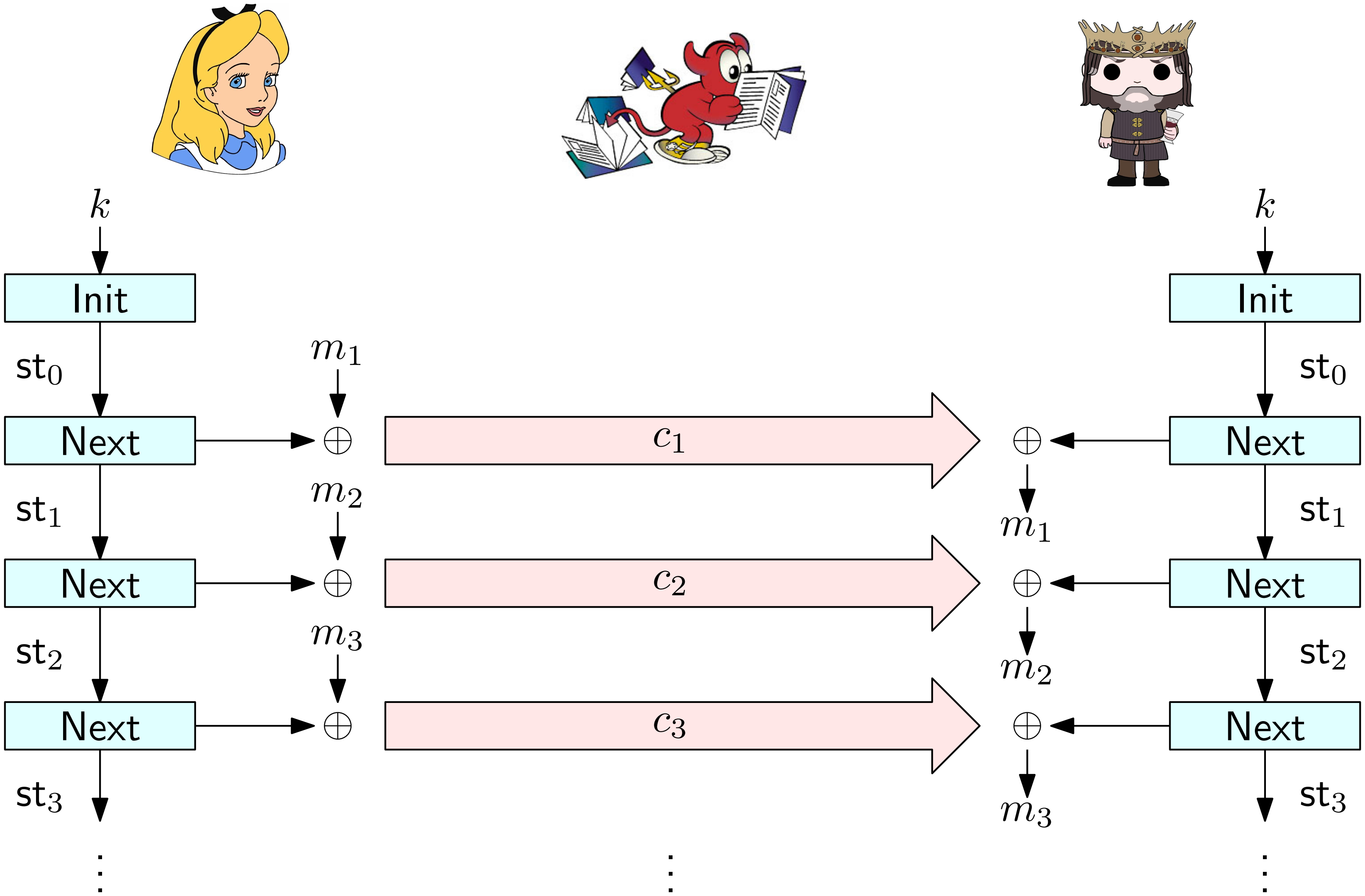
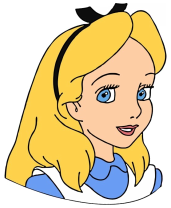
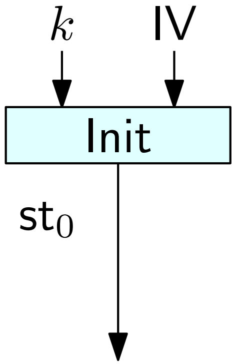


Alice & Bob need to keep track of the last state for as long as they wish to communicate

Unsynchronized mode

Alice picks a random IV





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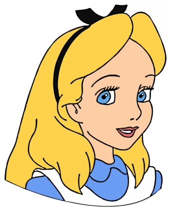

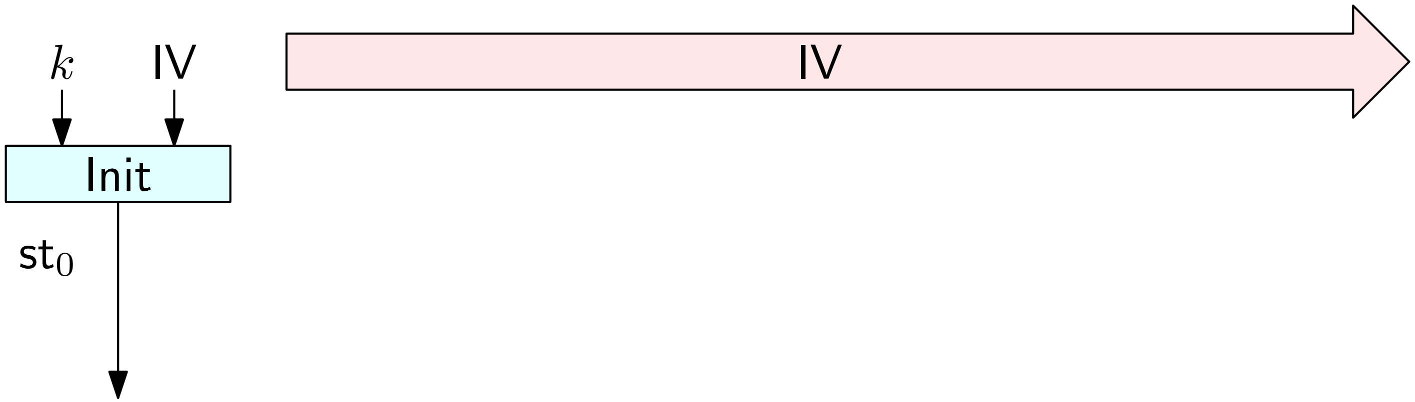

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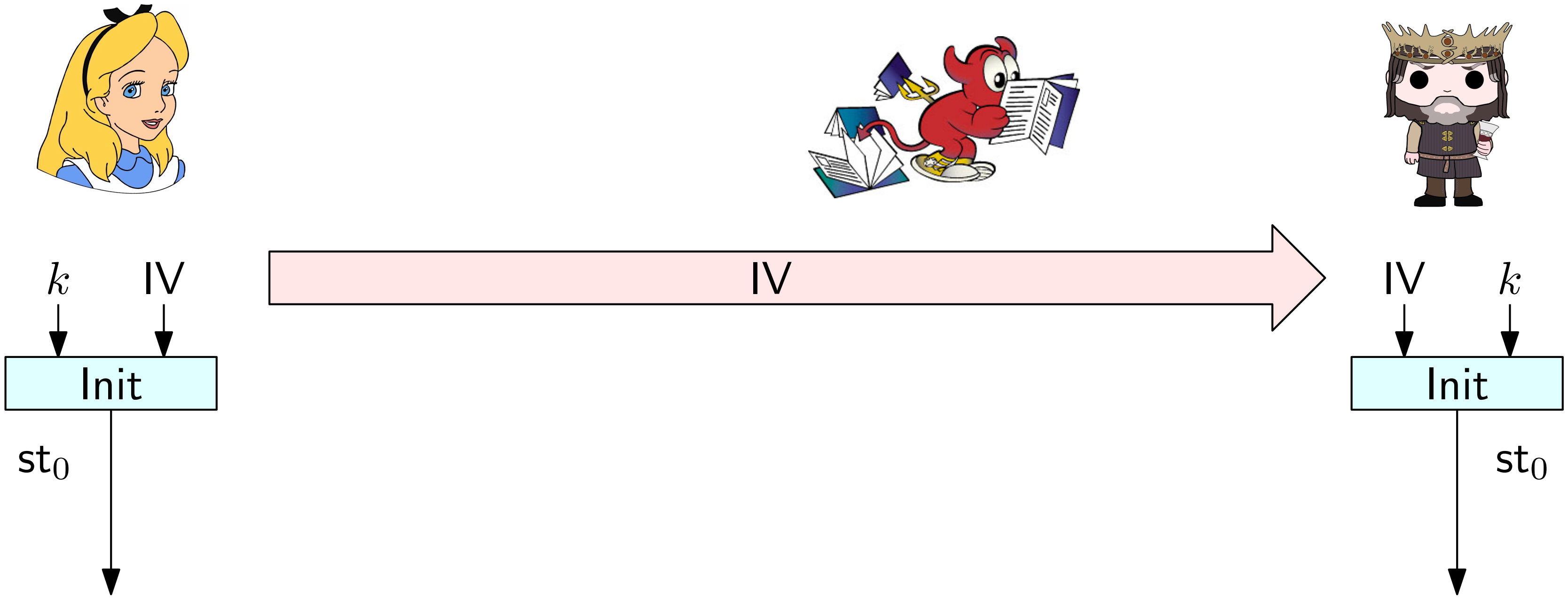

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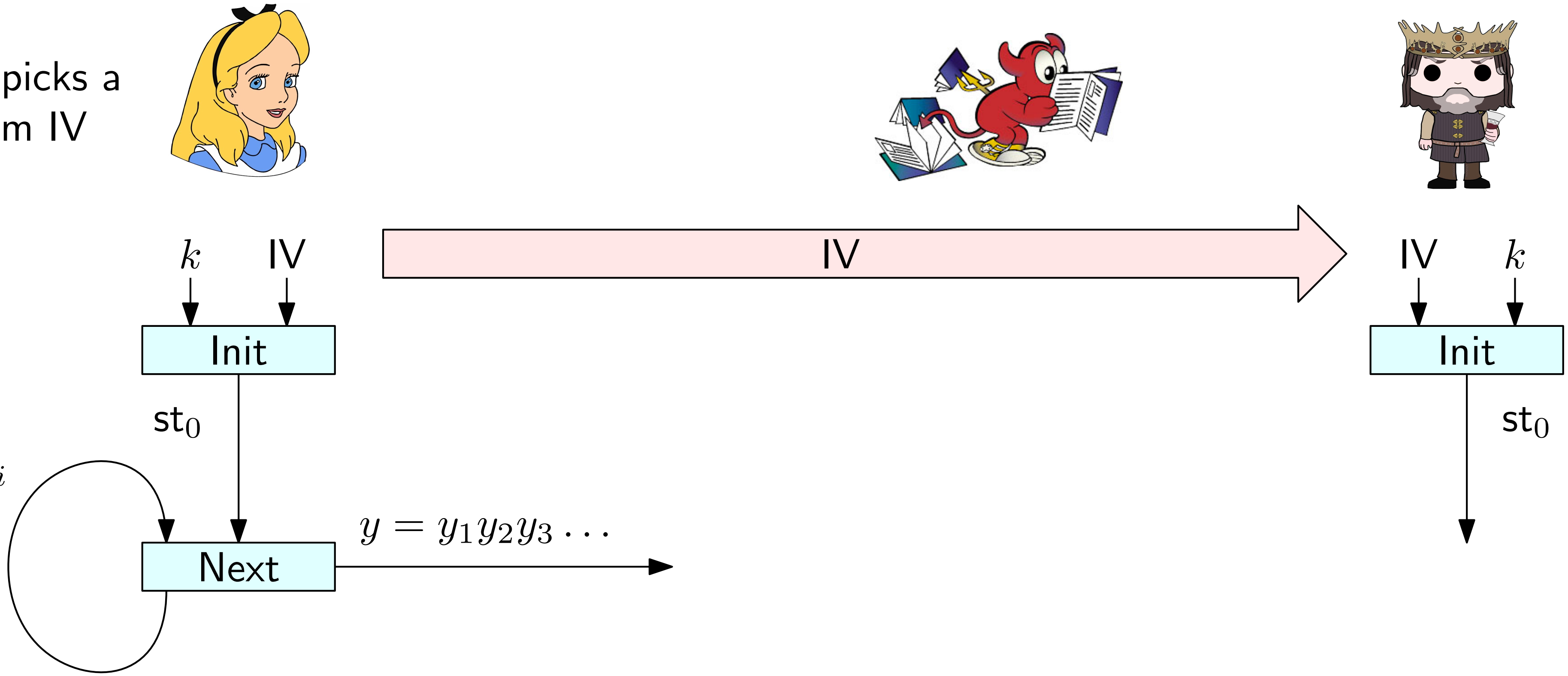

Generate as many bits
$y_1 y_2 y_3 \ldots$ as needed

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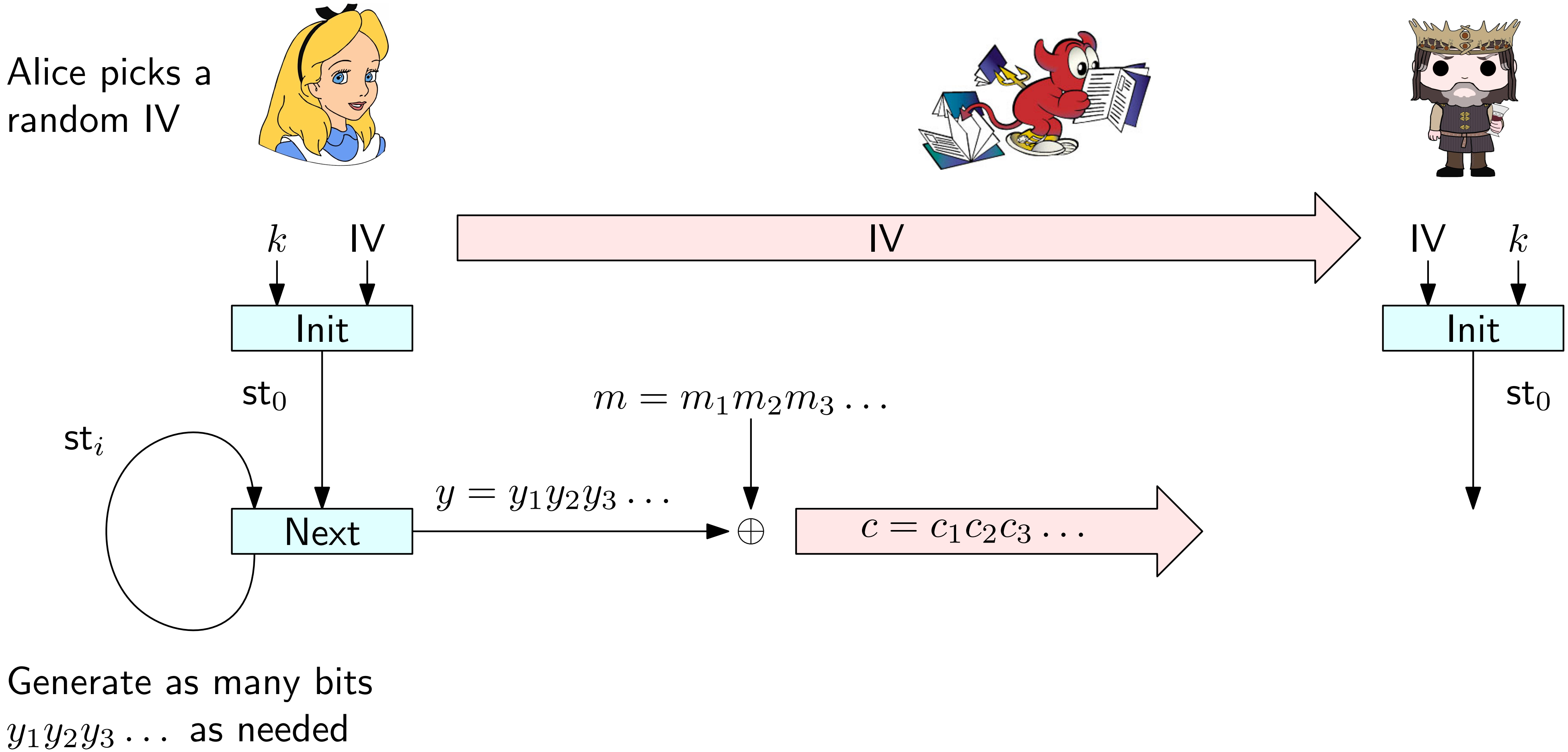

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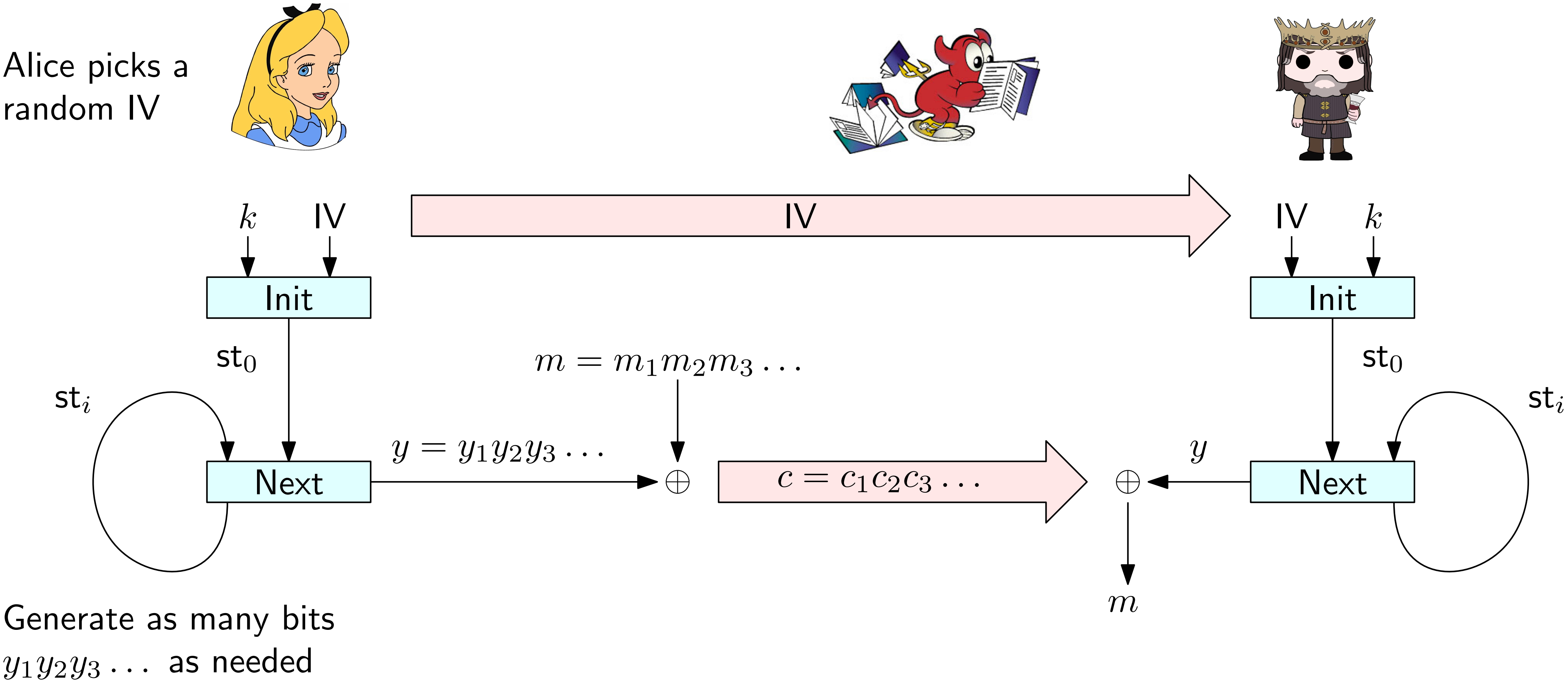

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- Some popular practical constructions of stream ciphers:
  - Trivium: optimized for hardware
  - RC4 (insecure): optimized for software
  - ChaCha20: replacement of RC4