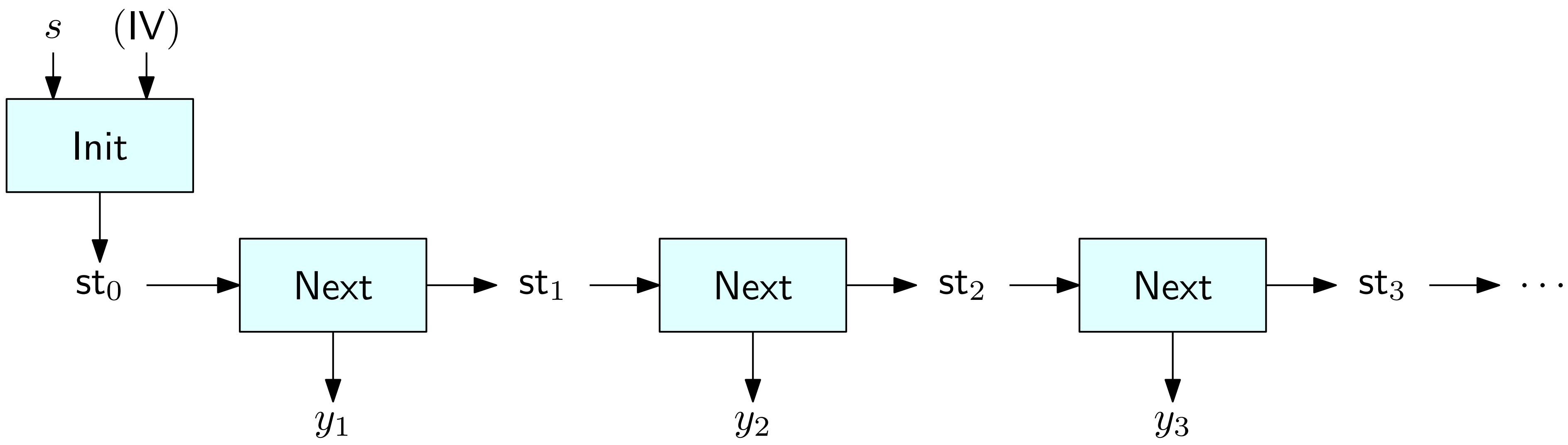
Stream ciphers (reminder)

A stream cipher is a pair of deterministic polynomial-time algorithms

- **Init:** takes a n-bit seed s, and possibly a n-bit *initialization vector* (IV), and outputs a *state* st
- **Next:** takes a state st and outputs a bit y and a new (updated) state st′

Idea: we can generate as many random bits as desired, by repeatedly calling Next



* In practice, **Next** can output multiple bits at once (e.g., a byte)

RC4

- Stands for Rivest Cipher 4
- Designed for performance in software

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WEP Encryption

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WEP Encryption

- We will see how to attack it

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- An array S of 256 bytes, which will always be a permutation of $\{0, \ldots, 255\}$
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Init(k : array of 16 bytes):

- $S \leftarrow [0, 1, 2, \dots, 255]$
- $k \leftarrow \underbrace{k \,\|\, k \,\|\, \dots \,\|\, k}_{16 \text{ times}}$
- $j \leftarrow 0$
- For $i \leftarrow 0, 1, \dots, 255$:
  - $j \leftarrow j + S[i] + k[i] \pmod{256}$
  - Swap $S[i]$ and $S[j]$
- Return $\langle S, i = 0, j = 0 \rangle$

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[Demo]

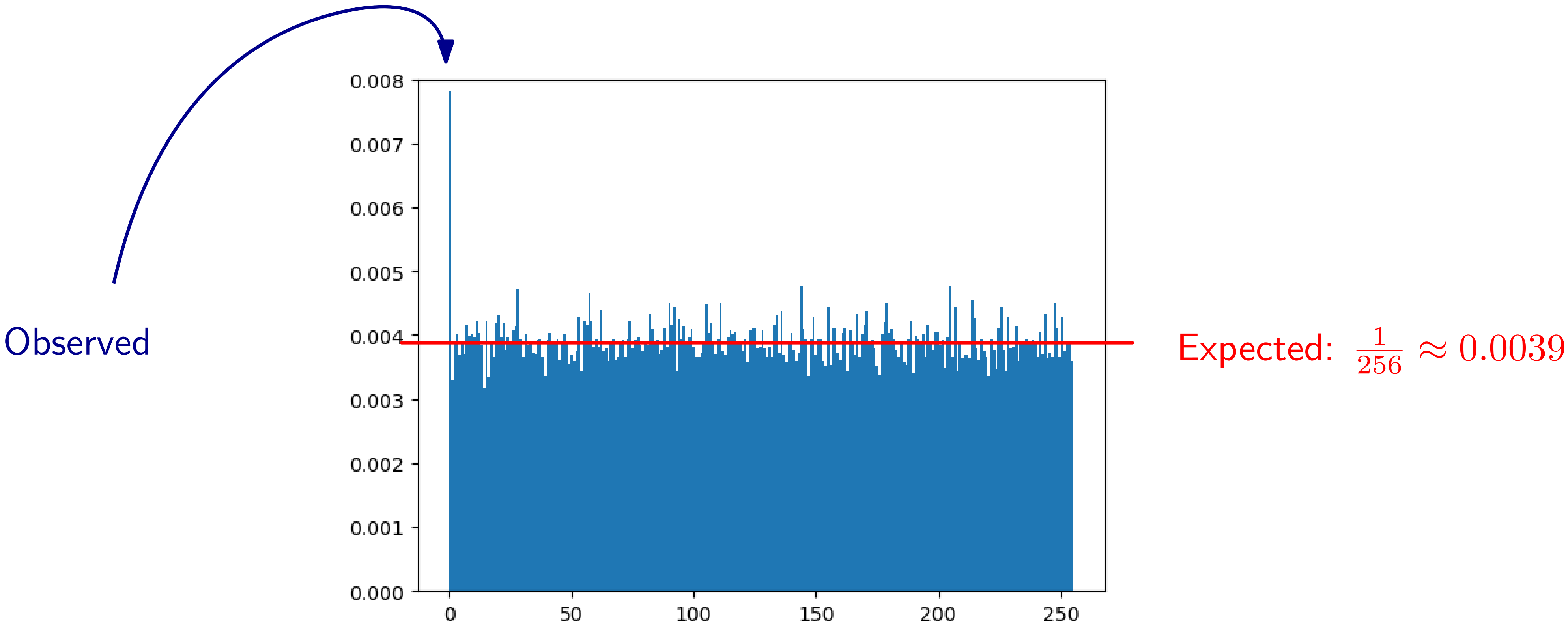
Test vectors

```
Key length: 128 bits.
key: 0x0102030405060708090a0b0c0d0e0f10

DEC    0 HEX    0:  9a c7 cc 9a  60 9d 1e f7   b2 93 28 99  cd e4 1b 97
DEC   16 HEX   10:  52 48 c4 95  90 14 12 6a   6e 8a 84 f1  1d 1a 9e 1c
DEC  240 HEX   f0:  06 59 02 e4  b6 20 f6 cc   36 c8 58 9f  66 43 2f 2b
DEC  256 HEX  100:  d3 9d 56 6b  c6 bc e3 01   07 68 15 15  49 f3 87 3f
DEC  496 HEX  1f0:  b6 d1 e6 c4  a5 e4 77 1c   ad 79 53 8d  f2 95 fb 11
DEC  512 HEX  200:  c6 8c 1d 5c  55 9a 97 41   23 df 1d bc  52 a4 3b 89
DEC  752 HEX  2f0:  c5 ec f8 8d  e8 97 fd 57   fe d3 01 70  1b 82 a2 59
DEC  768 HEX  300:  ec cb e1 3d  e1 fc c9 1c   11 a0 b2 6c  0b c8 fa 4d
DEC 1008 HEX  3f0:  e7 a7 25 74  f8 78 2a e2   6a ab cf 9e  bc d6 60 65
DEC 1024 HEX  400:  bd f0 32 4e  60 83 dc c6   d3 ce dd 3c  a8 c5 3c 16
DEC 1520 HEX  5f0:  b4 01 10 c4  19 0b 56 22   a9 61 16 b0  01 7e d2 97
DEC 1536 HEX  600:  ff a0 b5 14  64 7e c0 4f   63 06 b8 92  ae 66 11 81
DEC 2032 HEX  7f0:  d0 3d 1b c0  3c d3 3d 70   df f9 fa 5d  71 96 3e bd
DEC 2048 HEX  800:  8a 44 12 64  11 ea a7 8b   d5 1e 8d 87  a8 87 9b f5
DEC 3056 HEX  bf0:  fa be b7 60  28 ad e2 d0   e4 87 22 e4  6c 46 15 a3
DEC 3072 HEX  c00:  c0 5d 88 ab  d5 03 57 f9   35 a6 3c 59  ee 53 76 23
DEC 4080 HEX  ff0:  ff 38 26 5c  16 42 c1 ab   e8 d3 c2 fe  5e 57 2b f8
DEC 4096 HEX 1000:  a3 6a 4c 30  1a e8 ac 13   61 0c cb c1  22 56 ca cc
```

Output bias

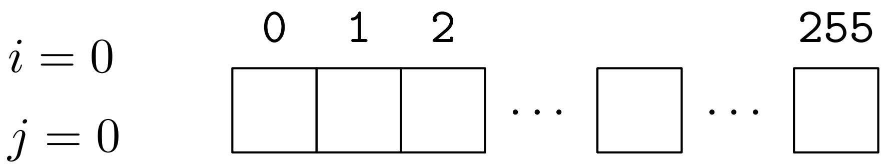
Empirical distribution of the value of the 2nd output byte over 50000 samples (with keys chosen u.a.r.)



There is a bias towards 0 in the second byte output by RC4 (about twice as likely to be 0)

Output bias: analysis

- Consider the state immediately after **Init**
- For simplicity, think of S as a uniform permutation over $\{0, 1, \ldots, 255\}$



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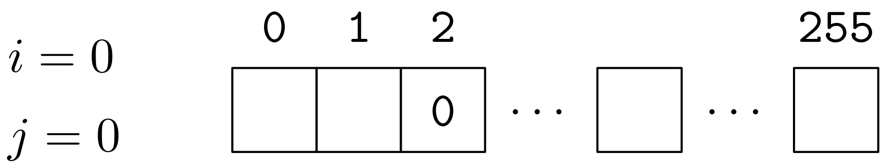

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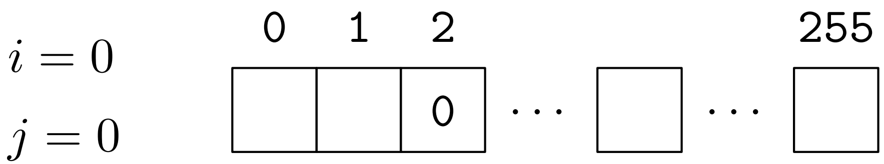

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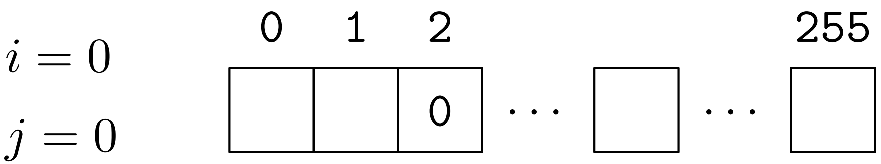

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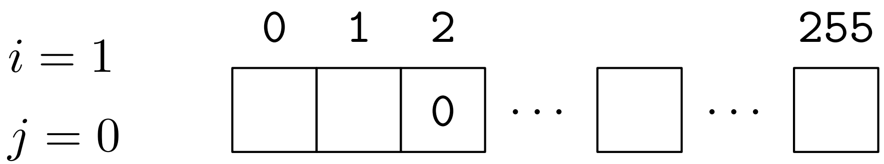

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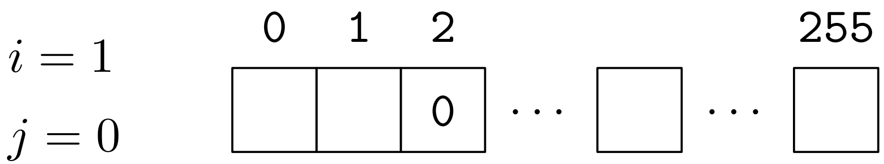

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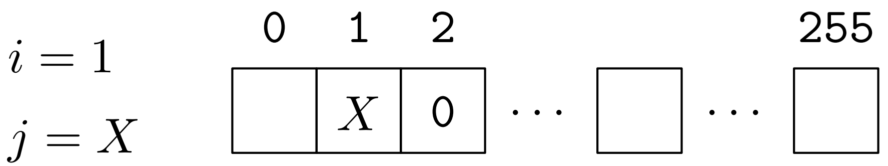

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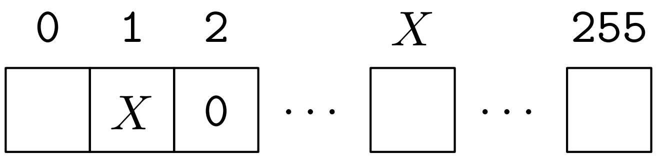

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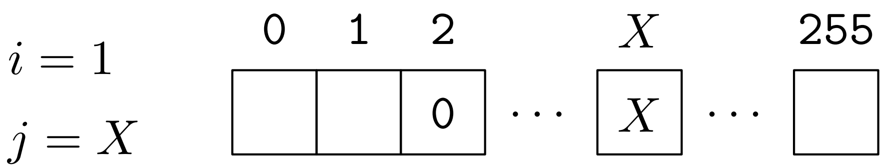

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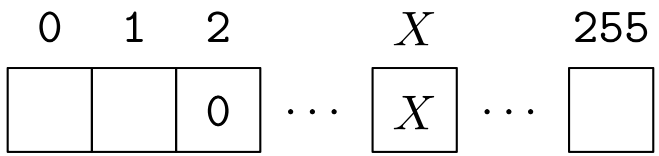

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The rest of the code does not modify the state

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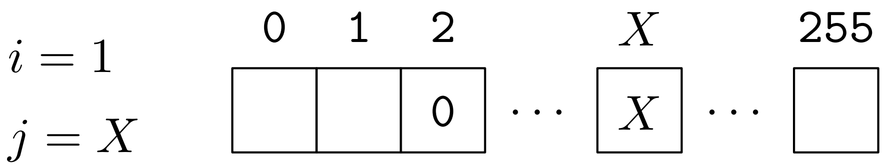

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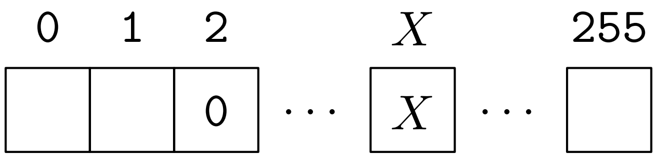

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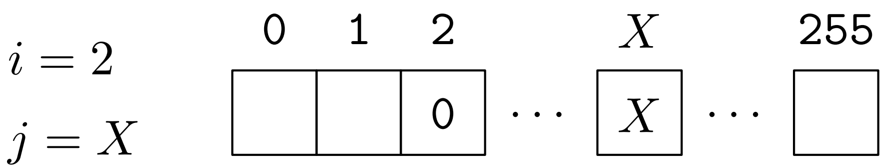

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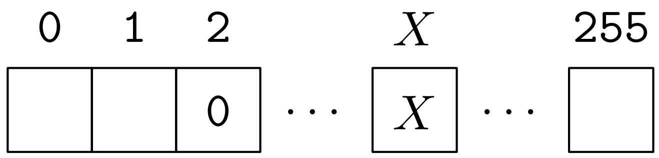

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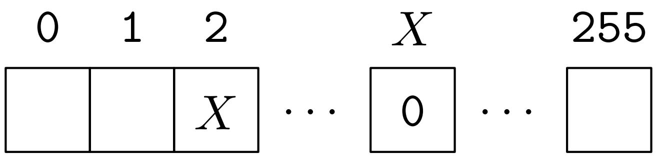

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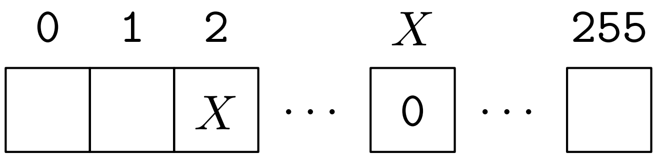

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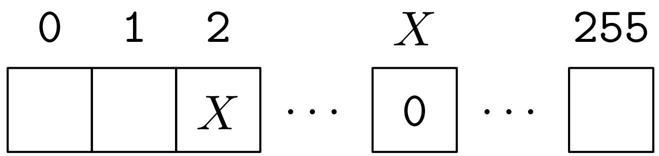

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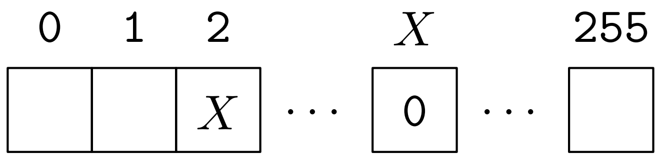

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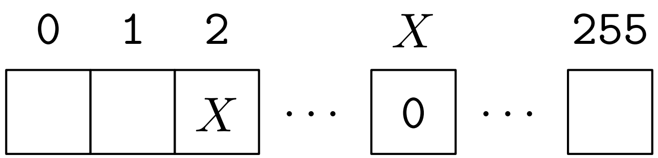

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- With probability $\approx \frac{255}{256} \approx 1$ we have that $S[2]$ is distributed "uniformly at random" after 2 iterations

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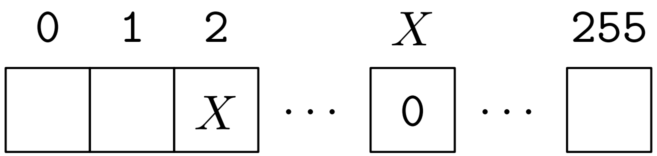

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Probability that the 2nd output byte is 0:

$\approx \frac{1}{256} + 1 \cdot \frac{1}{256} = \frac{2}{256}$

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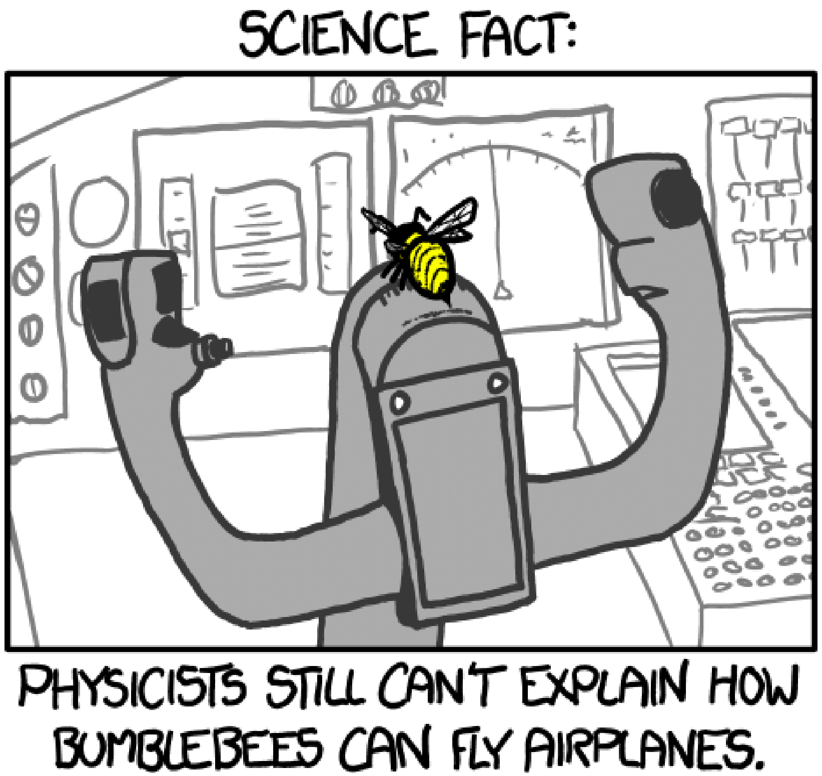
In summary: Do not use RC4!

RC4 and IVs

RC4 is **not** designed to take an IV . . . but programmers don’t know it and use an IV anyway



xkcd.com

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- We show a simplified attack that recovers the first byte of the key (i.e., $k[3]$)

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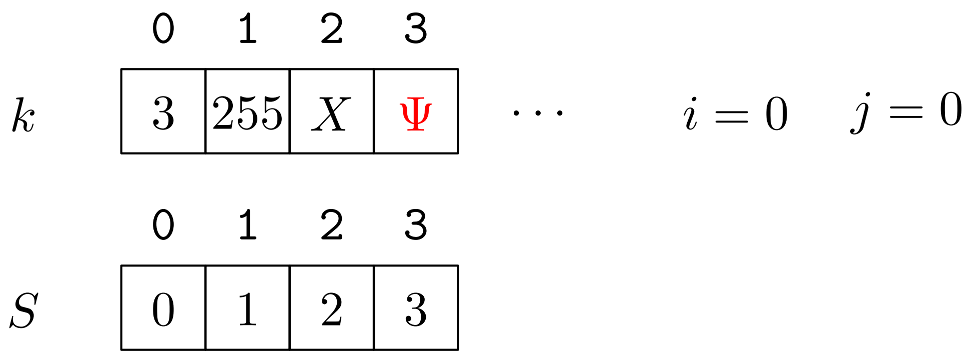

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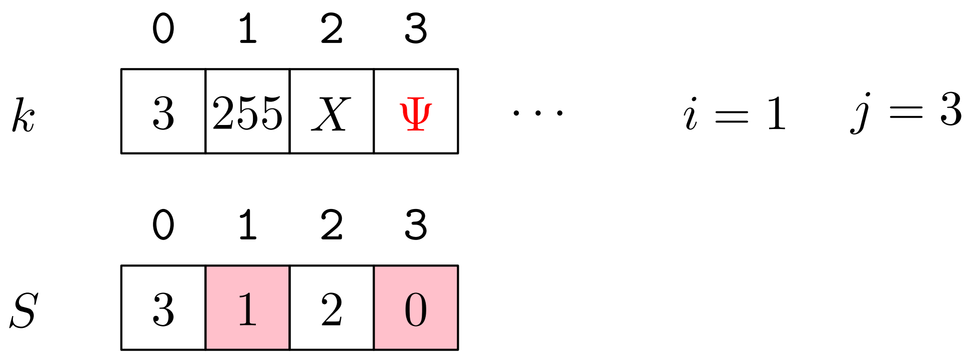

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S	3	0	2	1

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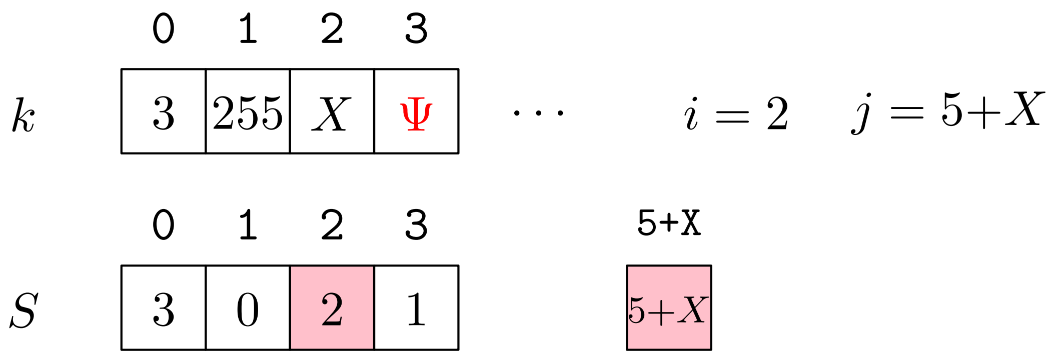

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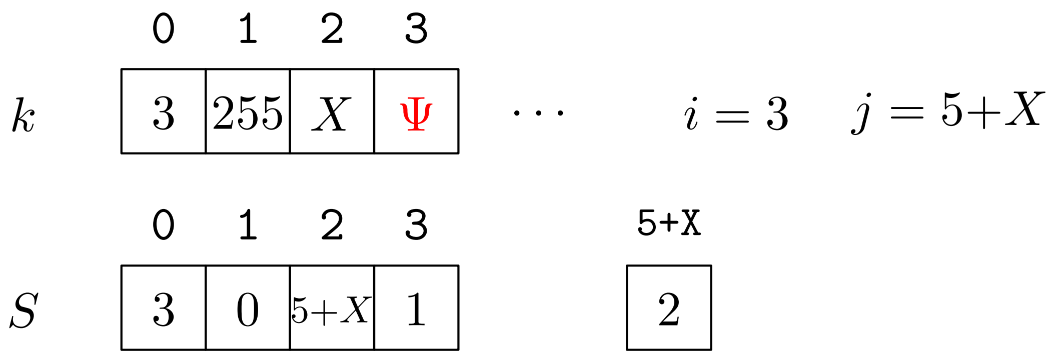

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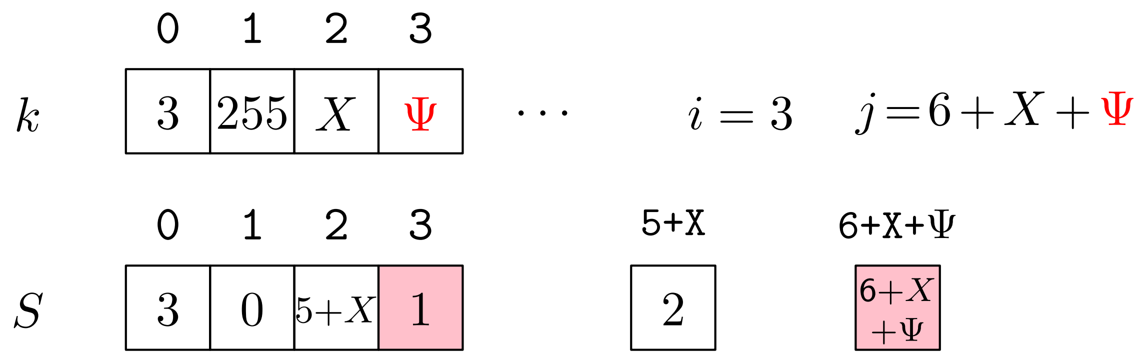

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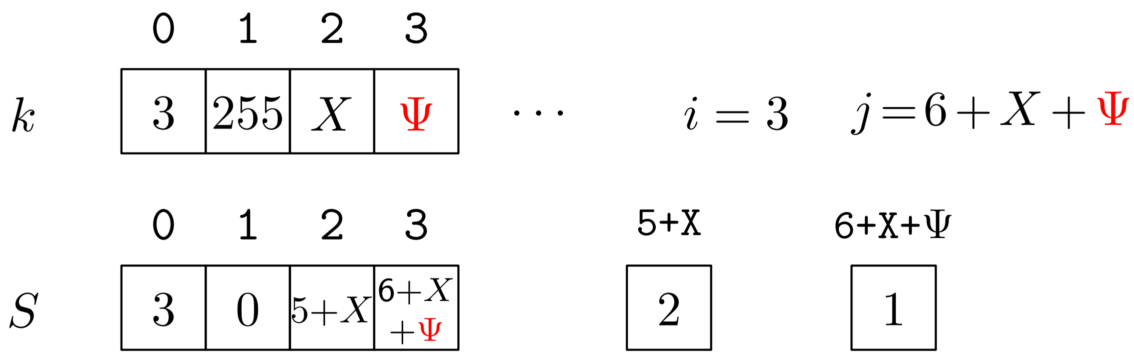

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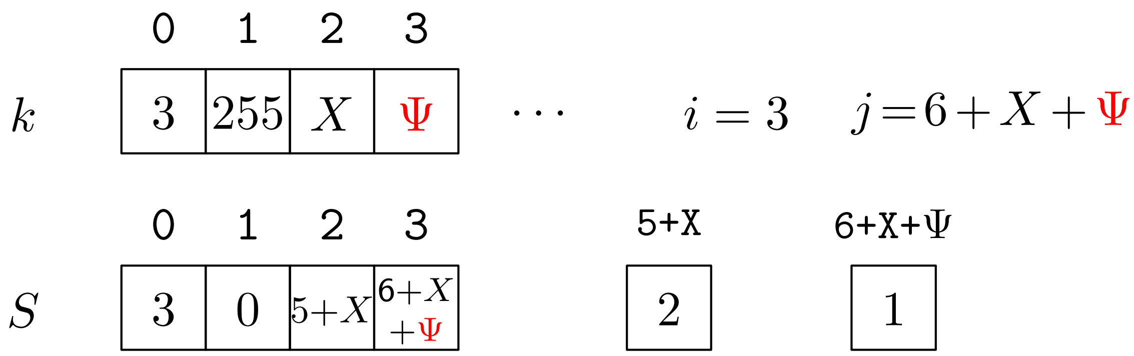

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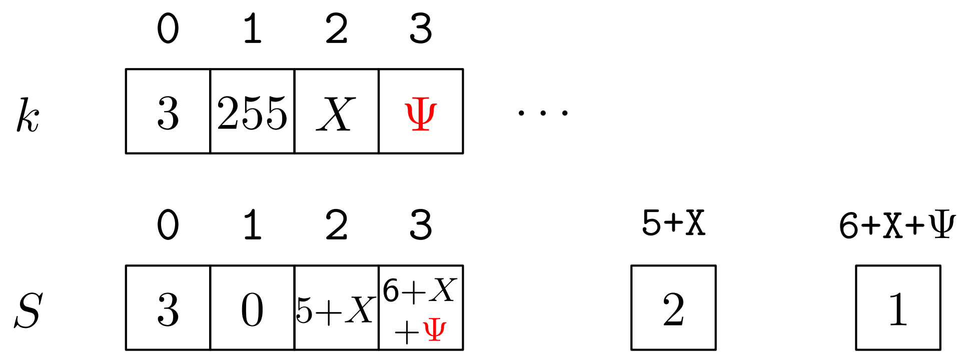

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	0	1	2	3	5+X	6+X+Ψ
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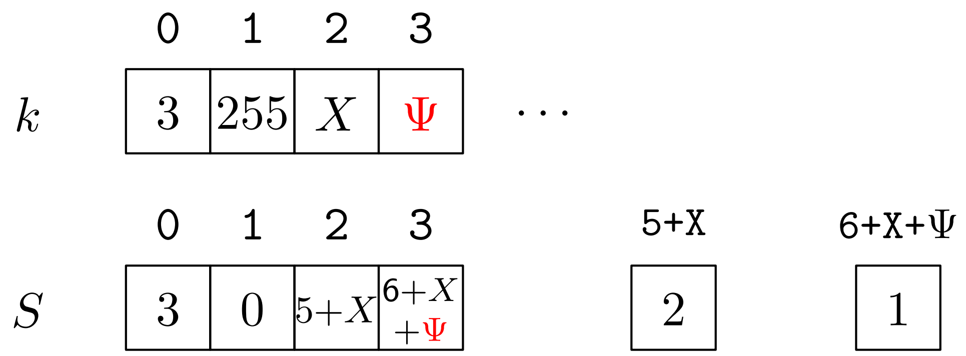

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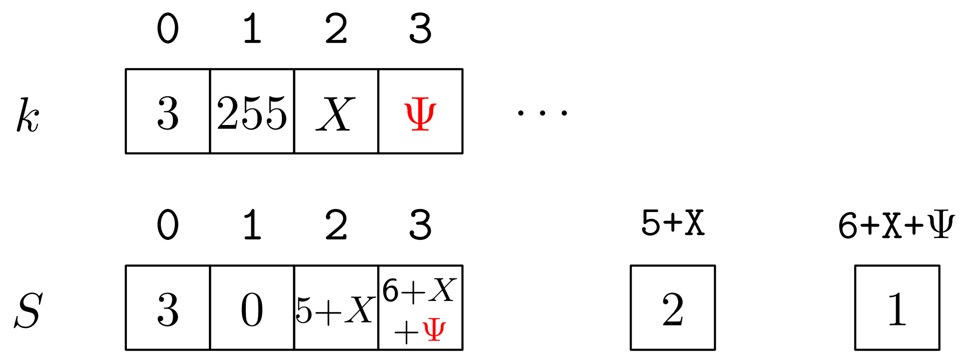

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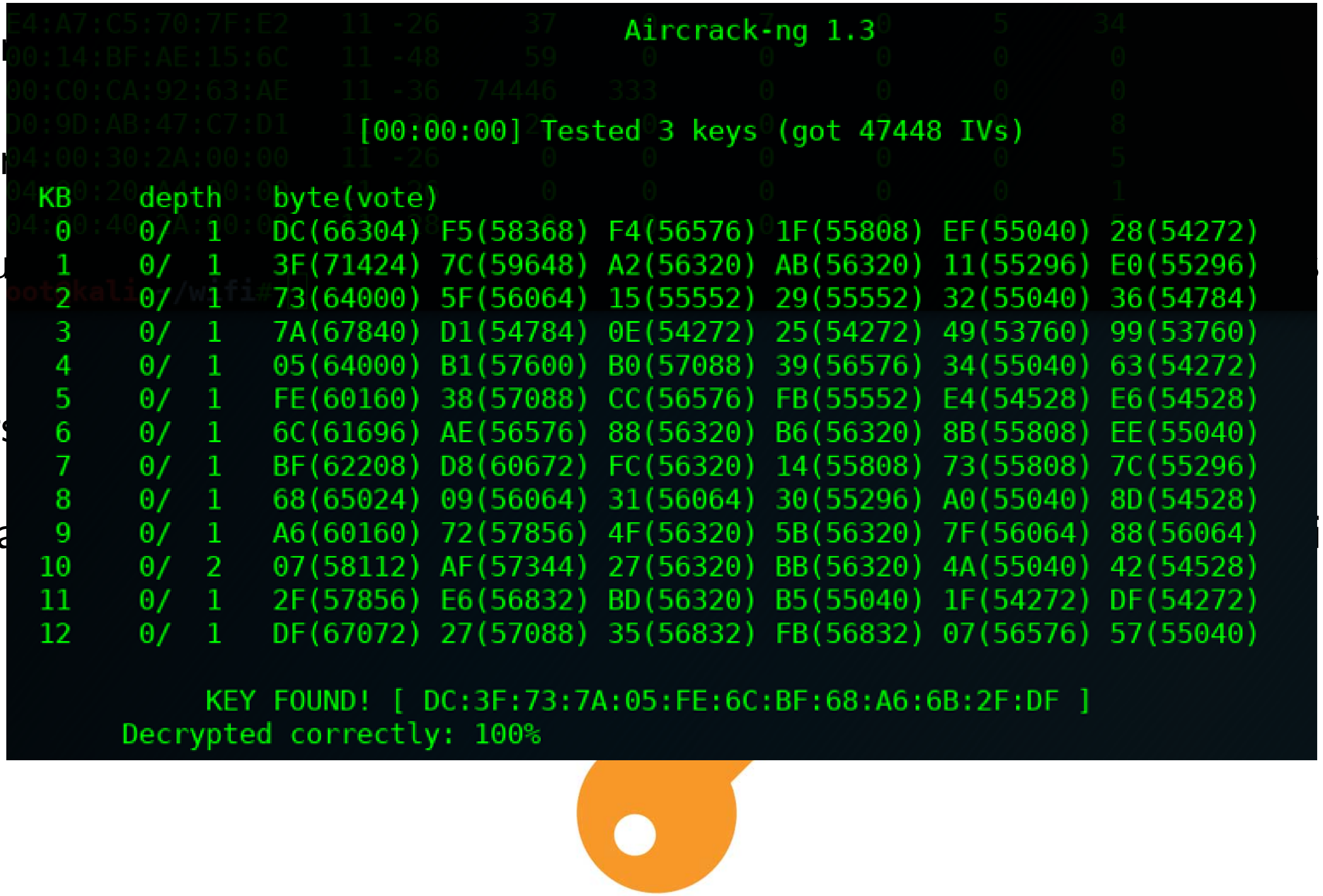

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Takes a 256-bit key k and a 64-bit IV

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The permutation P is used to construct a keyed function with a 256-bit key, 128-bit inputs and 512-bit outputs

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Not patented. Several public domain implementations available

Block Ciphers

A block cipher is. . .

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A block cipher is. . . just another name for a (possibly strong) pseudorandom permutation

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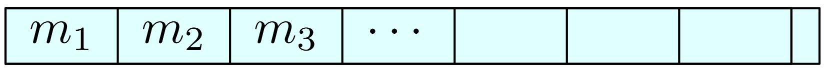

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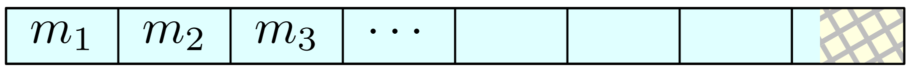
$m =$ 

What if the length of m is not a multiple of n? Padding (with care)

Block Ciphers

Recall that we can always build a stream cipher from a block cipher

For example:

Init(s, IV):

- Output $(s, \text{IV}, 0)$

Next(st):

- Unpack the state in $(s, \text{IV}, \langle i \rangle)$
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$\langle i \rangle$ = Binary encoding of i using $n/4$ bits

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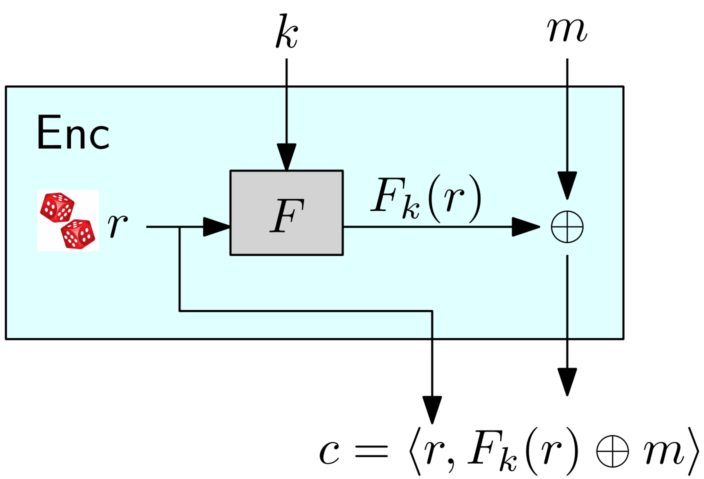

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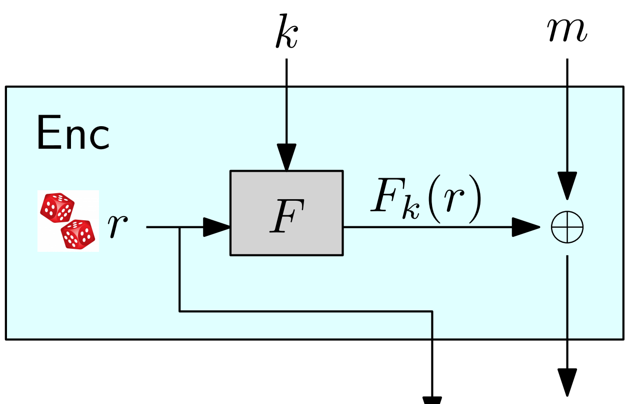

$$c = \langle r, F_k(r) \oplus m \rangle$$

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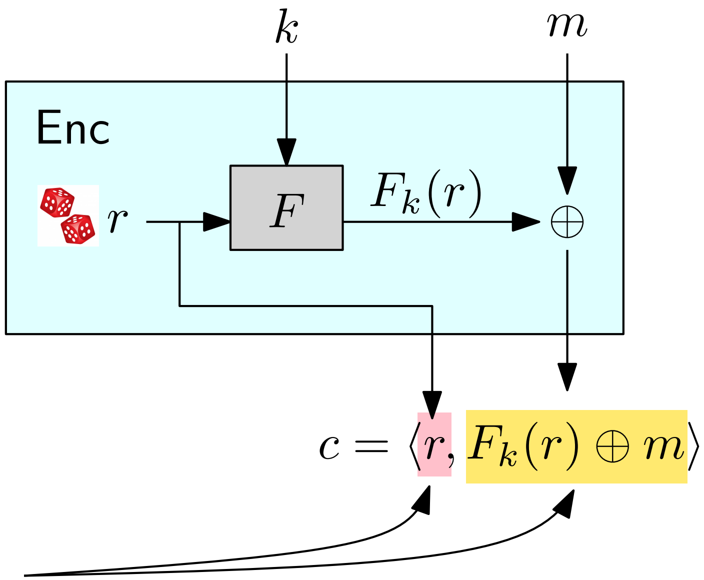

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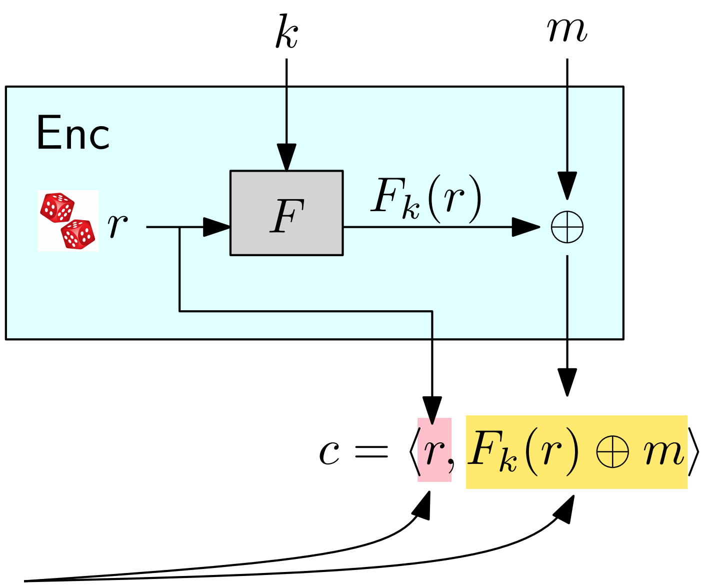

- The ciphertext is (at least) twice as long as the plaintext
- Can we do better? Several options (modes of operations)

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Electronic Code Book (ECB) mode

First idea:

- Encrypt each block of the message independently

$m =$

m_1	m_2	m_3	$\cdots$				

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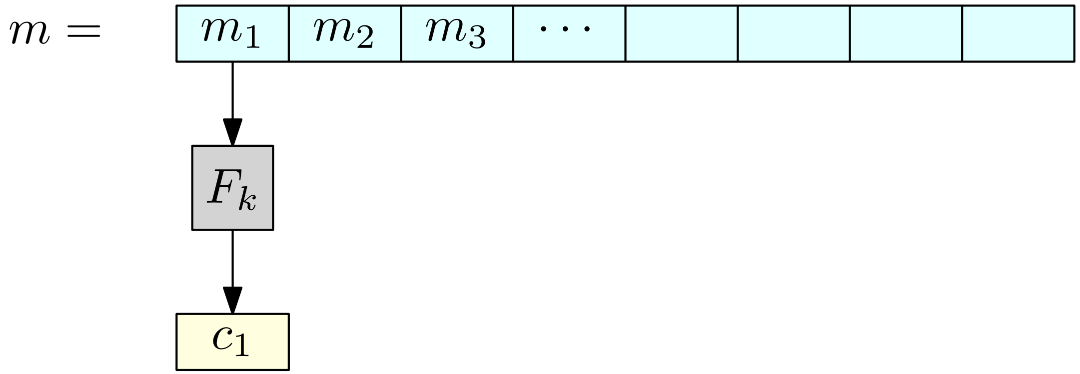

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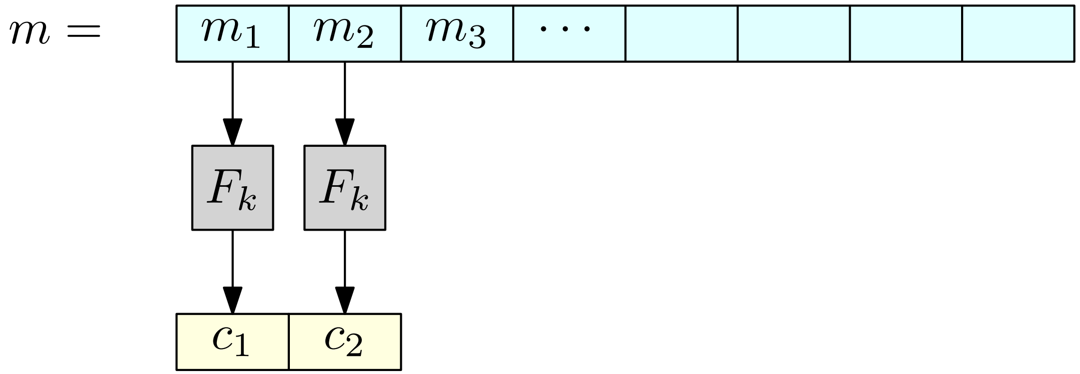

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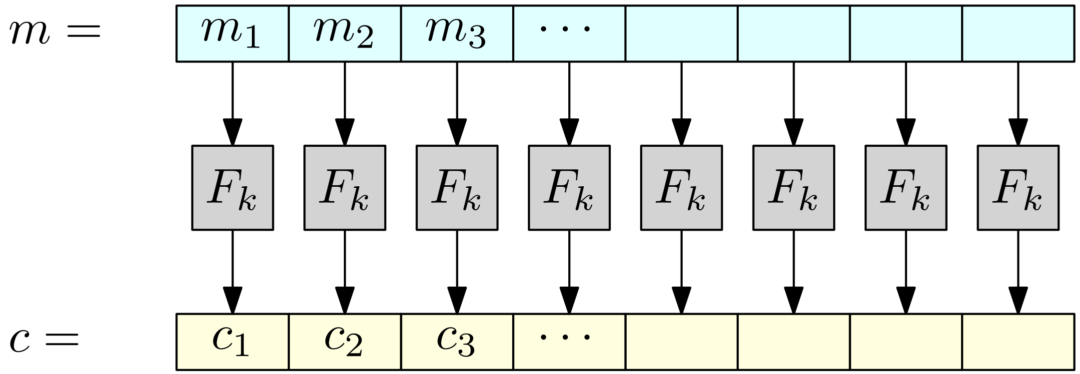

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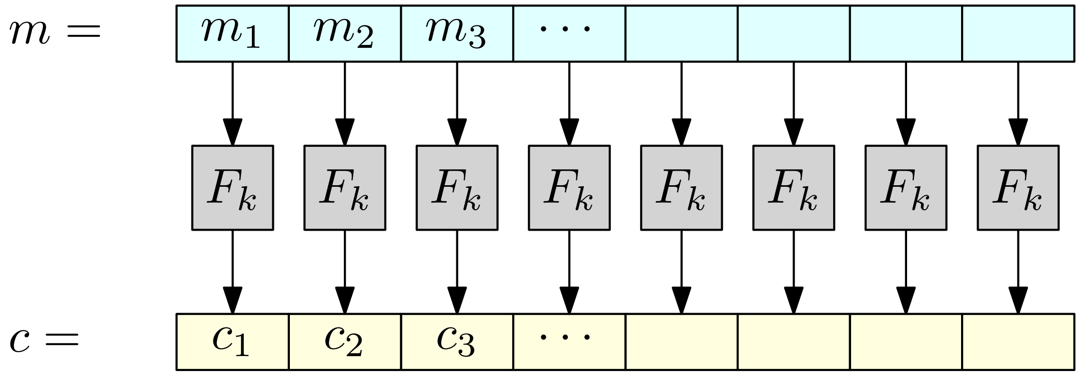

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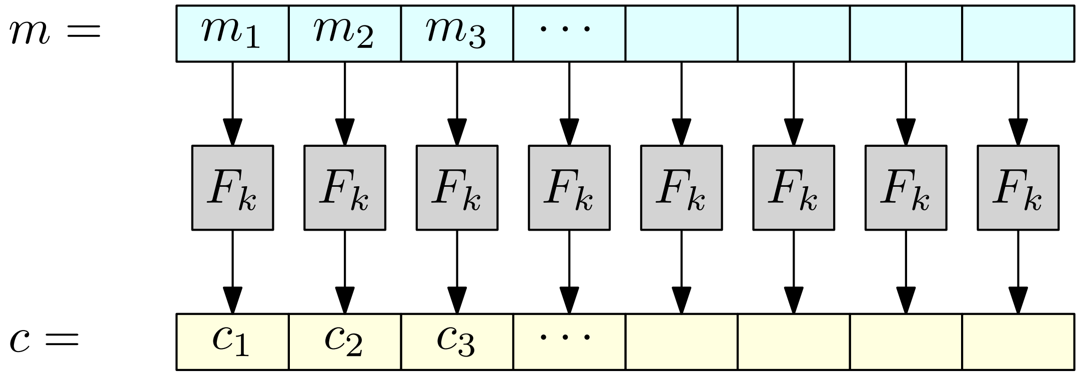

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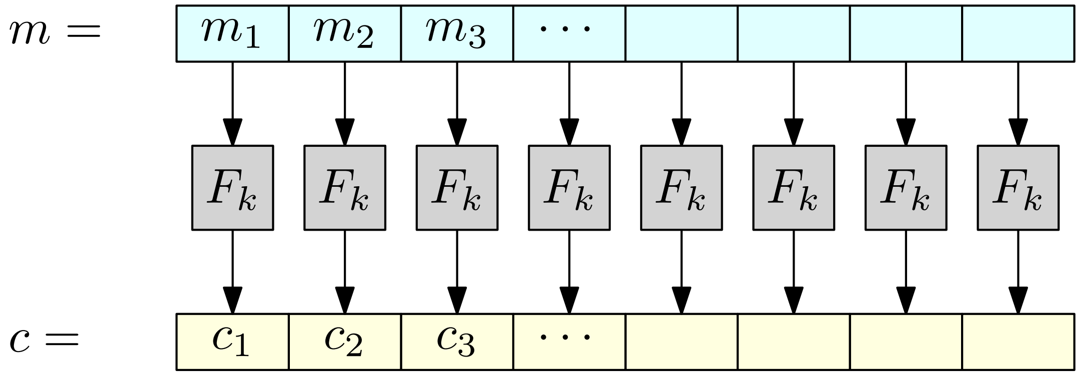

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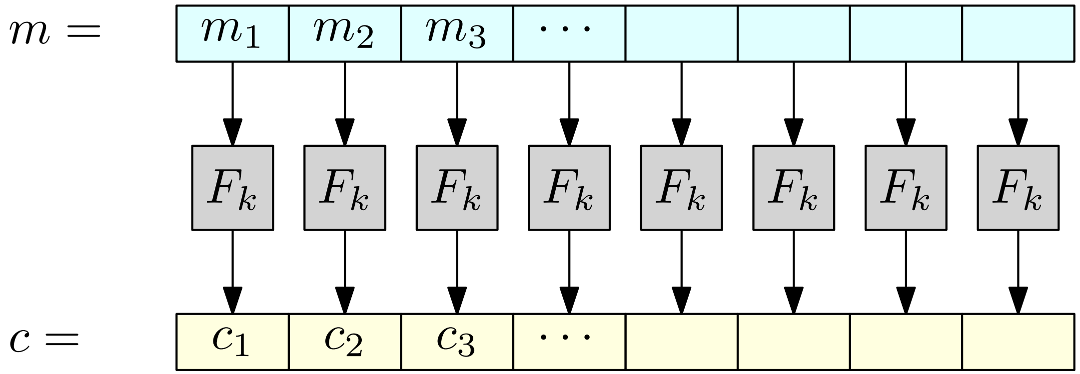

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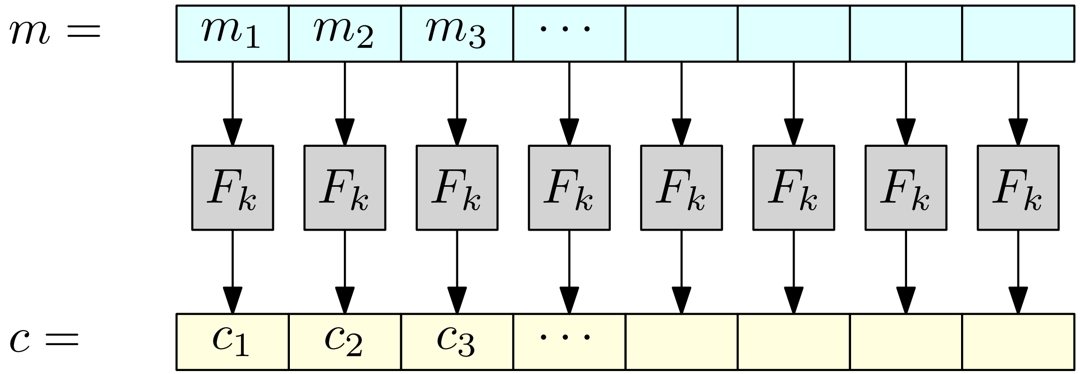

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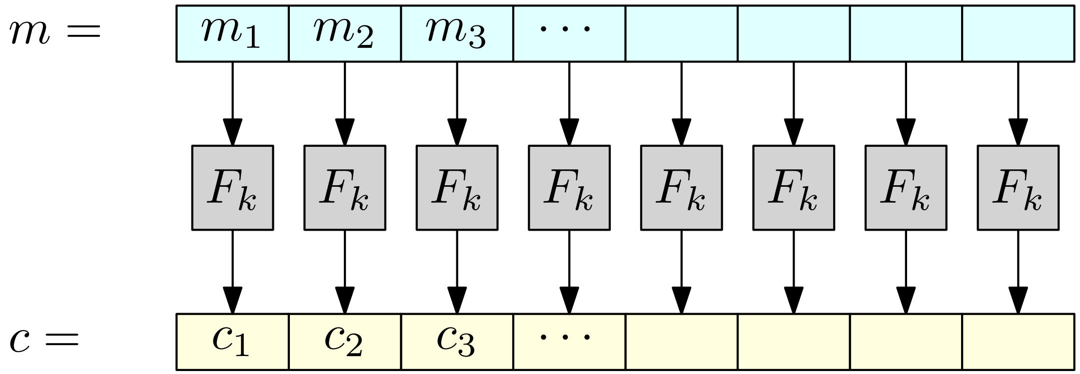

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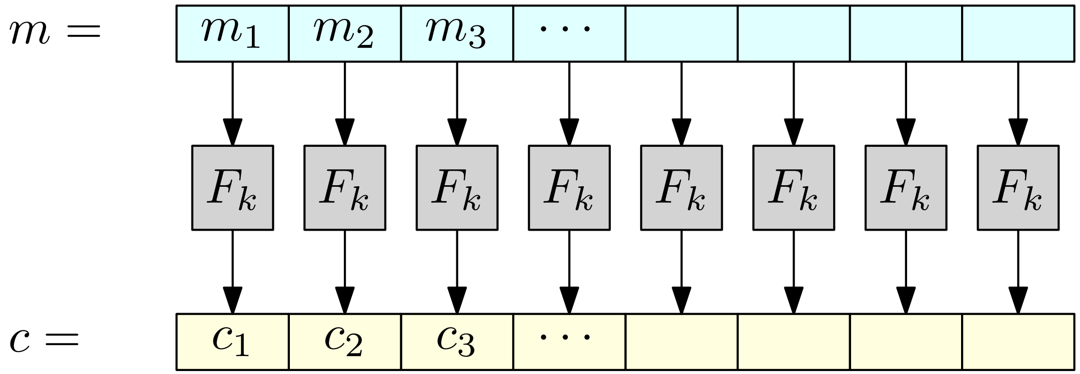

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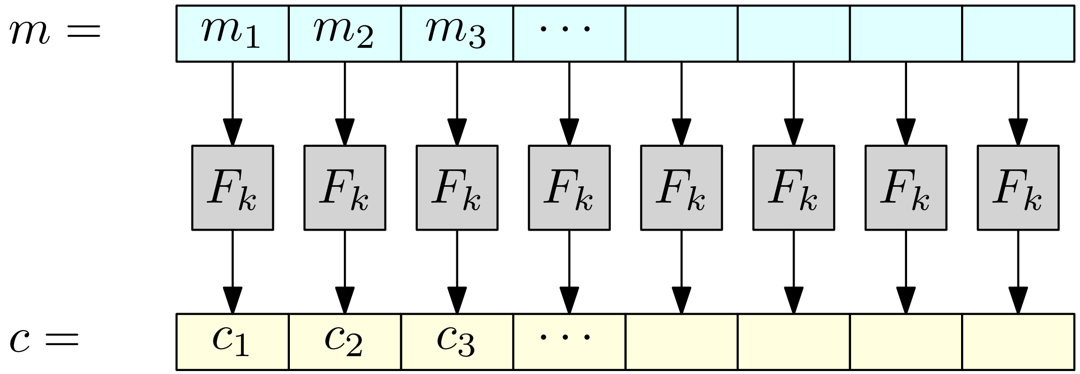

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Never use ECB!

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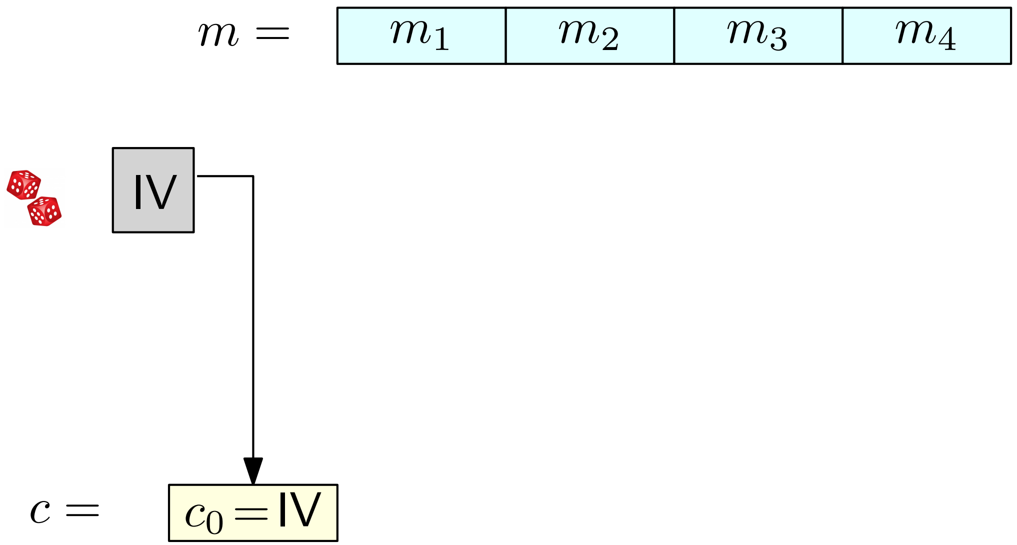
Cipher Block Chaining (CBC) mode



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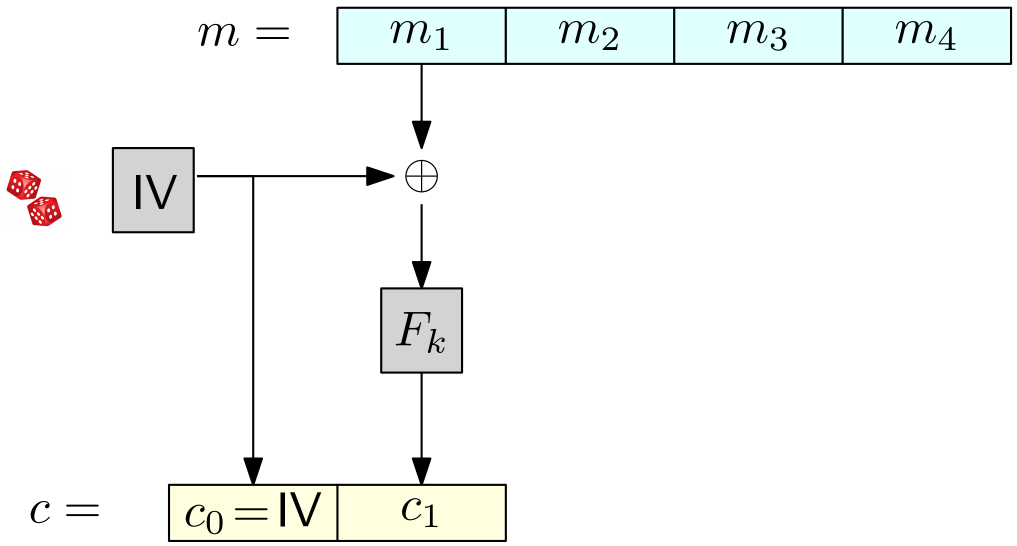

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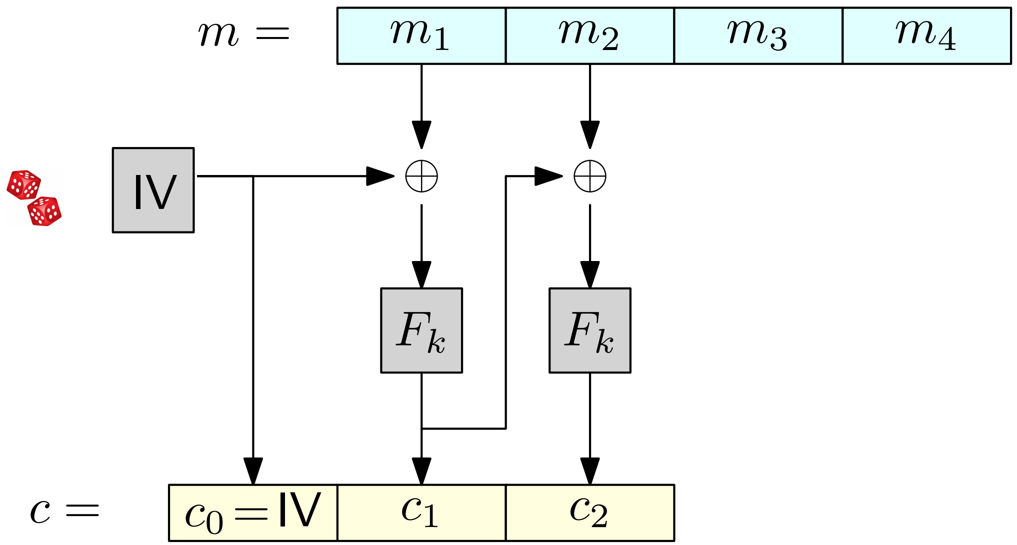

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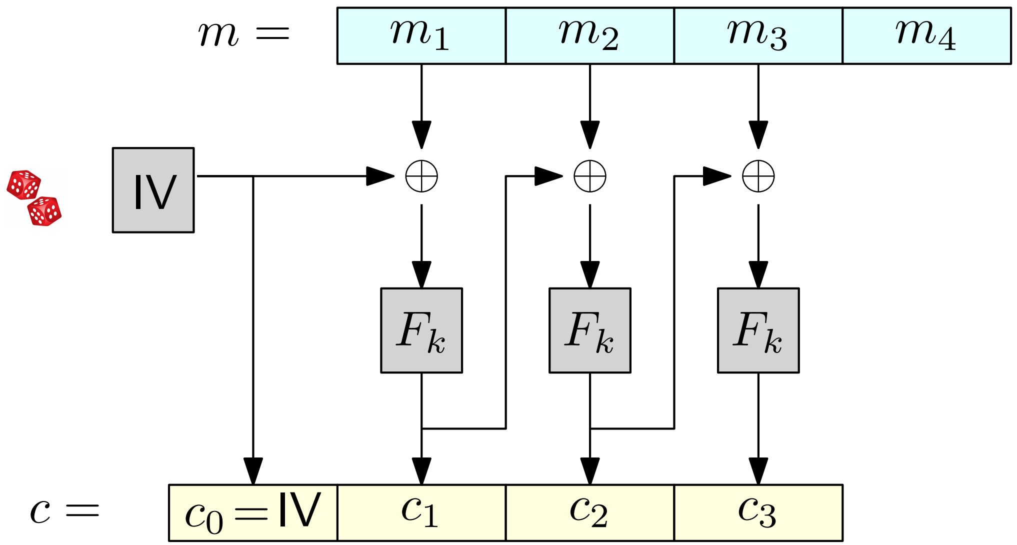

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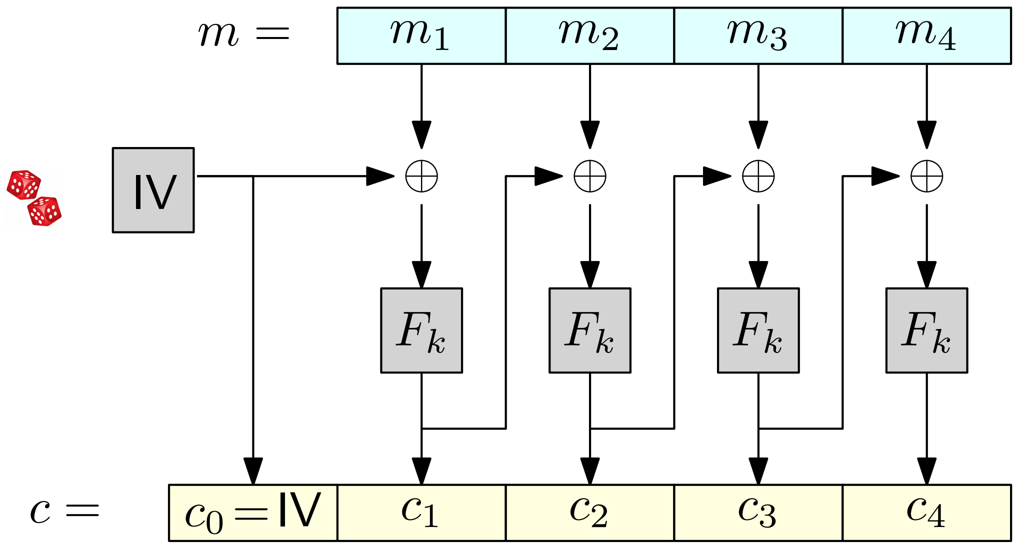

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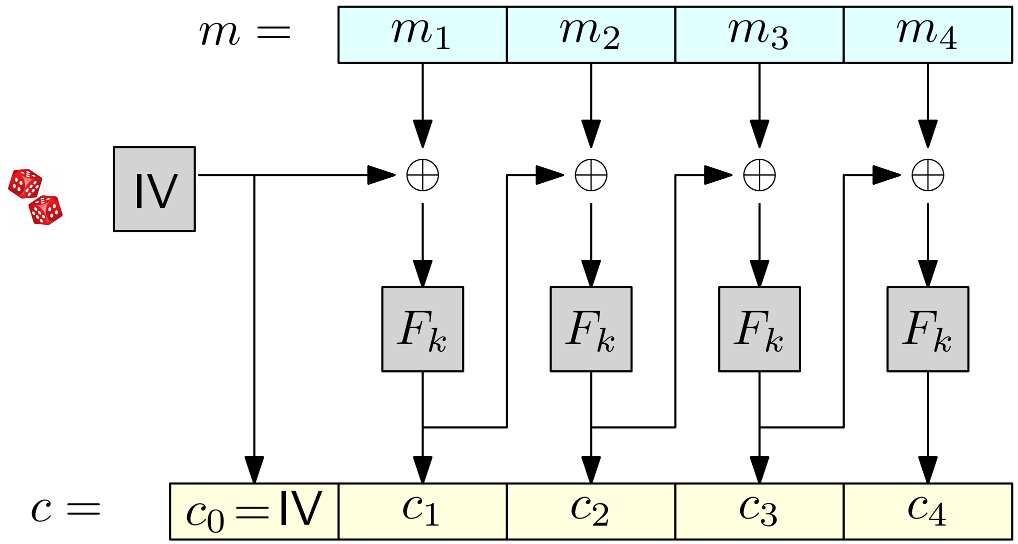

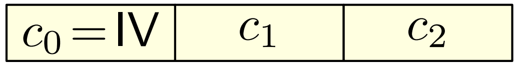

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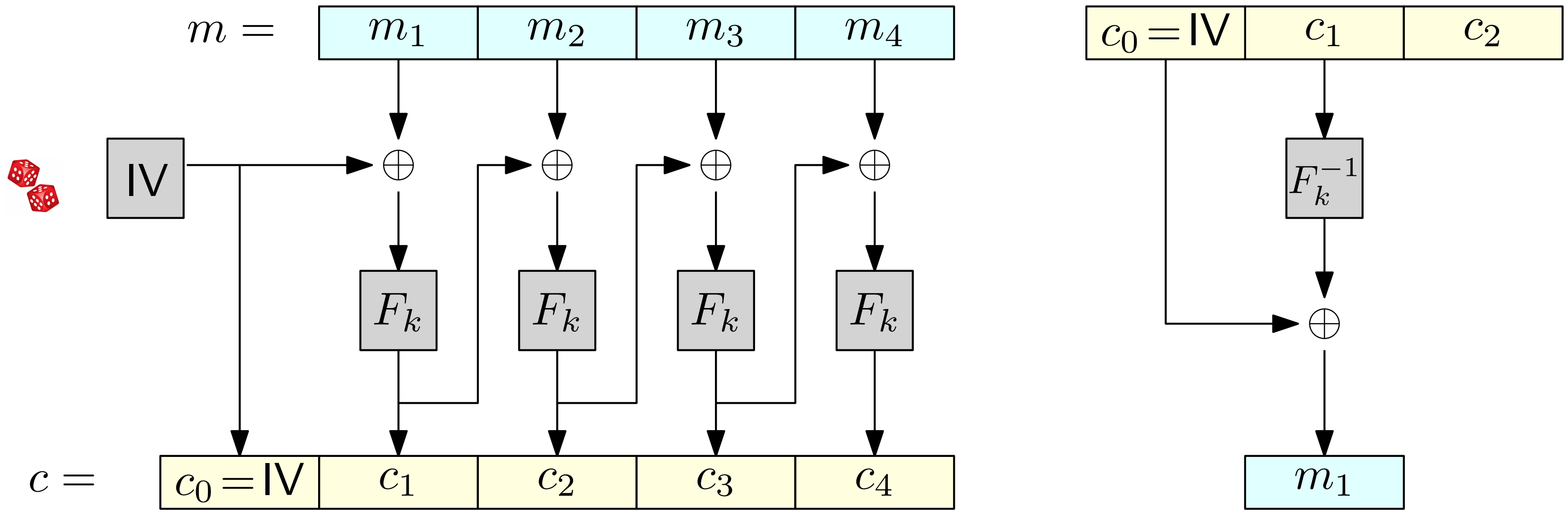

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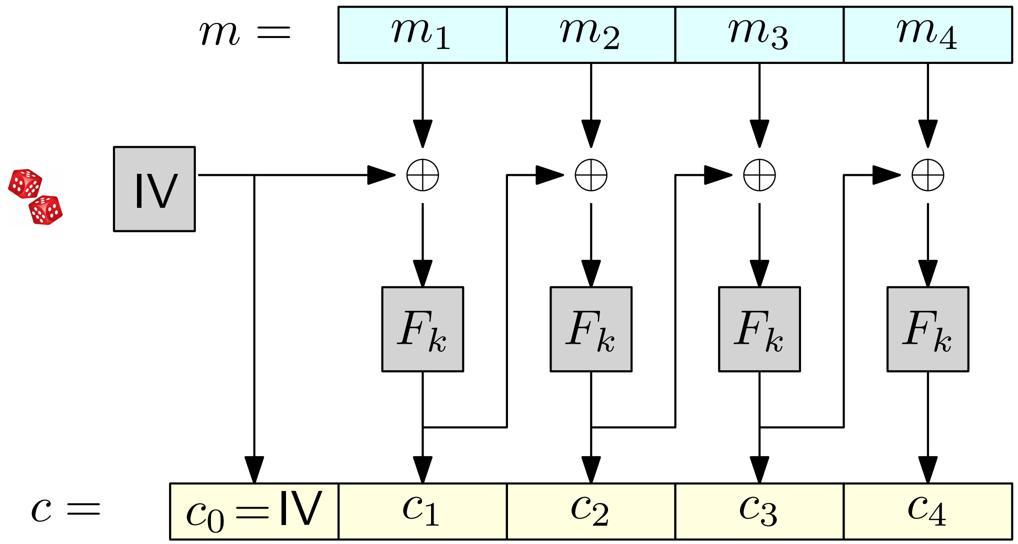

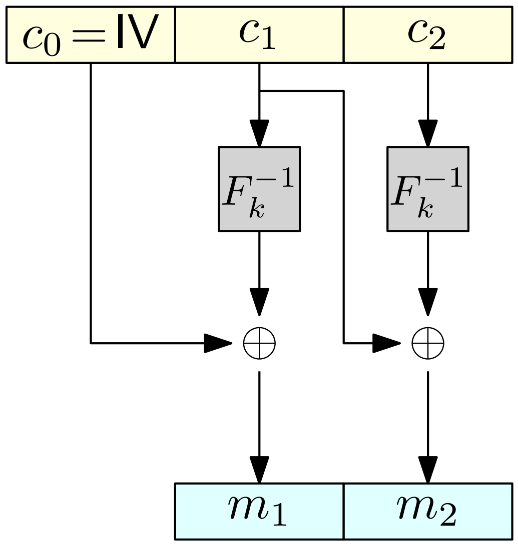

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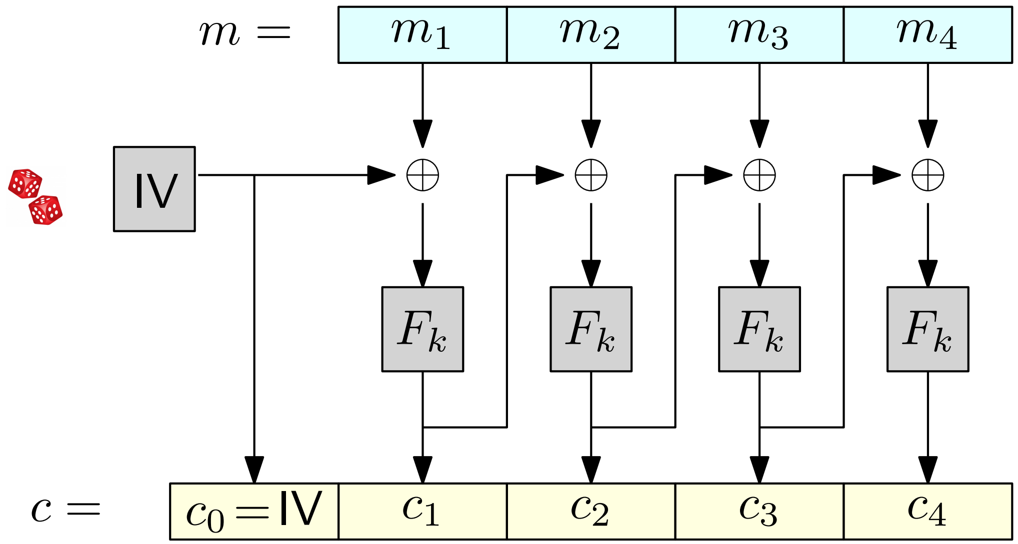

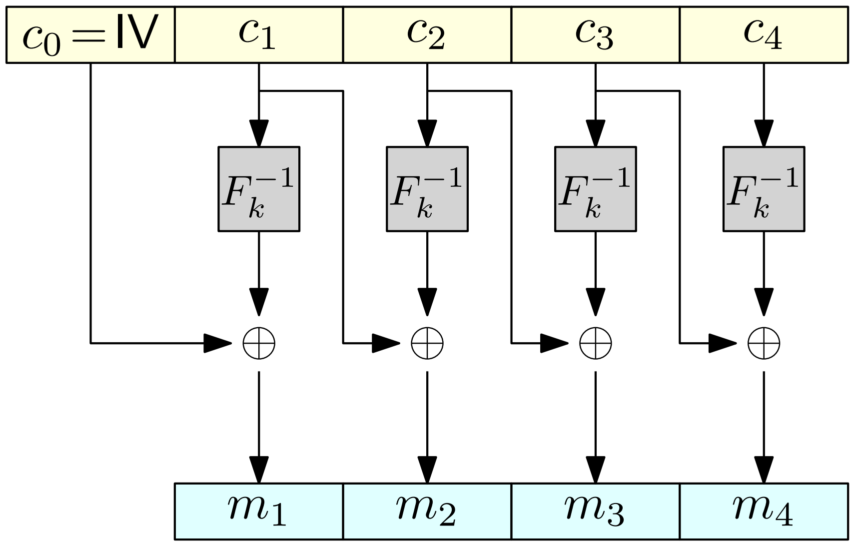

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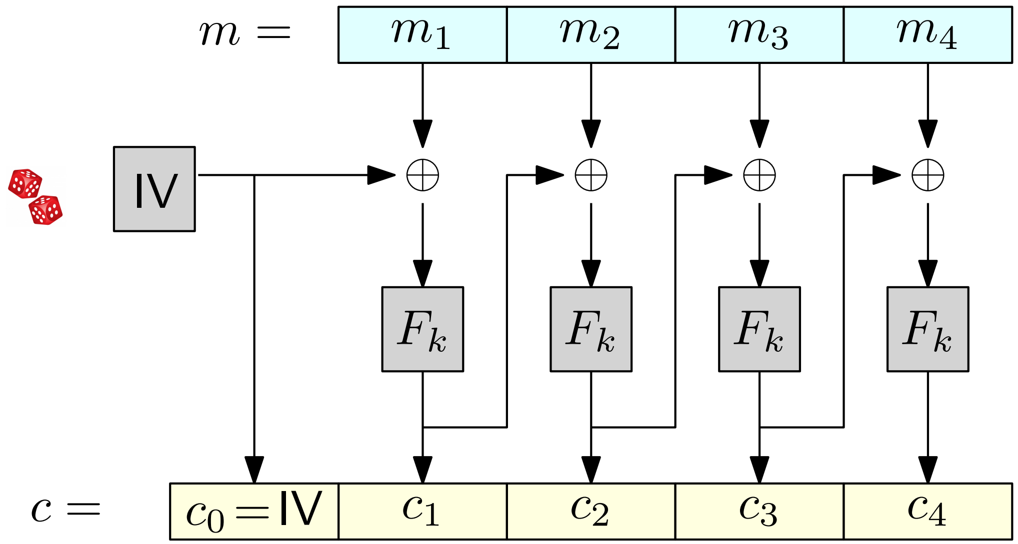

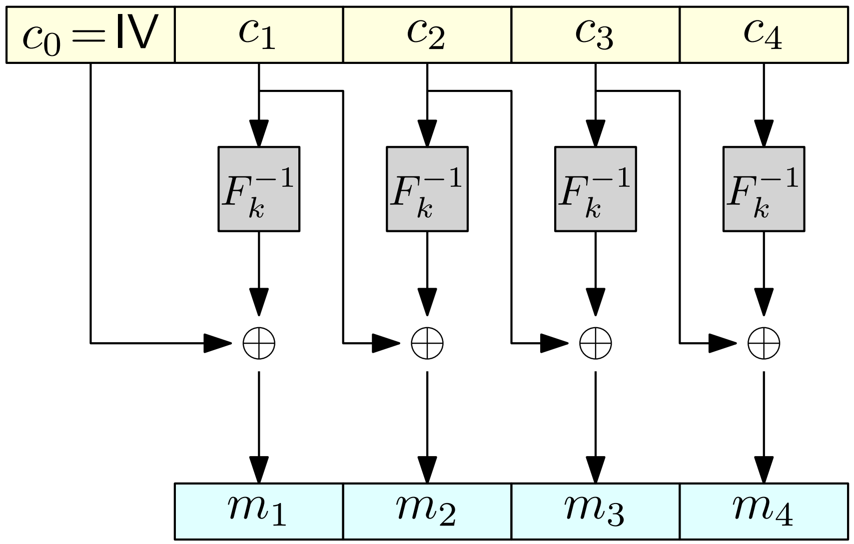

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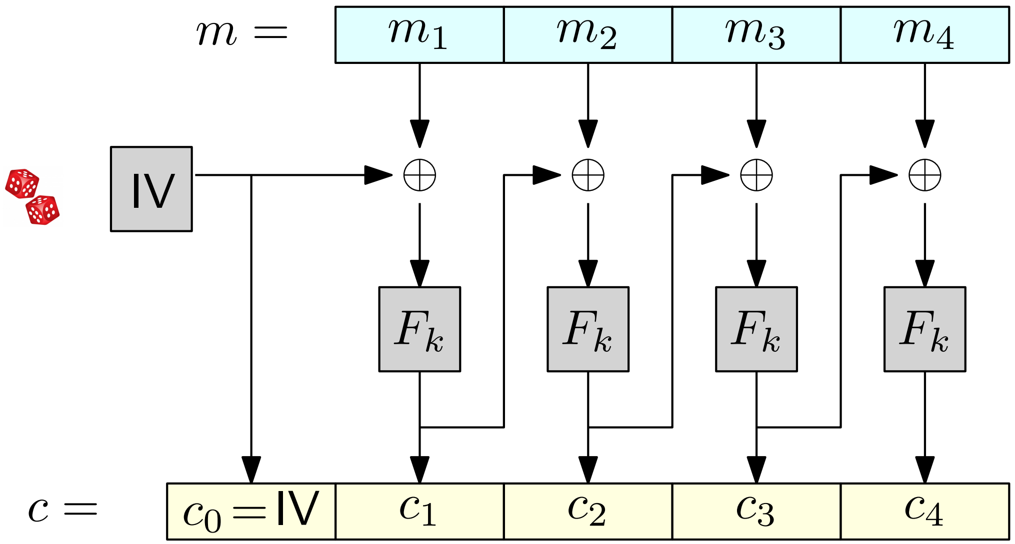

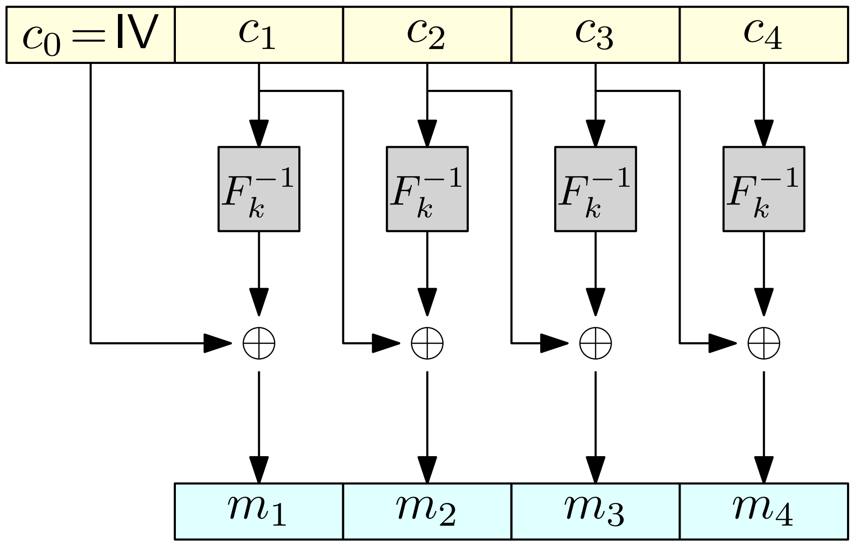

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Cipher Block Chaining (CBC) mode

Is CBC mode CPA secure?

Cipher Block Chaining (CBC) mode

Is CBC mode CPA secure? Yes!*

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Theorem: If F is a pseudorandom permutation, then CBC mode is CPA-secure.

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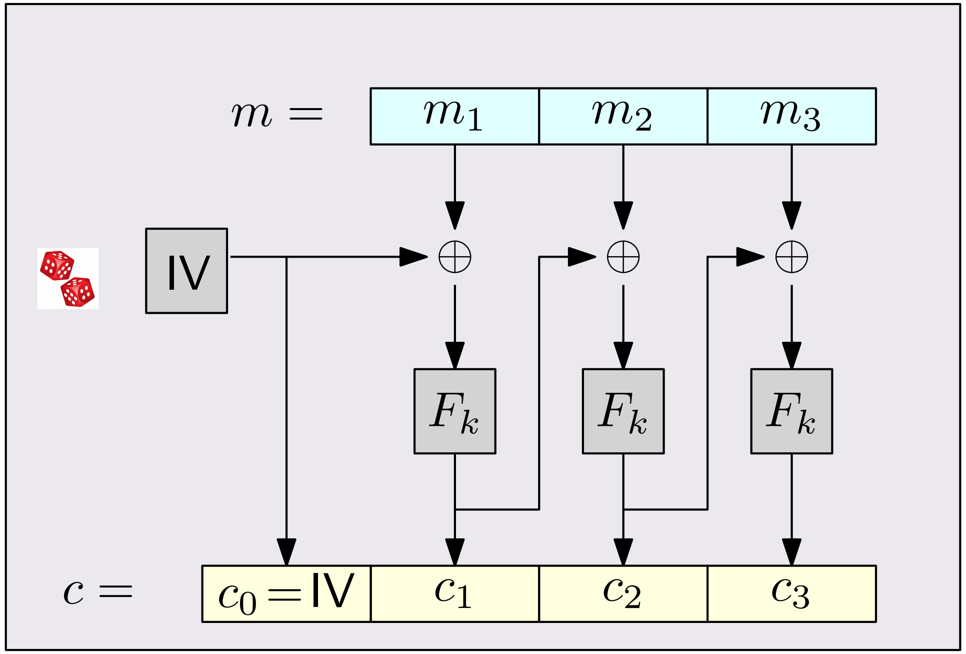
Theorem: If F is a pseudorandom permutation, then CBC mode is CPA-secure.

*But, depending on the implementation, it might be vulnerable to some subtle attacks
(not really a fault of the encryption scheme, but something to be aware of)

Chained CBC mode

There is a stateful variant of CBC called **chained CBC** that handles multiple messages as follows:

- When the first message is encrypted a random IV is chosen (like in CBC mode)



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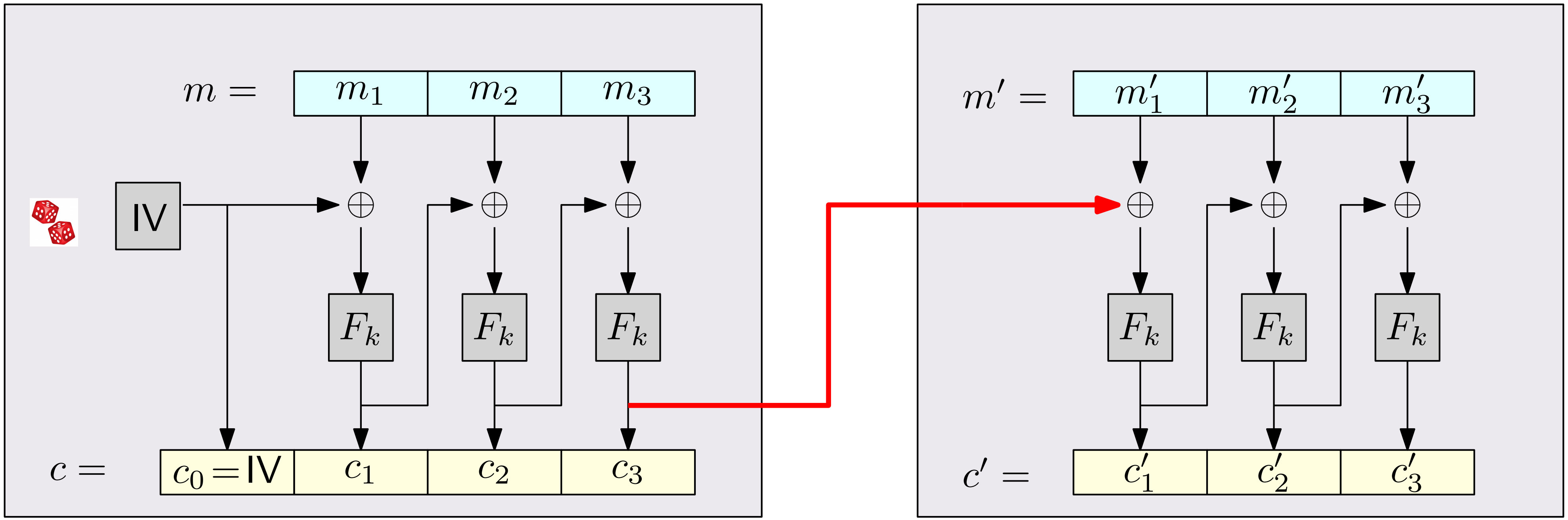
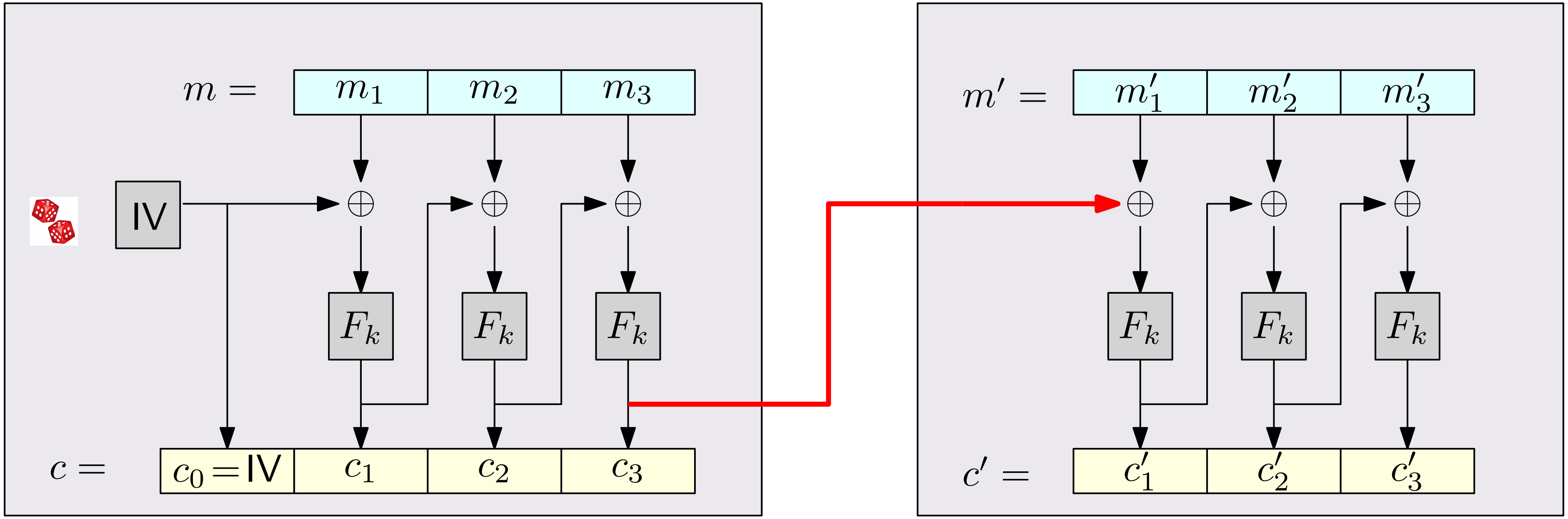


Security of Chained CBC mode



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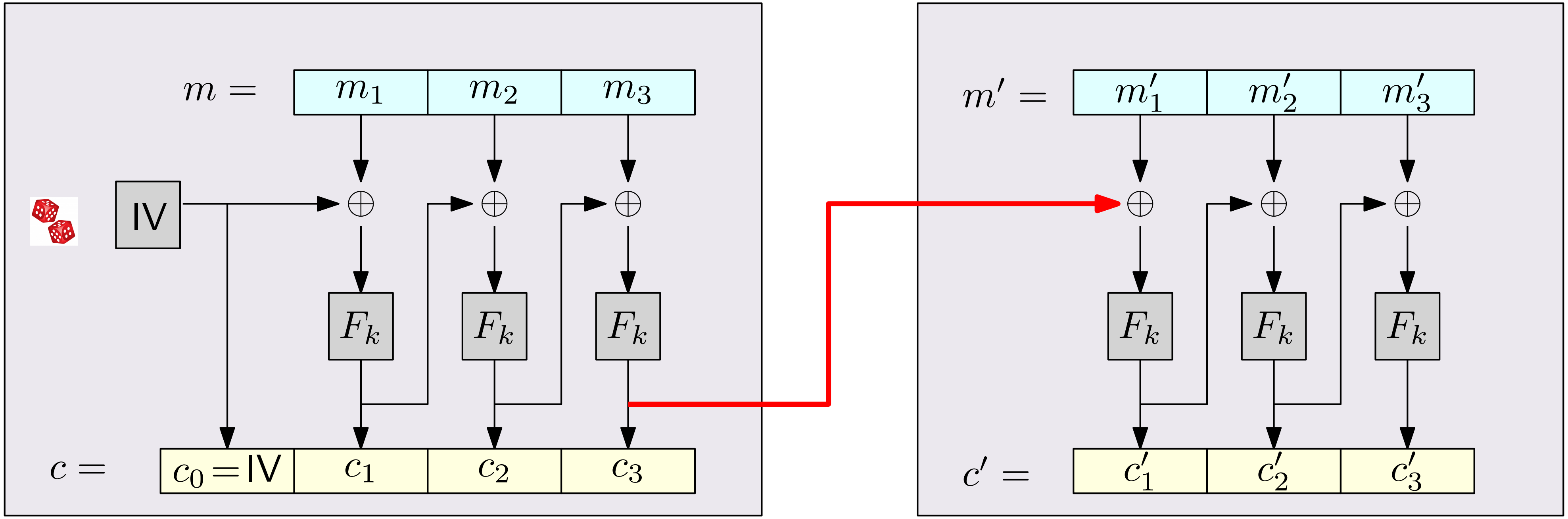

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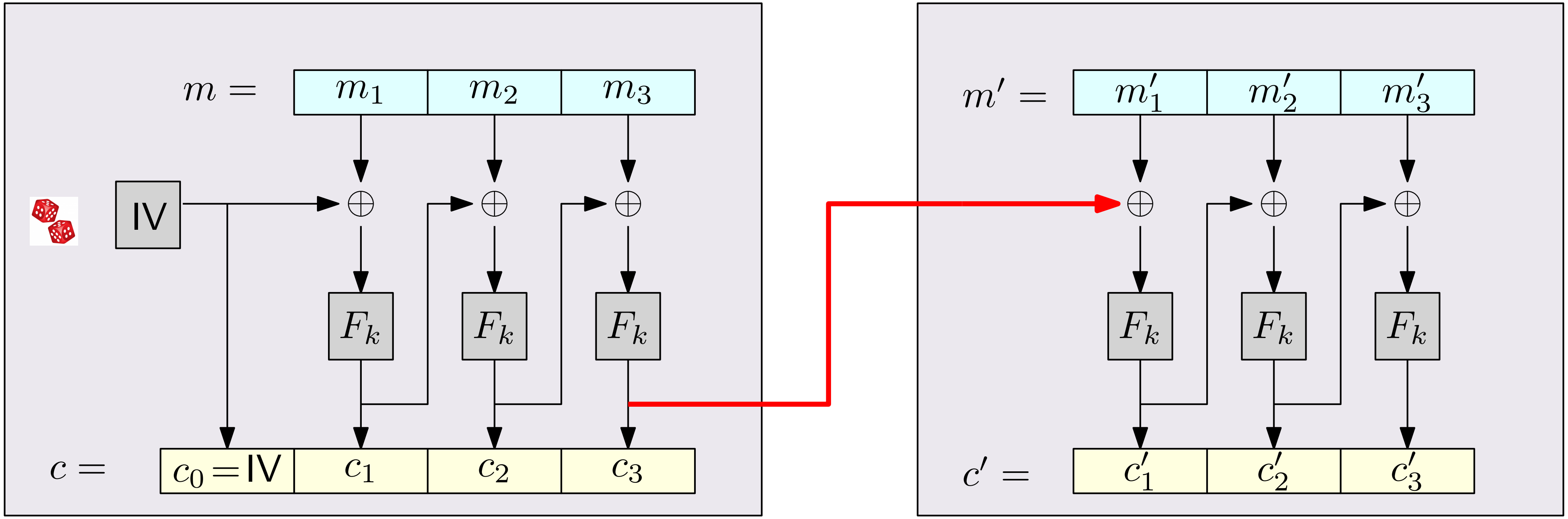

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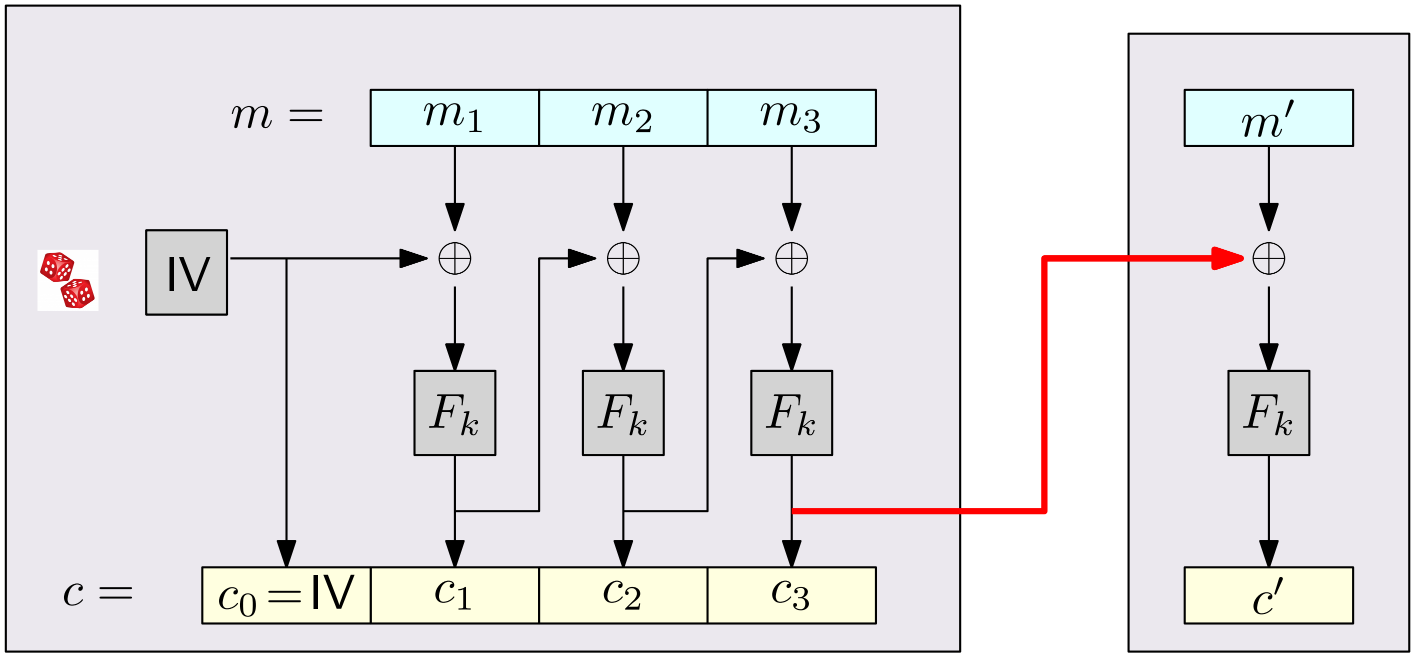

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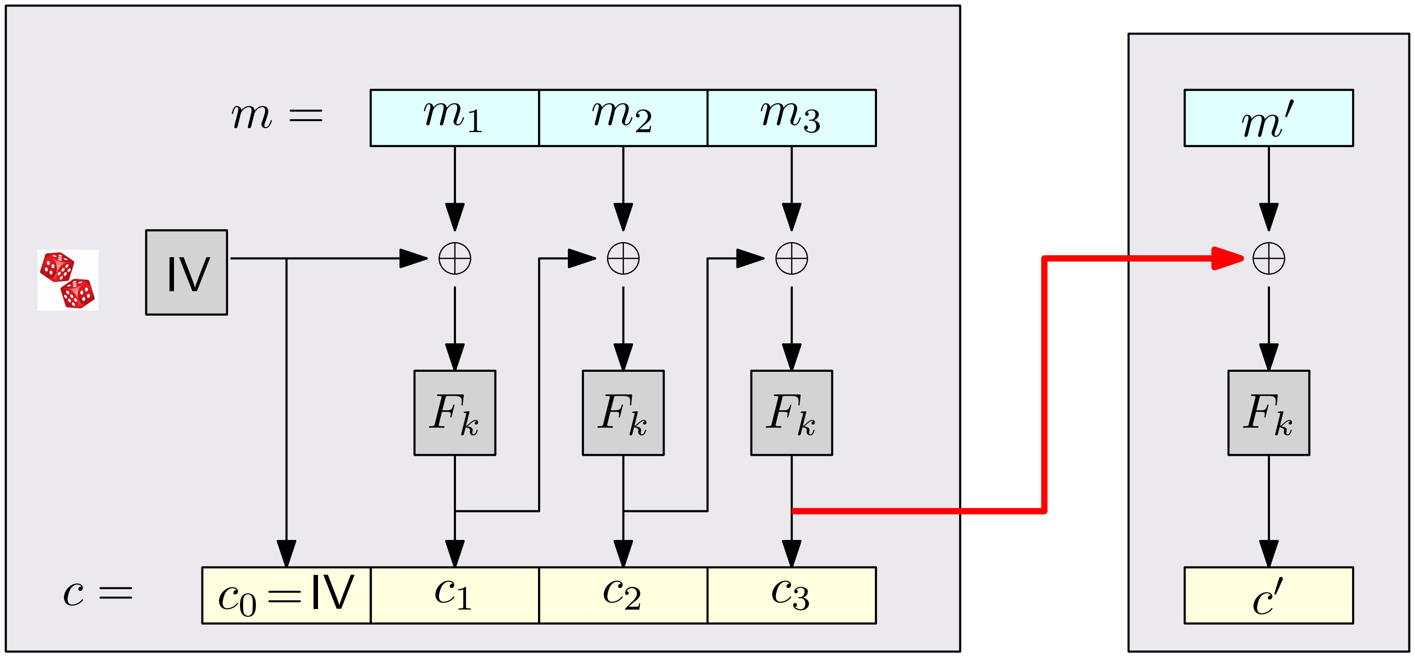
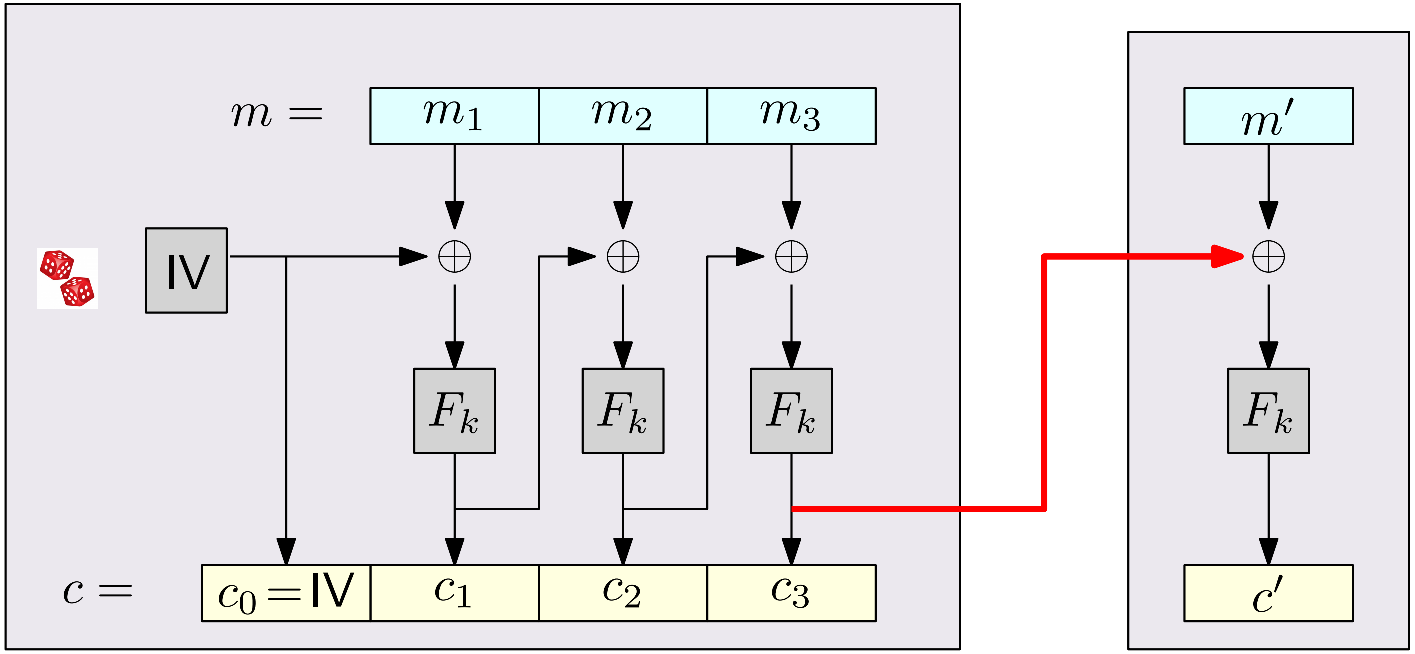


Suppose that the adversary observes c and knows that m_1 is either x or y (e.g., $x = \texttt{ATTACK!}$ and $y = \texttt{RETREAT}$)

The adversary convinces Alice to encrypt $m' = c_0 \oplus x \oplus c_3$

Security of Chained CBC mode



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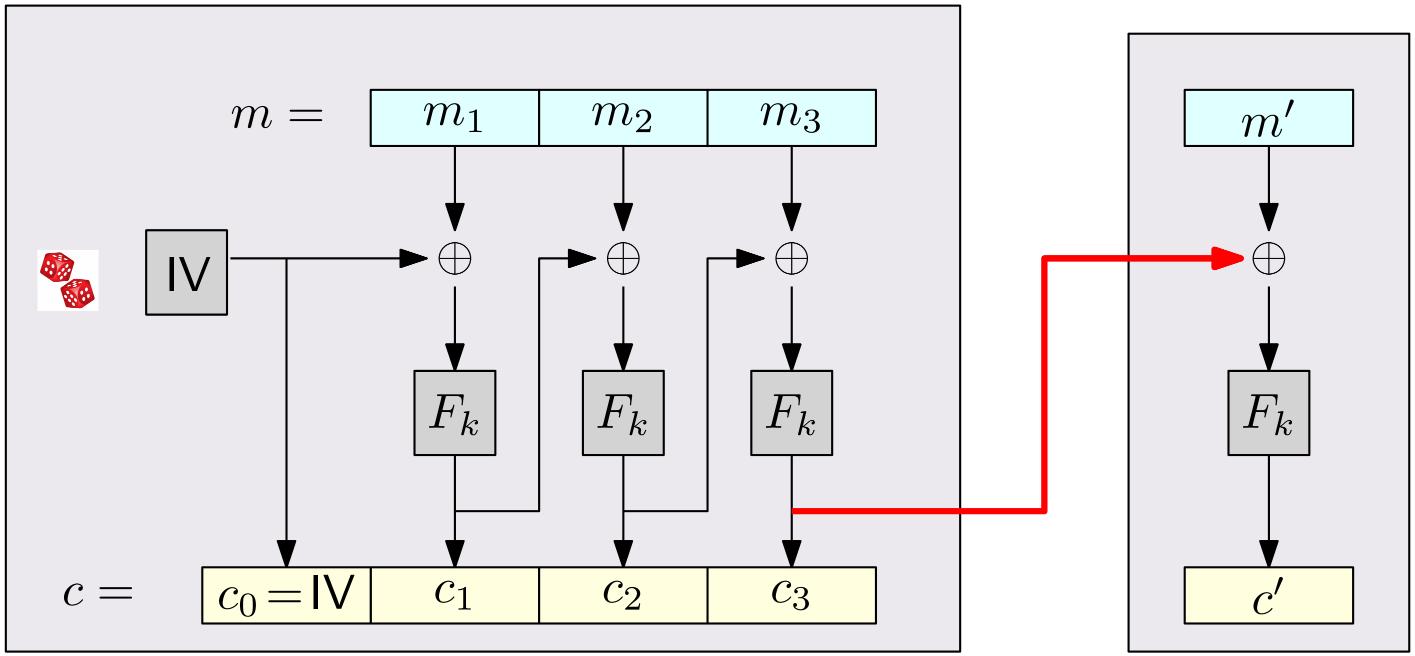

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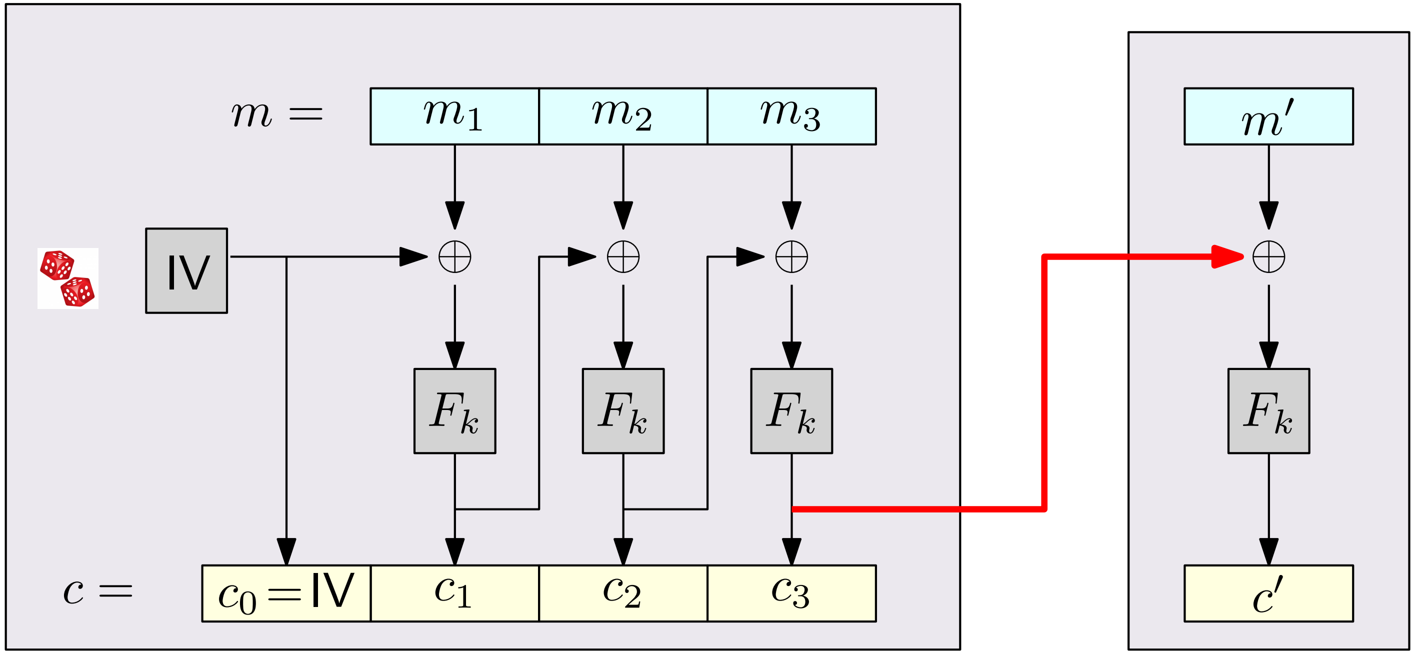

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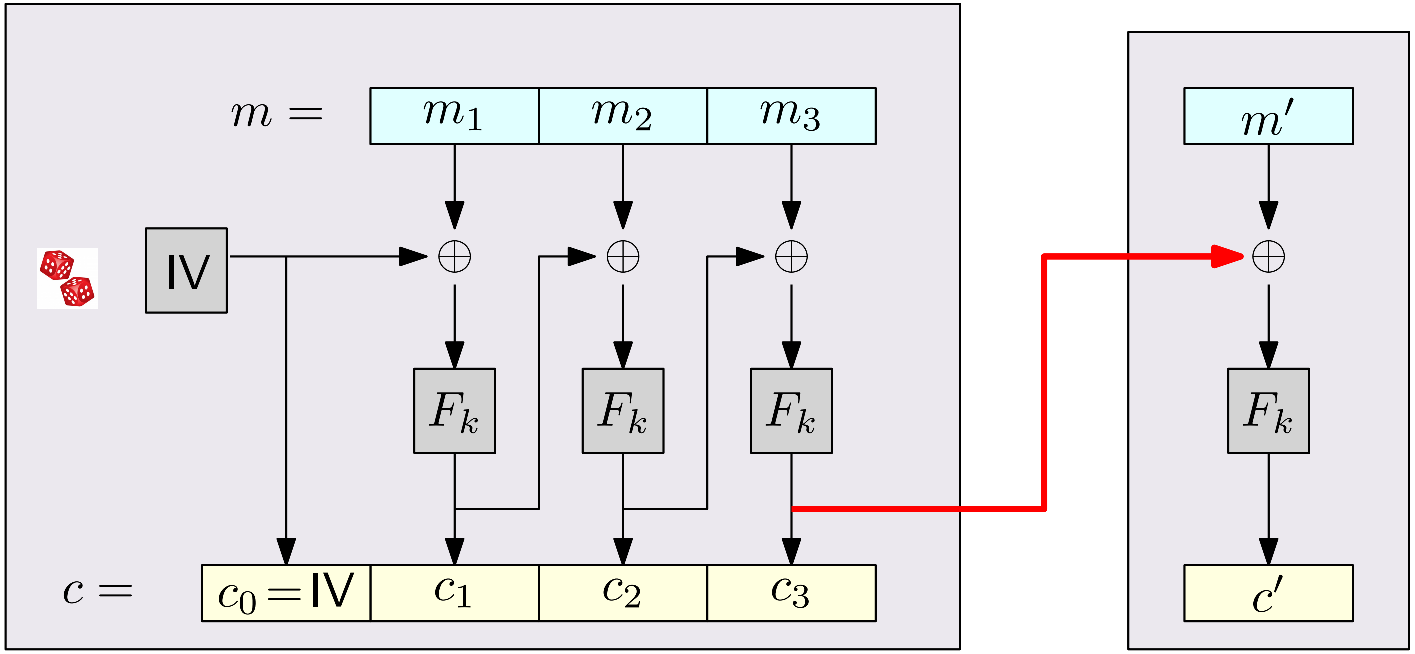

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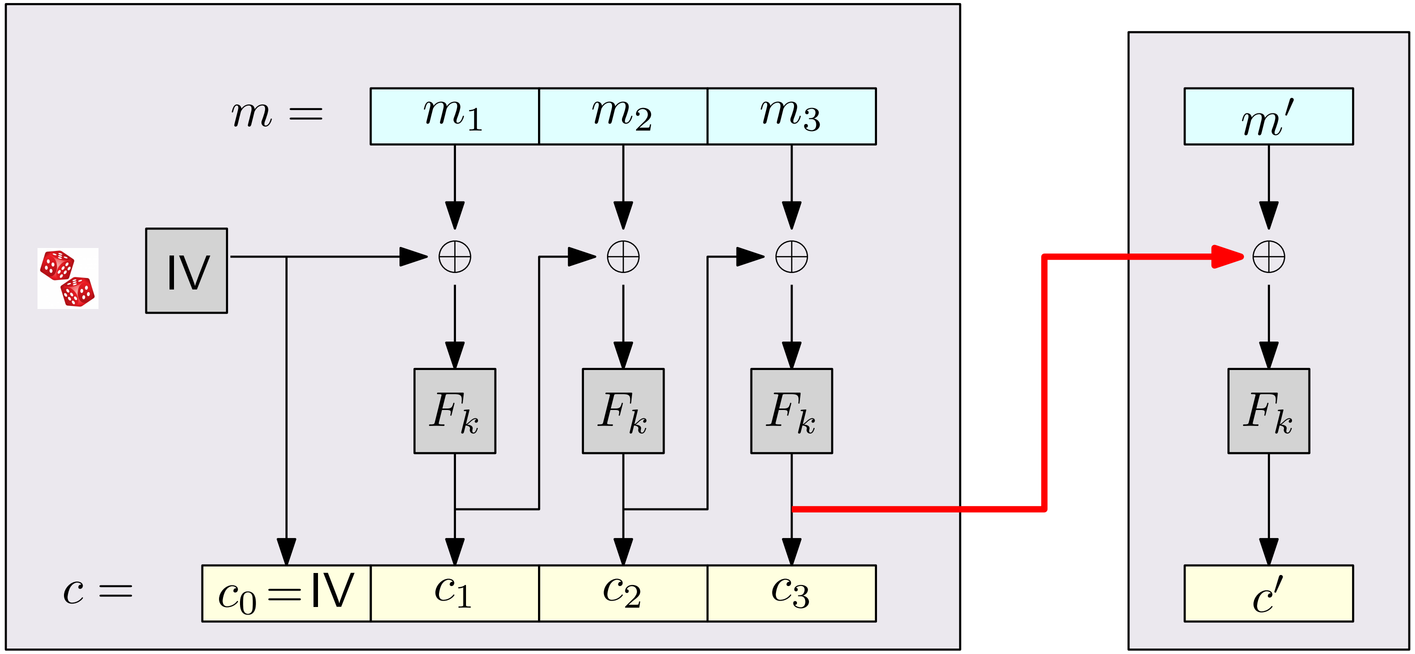

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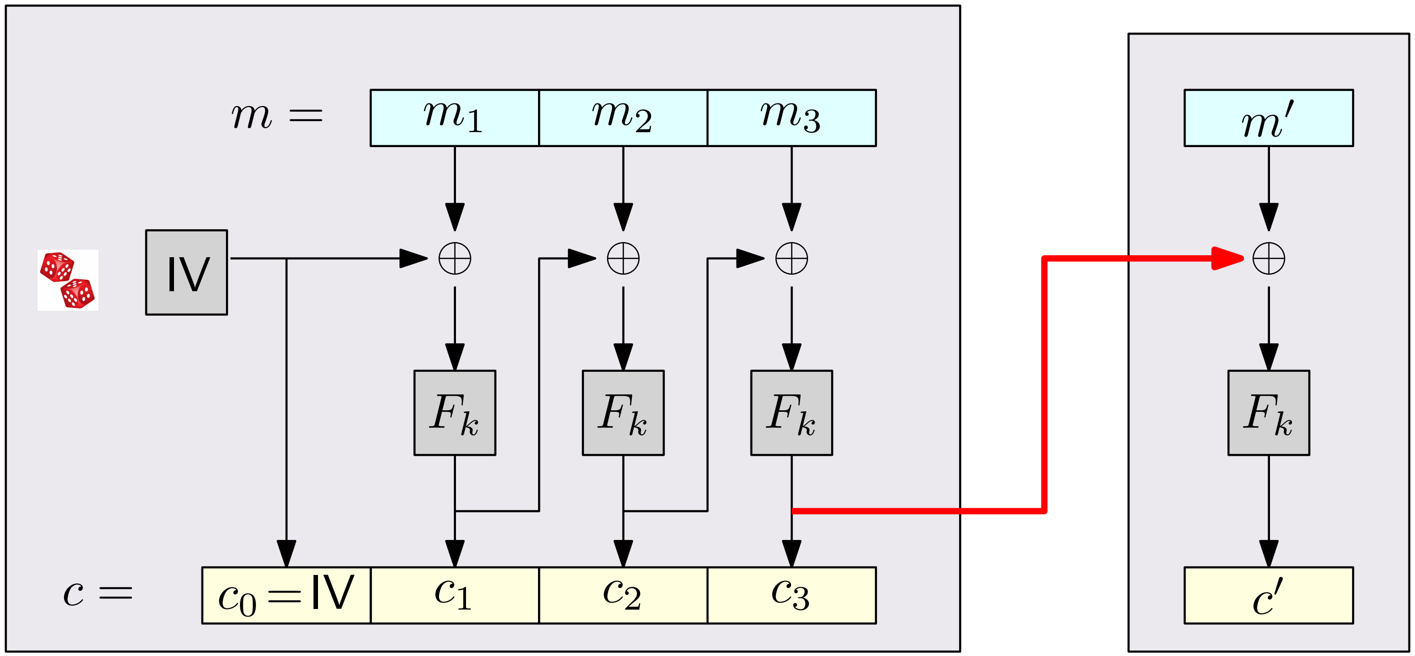

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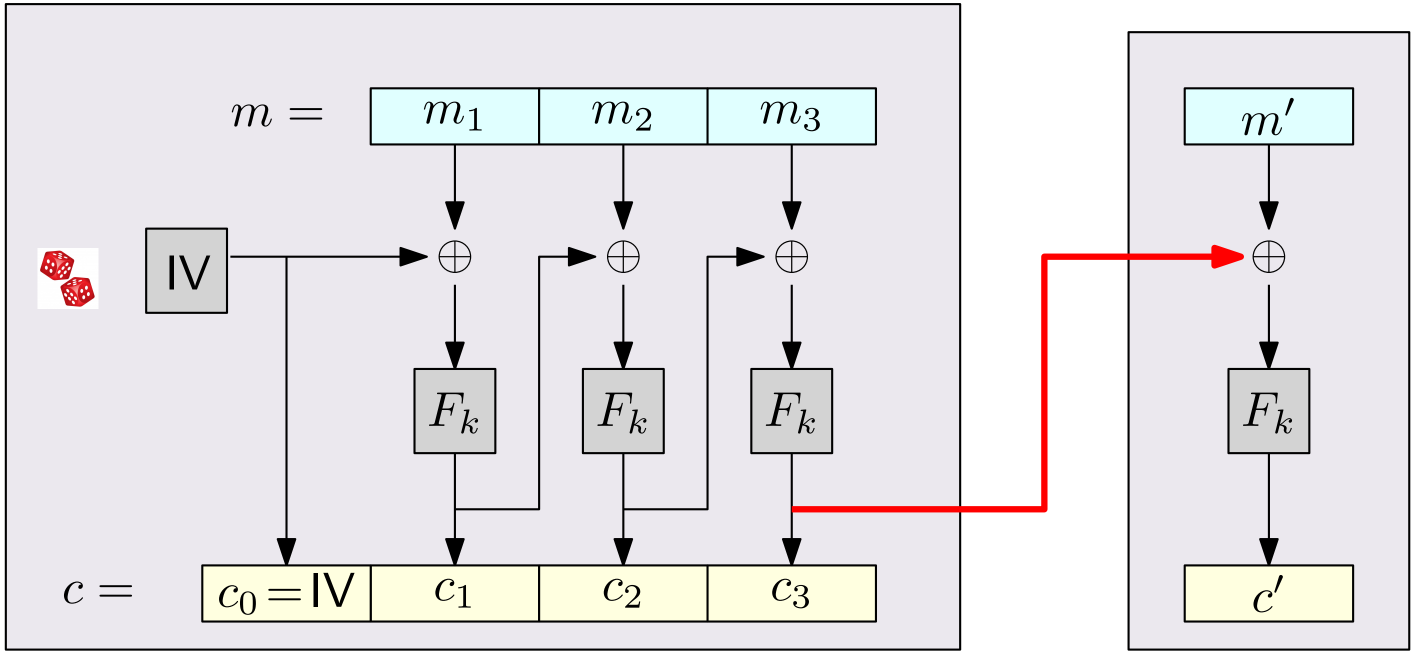

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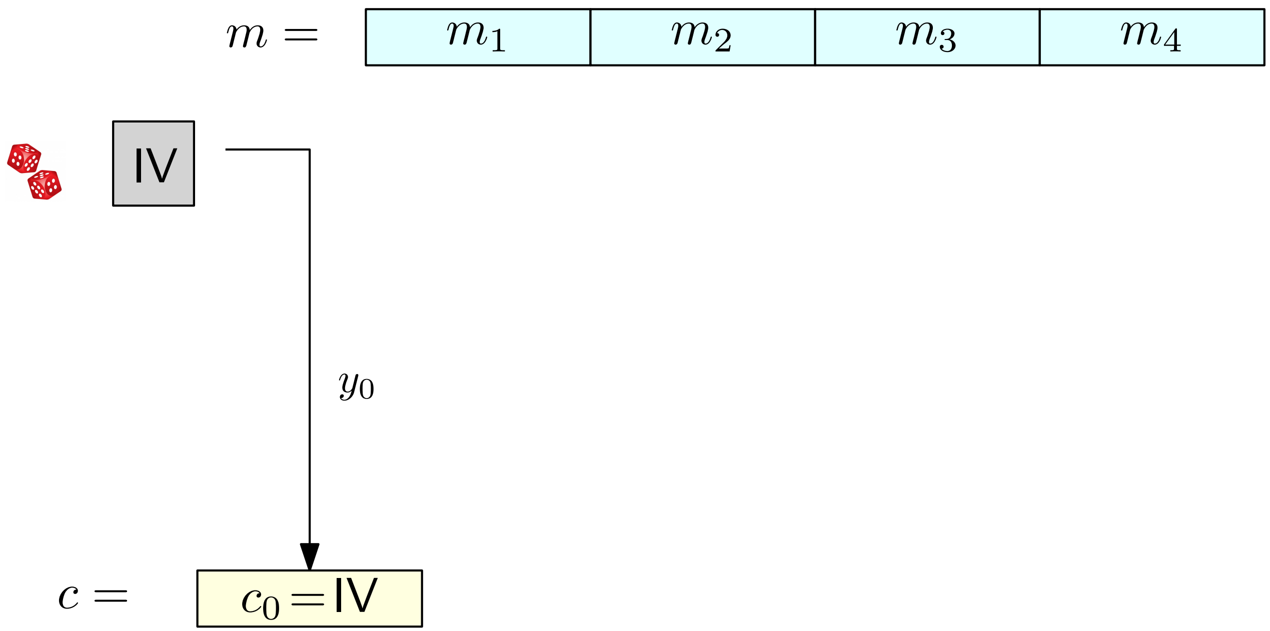

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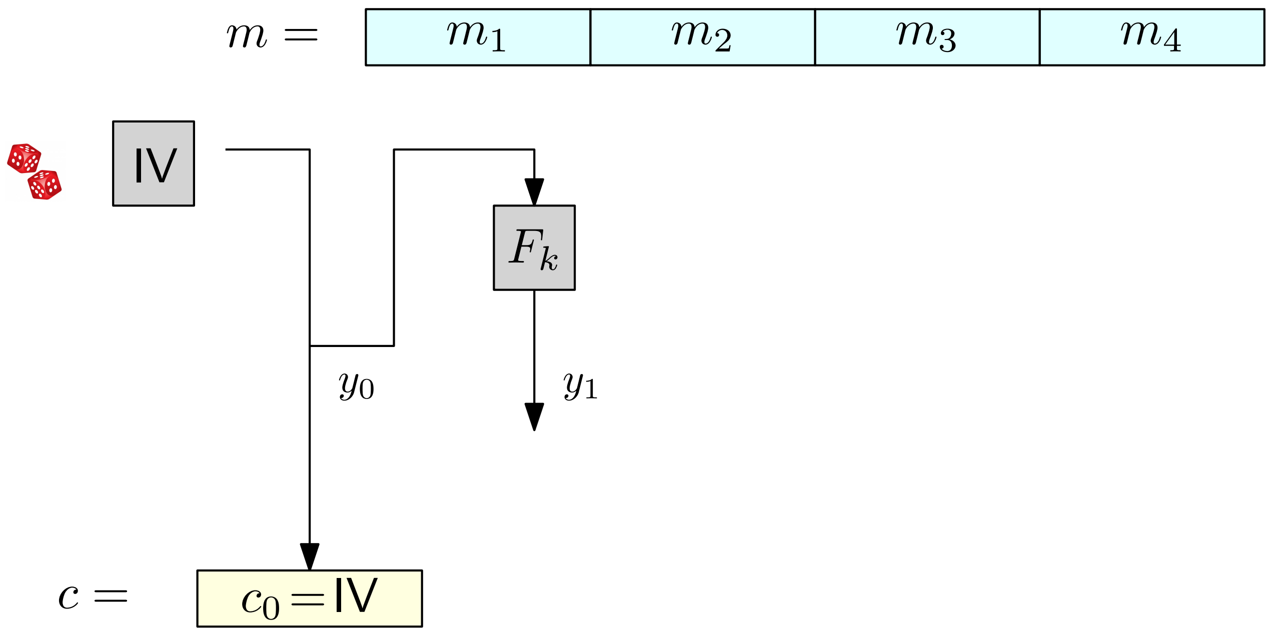

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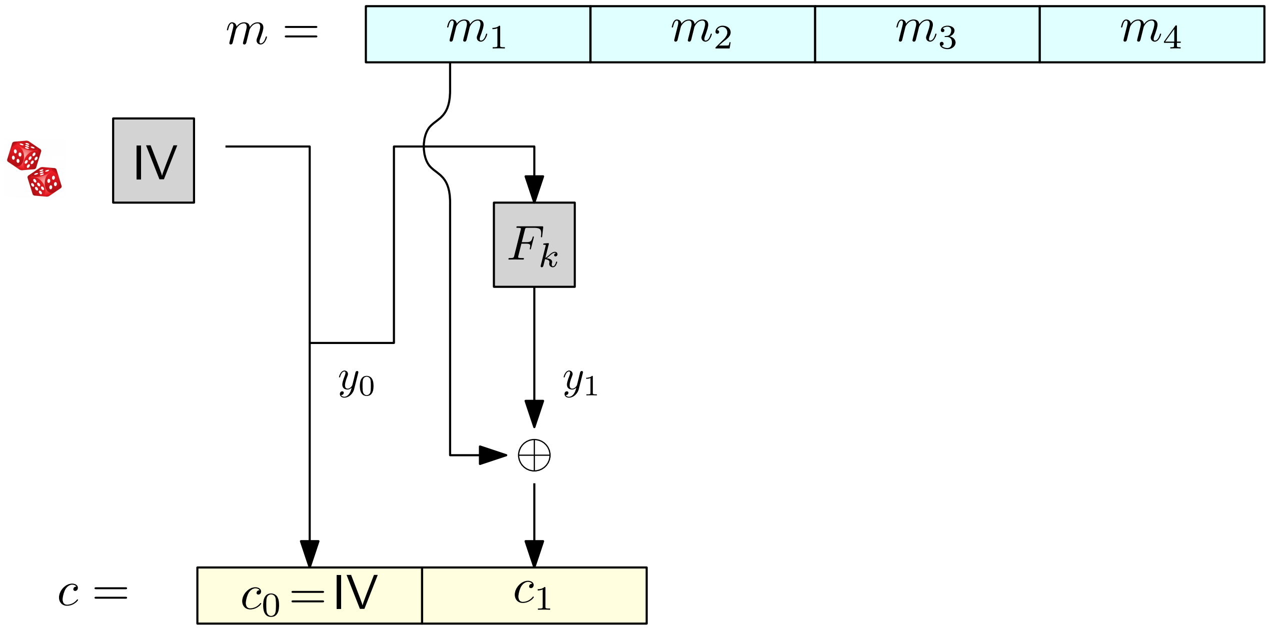

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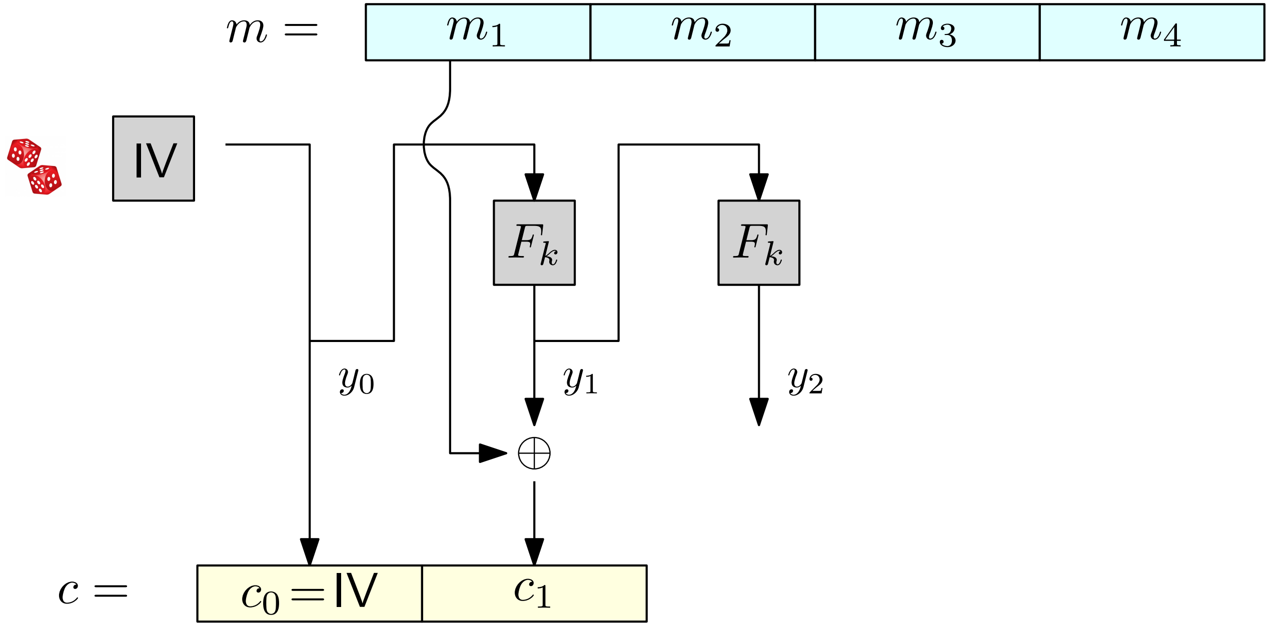

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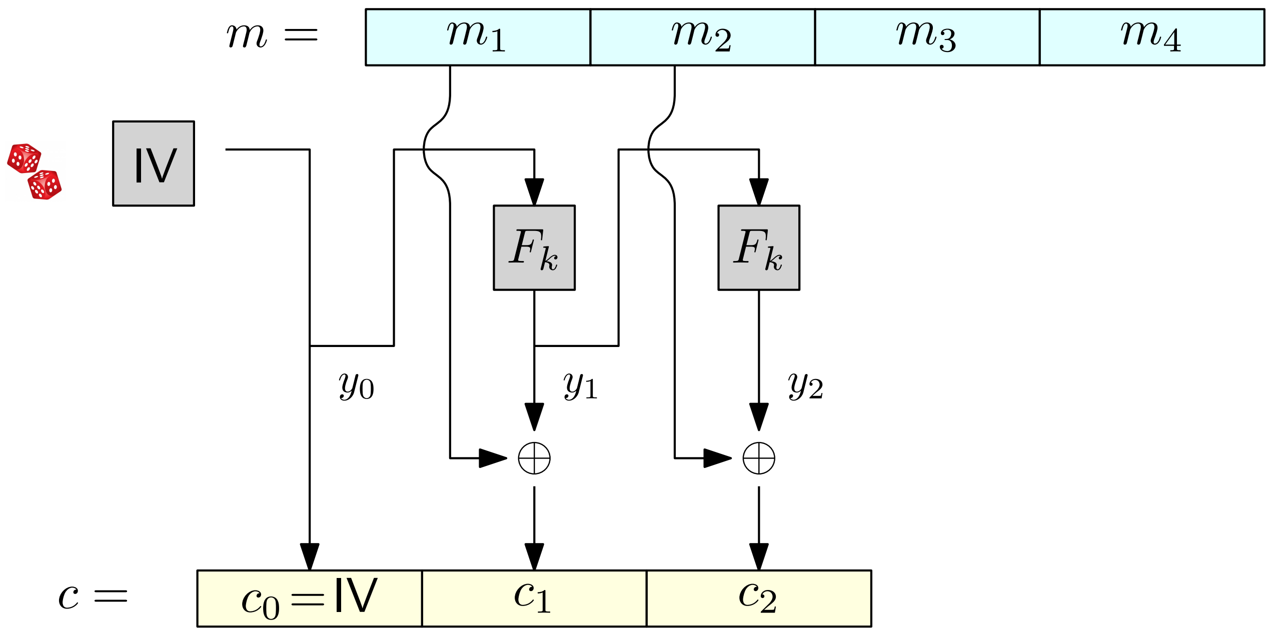

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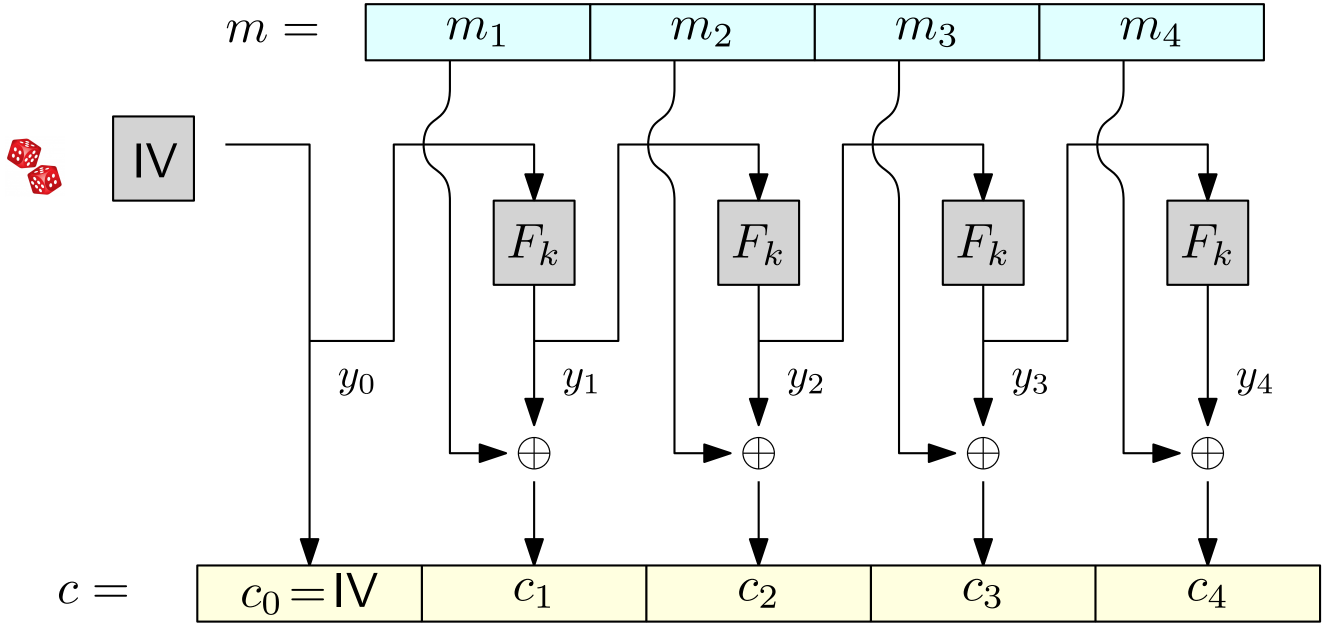

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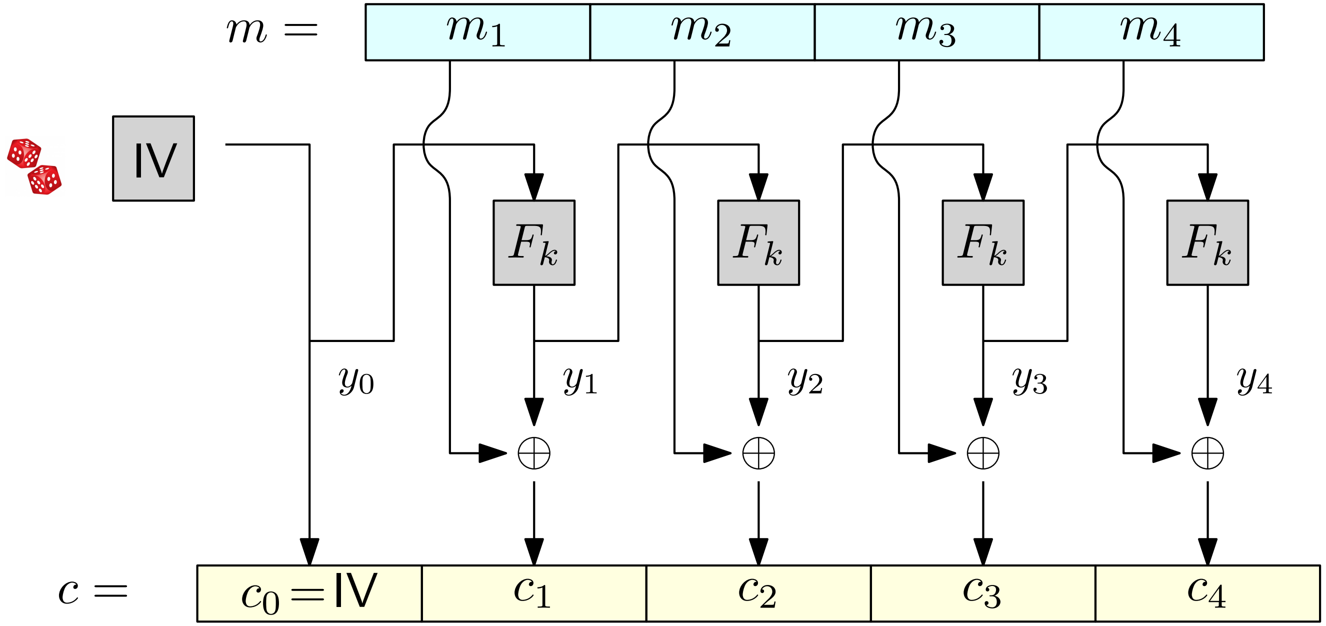

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Can be thought of as a stream cipher (generate $y_1, y_2, \ldots$ and XOR it with the message)

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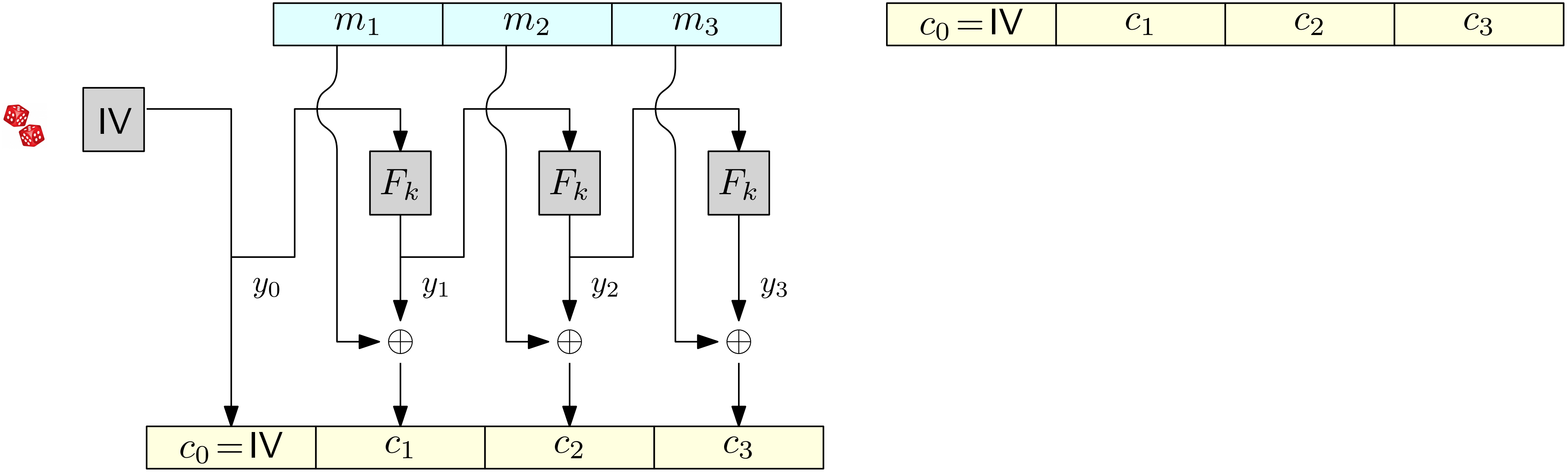

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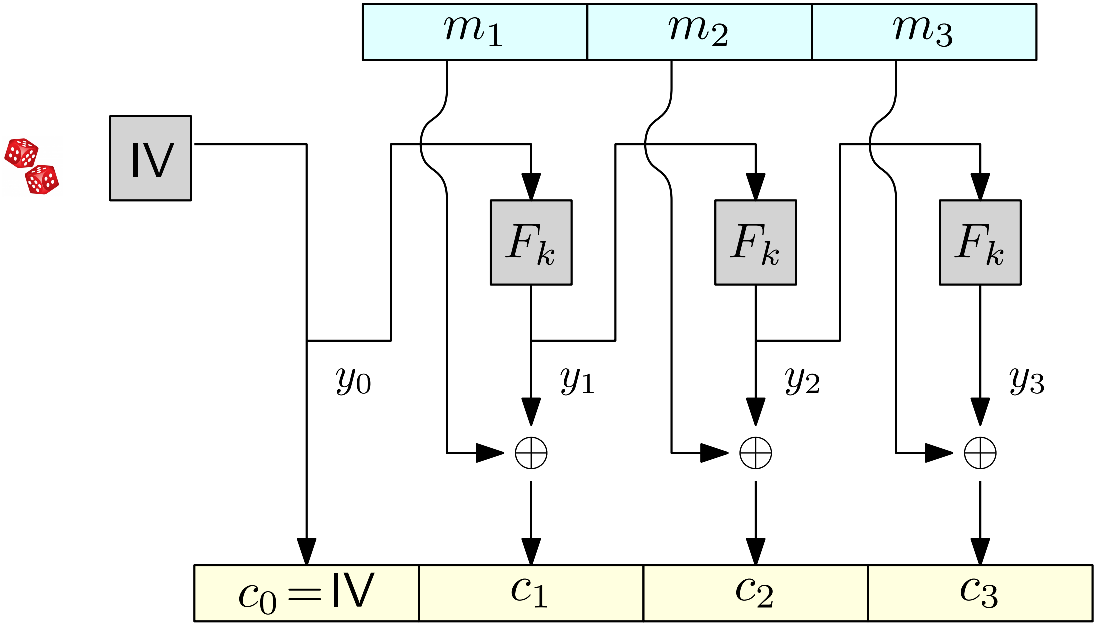

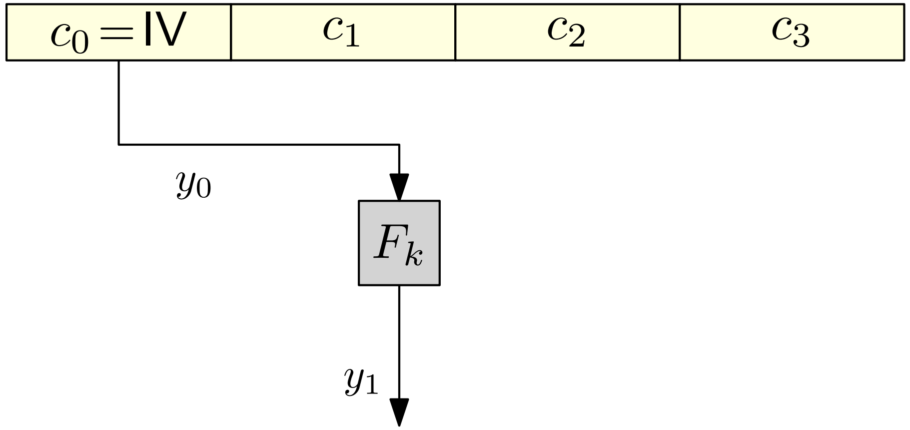

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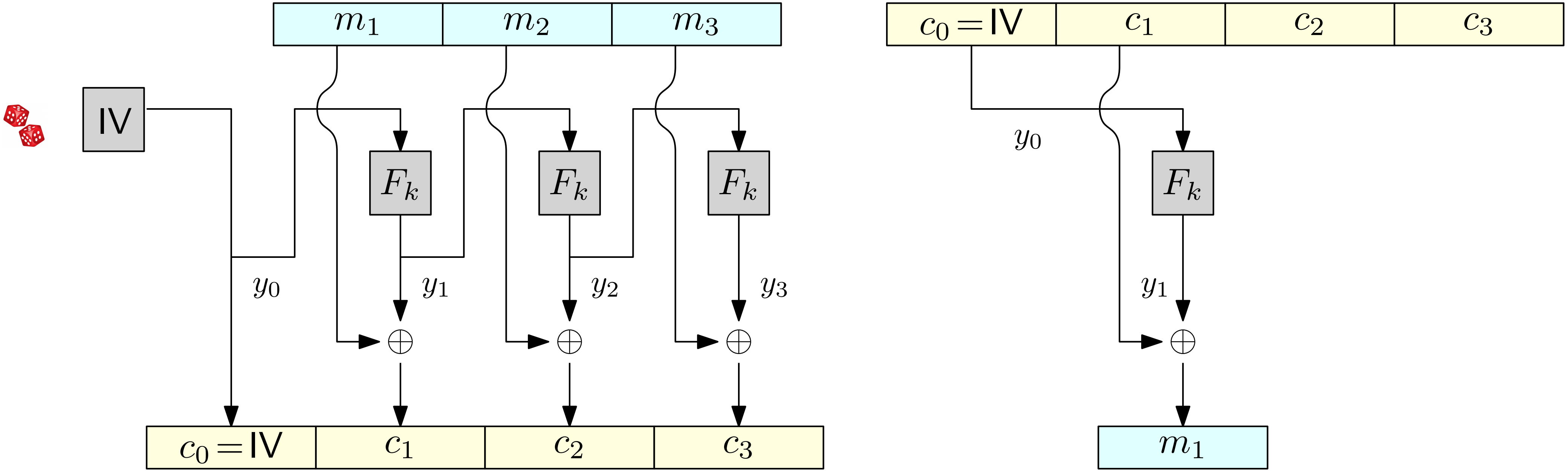

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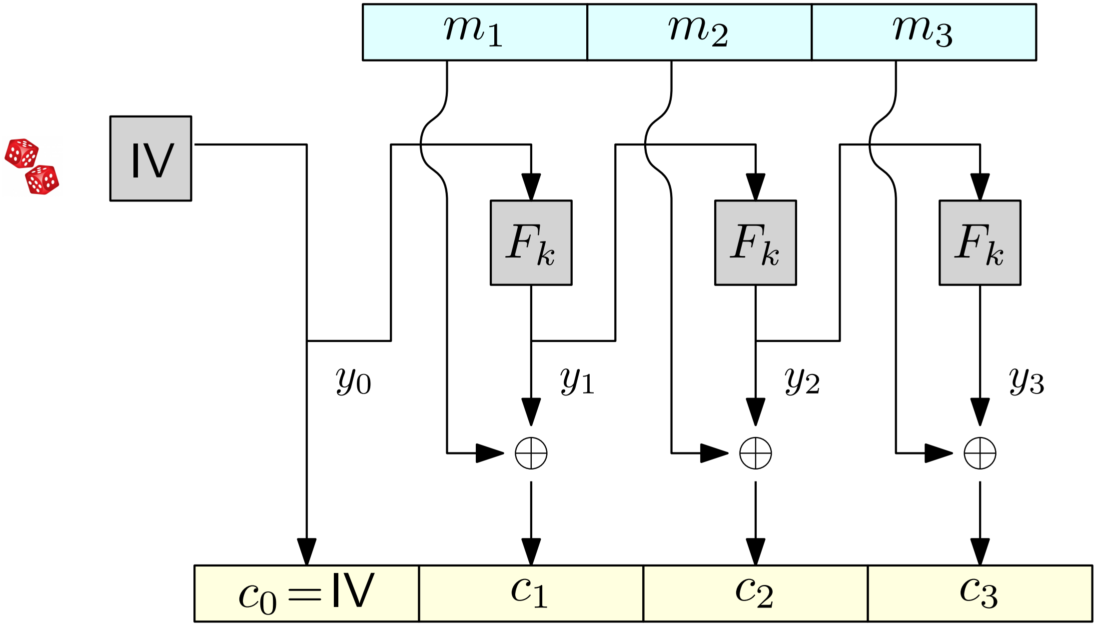

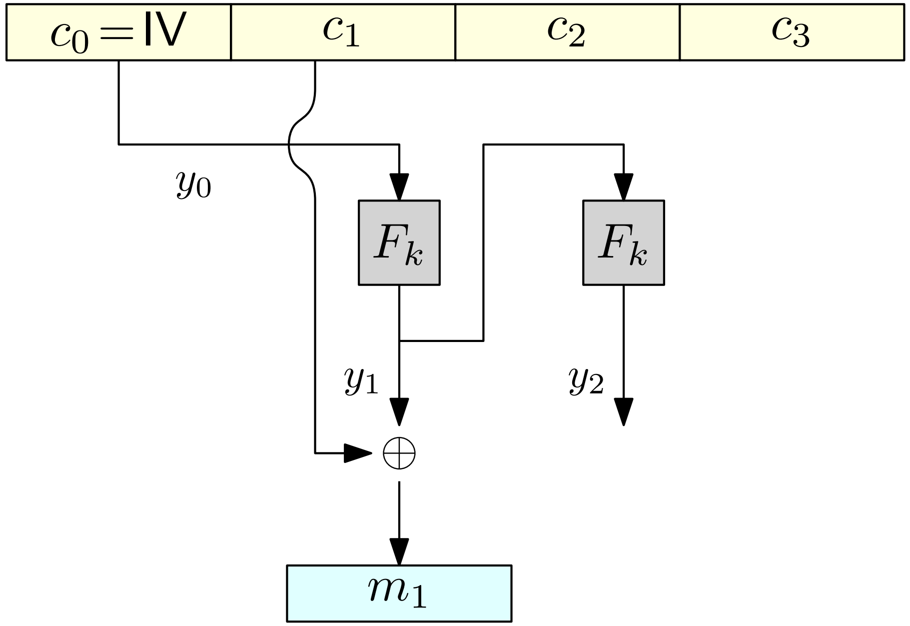

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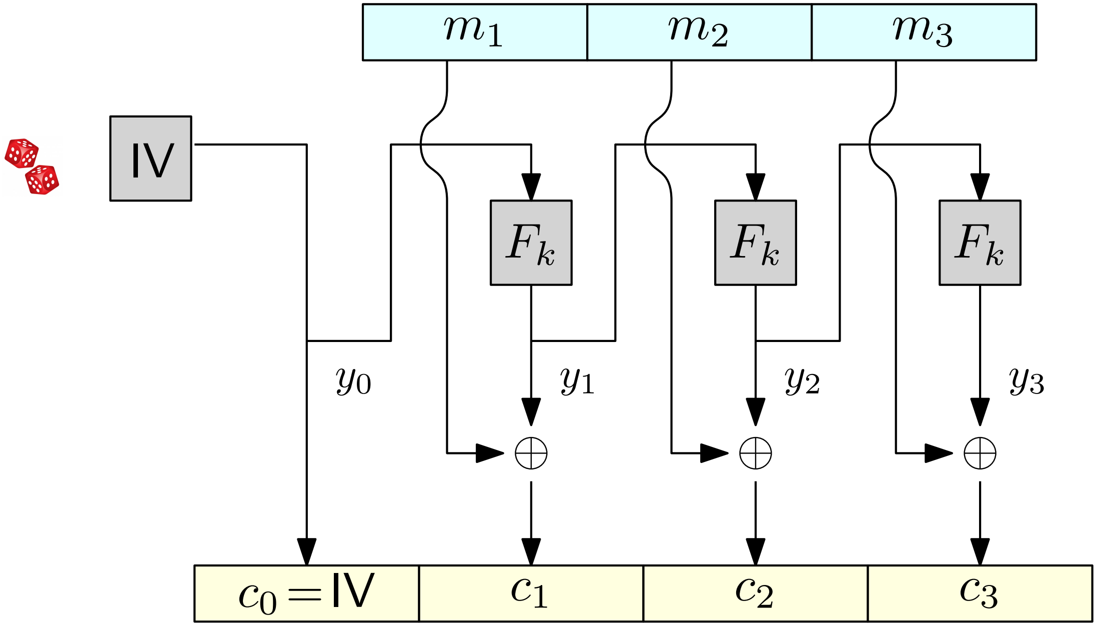

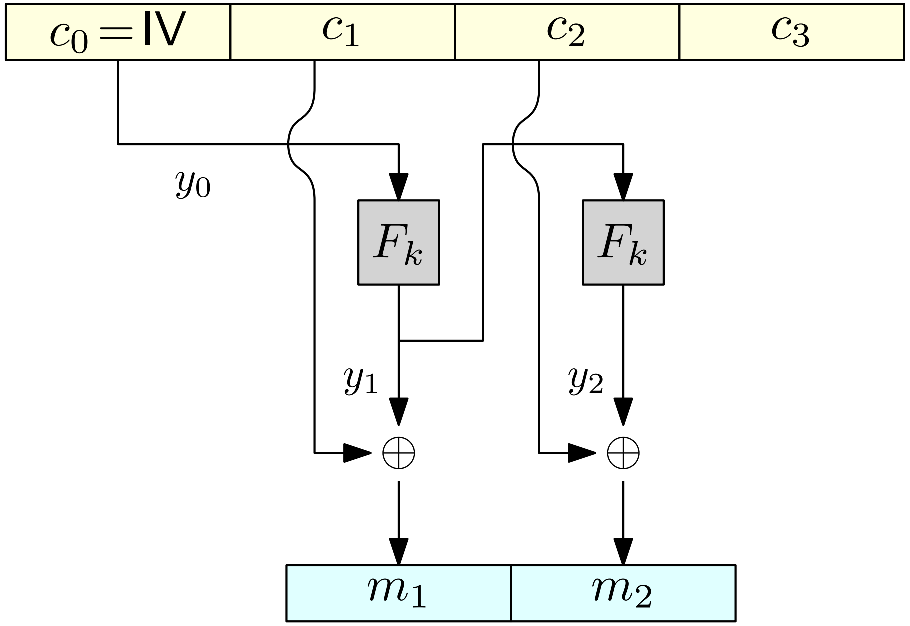

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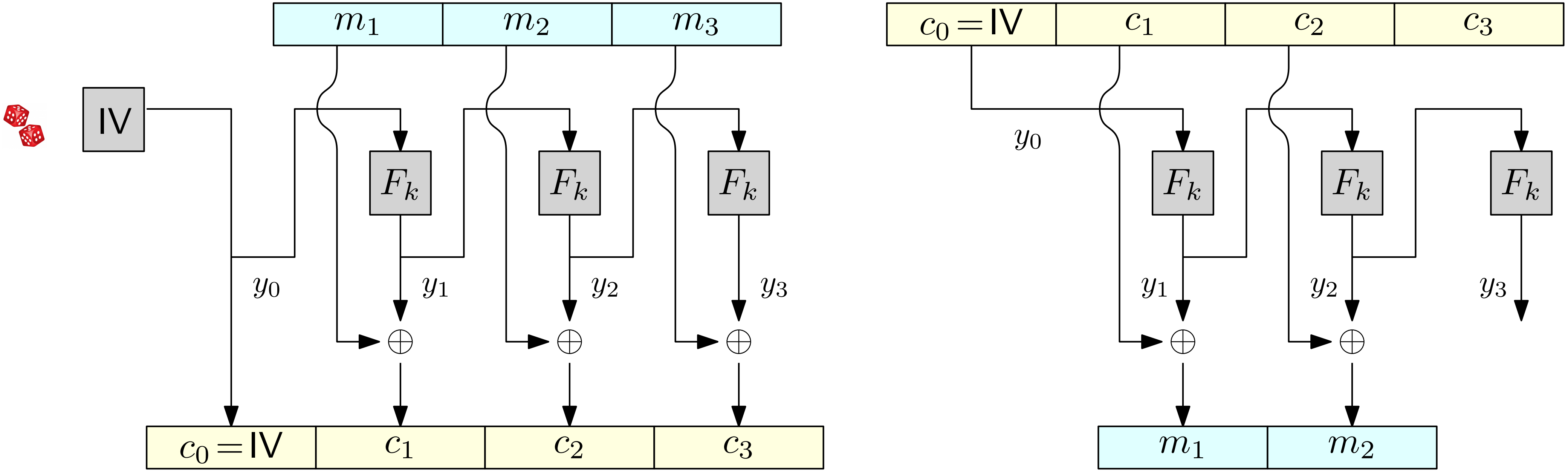

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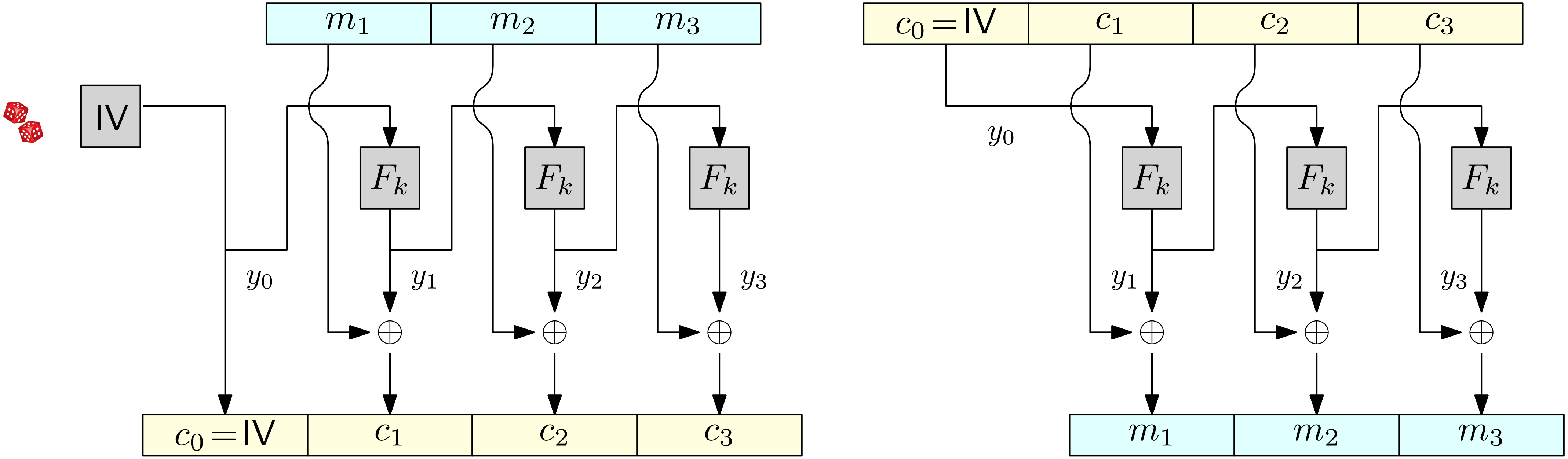

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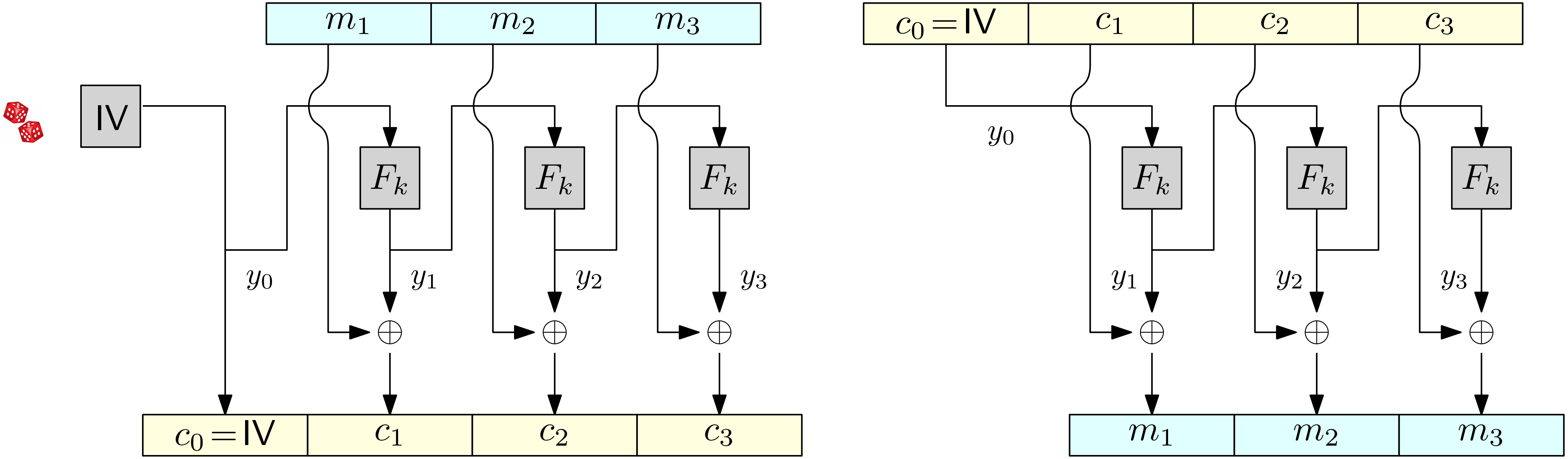

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Is OFB mode CPA-secure?

Theorem: If F is a pseudorandom function, then OFB mode is CPA-secure.

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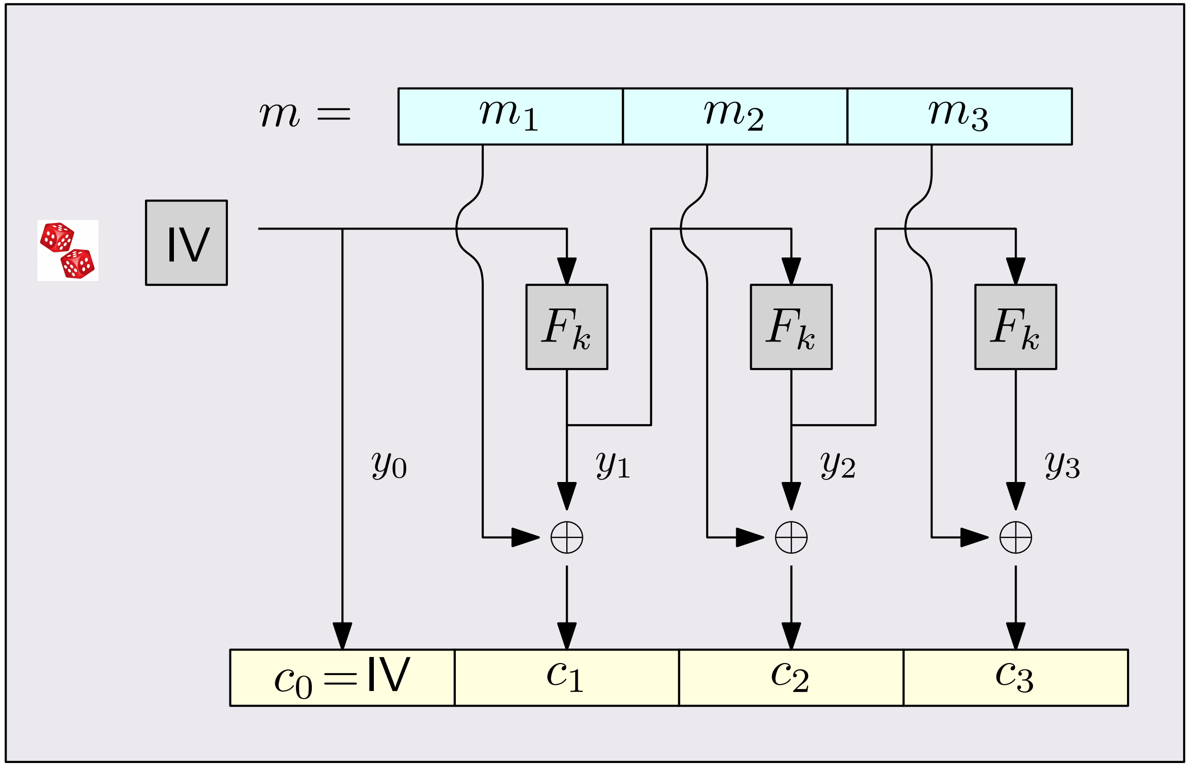
The stateful variant of OFB (the final value y_i is used in place of y_0 when the next message needs to be encrypted) is also **CPA-secure**



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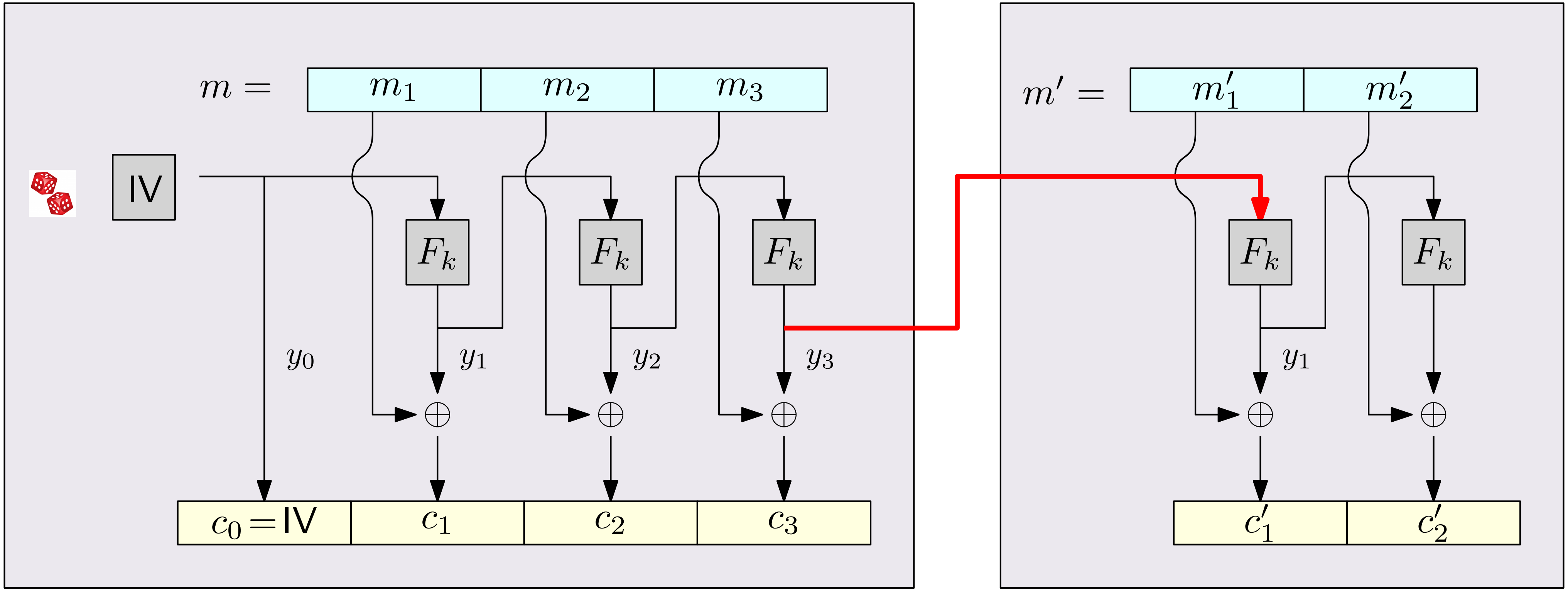

Counter (CTR) mode

Can be viewed as a stream cipher

$m =$

m_1	m_2	m_3	m_4

- Split the input to F into an IV and a counter

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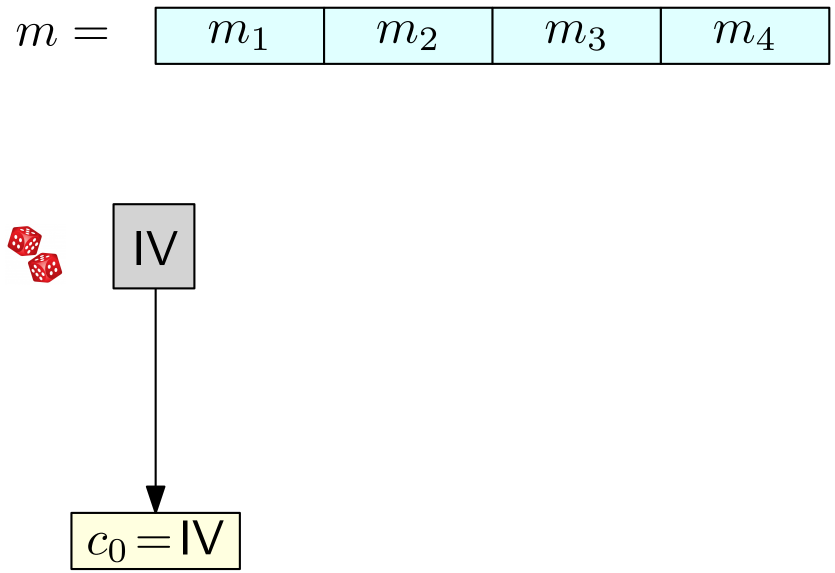

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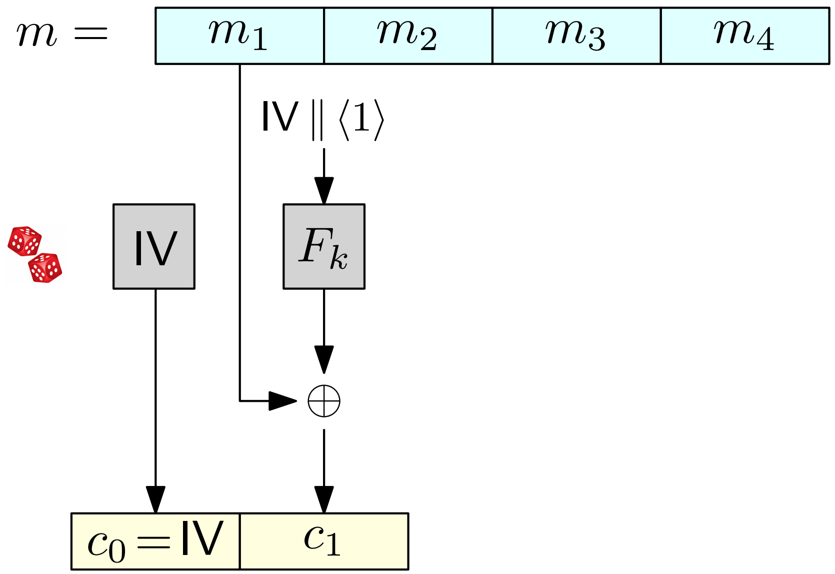

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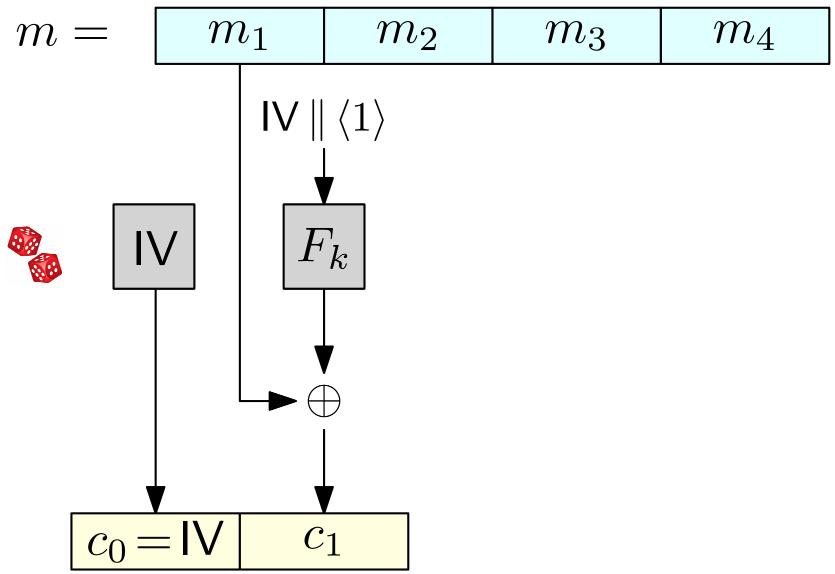

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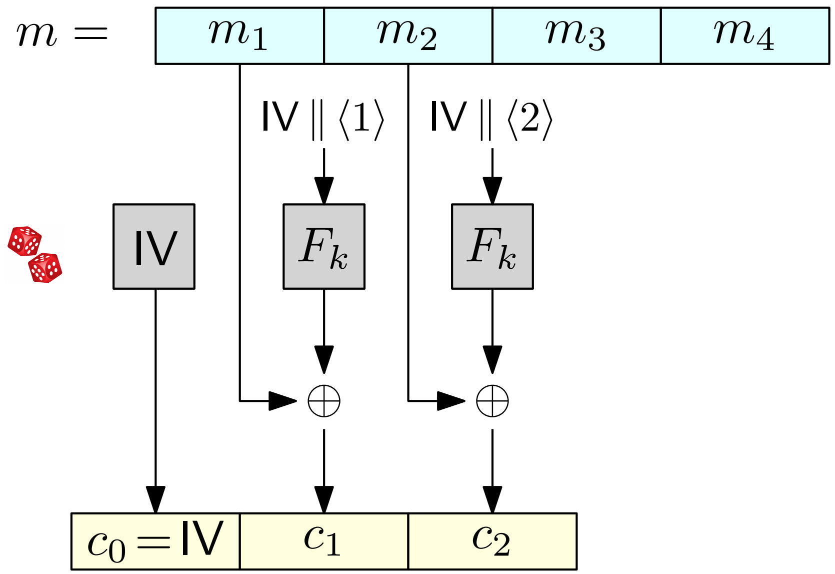

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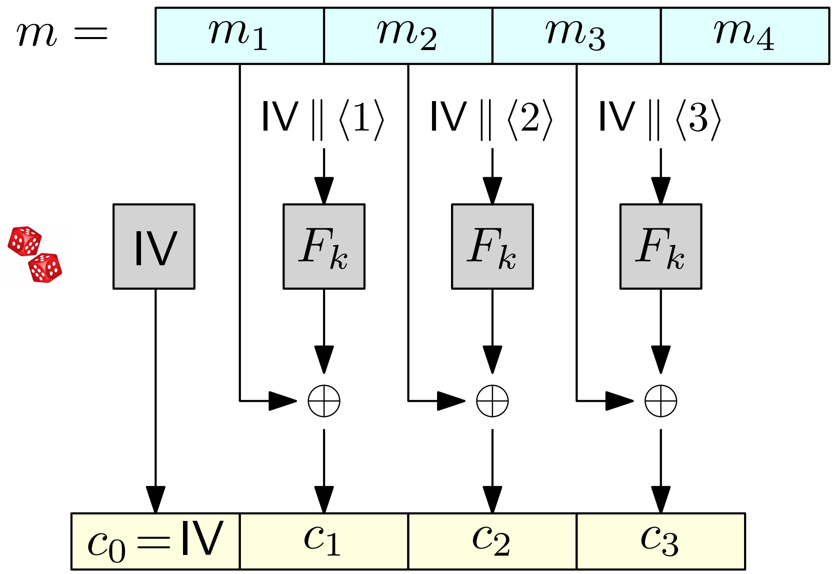

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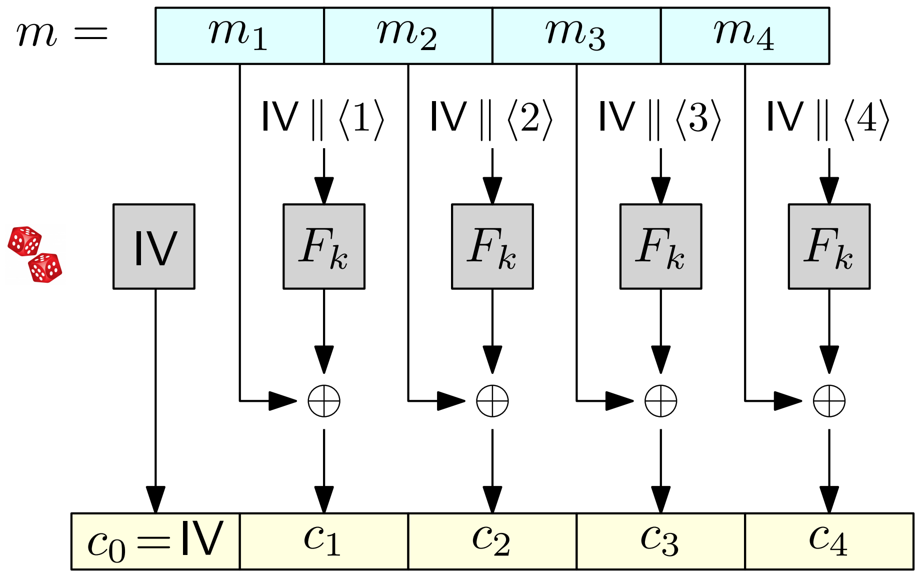

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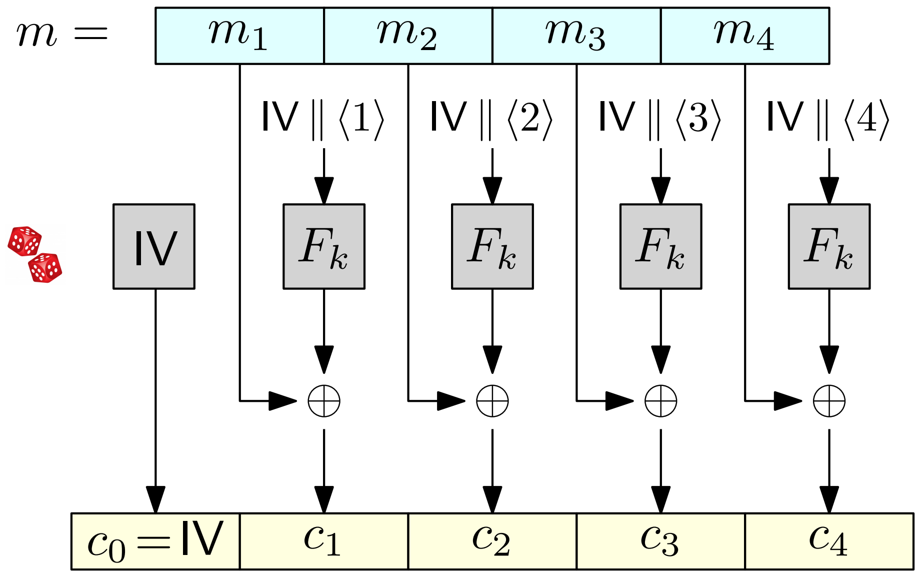

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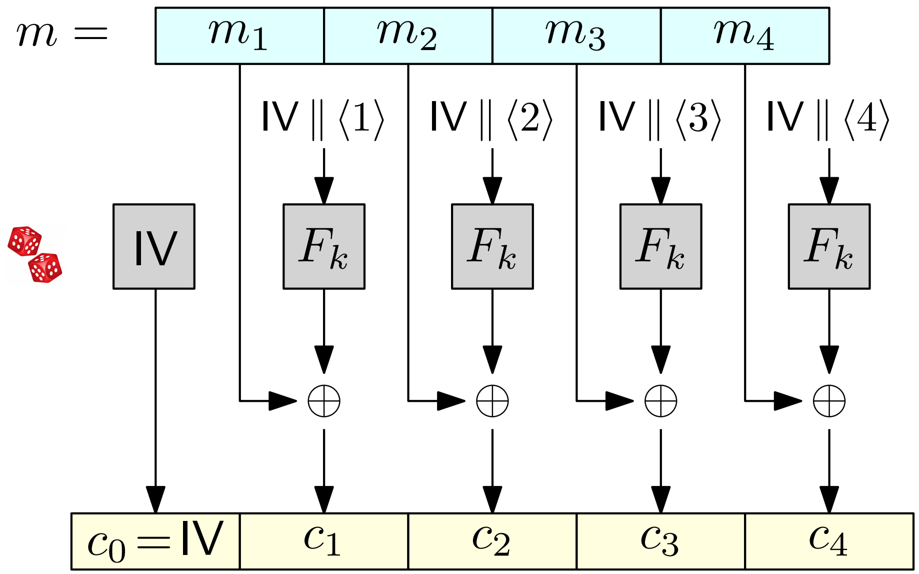

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- Remains secure even if IVs are not chosen u.a.r., in fact it suffices that IVs never repeat

$$\text{IV} = 00\ldots000,\ 00\ldots001,\ 00\ldots010,\ 00\ldots011,\ \ldots$$