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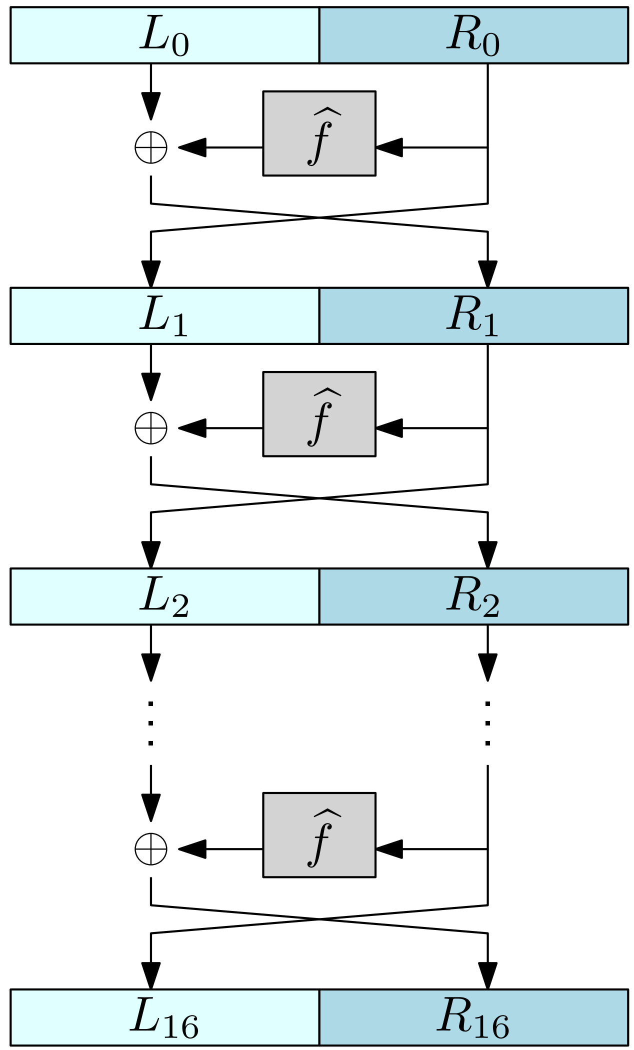
- Still considered *insecure* nowadays

Structure of DES

- From a high-level perspective, DES is a Feistel network
- The number of rounds is 16



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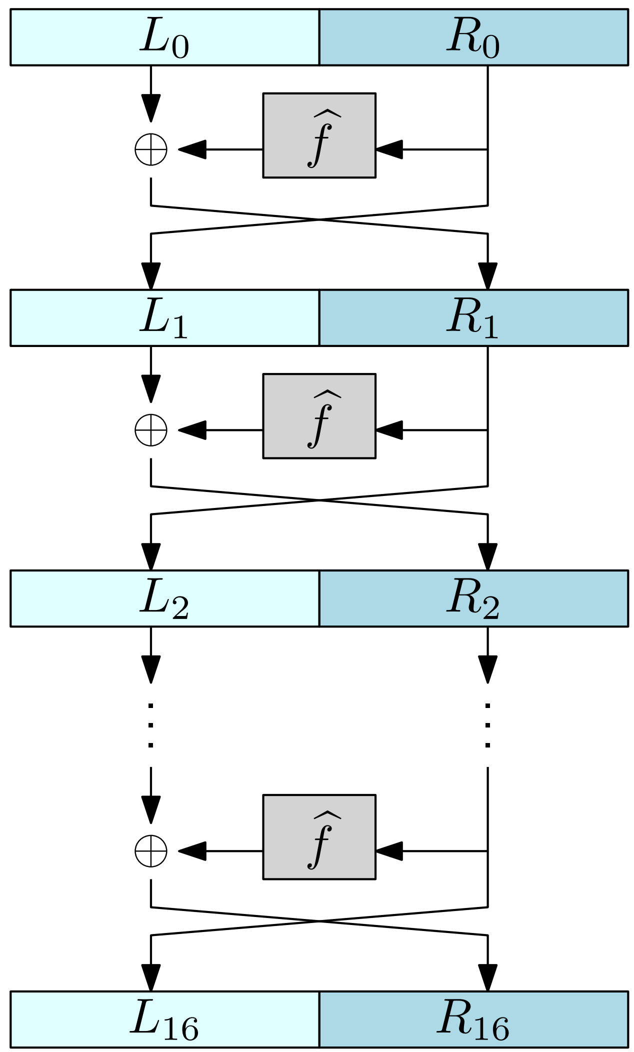

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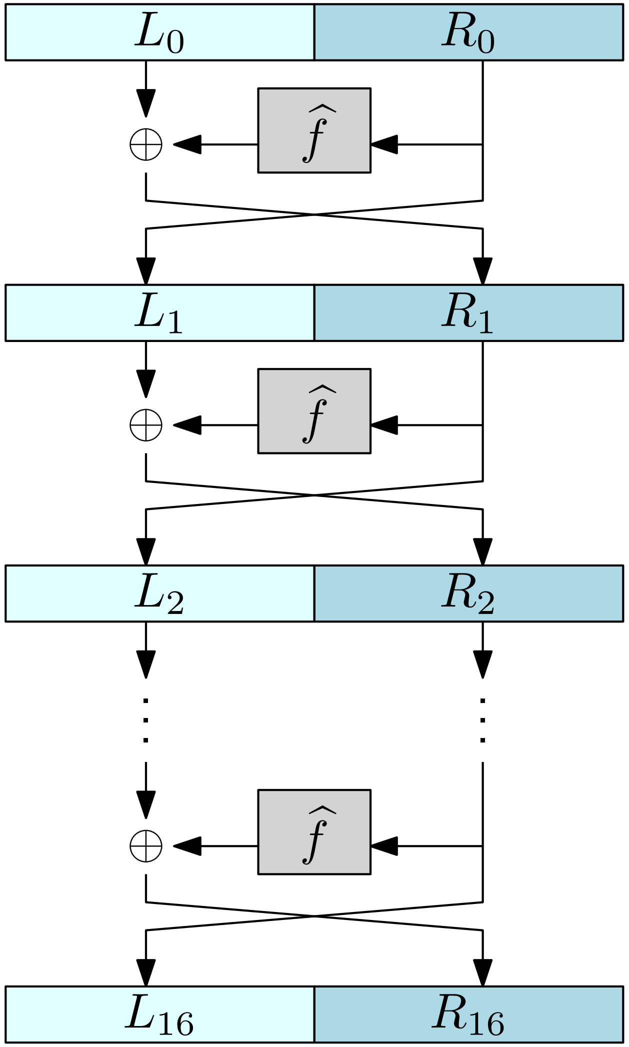

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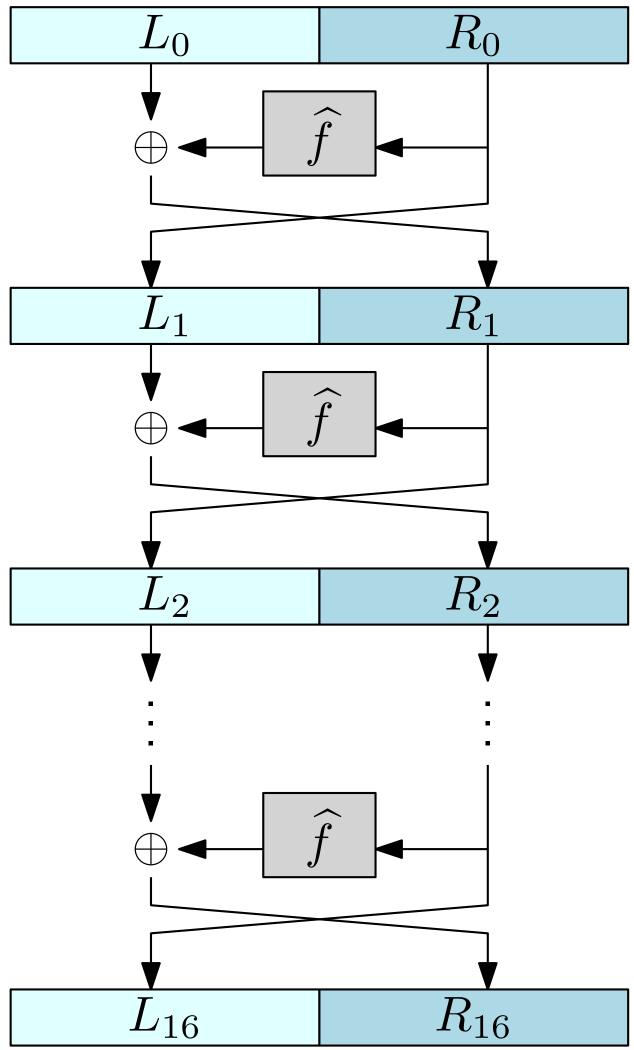
- The sub-keys are are formed by selecting and permuting a subset of 48-bit from the 56-bit master key
- The bit selection rule and the permutations are public, the only secret information is the master key itself



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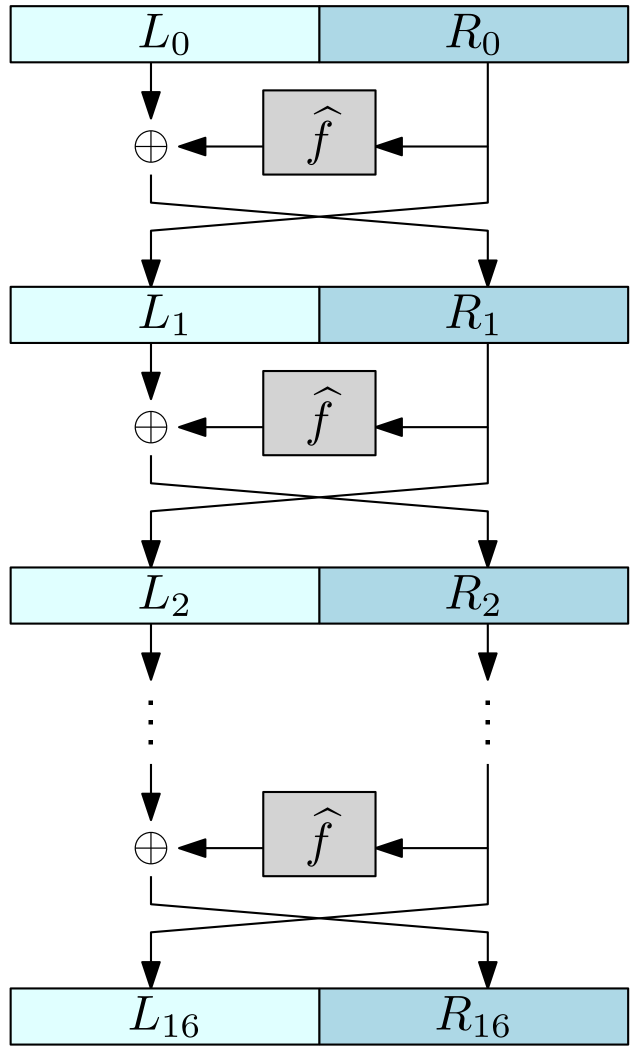

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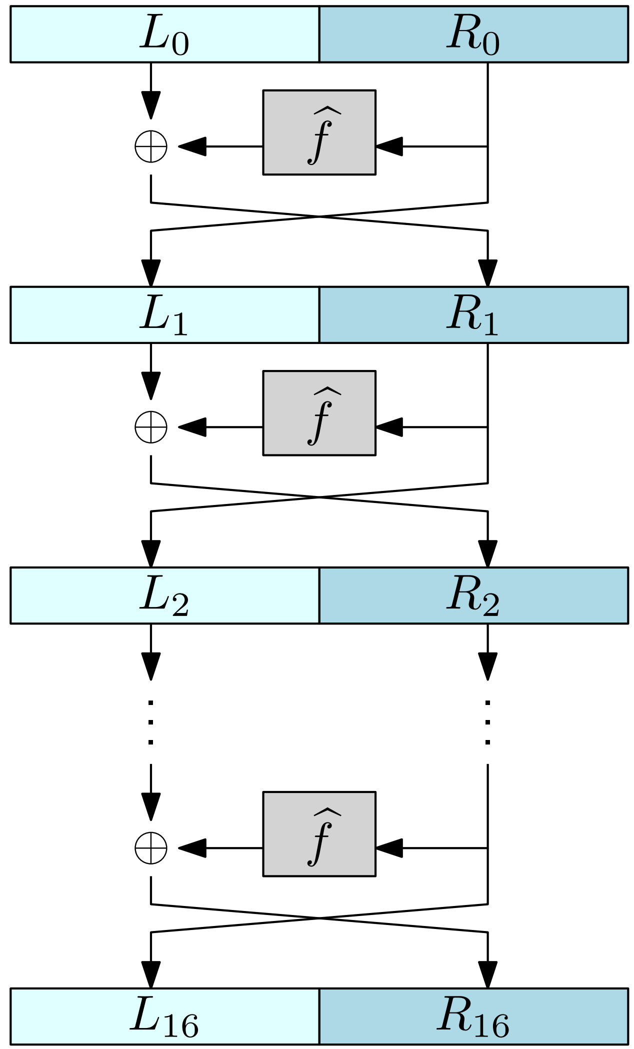

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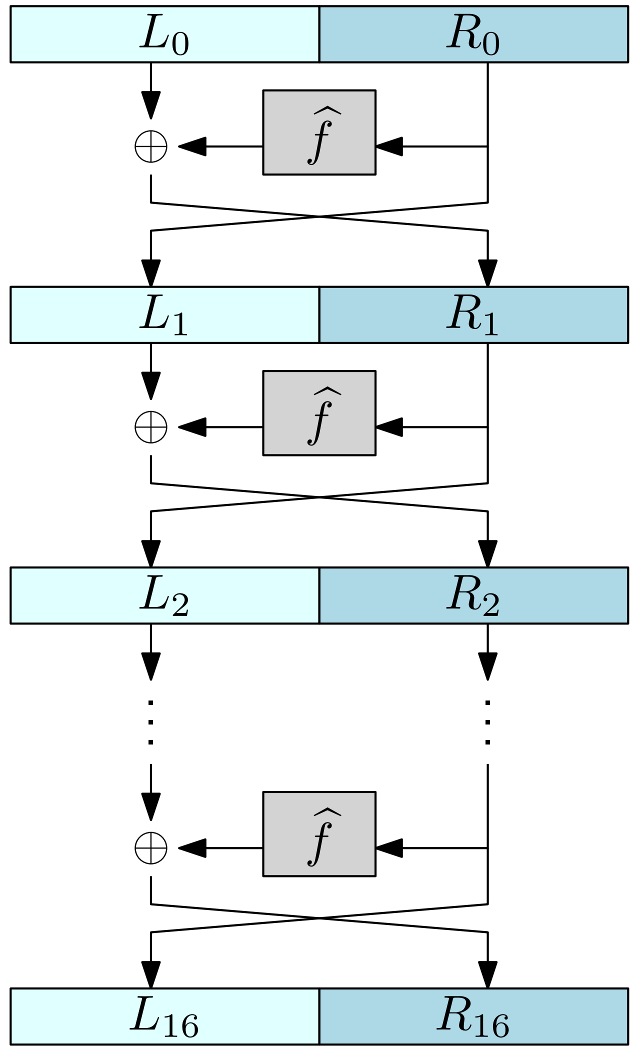

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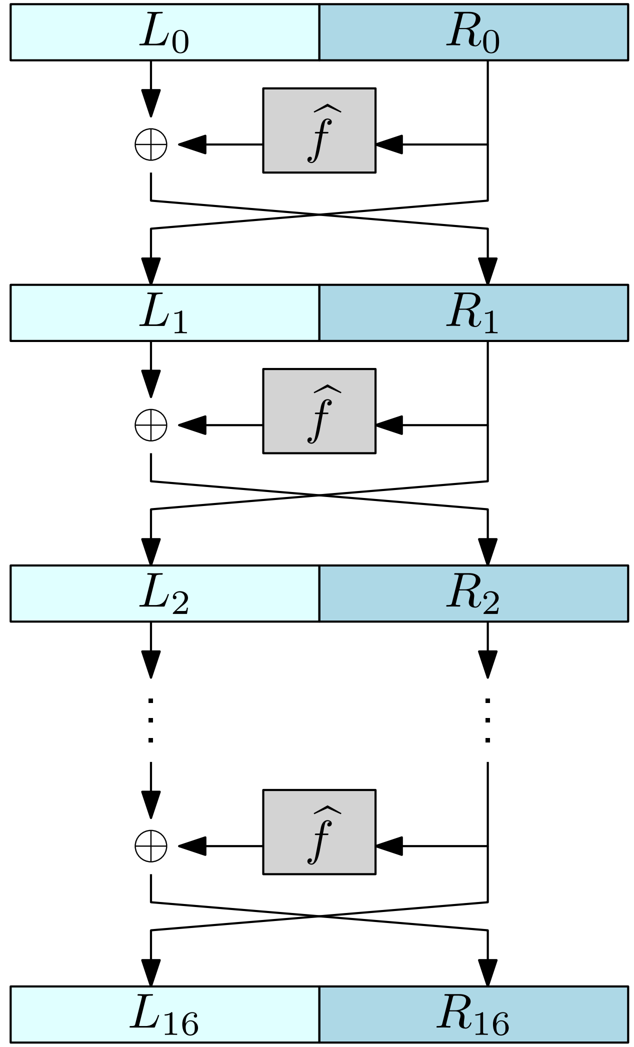

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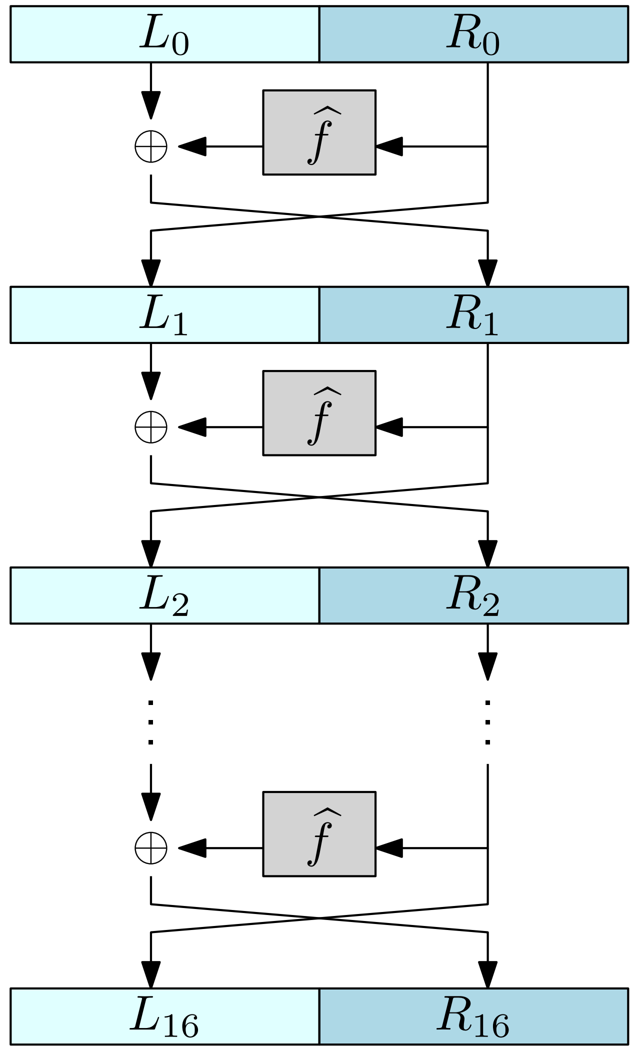
  This is not a problem, since Feistel networks do not require the round function to be PRP



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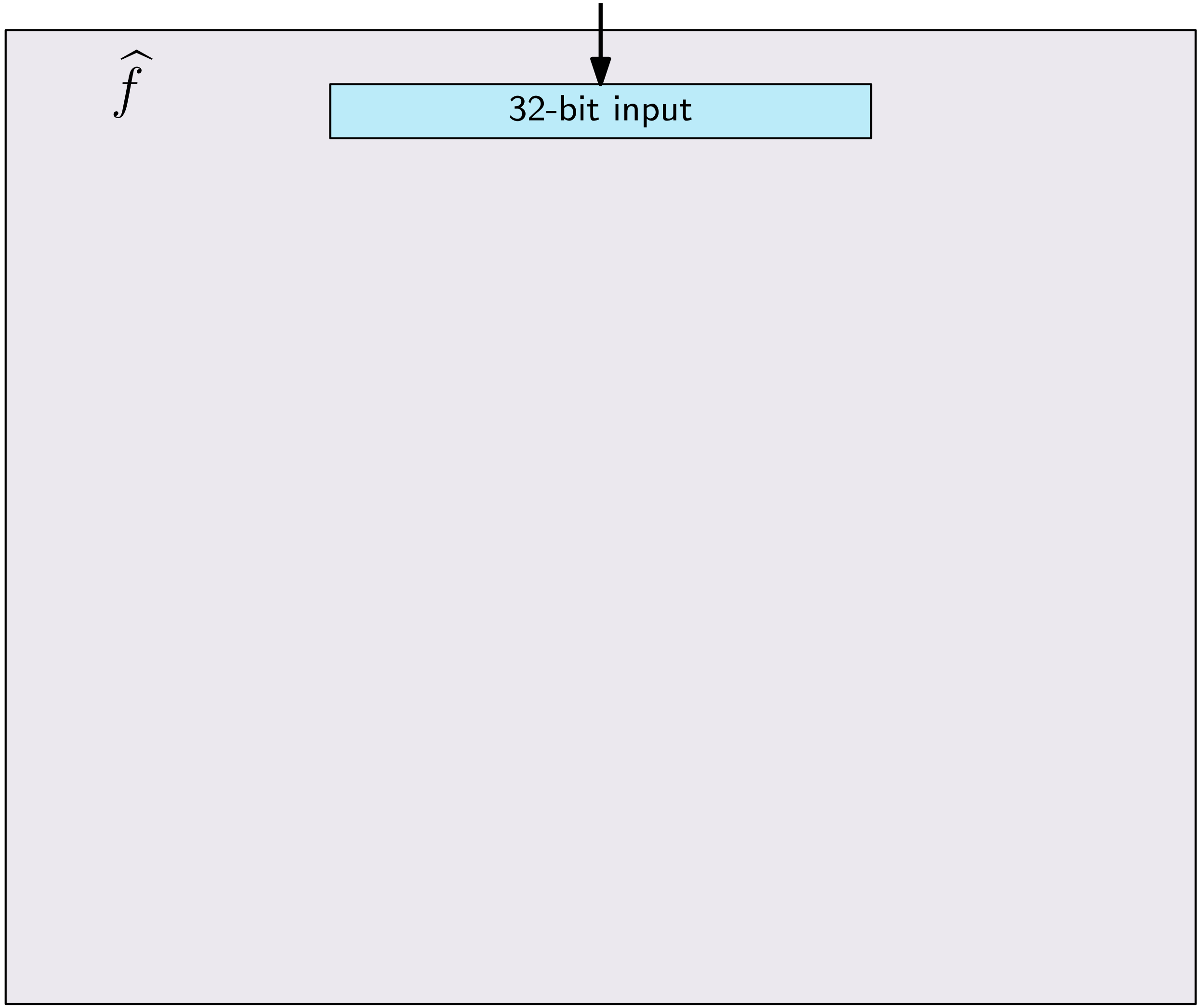

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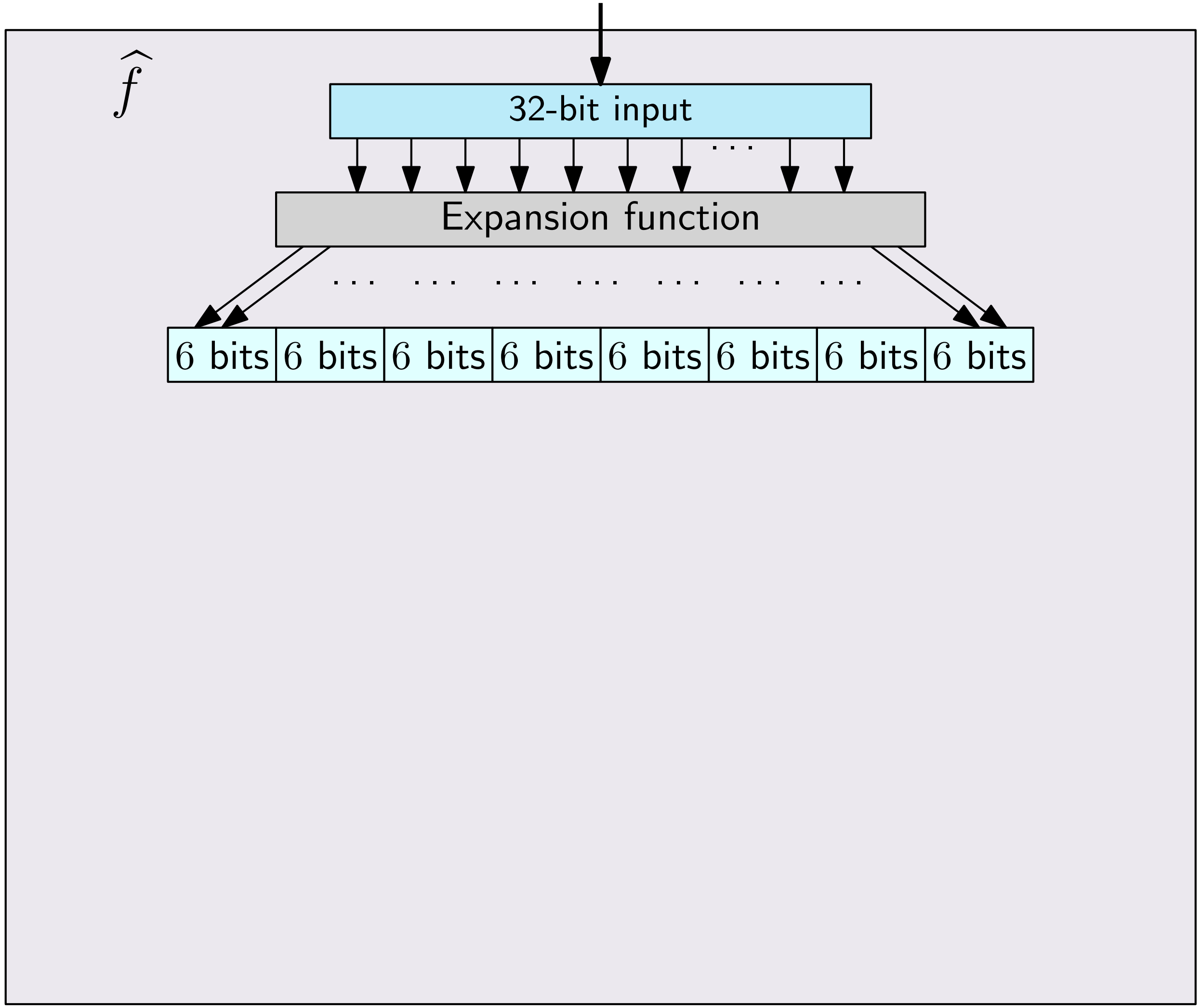

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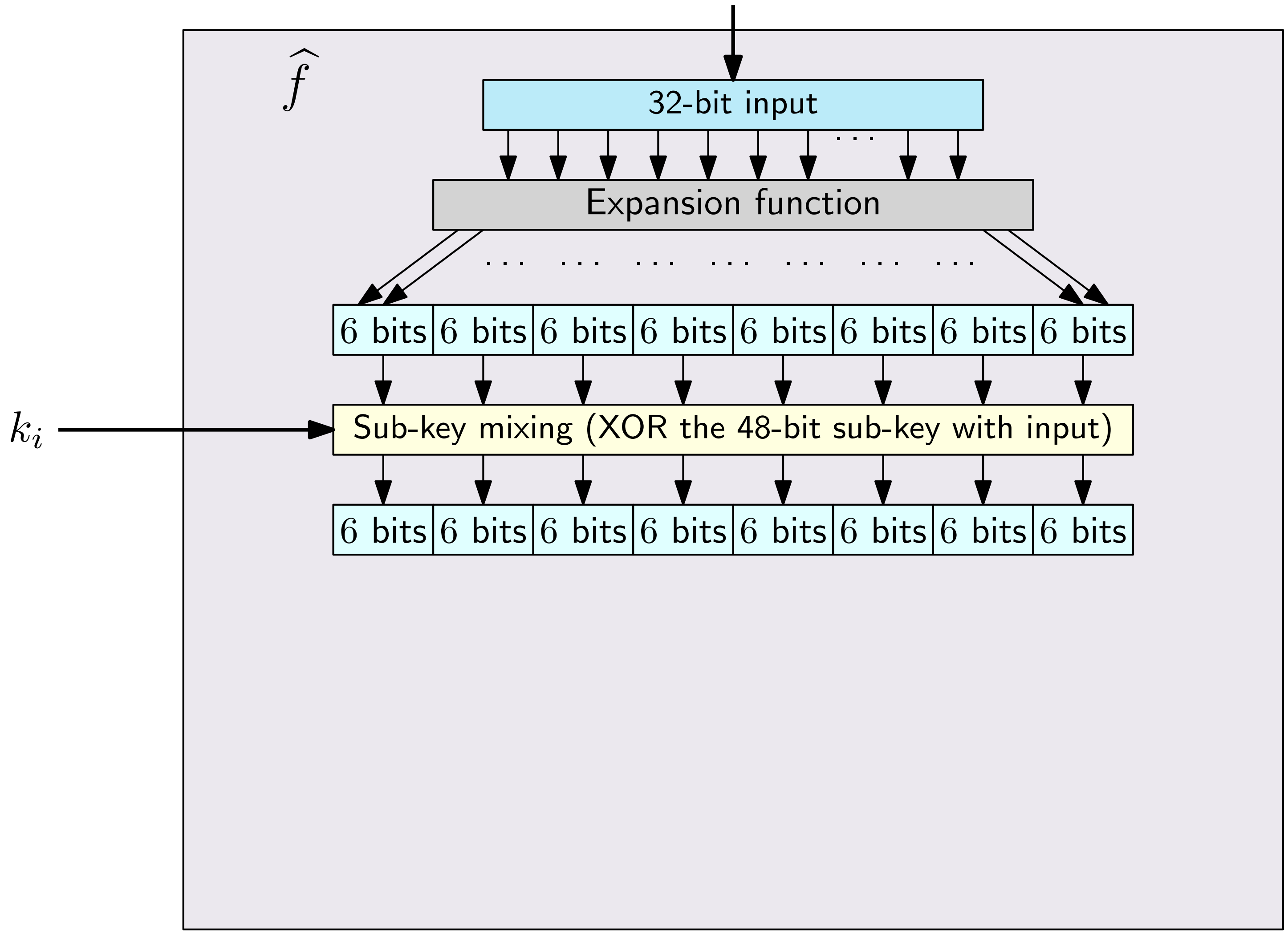

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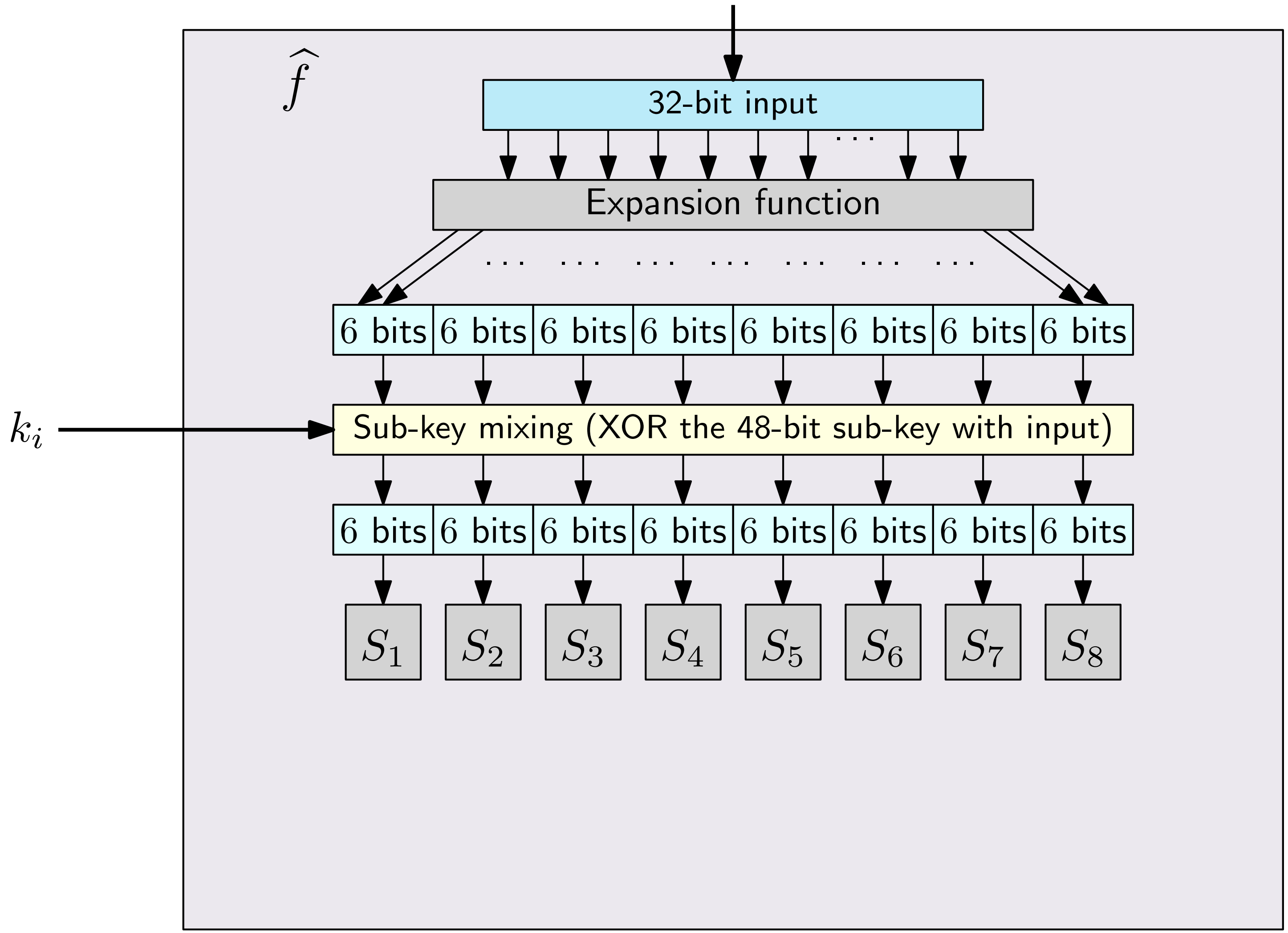

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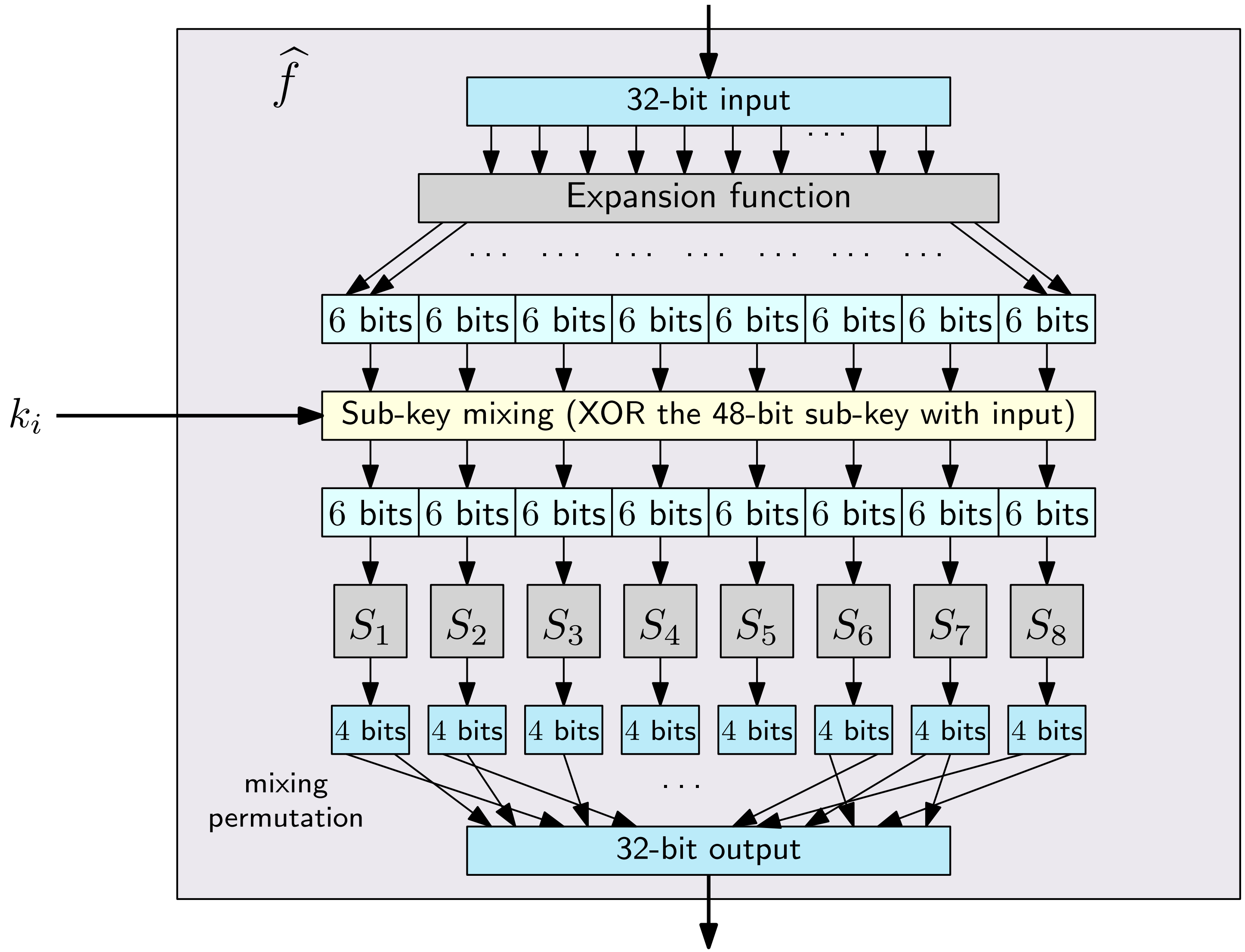

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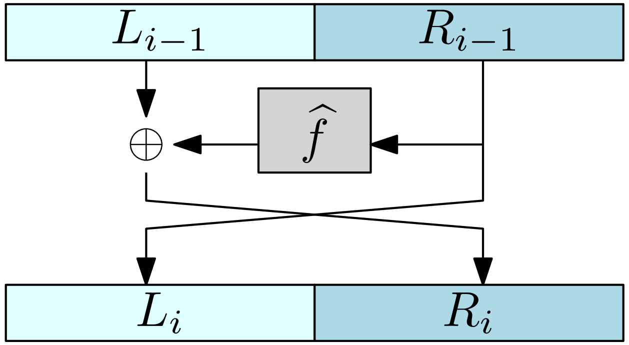

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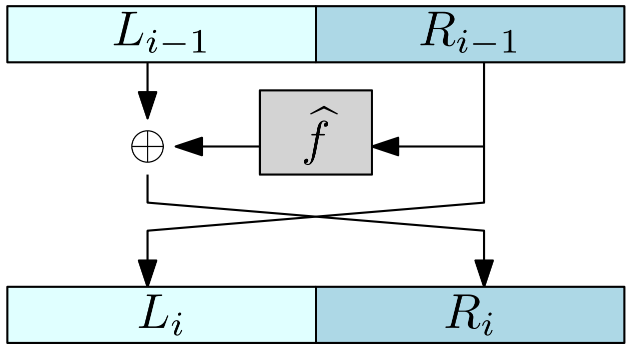

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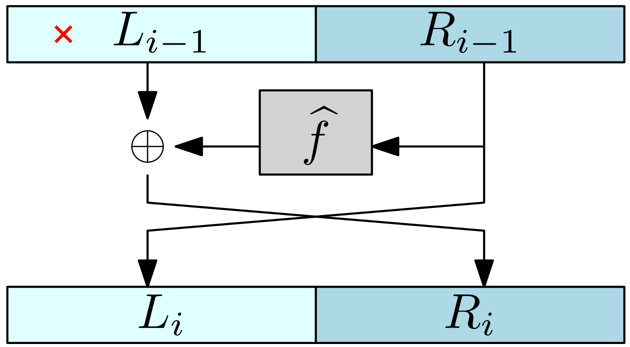

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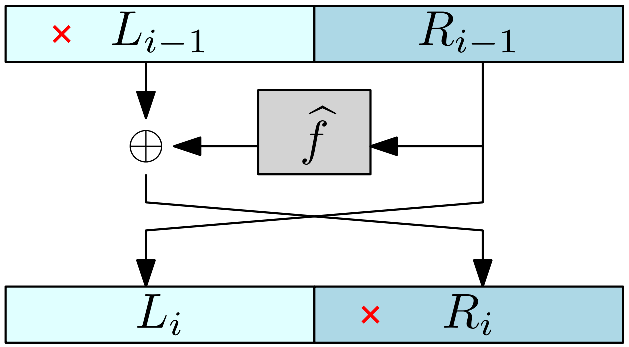

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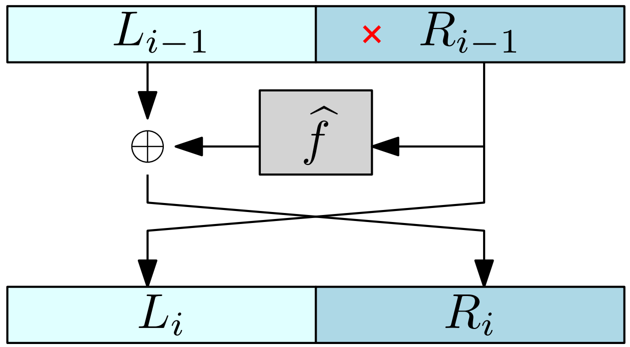

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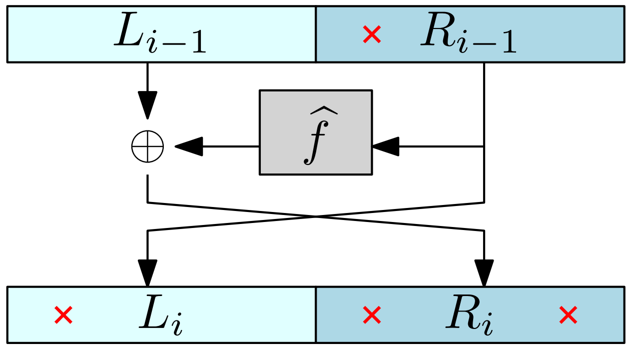

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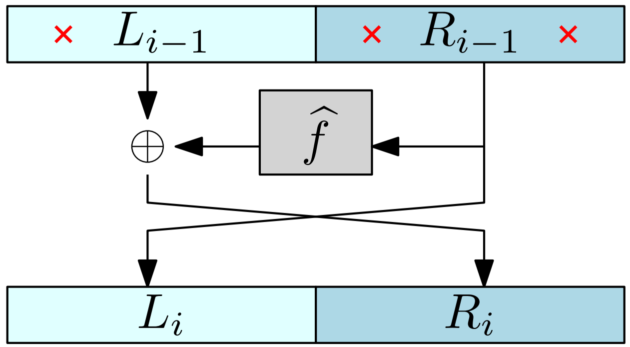

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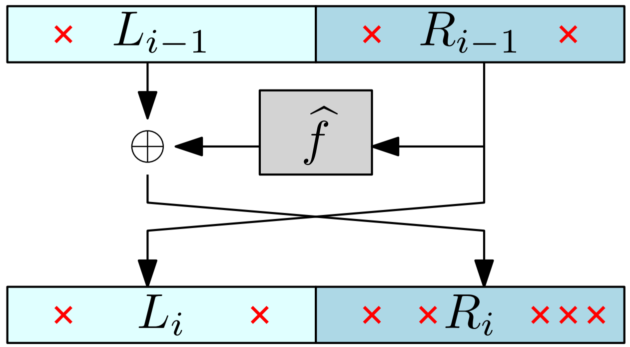

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- Nowadays: 22 hours using 48 FPGAs (crack.sh), $> 100\,000$ $

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- Probability of collision $> 60\%$ after encrypting 8TB

  (think, e.g., of full-disk encryption)

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  E.g., double encryption? Triple encryption?

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Double Encryption

Let $F : \{0,1\}^n \times \{0,1\}^\ell \to \{0,1\}^\ell$ be a block-cipher (with key length n and block length ℓ)

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Is F' "twice as strong" as F?

If the best attack on F takes time $\approx 2^n$, does the best attack on F' take time $\approx 2^{2n}$?

Meet-in-the-middle attack

There is a weakness that stems from the fact that F' can be "factored" into two independent components

Given a single input output pair (x, y), with $y = F'_{k_1^* \| k_2^*}(x)$, the adversary can:

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    (easy solution: append all pairs (z, k_1) to a list, then sort the list in time $O(2^n \text{poly}(n))$)

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- Try all possible 2^n choices for k_2
  - For each k_2, compute $z = F_{k_2}^{-1}(y)$
  - Check whether z is in the dictionary. If z is found retrieve the satellite data k_1 and output $k_1 \| k_2$ as a candidate key for F'

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This is not enough...

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$$2^{2n} \cdot 2^{-2\ell} = 2^{2n-2\ell} \qquad < 1 \text{ for Double-DES}$$

Triple Encryption

Double encryption is not more secure than a single encryption...

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Two ways to define triple encryption:

- **Using three keys:** Pick three independent keys $k_1, k_2, k_3 \in \{0,1\}^n$ and let:

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  Time: 2^{2n} (still an improvement over double encryption)

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There are better attacks when many input-output pairs are known. If 2^t pairs are known then the key can be recovered in time

$$\approx 2^{n+\ell-t}$$

3DES

Triple encryption DES has been standardized in 1999 to try to overcome the small key-length of DES

- Two-key 3DES is no longer recommended (also due to the $\approx 2^{n+\ell-t}$ time known-plaintext attack)
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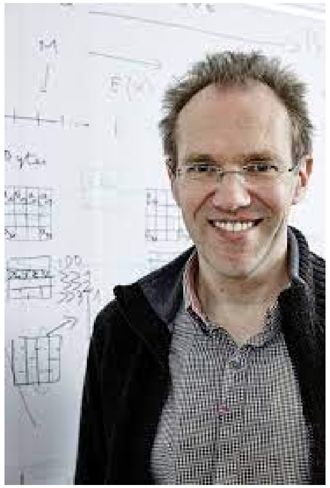
DES and 3DES have been superseded by the **Advanced Encryption Standard (AES)**

Advanced Encryption Standard (AES)

- Winner of a public competition by NIST (National Institute of Standards and Technology) in 1997
- The public and each team that submitted a cipher tried to find vulnerabilities in the (other) ciphers
- 5 finalist were selected, any of them would have been an excellent choice for the winner
- AES (whose name was Rijndael) has been selected based in part on properties such as efficiency, performance in hardware, flexibility, etc.

Vincent Rijmen



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No significant weaknesses currently known!

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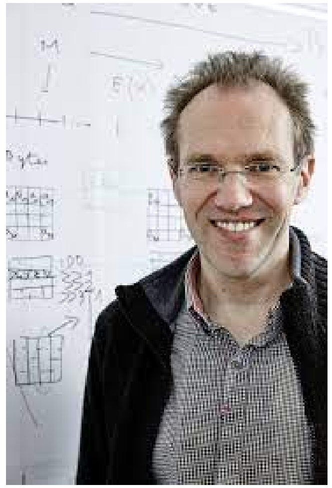

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$$x = b_0\, b_1\, b_2\, b_3\, b_4\, b_5\, b_6\, b_7\, b_8\, b_9\, b_{10}\, b_{11}\, b_{12}\, b_{13}\, b_{14}\, b_{15} \qquad b_i \in \{0,1\}^8$$

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b_3	b_7	b_{11}	b_{15}

Advanced Encryption Standard (AES)

Each round of the SPN modifies the state by performing the following operations:

1) AddRoundKey: A 128-bit subkey is derived from the master key, viewed as a 4×4 matrix and XOR-ed with the state. This is the only step that depends on the key.

$$\begin{array}{|c|c|c|c|}\hline b_0 & b_4 & b_8 & b_{12} \\ \hline b_1 & b_5 & b_9 & b_{13} \\ \hline b_2 & b_6 & b_{10} & b_{14} \\ \hline b_3 & b_7 & b_{11} & b_{15} \\ \hline \end{array} \quad \leftarrow \quad \begin{array}{|c|c|c|c|}\hline k_0 & k_4 & k_8 & k_{12} \\ \hline k_1 & k_5 & k_9 & k_{13} \\ \hline k_2 & k_6 & k_{10} & k_{14} \\ \hline k_3 & k_7 & k_{11} & k_{15} \\ \hline \end{array} \quad \oplus \quad \begin{array}{|c|c|c|c|}\hline b_0 & b_4 & b_8 & b_{12} \\ \hline b_1 & b_5 & b_9 & b_{13} \\ \hline b_2 & b_6 & b_{10} & b_{14} \\ \hline b_3 & b_7 & b_{11} & b_{15} \\ \hline \end{array}$$

The generic entry b_i is updated to $b_i \oplus k_i$

Advanced Encryption Standard (AES)

Each round of the SPN modifies the state by performing the following operations:

2) SubBytes: Each byte b_i is replaced by another byte $S(b_i)$ where S is a **single, fixed** permutation on $\{0,1\}^8$

$$\begin{array}{|c|c|c|c|}\hline b_0 & b_4 & b_8 & b_{12} \\ \hline b_1 & b_5 & b_9 & b_{13} \\ \hline b_2 & b_6 & b_{10} & b_{14} \\ \hline b_3 & b_7 & b_{11} & b_{15} \\ \hline \end{array} \quad \longleftarrow \quad \begin{array}{|c|c|c|c|}\hline S(b_0) & S(b_4) & S(b_8) & S(b_{12}) \\ \hline S(b_1) & S(b_5) & S(b_9) & S(b_{13}) \\ \hline S(b_2) & S(b_6) & S(b_{10}) & S(b_{14}) \\ \hline S(b_3) & S(b_7) & S(b_{11}) & S(b_{15}) \\ \hline \end{array}$$

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Each round of the SPN modifies the state by performing the following operations:

3) ShiftRows: The bytes in each row in the matrix undergo a cyclic left shift. The i-th row, counting from 0, is shifted by i places (row 0 is unaffected).

b_0	b_4	b_8	b_{12}
b_1	b_5	b_9	b_{13}
b_2	b_6	b_{10}	b_{14}
b_3	b_7	b_{11}	b_{15}

b_0	b_4	b_8	b_{12}
b_5	b_9	b_{13}	b_1
b_{10}	b_{14}	b_2	b_6
b_{15}	b_3	b_7	b_{11}

Advanced Encryption Standard (AES)

Each round of the SPN modifies the state by performing the following operations:

4) MixColumns: An invertible linear transformation is applied to each column. This transformation has the property that if two inputs differ in $b > 0$ bytes, then the resulting outputs differ in at least $5 - b$ bytes.

b_0	b_4	b_8	b_{12}
b_1	b_5	b_9	b_{13}
b_2	b_6	b_{10}	b_{14}
b_3	b_7	b_{11}	b_{15}

$$\begin{bmatrix} b_0 \\ b_1 \\ b_2 \\ b_3 \end{bmatrix} \leftarrow \begin{bmatrix} 2 & 3 & 1 & 1 \\ 1 & 2 & 3 & 1 \\ 1 & 1 & 2 & 3 \\ 3 & 1 & 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} b_0 \\ b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

Multiplication and additions are done over the finite field GF(2^8)

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In the final round, the **MixColumns** step is replaced with **AddRoundKey**

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This is because the **SubBytes**, **MixRows**, and **MixColumns** do not depend on the key

Without the final **AddRoundKey** step, an adversary could simply invert the last three steps of the last round

Advanced Encryption Standard (AES)