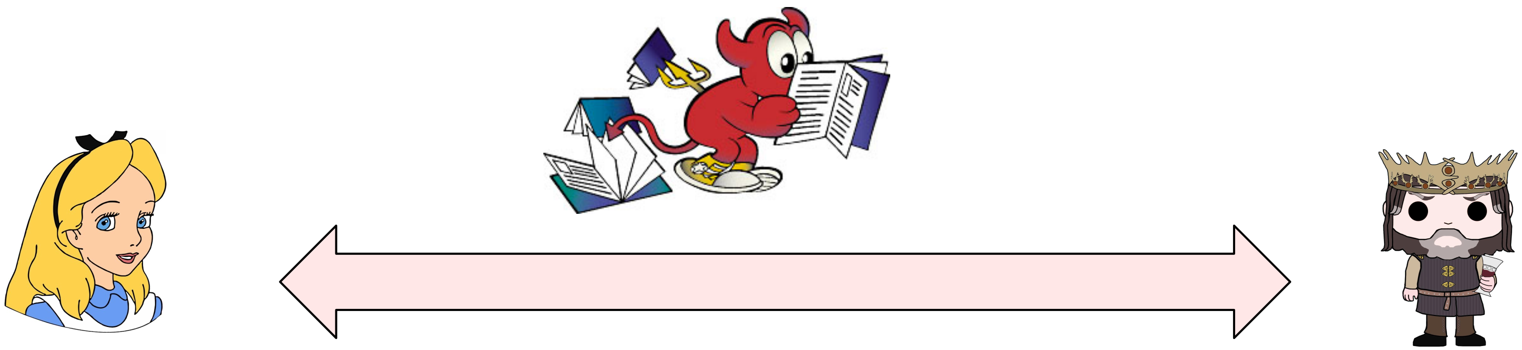
Passive vs Active Attacks

So far we have mainly considered passive attacks

- The attacker simply observed the ciphertexts transmitted over the communication channel
- At best, it influences Alice and Bob's choice of the plaintexts , but it never tampers with the data in transit



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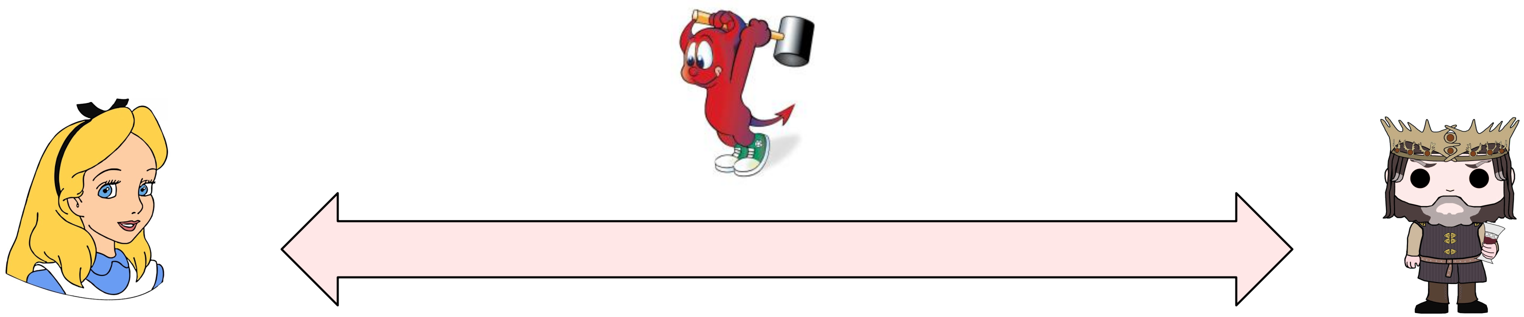

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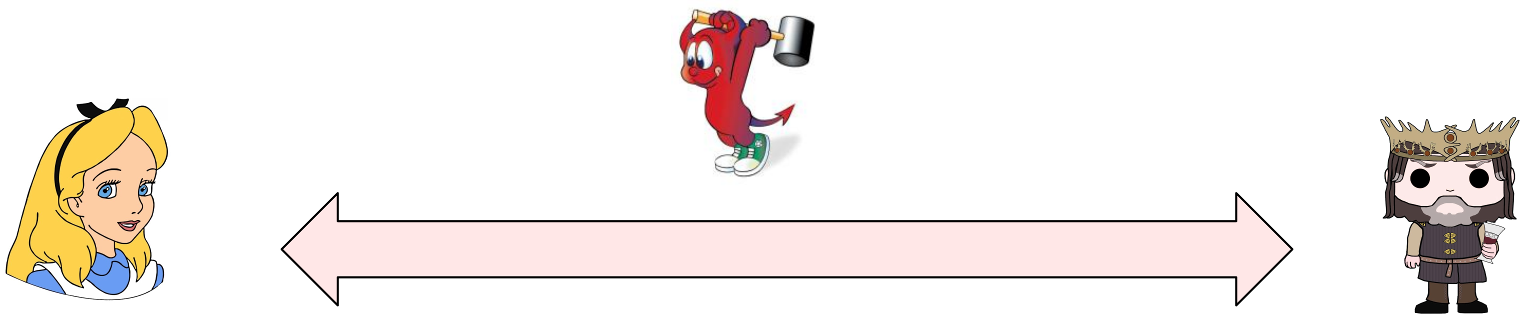

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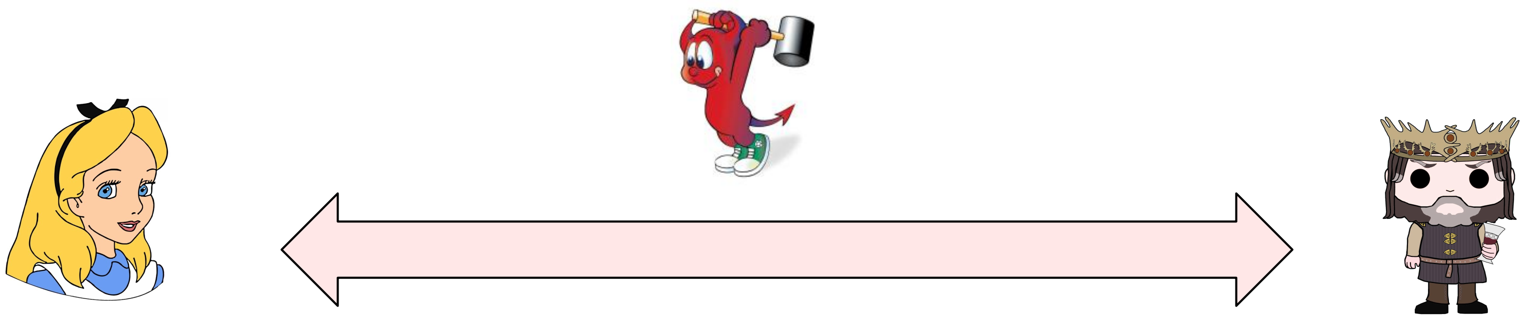

Passive vs Active Attacks

We now consider **active** attacks:

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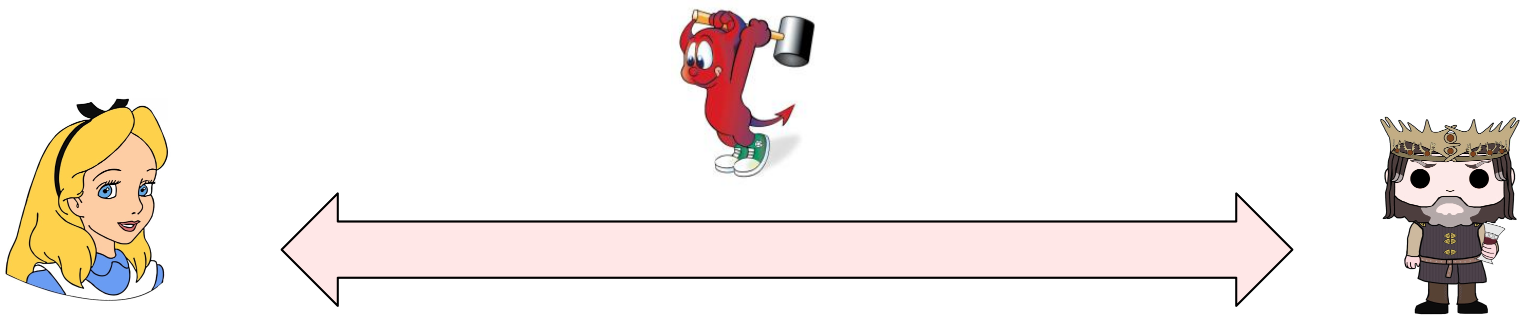
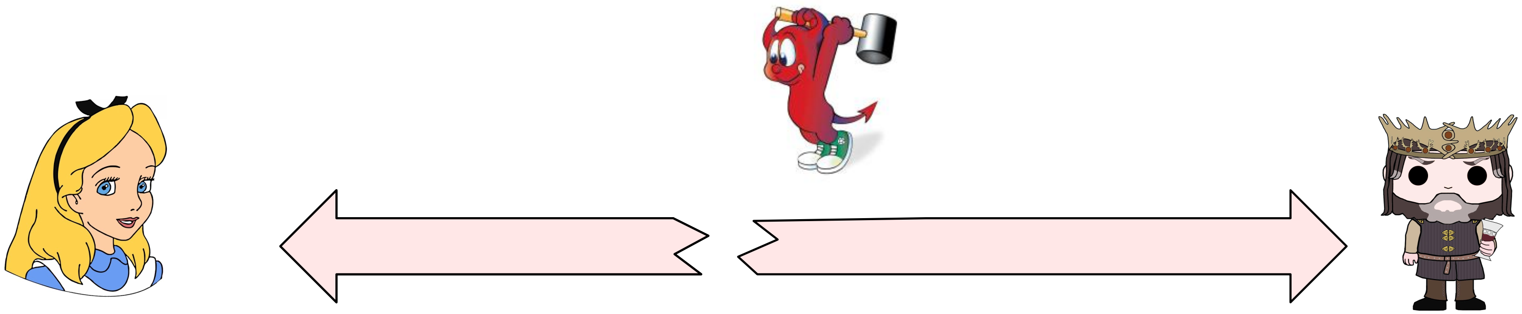
- Can alter the message contents
- Can drop messages
- Can forge new messages



Denial of Service

An adversary this powerful can always stop any communication between Alice and Bob (by simply dropping all messages)...



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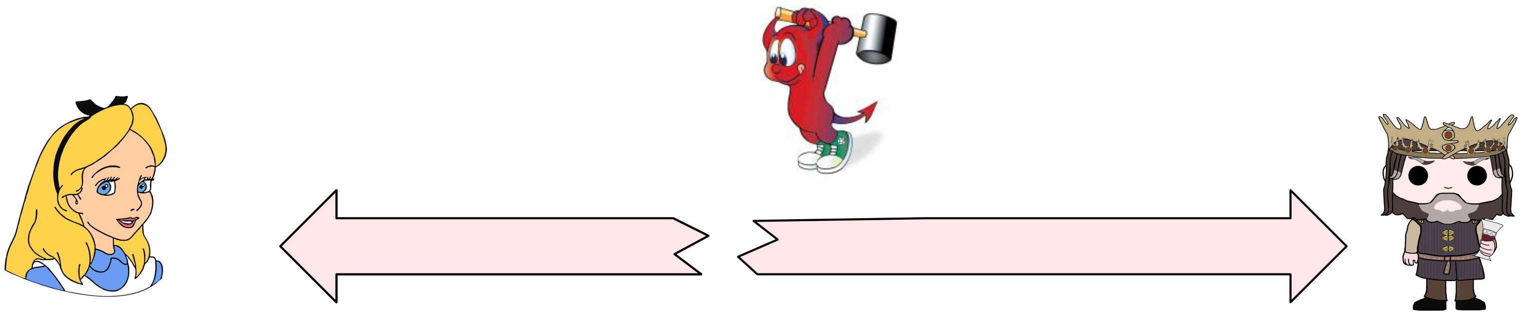

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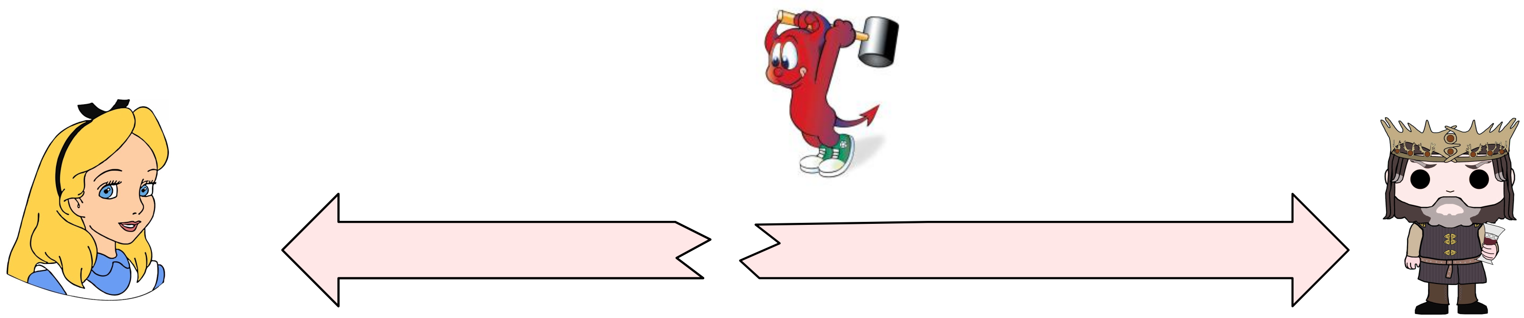

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We are interested in what security guarantees we can achieve *when communication does happen*

Secrecy vs Integrity

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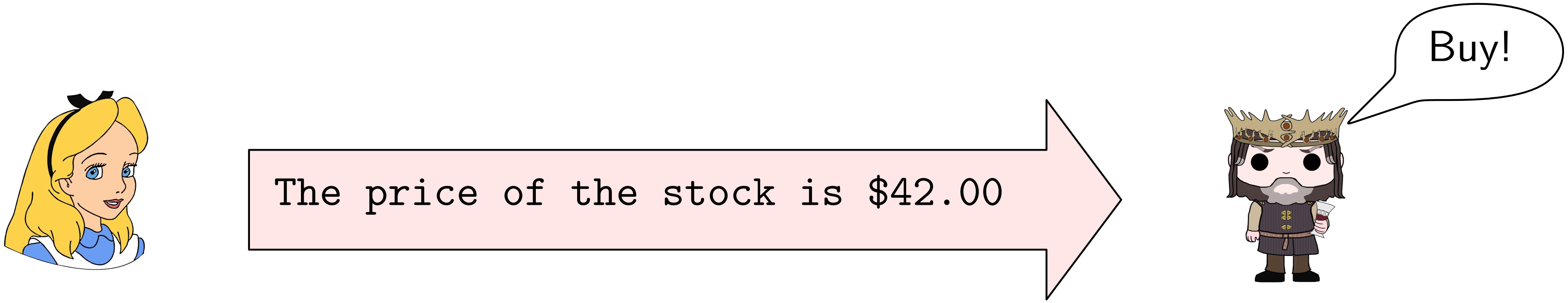
Integrity and Secrecy are **orthogonal** concerns

Secrecy vs Integrity



- Not a secret information!
- No need to encrypt
- Need to check that it comes from a trusted party
- Need to check that the amount has not been tampered with

Encryption schemes for Integrity?

In all the schemes we have seen so far:

- A modified ciphertext can be decrypted without any issue (and it yields a different plaintext)
- Any random string is a valid ciphertext!
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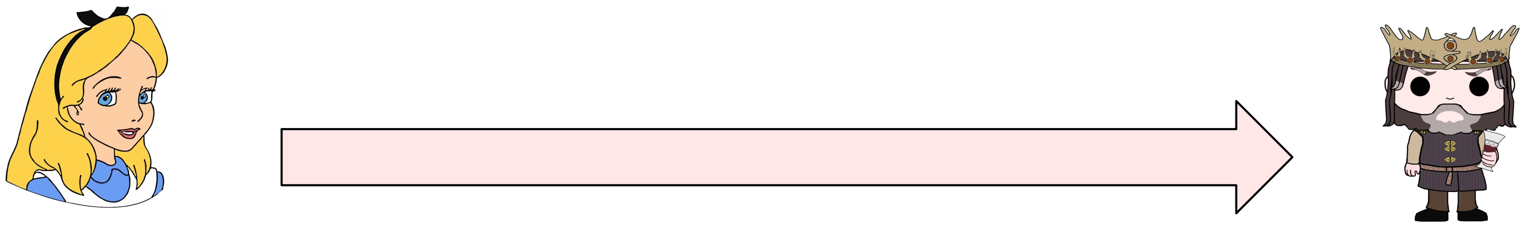
Encryption schemes are not the right tool to guarantee integrity

Message Authentication Codes (MACs)

The right tool are **Message Authentication Codes**



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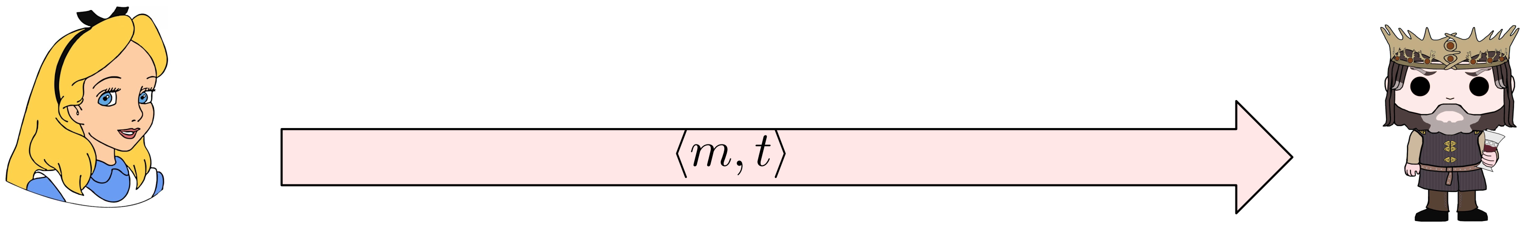

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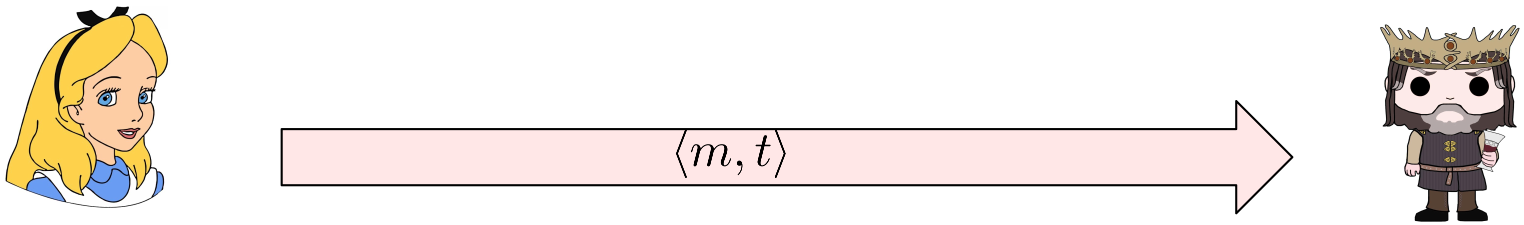

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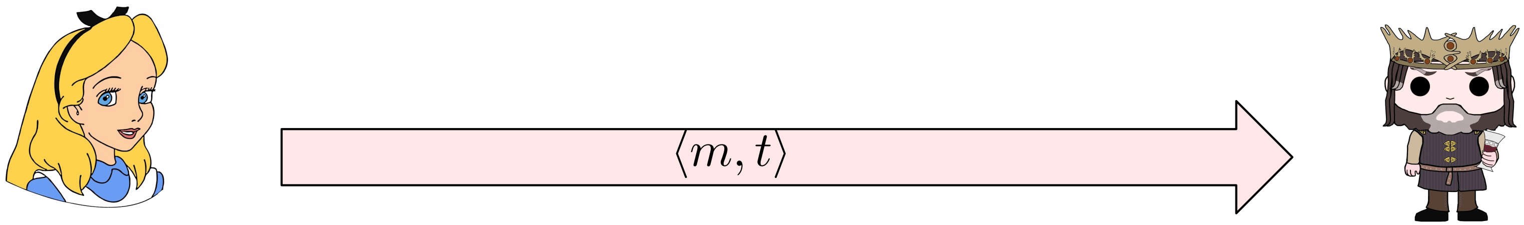

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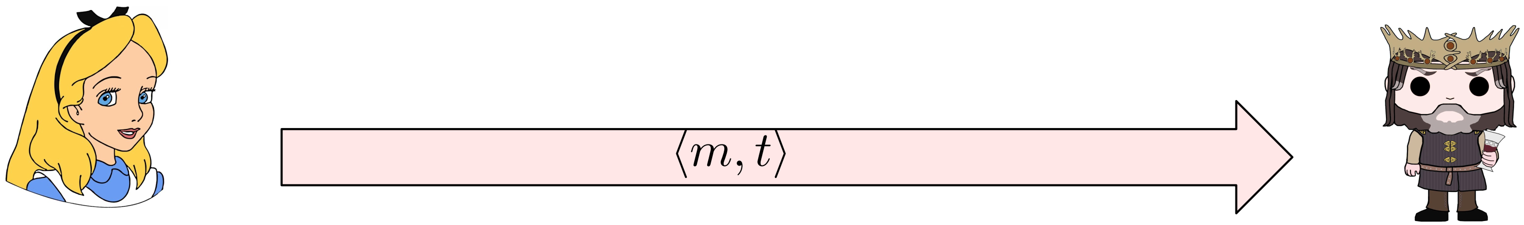

Security?

- Intuitively, no (efficient) adversary can forge t

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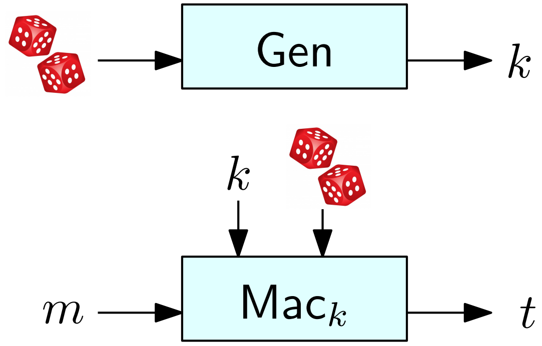

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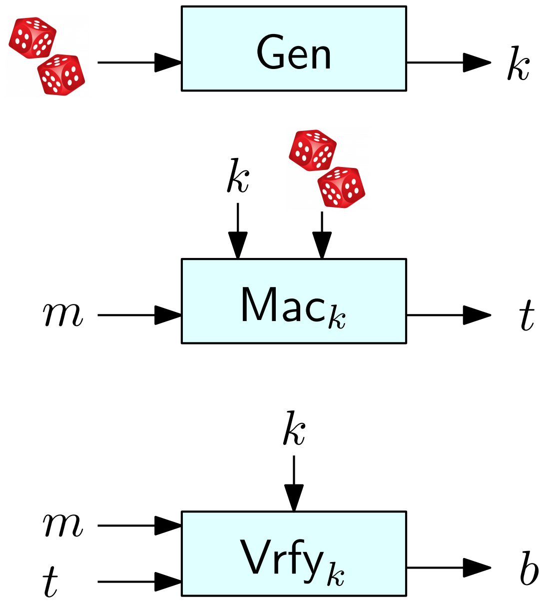

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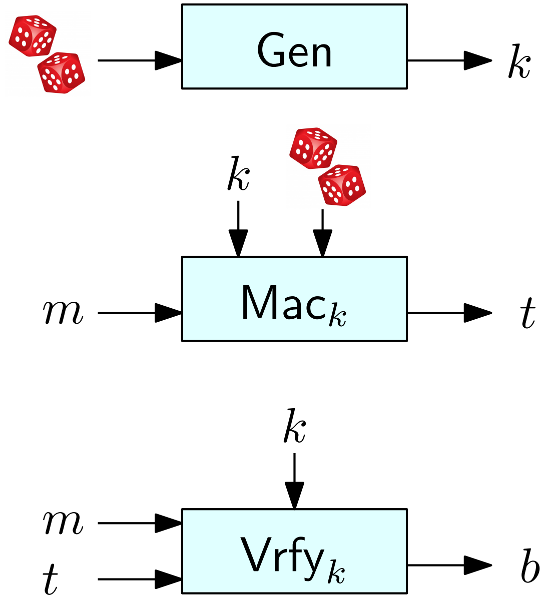

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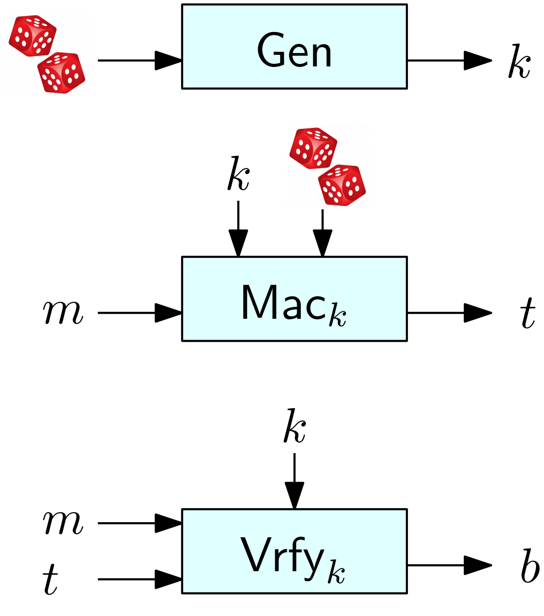

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If Mac is only defined for messages $m \in \{0,1\}^{\ell(n)}$ we call $(\mathsf{Gen}, \mathsf{Mac}, \mathsf{Vrfy})$ a **fixed-length** MAC for messages of length $\ell(n)$.

Message Authentication Codes (MACs)

In the special case in which Mac is a deterministic algorithm, we can use the following **canonical verification** algorithm:

Vrfy$_k$(m, t):

- $\widetilde{t} \leftarrow \mathsf{Mac}_k(m)$
- If $\widetilde{t} = t$:
    - Return $b = 1$
- Else:
    - Return $b = 0$

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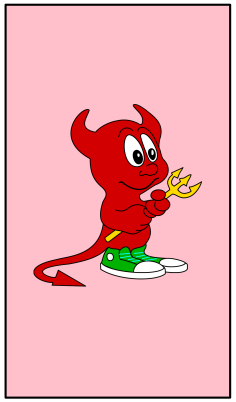
Security Goal: Existential unforgeability

- No efficient attacker should be able to provide a valid tag for any message that was not previously authenticated by the sender, except with negligible probability.

The Message Authentication Experiment

Let $\Pi = (\mathsf{Gen}, \mathsf{Mac}, \mathsf{Vrfy})$ be a MAC. We name the following experiment $\mathsf{Mac\text{-}forge}_{\mathcal{A},\Pi}(n)$:

- A key k is generated using $\mathsf{Gen}(1^n)$



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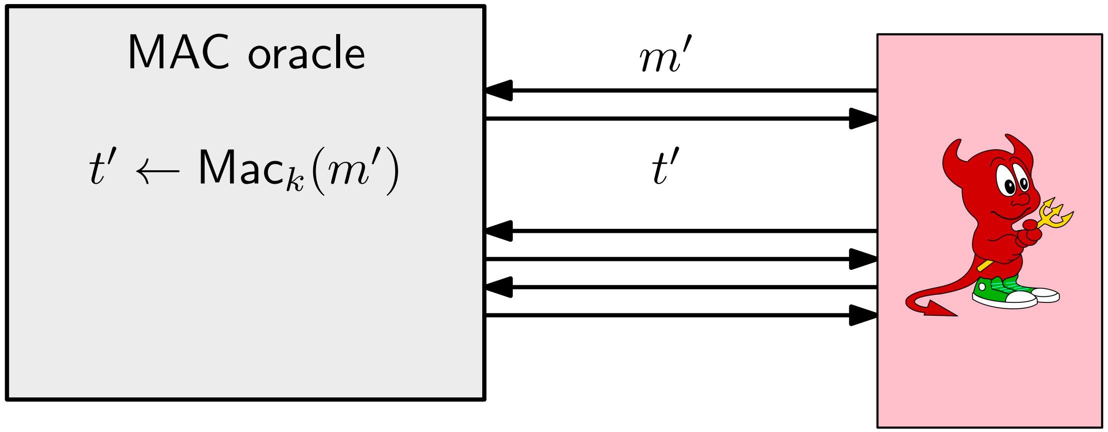

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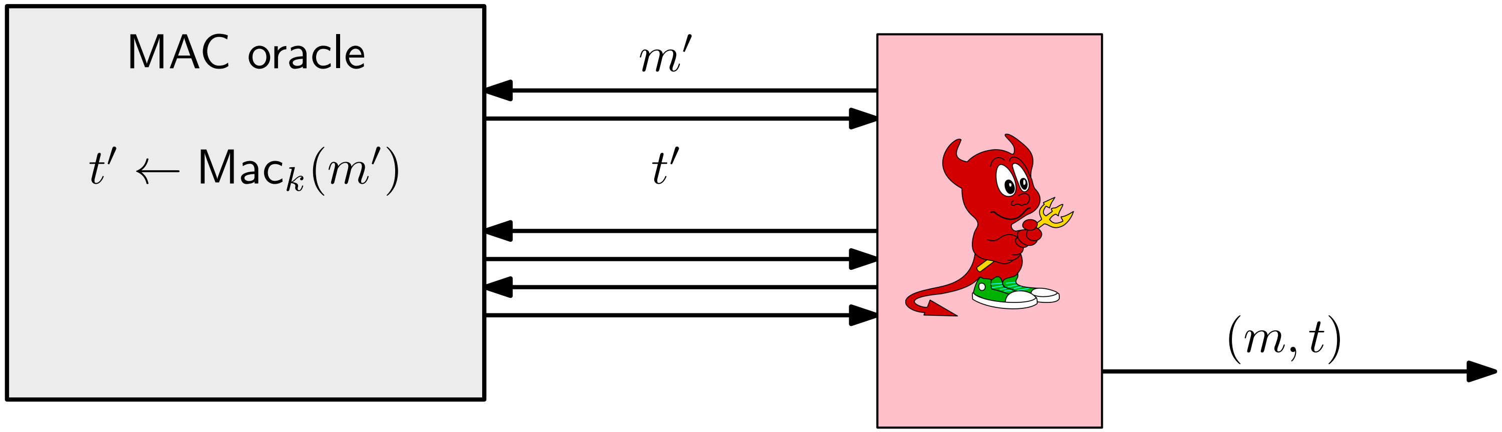

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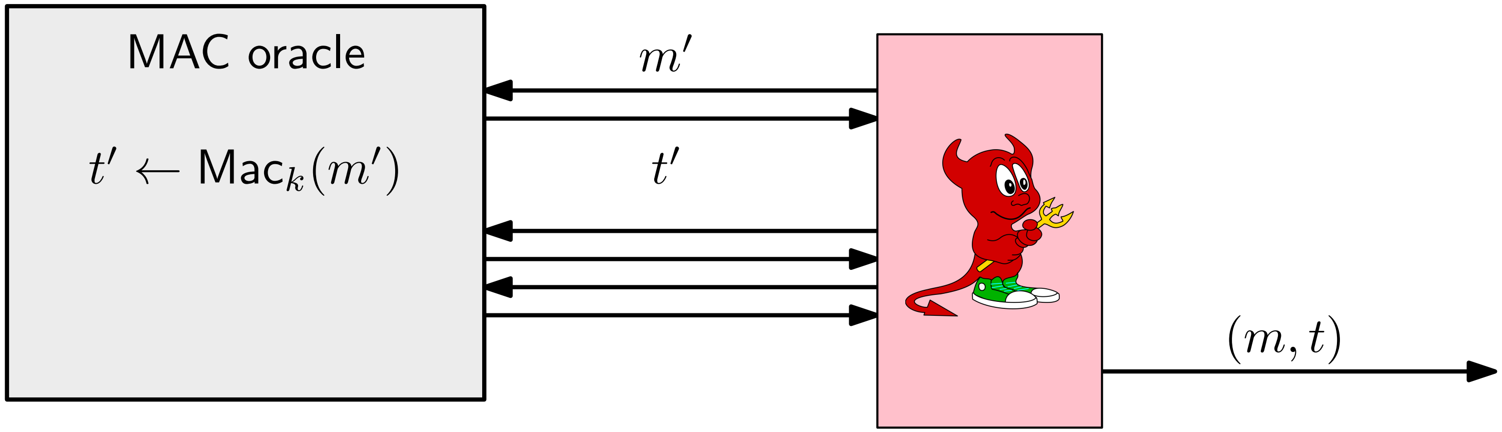
- The outcome of the experiment is 1 if (*) holds and $\mathsf{Vrfy}_k(m, t) = 1$. Otherwise the outcome is 0.

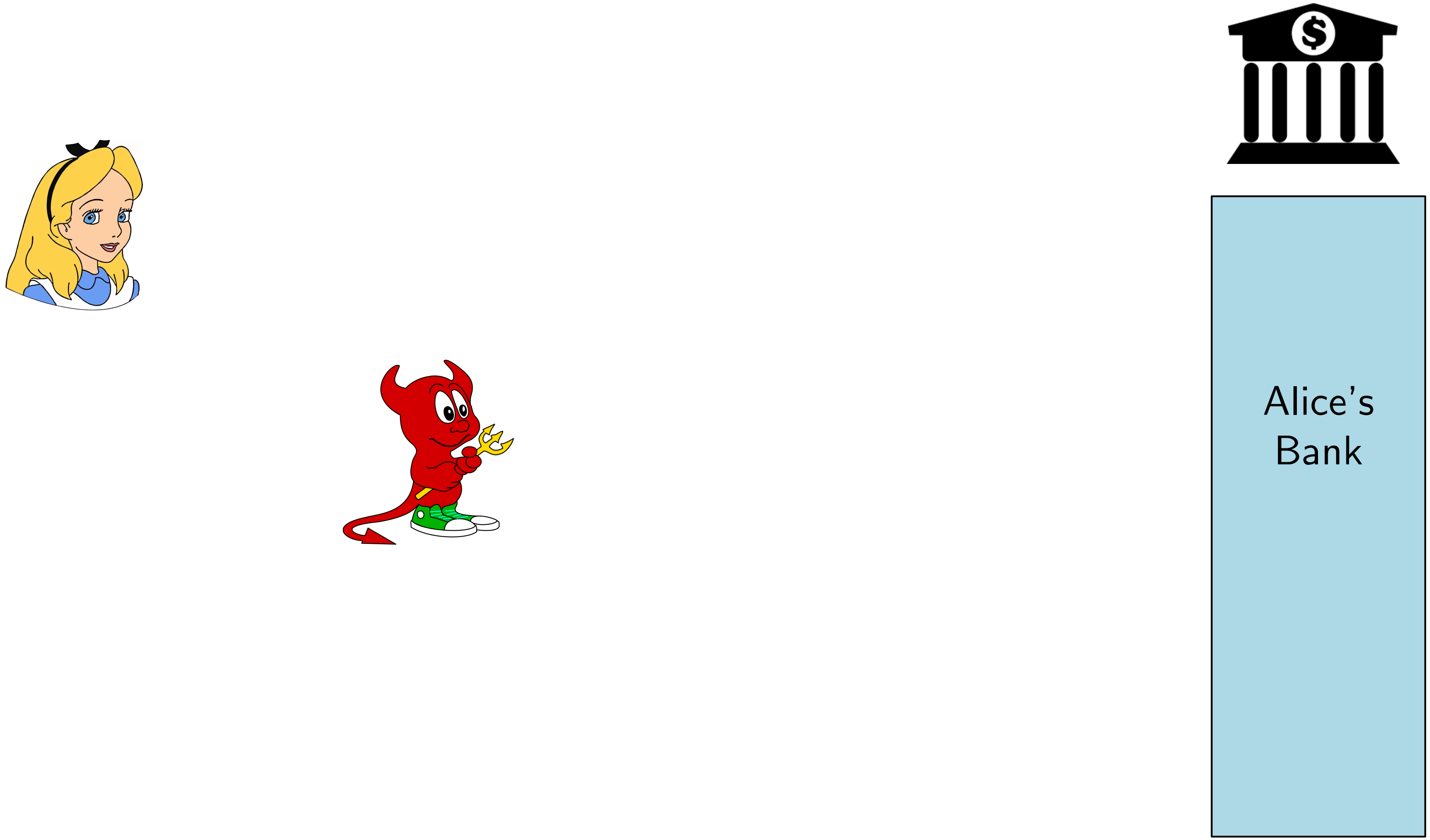


Secure MACs

Definition: *A message authentication code* Π *is existentially unforgeable under an adaptive chosen-message attack (is* **secure***) if, for every probabilistic polynomial-time adversary* $\mathcal{A}$*, there is a negligible function* ε *such that:*

$$\Pr[\textit{Mac-forge}_{\mathcal{A},\Pi}(n) = 1] \leq \varepsilon(n)$$

Replay attacks



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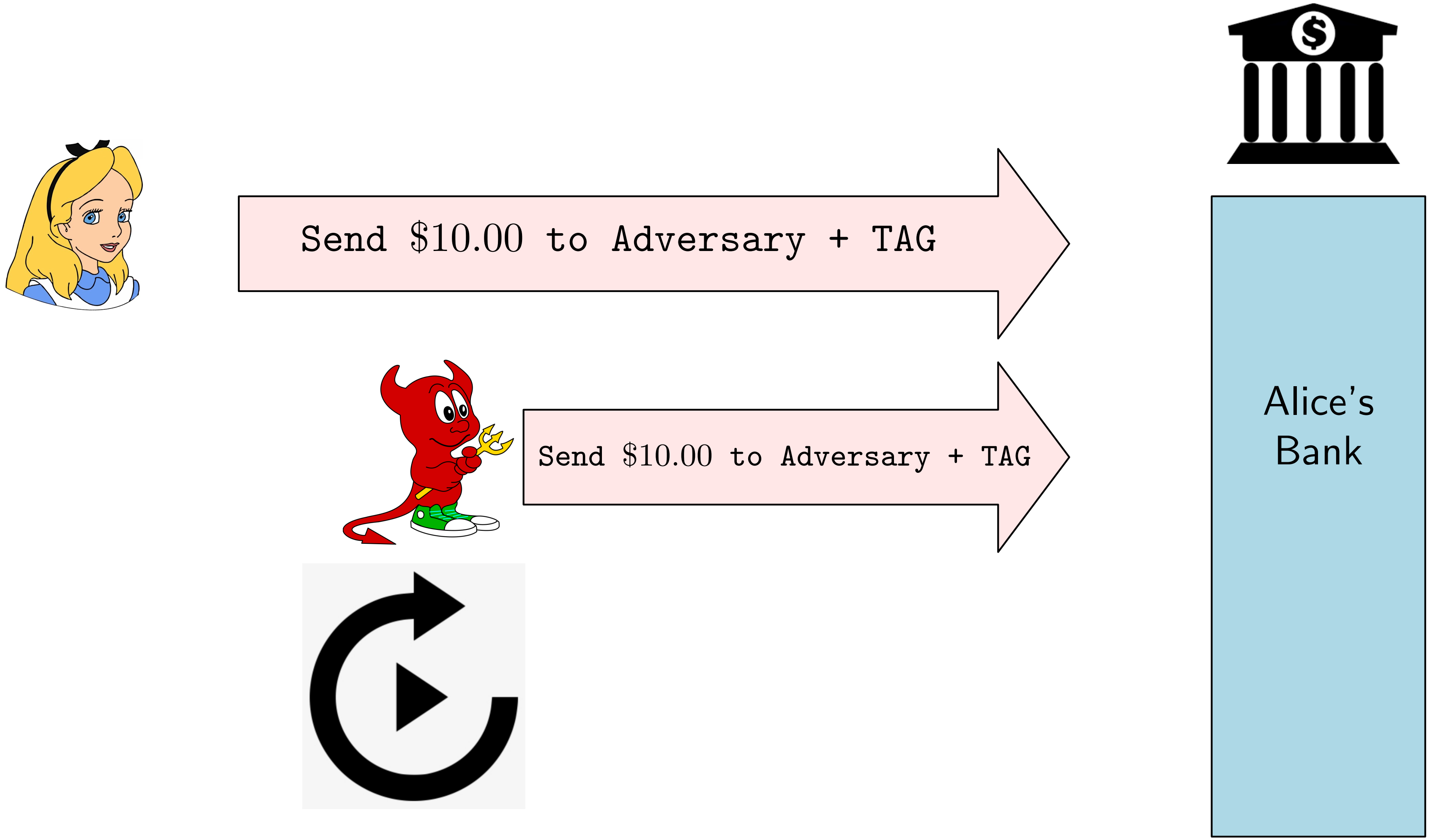

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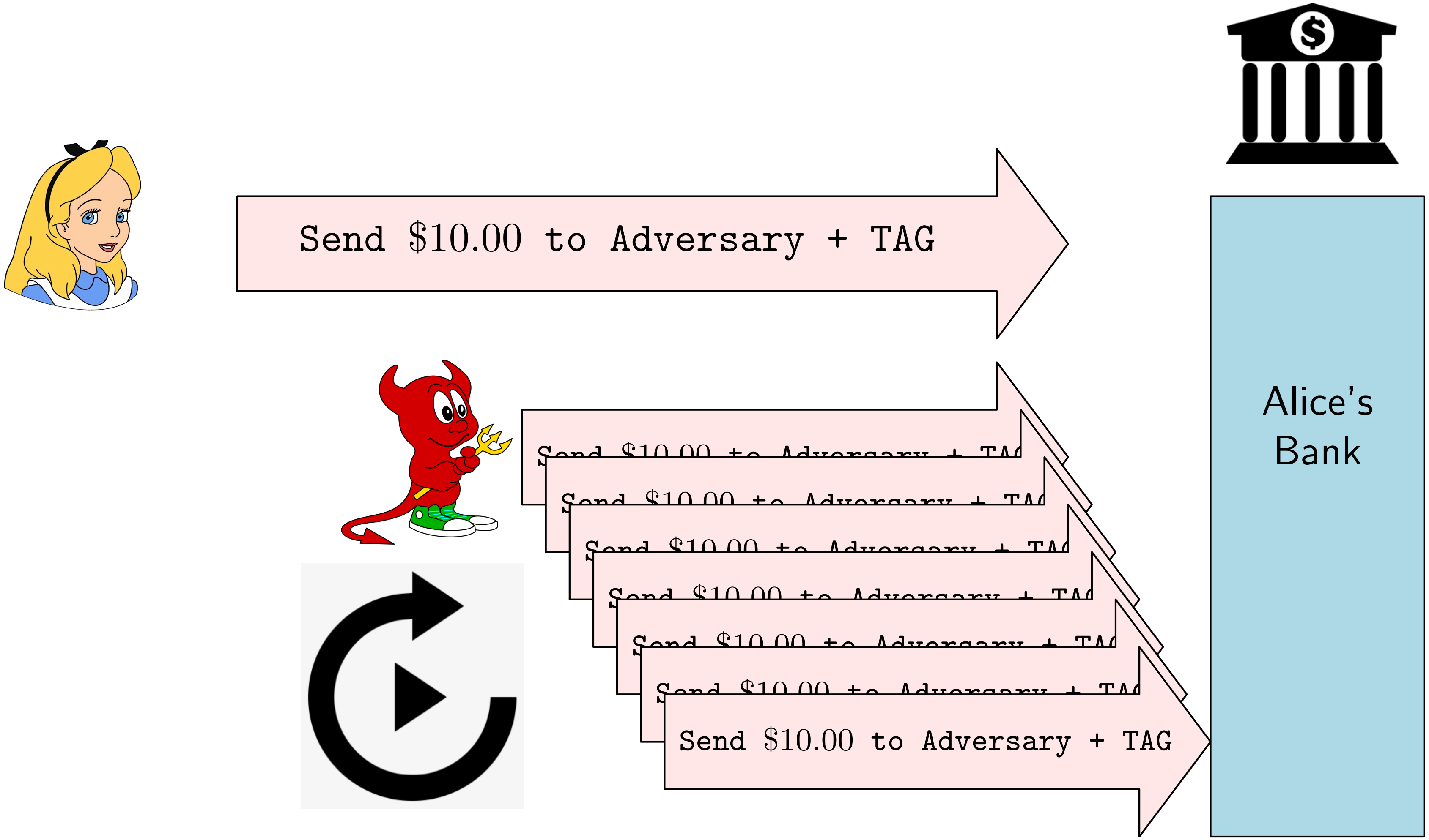

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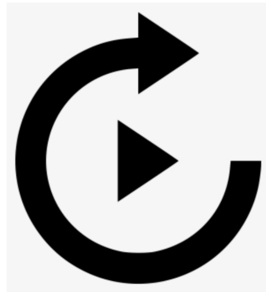

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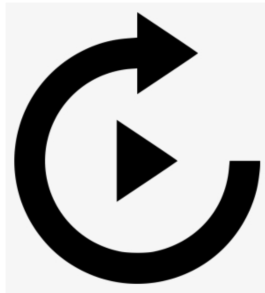

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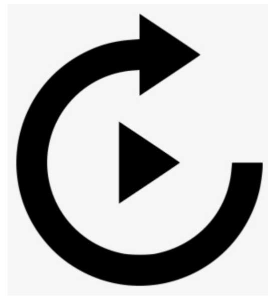

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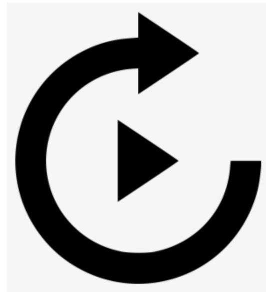

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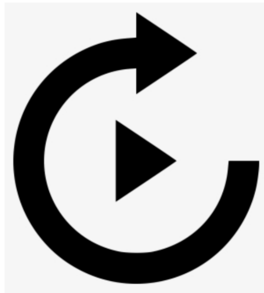

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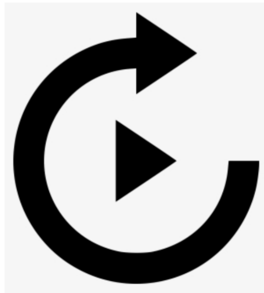

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A fixed-length MAC

Intuition: We want some keyed function $\mathsf{Mac}_k(\cdot)$ such that, even if we know $m_1, m_2, \ldots$, and $\mathsf{Mac}_k(m_1), \mathsf{Mac}_k(m_2), \ldots$ it is infeasible to predict $\mathsf{Mac}_k(m)$ for some $m \notin \{m_1, m_2, \ldots\}$

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Given a length-preserving keyed function F, we can build the following MAC Π:

- $\mathsf{Gen}(1^n)$ returns a random key for F
- $\mathsf{Mac}_k(m)$ returns $F_k(m)$
- $\mathsf{Vrfy}_k(m, t)$ is the canonical verification algorithm (output 1 iff $F_k(m) = t$ and 0 otherwise)

A fixed-length MAC

Intuition: We want some keyed function $\mathsf{Mac}_k(\cdot)$ such that, even if we know $m_1, m_2, \ldots$, and $\mathsf{Mac}_k(m_1), \mathsf{Mac}_k(m_2), \ldots$ it is infeasible to predict $\mathsf{Mac}_k(m)$ for some $m \notin \{m_1, m_2, \ldots\}$

We already have a function with this property...

Let **Mac**$_k$ be a pseudorandom function!

Given a length-preserving keyed function F, we can build the following MAC Π:

- $\mathsf{Gen}(1^n)$ returns a random key for F
- $\mathsf{Mac}_k(m)$ returns $F_k(m)$
- $\mathsf{Vrfy}_k(m, t)$ is the canonical verification algorithm (output 1 iff $F_k(m) = t$ and 0 otherwise)

Theorem: *If F is a pseudorandom function, then the MAC Π constructed from F as above is secure.*

Proof of security

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Usual proof strategy:

- Assume that there is some polynomial-time adversary $\mathcal{A}$ that wins Mac-forge$_{\mathcal{A},\Pi}(n)$ with non-negligible probability
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Reminder:

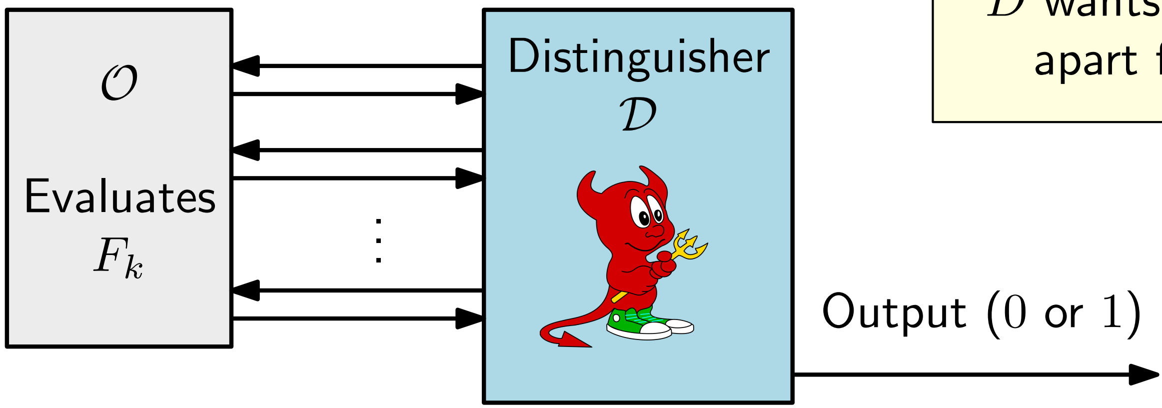
Definition: An efficient, length preserving, keyed function $F : \{0,1\}^n \times \{0,1\}^n \to \{0,1\}^n$ is a **pseudorandom function** if for all probabilistic polynomial-time distinguishers D, there is a negligible function ε such that:

$$\left|\ \Pr[D^{F_k(\cdot)}(1^n) = 1] - \Pr[D^{f(\cdot)}(1^n) = 1]\ \right| \leq \varepsilon(n)$$

Reminder: distinguishers for pseudorandom functions

"World 0":

k is chosen u.a.r. in $\{0,1\}^n$

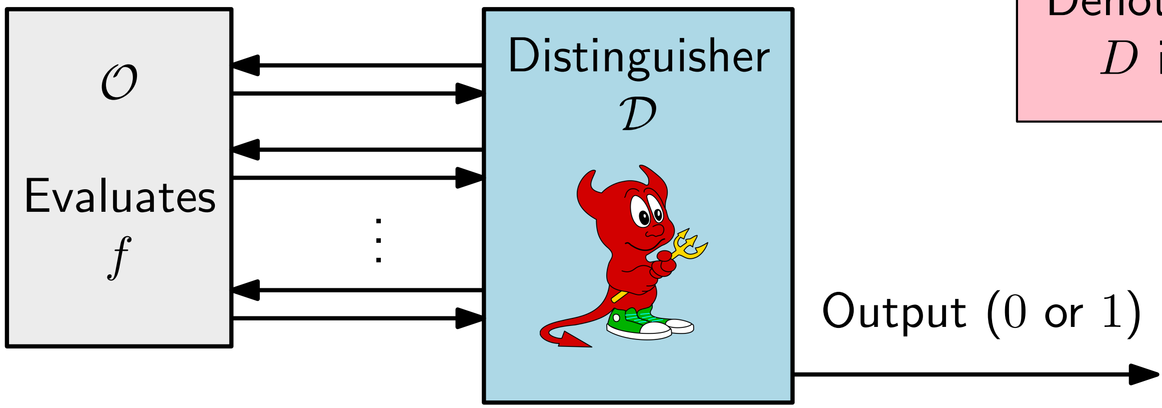


$D^{F_k(\cdot)}(1^n)$

D wants to tell "World 0" apart from "World 1"

"World 1":

f is chosen u.a.r. in Func_n



$D^{f(\cdot)}(1^n)$

Denotes the kind of oracle D is interacting with

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Distinguisher $\mathcal{D}^{\Phi}(1^n)$:

- Simulate the execution of $\mathcal{A}$

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- Whenever A queries its oracle with a message m':
    - Query Φ with m' and obtain a response t'
    - Answer t' to $\mathcal{A}$ (say that t' is a tag for m')
- Whenever A outputs (m, t) (at the end of its execution):
    - Query Φ with m and obtain a response t^*
    - Return 1 iff $t^* = t$ (return 0 otherwise)

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Non-negligible!

$\implies$ F is not a pseudorandom function! $\lightning$ $\square$

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This construction only works for messages having the same length as the inputs to F

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Existing practical construction of pseudorandom functions (i.e., block ciphers) take short, fixed-length, inputs

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How do we get a MAC for arbitrary length messages?

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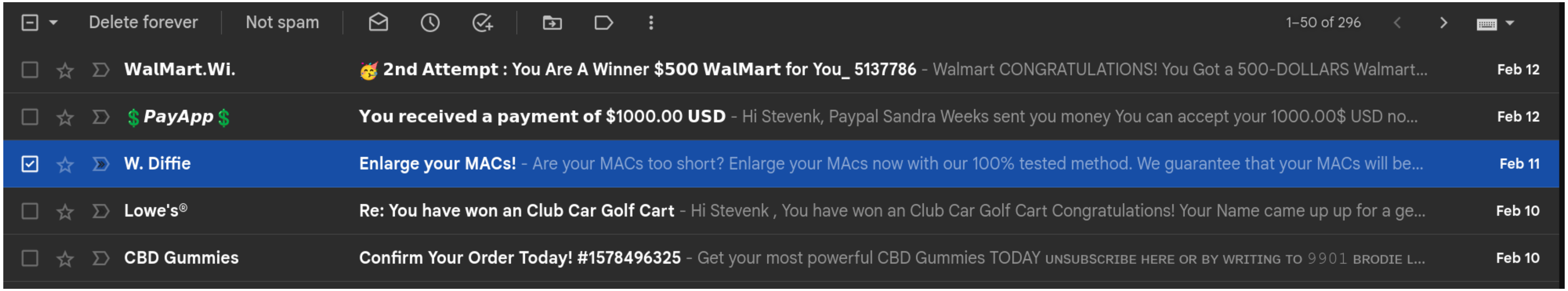
How do we get a MAC for arbitrary length messages?

Domain extension for MACs



Domain Extension for MACS

A first idea:

Split the message into blocks $m_1, m_2 \ldots$ of length ℓ

$$m = \boxed{m_1}\boxed{m_2}\boxed{m_3}\boxed{m_4}\boxed{m_5}$$

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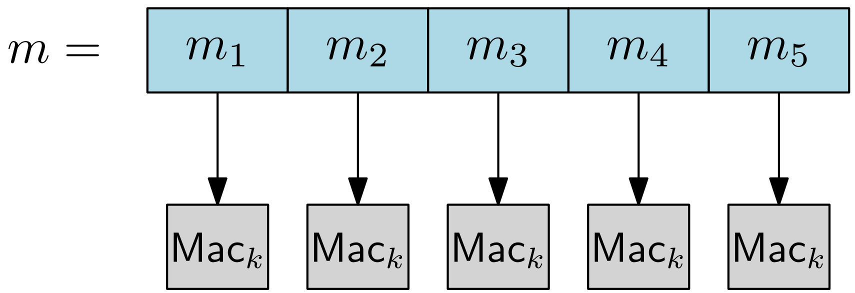

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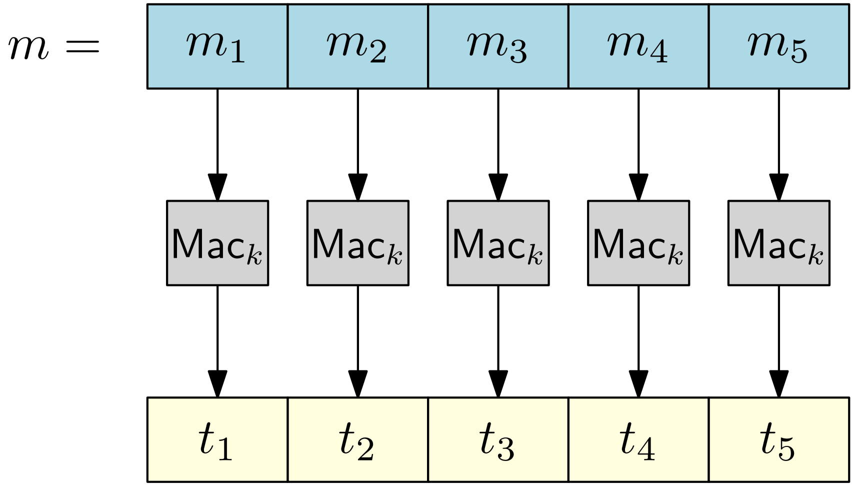

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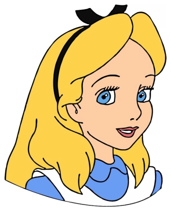
Output $t_1 \,\|\, t_2 \| \, t_3 \,\| \ldots$

Domain Extension for MACS

Does it work?



	m_1	m_2	m_3	m_4				
$m =$	`Do not`	`attack.`	`Commence`	`retreat.`	t_1	t_2	t_3	t_4

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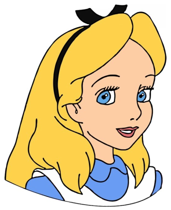

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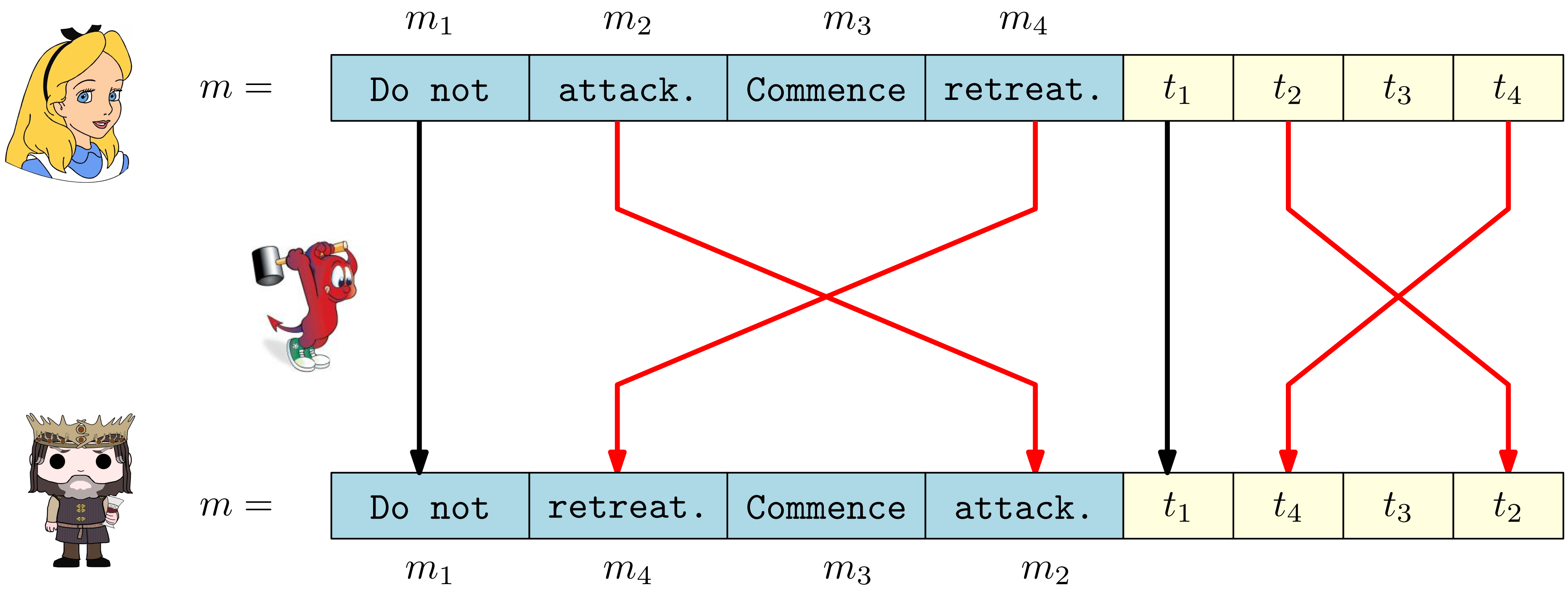

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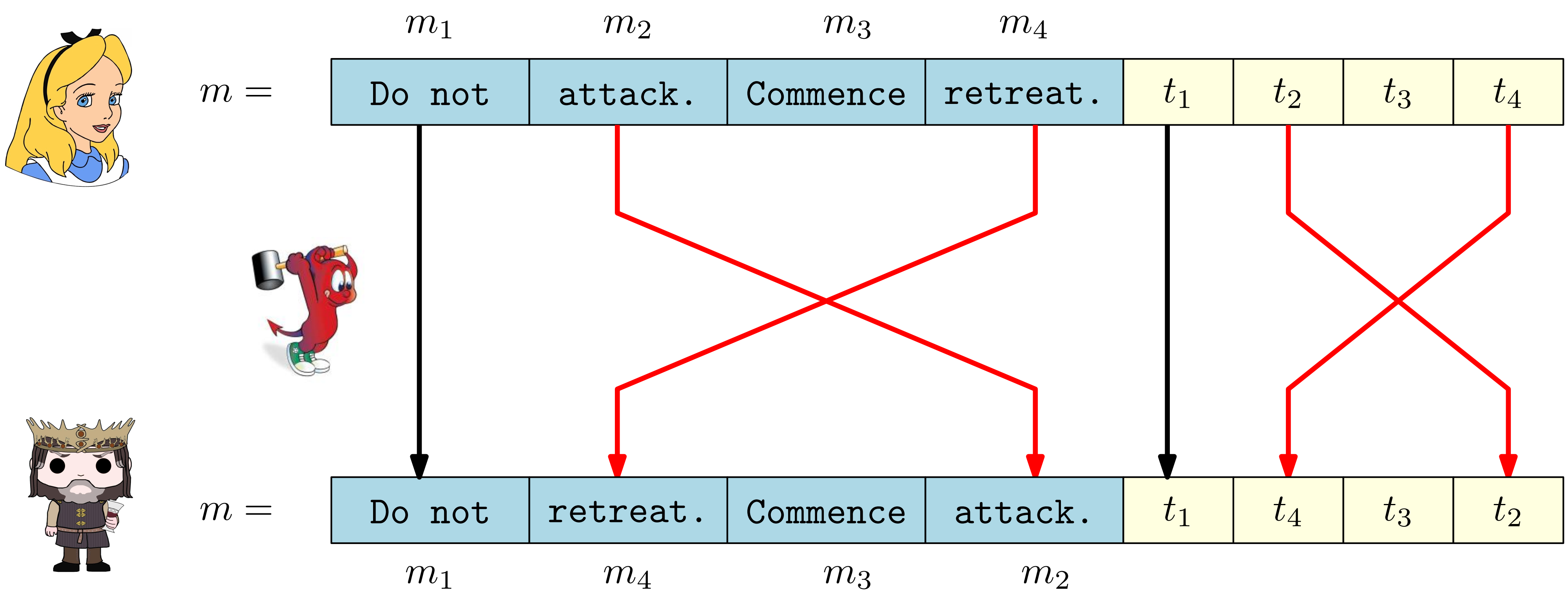

- Vulnerable to **block re-ordering attacks**

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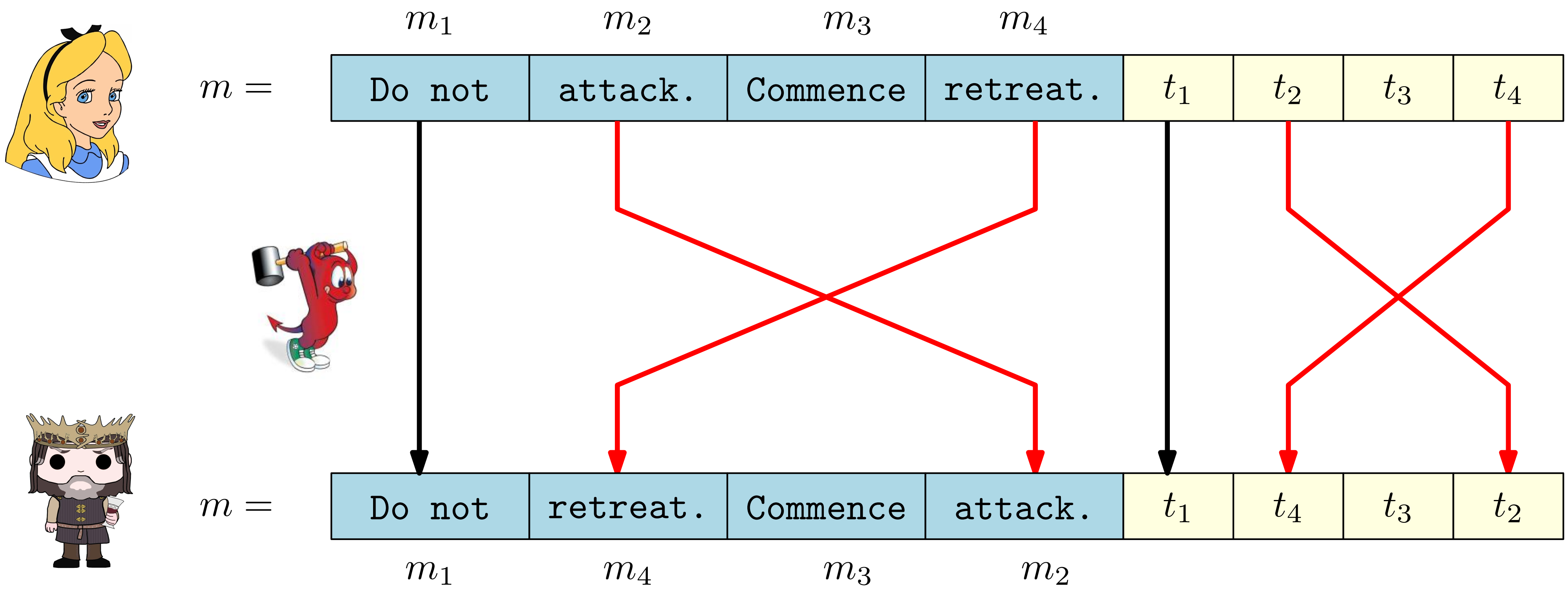
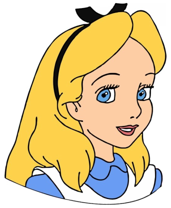
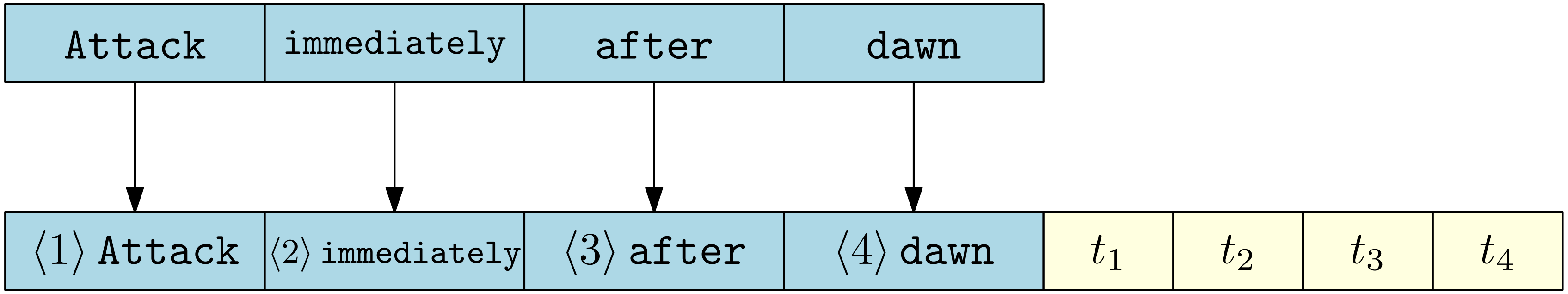


- Vulnerable to **block re-ordering attacks**
- We can prevent such attacks by adding a block index to each block

Domain Extension for MACS

Is the resulting MAC secure?





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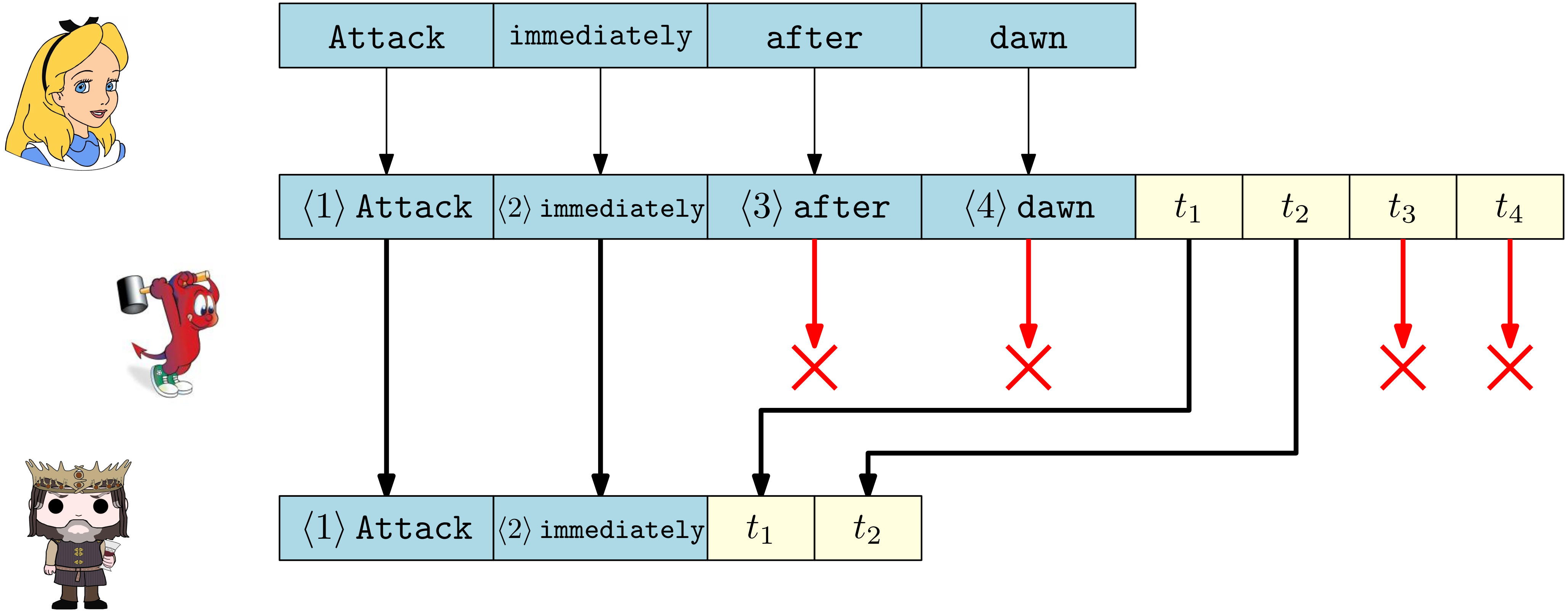

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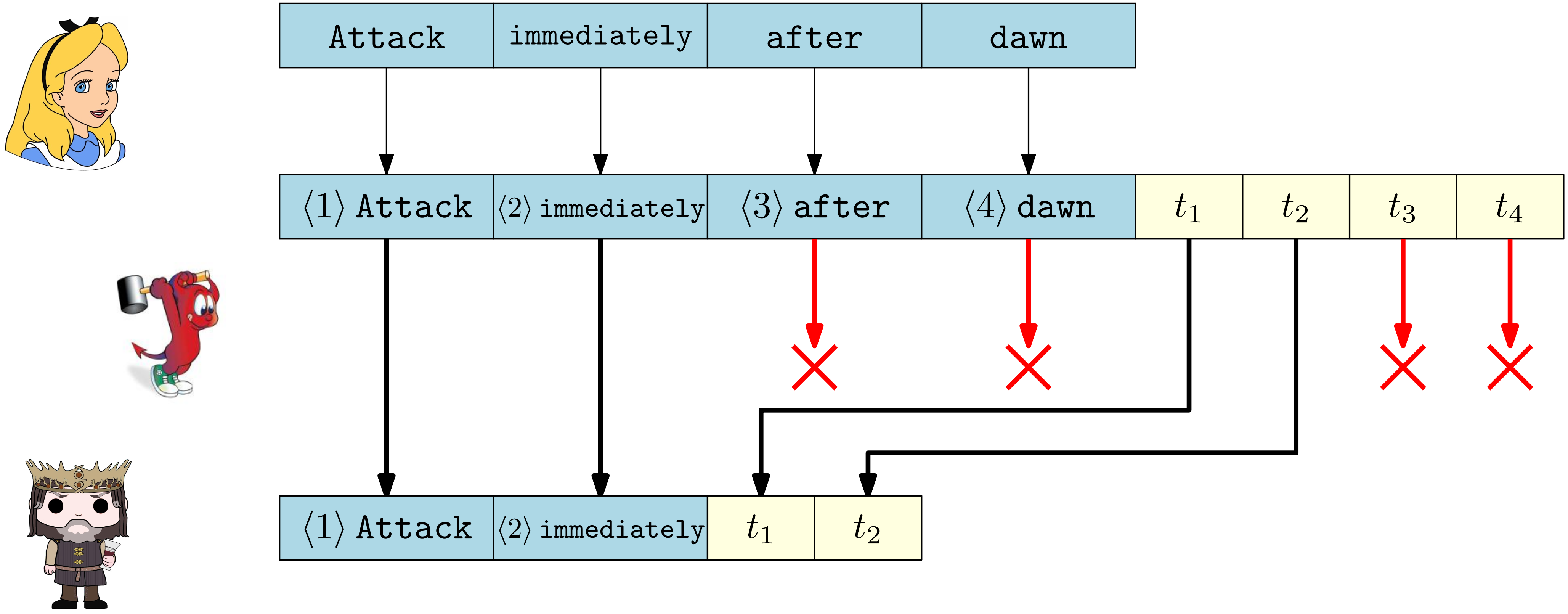

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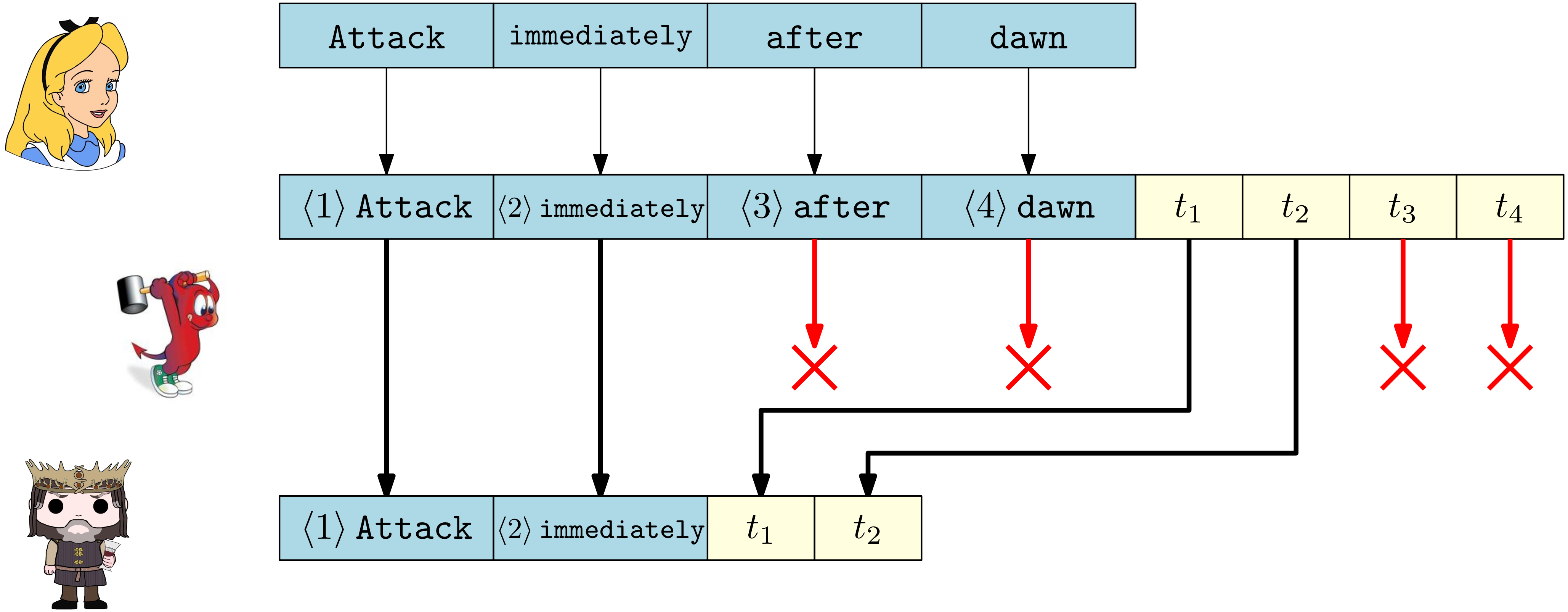

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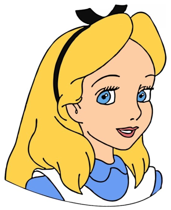

$\langle 27, 1\rangle$ `Fire`	$\langle 27, 2\rangle$ `John Doe`	$\langle 27, 3\rangle$ `for his`	$\langle 27, 4\rangle$ `theft`	t_1^1	t_2^1	t_3^1	t_4^1

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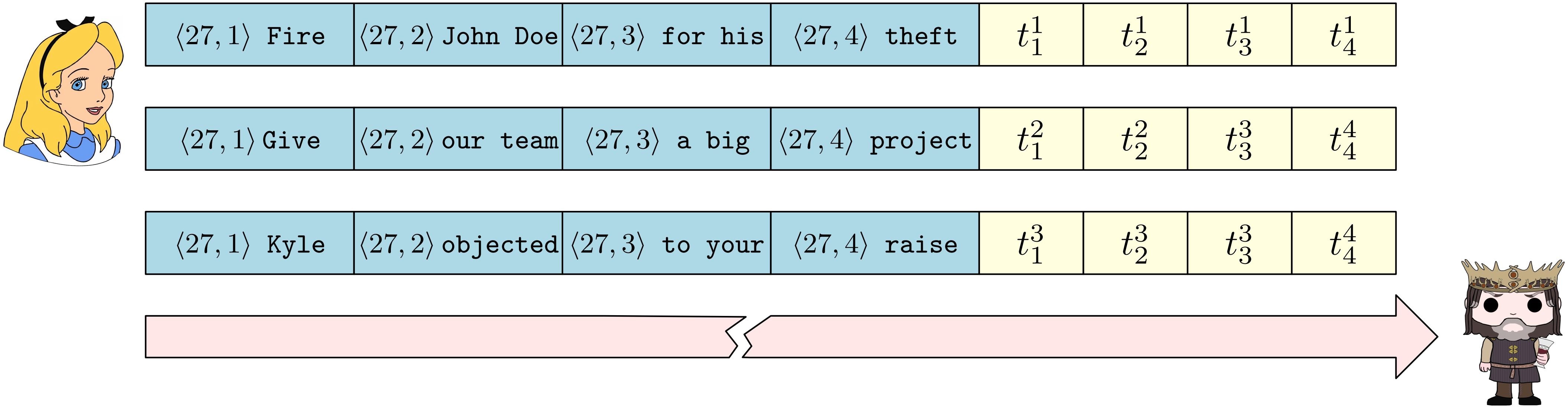

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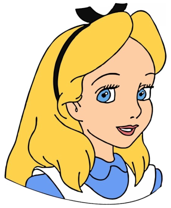

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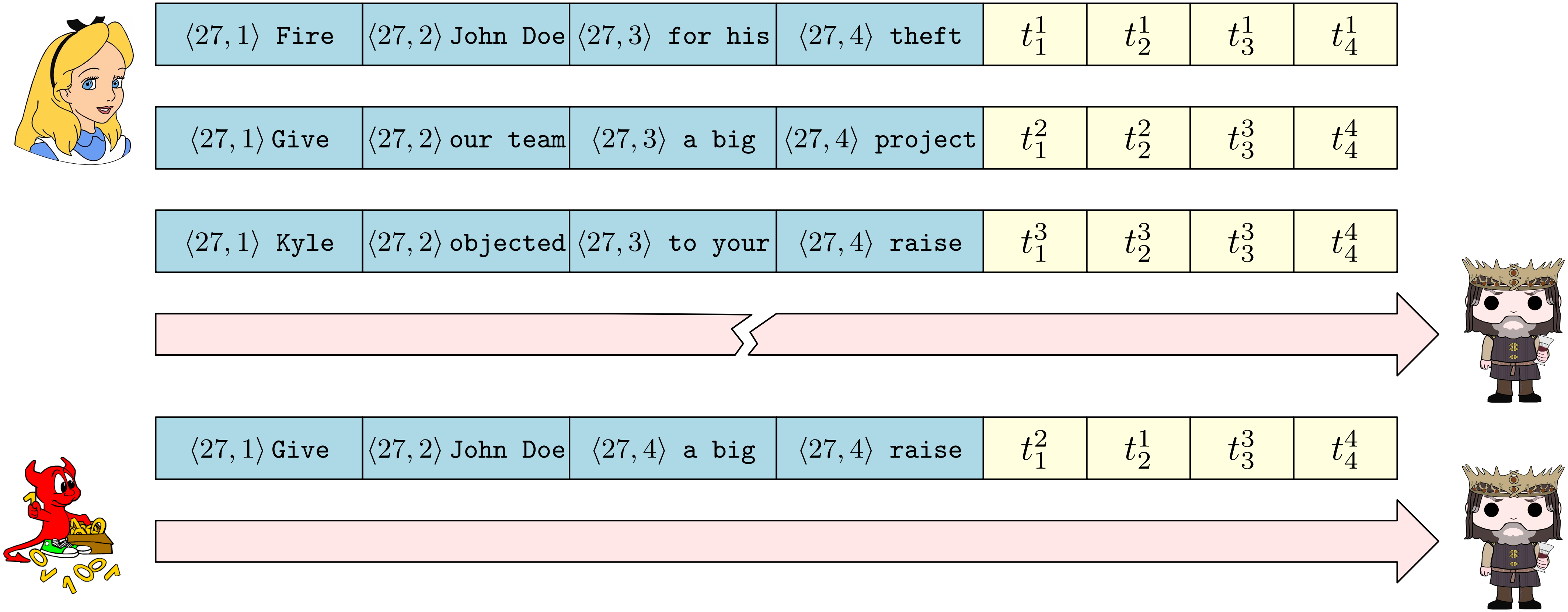

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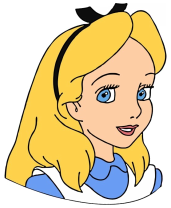

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We can use $\ell/4$ bits for for each of the message ID, message length, block index, and for the actual payload:

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Mac$'_k(m)$: (with $|m| < 2^{\ell/4}$)

- Choose r uniformly at random from $\{0,1\}^{\ell/4}$
- Split m into blocks $m_1, m_2, m_3, \ldots, m_d$ of $\ell/4$ bits each (pad the final block, if needed)
- For each $i = 1, 2, \ldots, d$
  - $t_i \leftarrow$ Mac$_k(\langle r \rangle \parallel \langle |m| \rangle \parallel \langle i \rangle \parallel m_i)$
- Output the tag $t = r \parallel t_1 \parallel t_2 \parallel \ldots \parallel t_d$

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    - $t_i \leftarrow$ Mac$_k(\langle r\rangle \,\|\, \langle |m| \rangle \,\|\, \langle i \rangle \,\|\, m_i)$
- Output the tag $t = r \,\|\, t_1 \,\|\, t_2 \,\|\, \ldots \,\|\, t_d$

Verfy$'_k$ (m, t):

- Parse t as $r \,\|\, t_1 \,\|\, t_2 \,\|\, \ldots \,\|\, t_d$
- Split m into blocks $m_1, m_2, m_3, \ldots, m_d$ of $\ell/4$ bits each
- For each $i = 1, 2, \ldots, d$
    - Check Vrfy$_k(\langle r\rangle \,\|\, \langle |m| \rangle \,\|\, \langle i \rangle \,\|\, m_i, t_i) = 1$
- Output 1 iff all checks passed (and 0 otherwise)

Domain Extension for MACS

Theorem: if Π is a secure fixed-length MAC for messages of length ℓ, then Π' is a secure MAC for arbitrary-length messages.

$\mathbf{Gen}'(1^n)$: return $\mathsf{Gen}(1^n)$

$\mathbf{Mac}'_k(m)$: (with $|m| < 2^{\ell/4}$)

- Choose r uniformly at random from $\{0,1\}^{\ell/4}$
- Split m into blocks $m_1, m_2, m_3, \ldots, m_d$ of $\ell/4$ bits each (pad the final block, if needed)
- For each $i = 1, 2, \ldots, d$
  - $t_i \leftarrow \mathsf{Mac}_k(\langle r \rangle \,\|\, \langle |m| \rangle \,\|\, \langle i \rangle \,\|\, m_i)$
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- Constructing a MAC Π for fixed-length messages from the block cipher
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Unfortunately this approach has some drawbacks in practice:

- To compute the tag for a message of length $|m|$, we need $\approx \frac{4|m|}{\ell}$ evaluations of the block cipher
- The computed tag is long (i.e., longer than $4|m|$ bits)

(Basic) CBC-MAC for fixed length messages

We can do better by using a construction similar to the ciphertext block chaining (CBC) mode used for block ciphers.

The construction only works for messages of some **fixed** length $n \cdot \ell(n)$, where n is the block length of F_k

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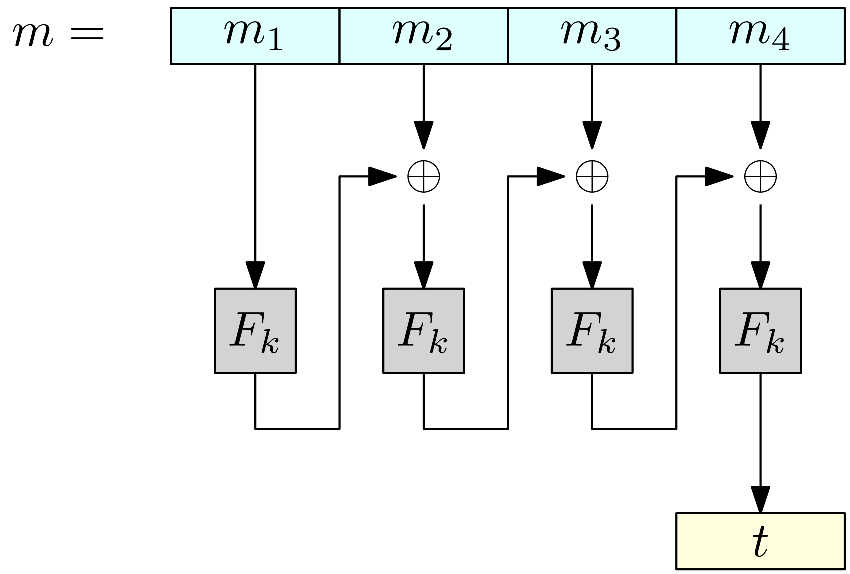
$\mathbf{Mac}_k(m)$:

- Parse m as $\ell(n)$ blocks $m_1, m_2, m_3, \ldots, m_{\ell(n)}$ of n bits each
- $t_0 \leftarrow 0^n$
- For $i = 1, \ldots, \ell(n)$
  - $t_i \leftarrow F_k(t_{i-1} \oplus m_i)$
- Output the tag $t = t_{\ell(n)}$



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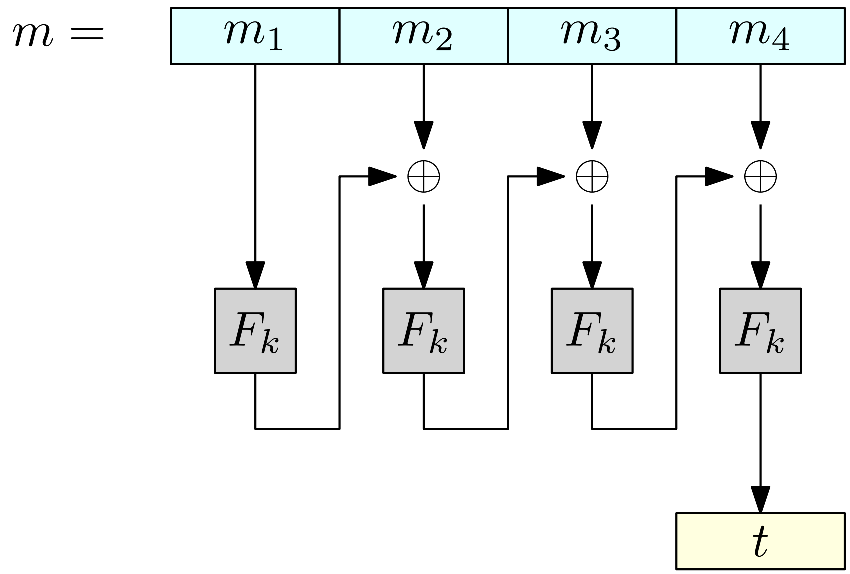

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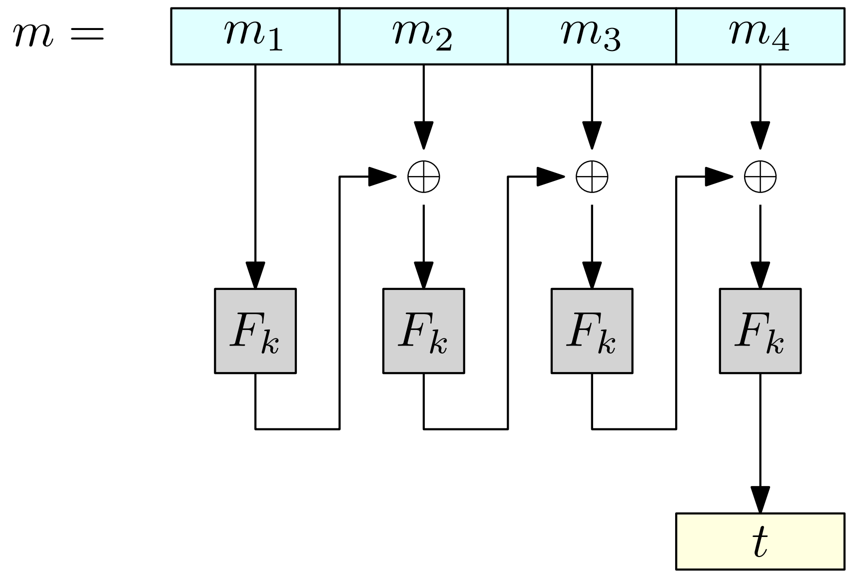

Some differences with CBC mode for block ciphers:

- No IV (notice that CBC-MAC is **deterministic**)
- Only the final invocation of the block cipher is output

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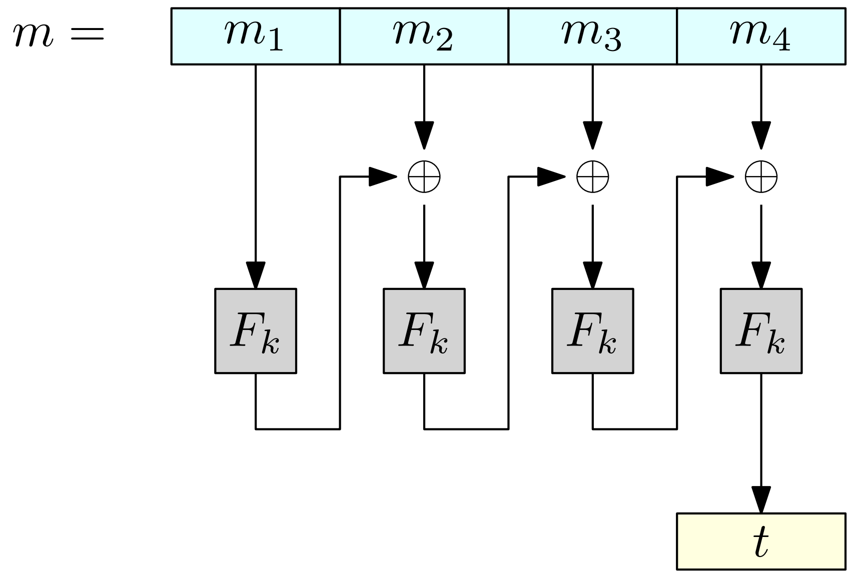
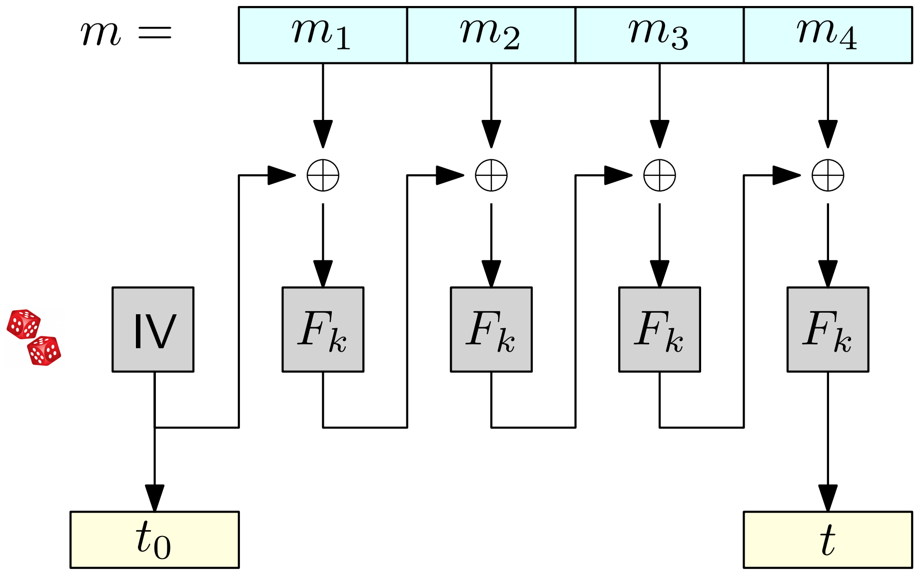


Theorem: Let ℓ be a polynomial. If F is a pseudorandom function with block length n, then Basic CBC-MAC is a secure MAC for messages of length $\ell(n) \cdot n$.

Basic CBC-MAC: some caveats (1/3)

If we modify the construction to take an IV, then the MAC is no longer secure!



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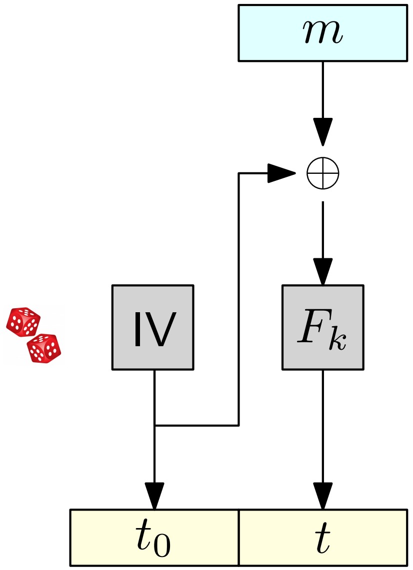

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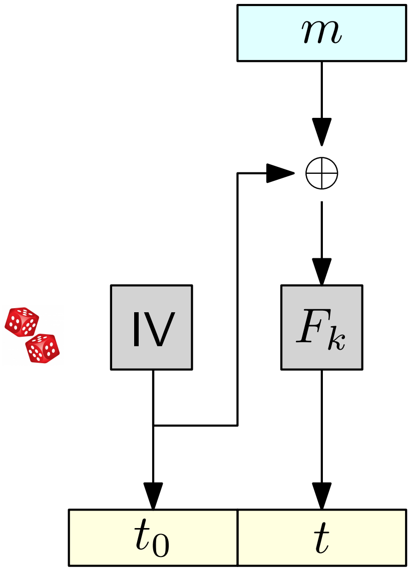
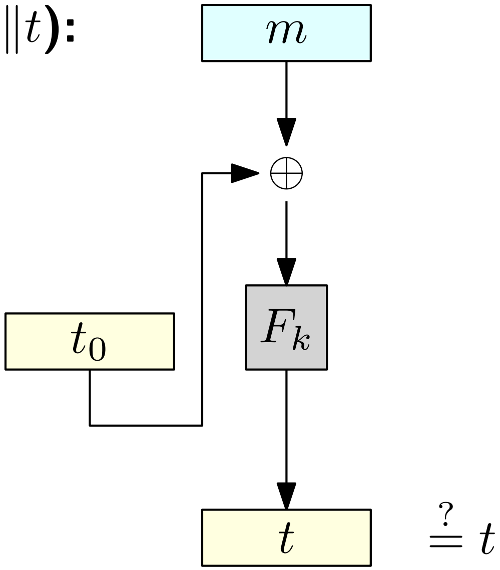


Vrfy($m, t_0 \| t$):



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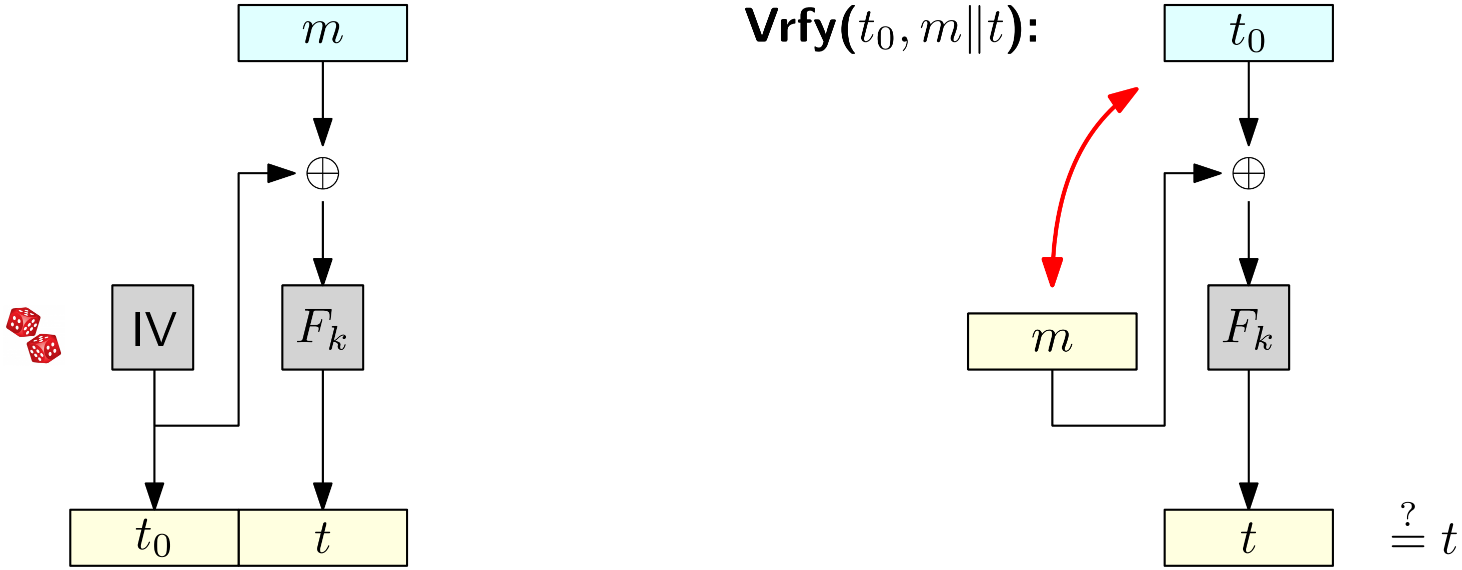

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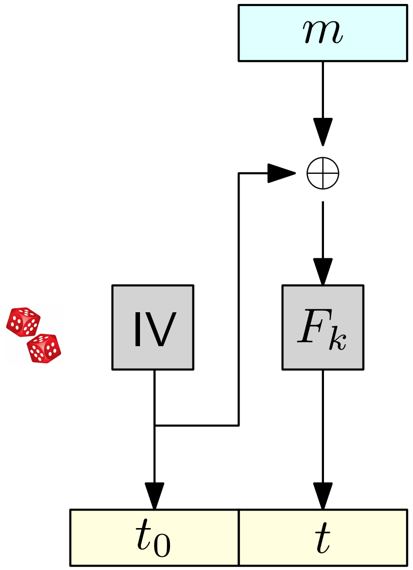

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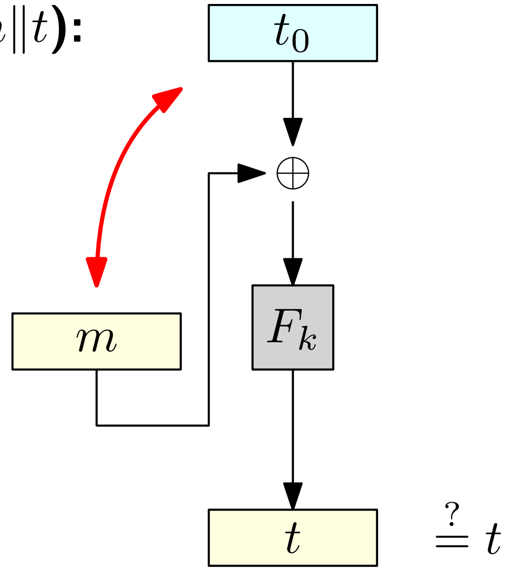

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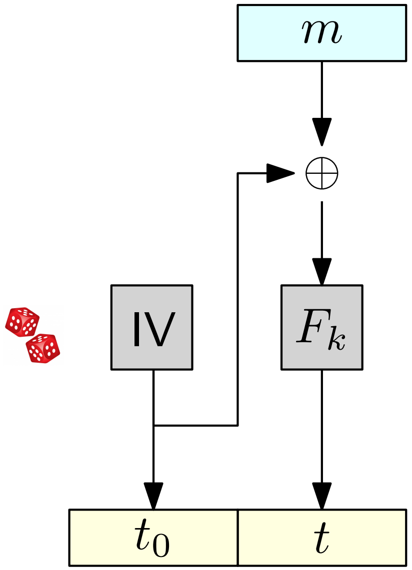

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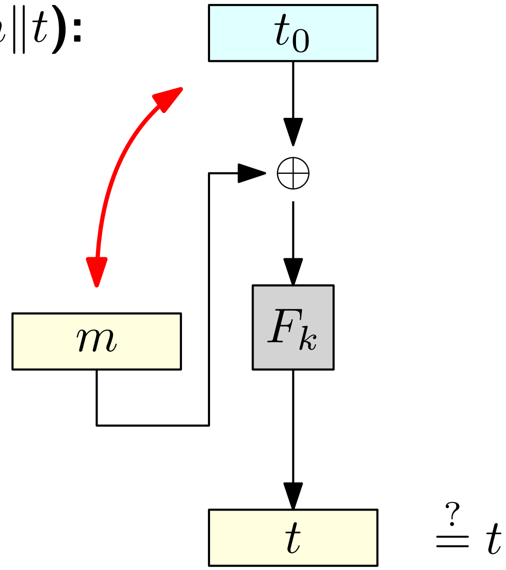

- Pick an arbitrary message m, and obtain the tag $t_0 \| t$
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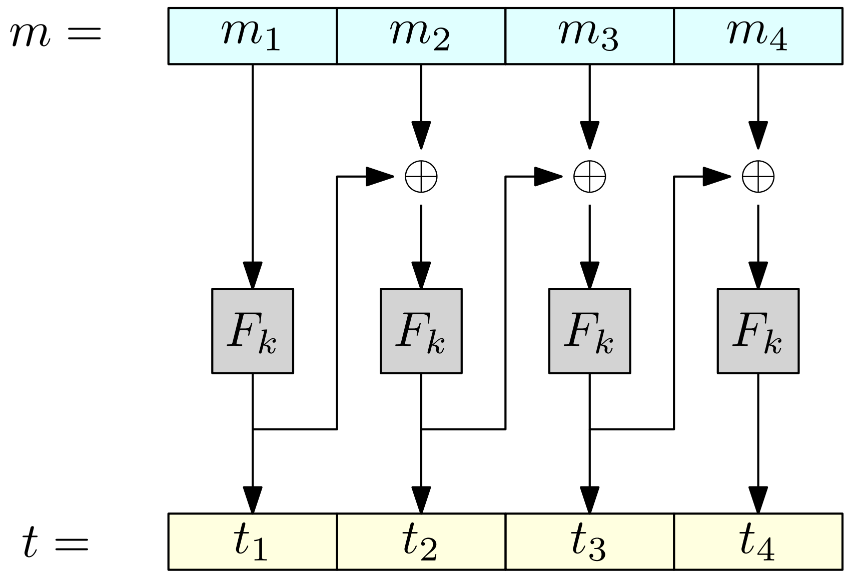
The forgery is successful

Basic CBC-MAC: some caveats (2/3)

If all invocations of F contribute to the output, then the MAC is no longer secure!



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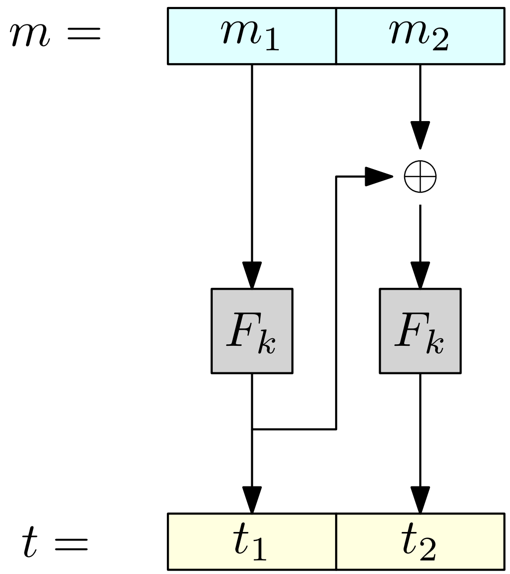

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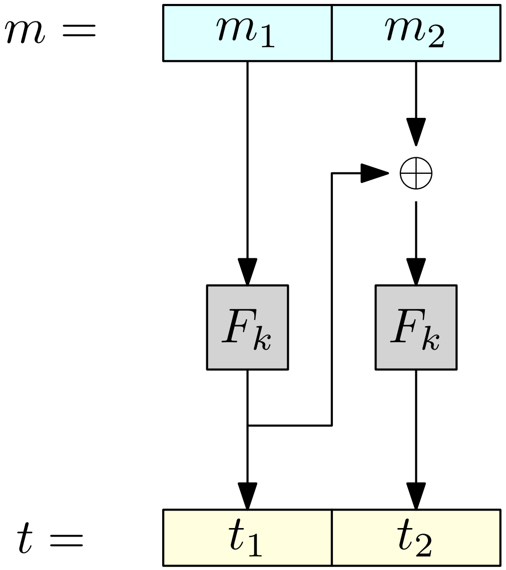

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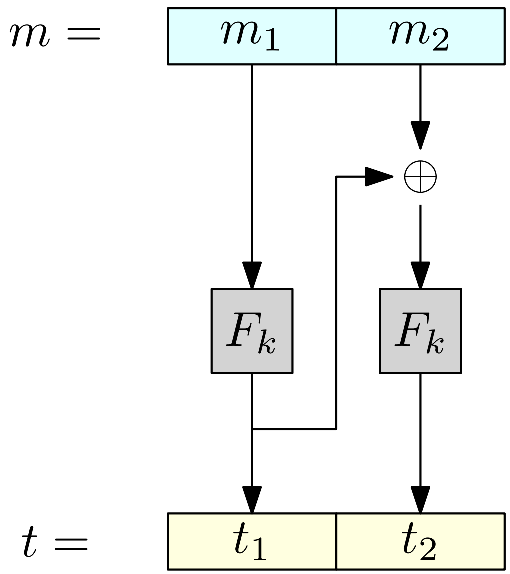

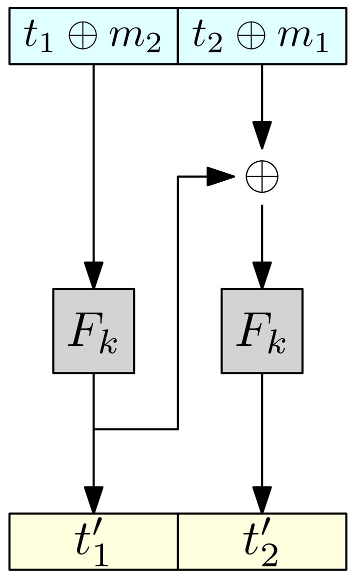

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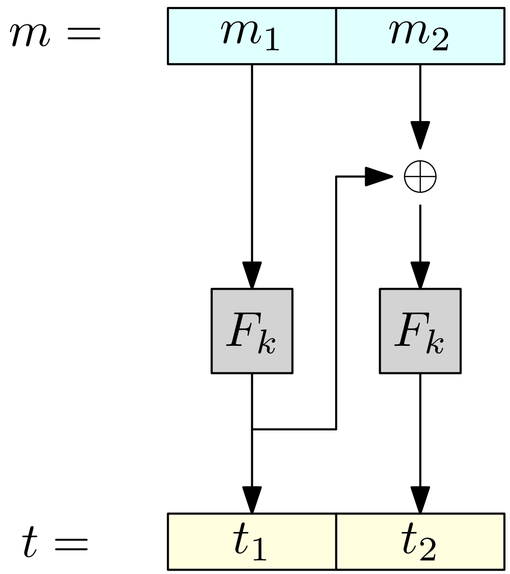

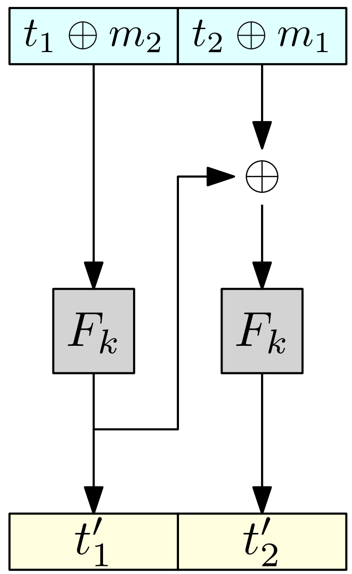

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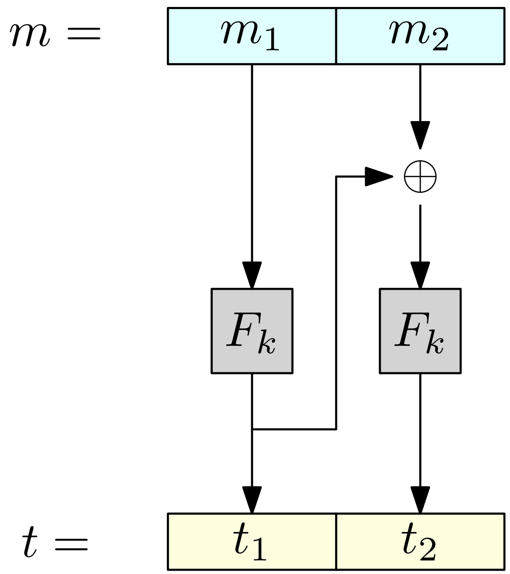

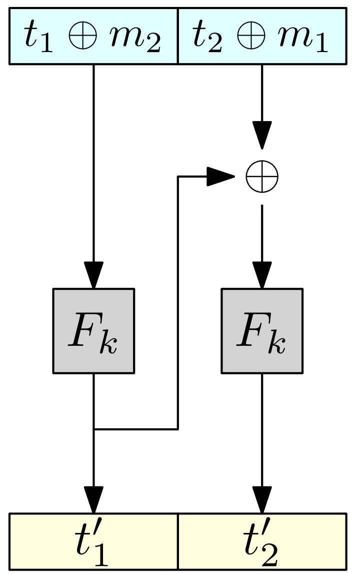

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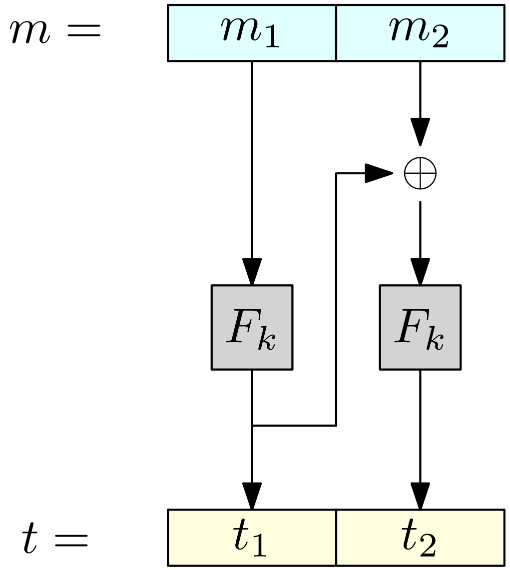

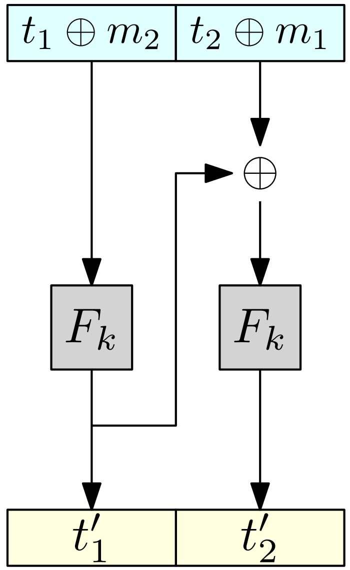

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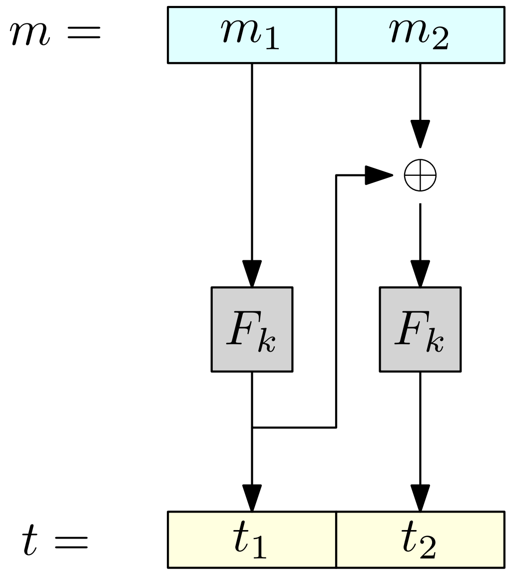

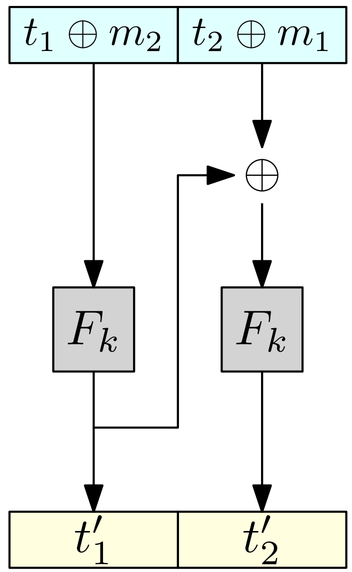

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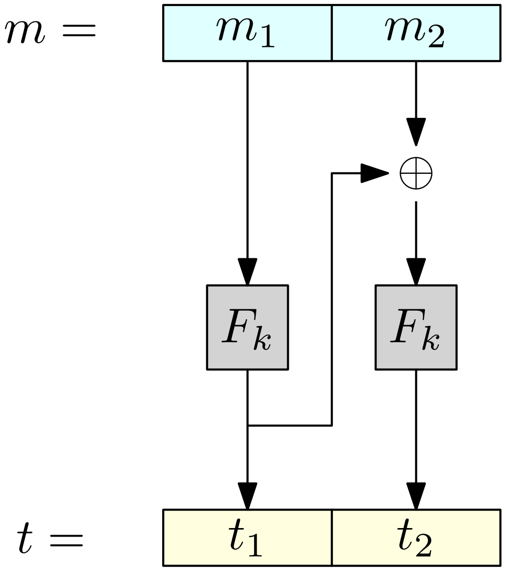

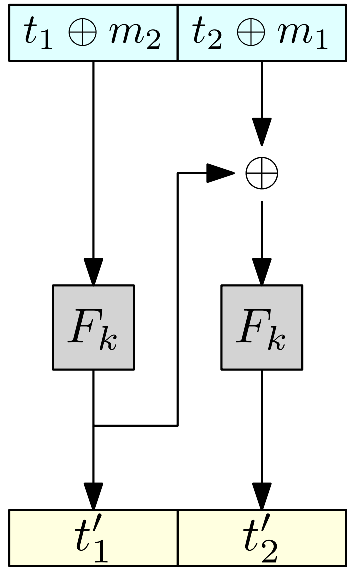

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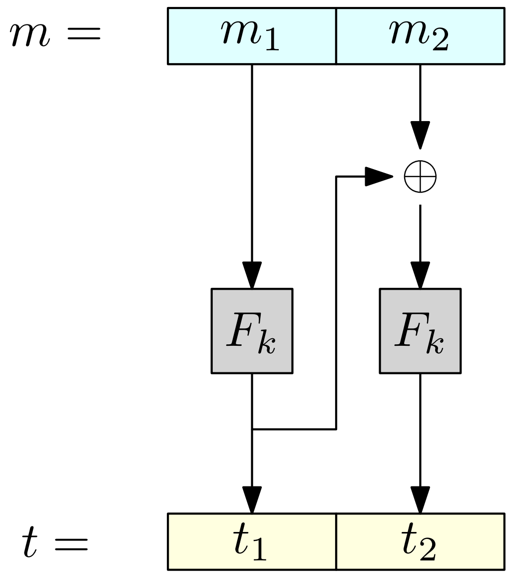

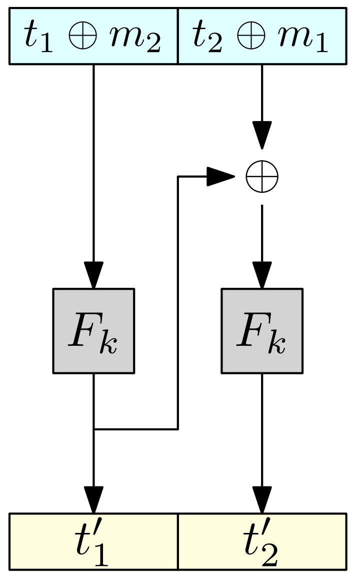

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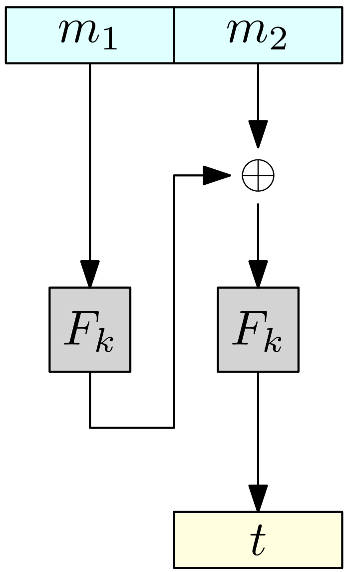

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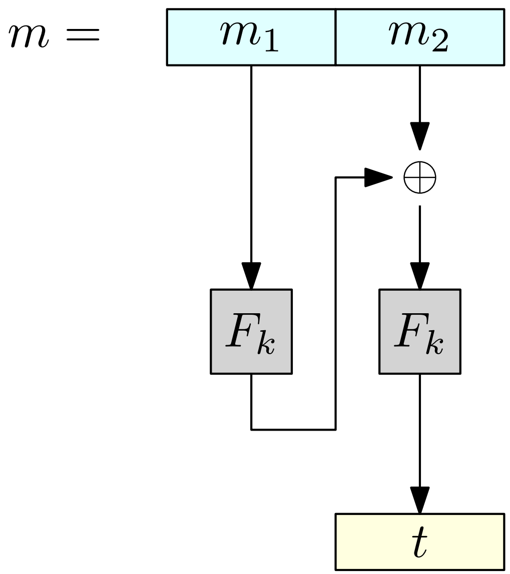

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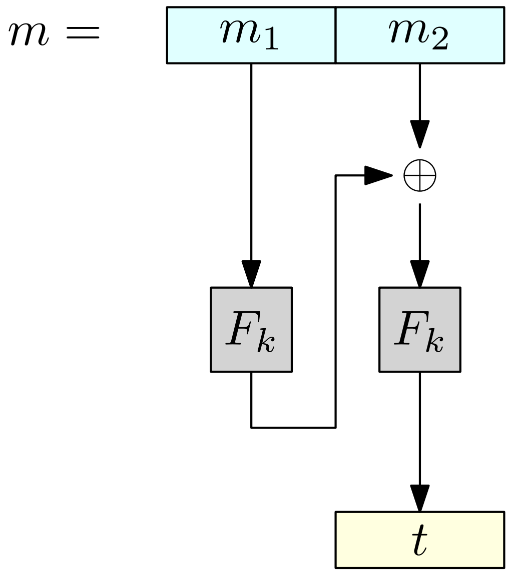

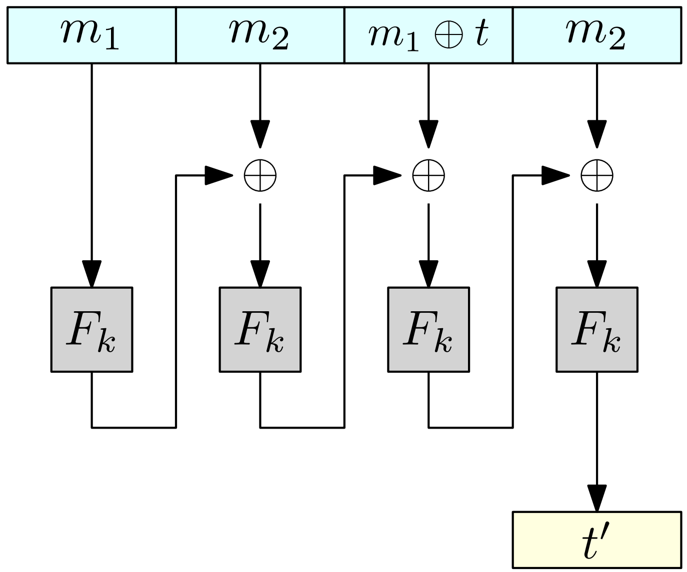

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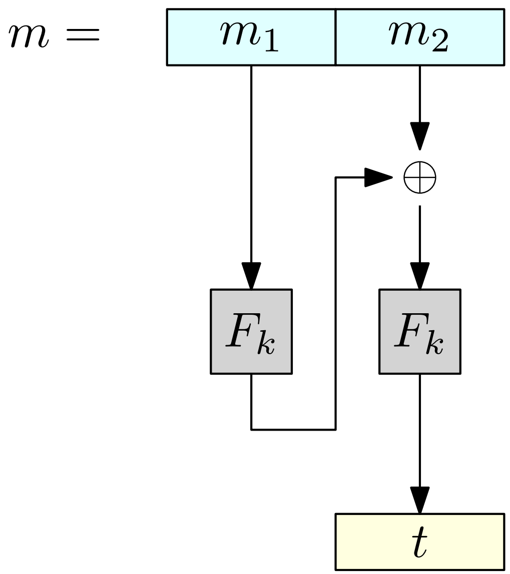

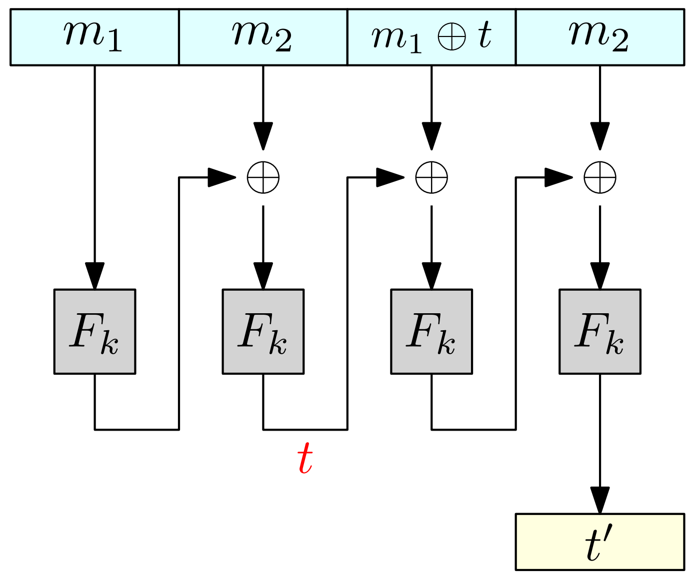

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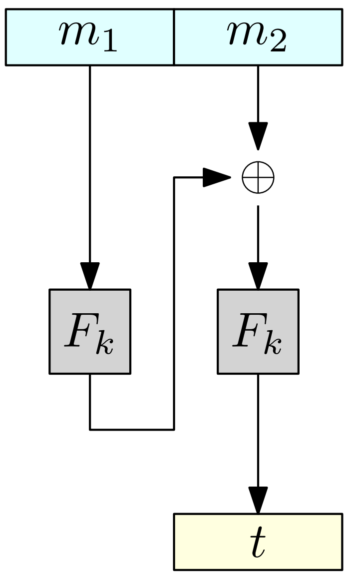

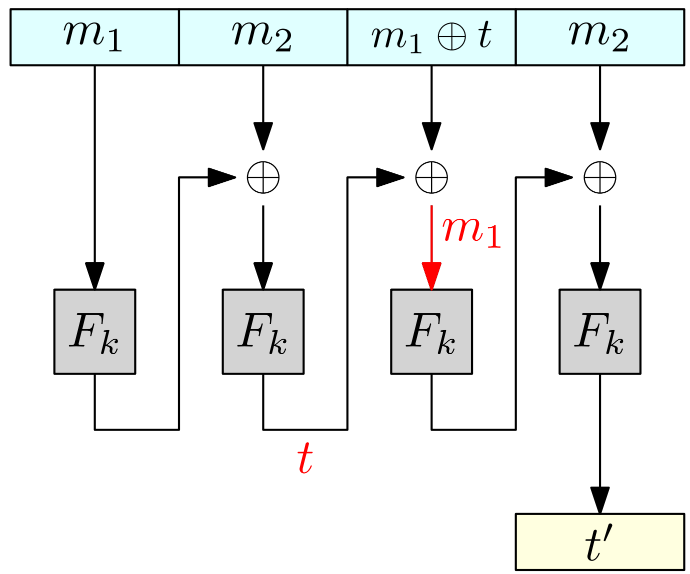

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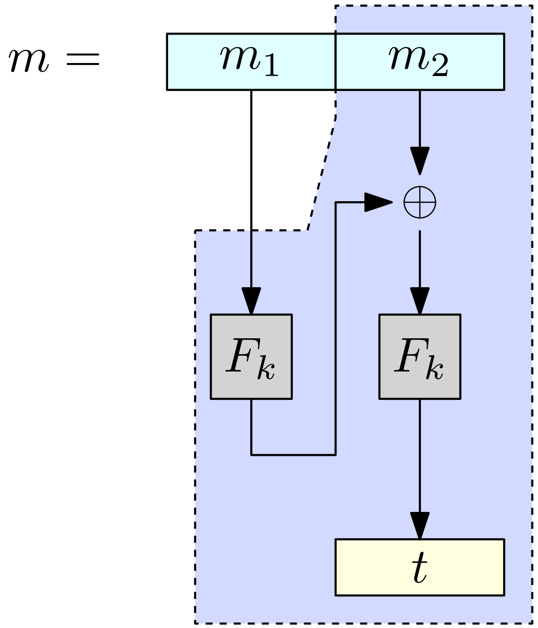

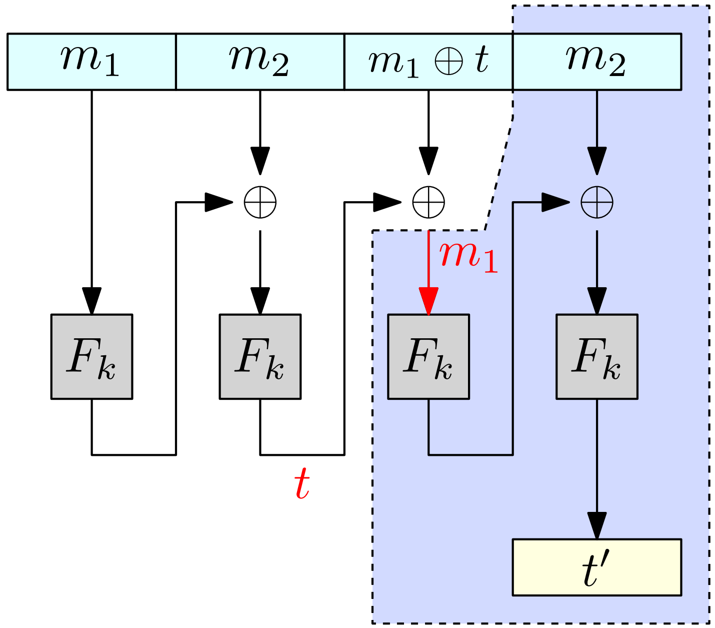

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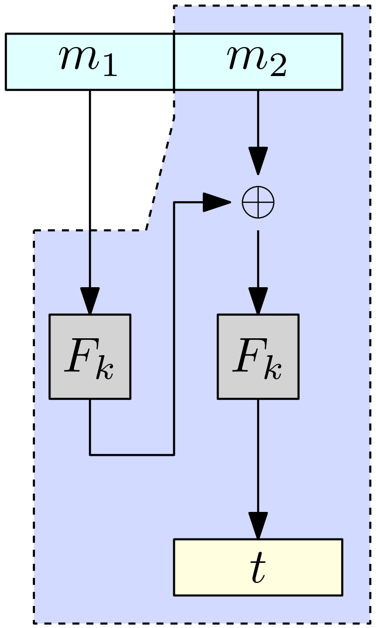

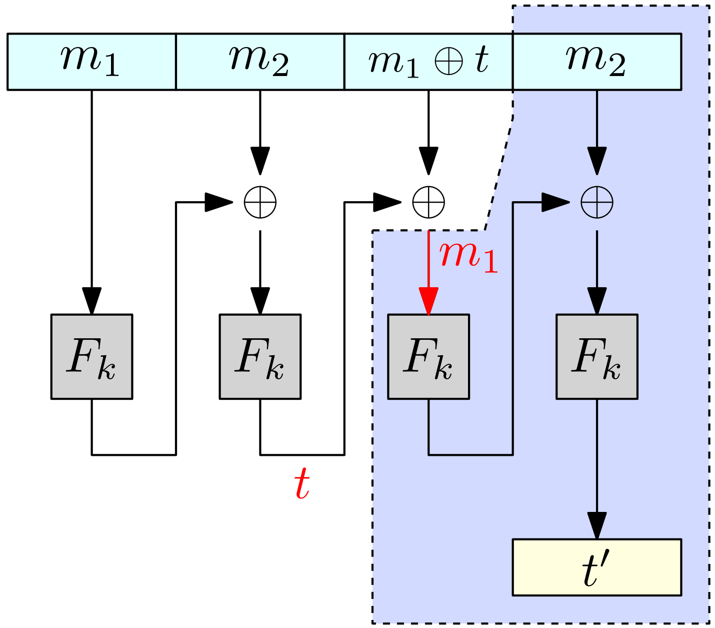

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$t' = t$
The forgery is successful

CBC-MAC for arbitrary length messages

Basic CBC-MAC can be extended to handle arbitrary-length messages

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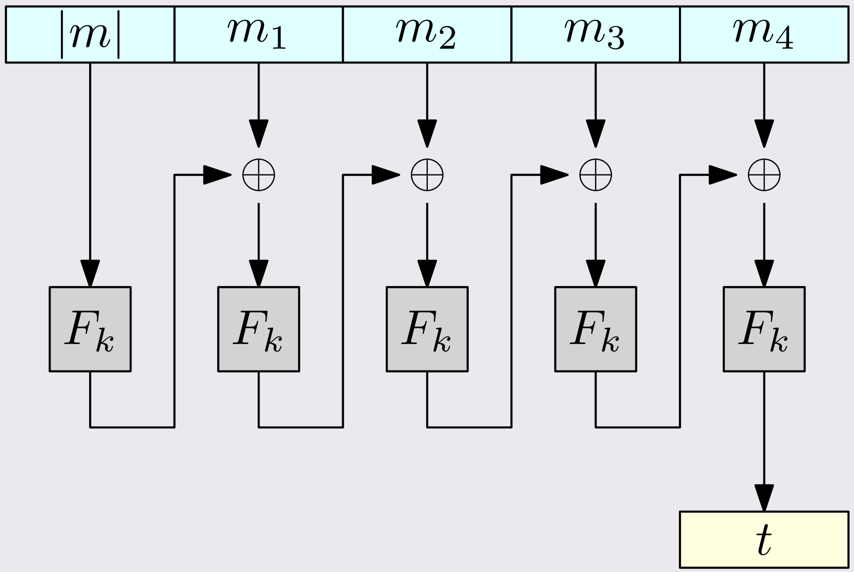
- Encode the message length m as a n-bit string, and **prepend** it to m



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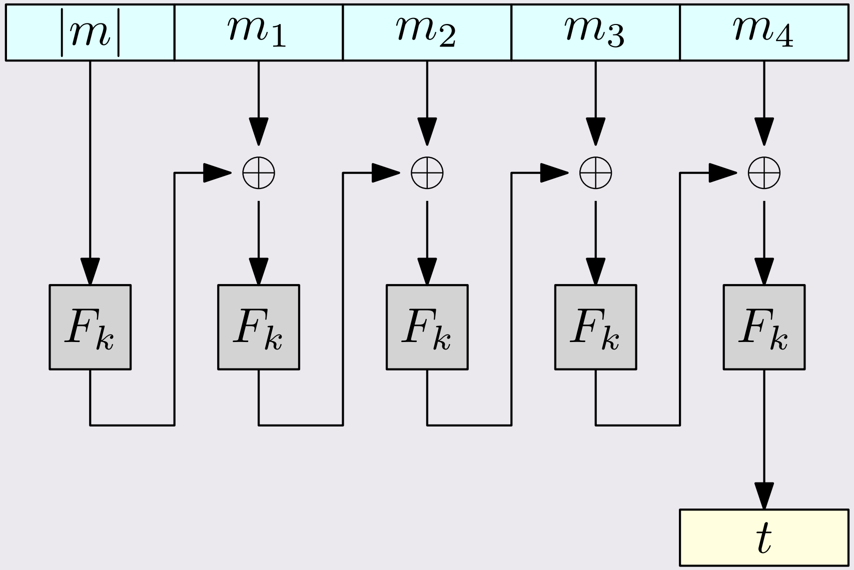

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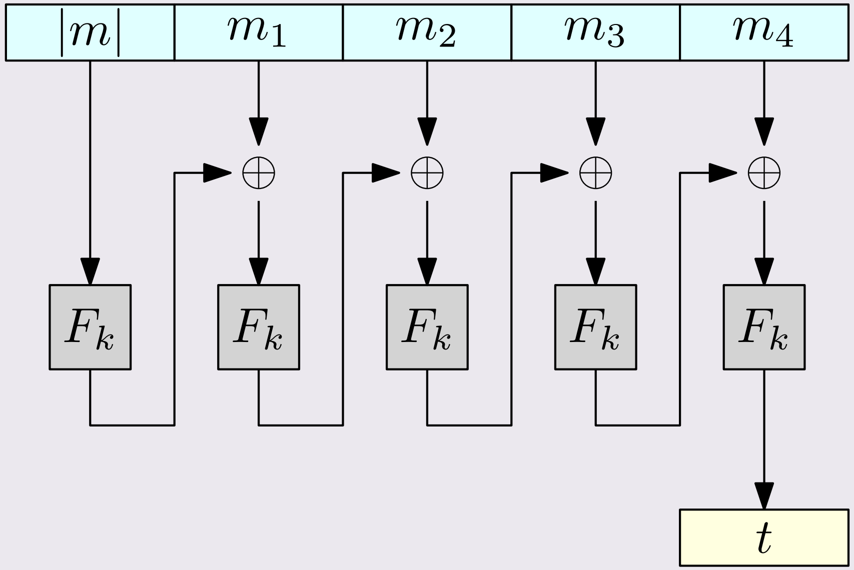

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Note that appending $|m|$ to m is **not secure**

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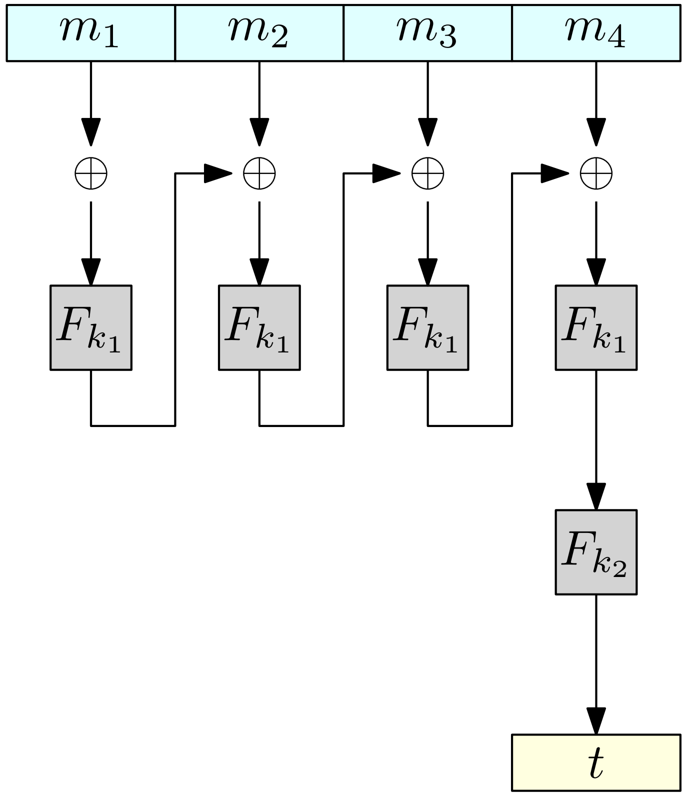

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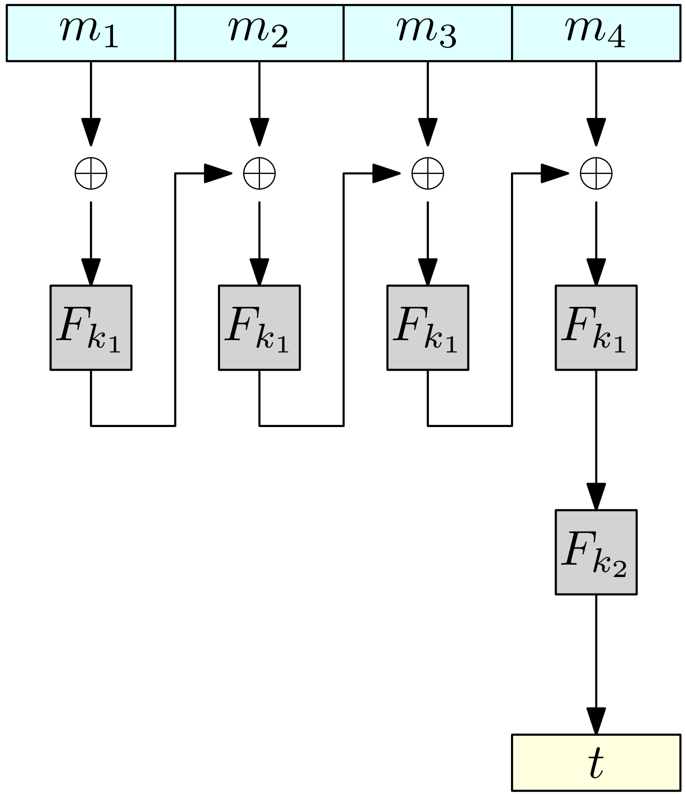

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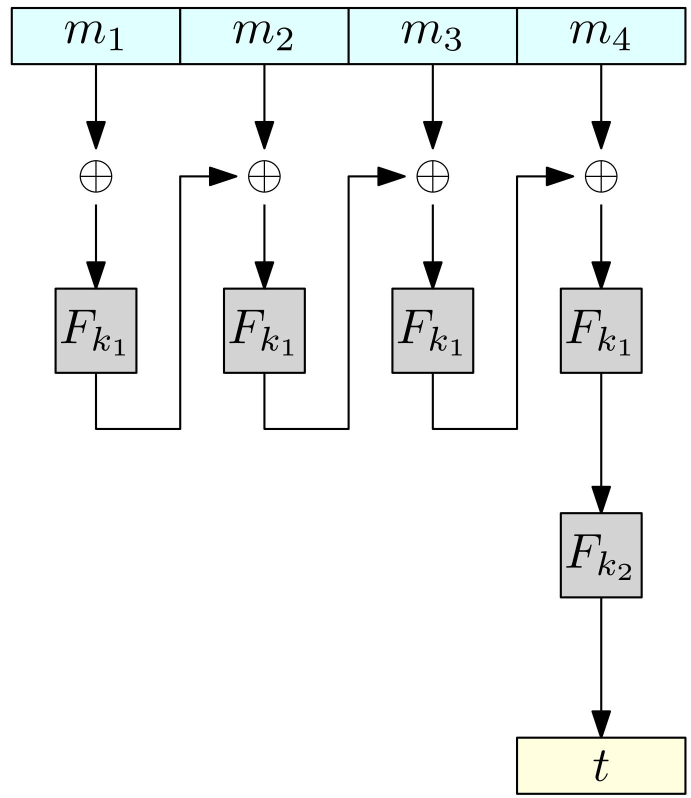

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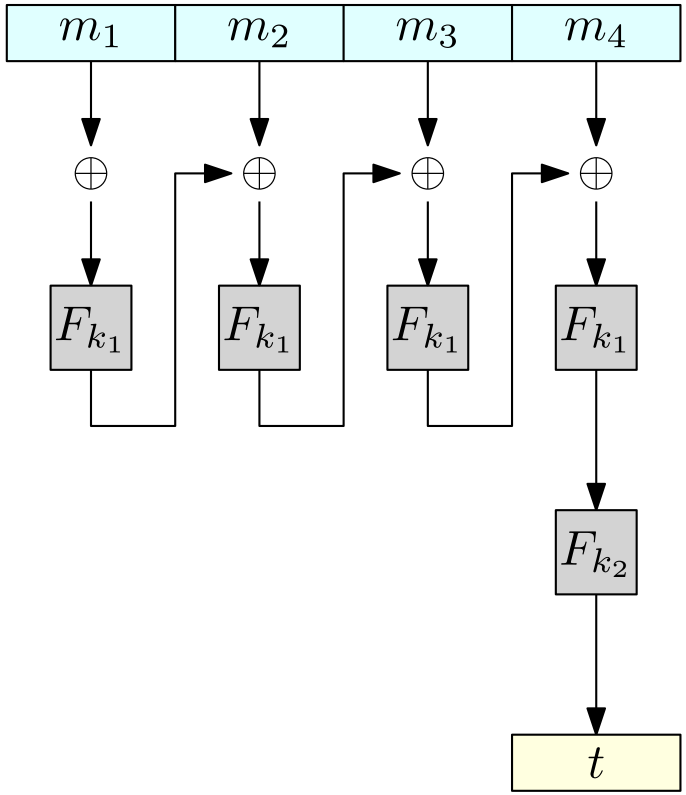

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Advantage: There is no need to know the length of m in advance (Mac$_k$ is a *streaming* algorithm)

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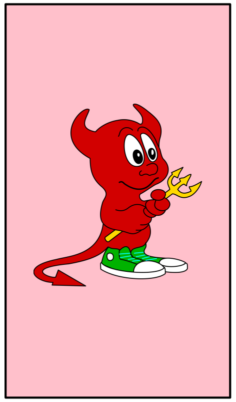
- We can modify our message authentication experiment (and security definition) to account for this

The **Strong** Message Authentication Experiment

Let $\Pi = (\mathsf{Gen}, \mathsf{Mac}, \mathsf{Vrfy})$ be a MAC. We name the following experiment Mac-sforge$_{\mathcal{A},\Pi}(n)$:

- A key k is generated using $\mathsf{Gen}(1^n)$



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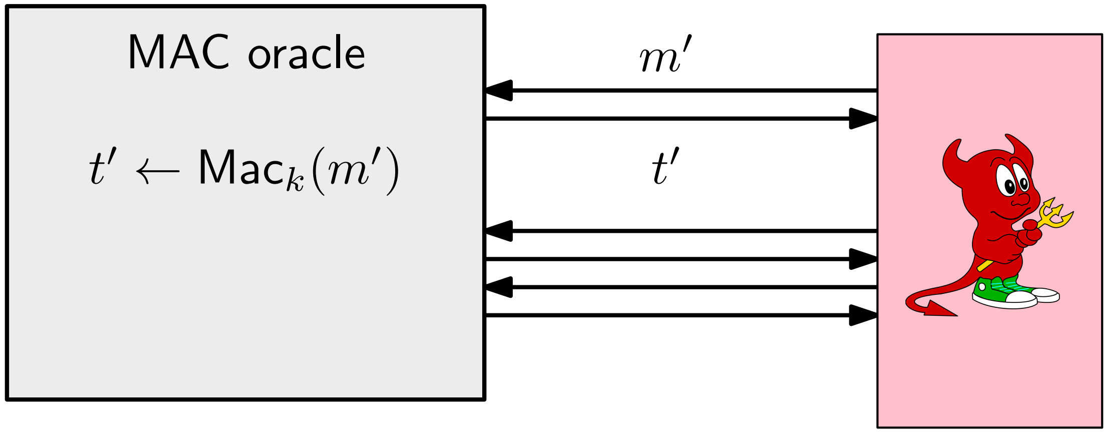

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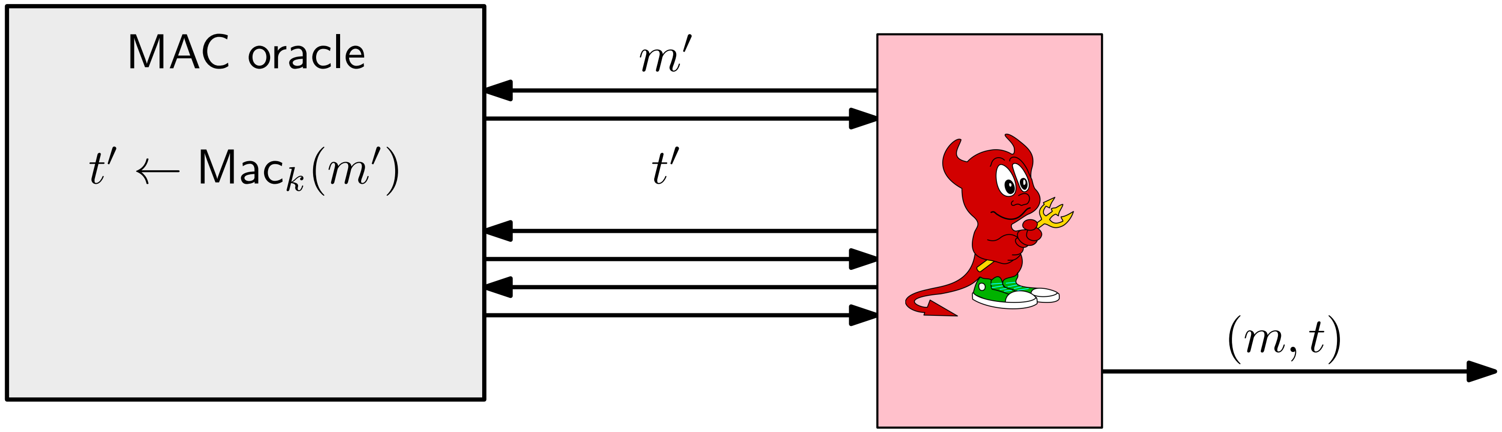

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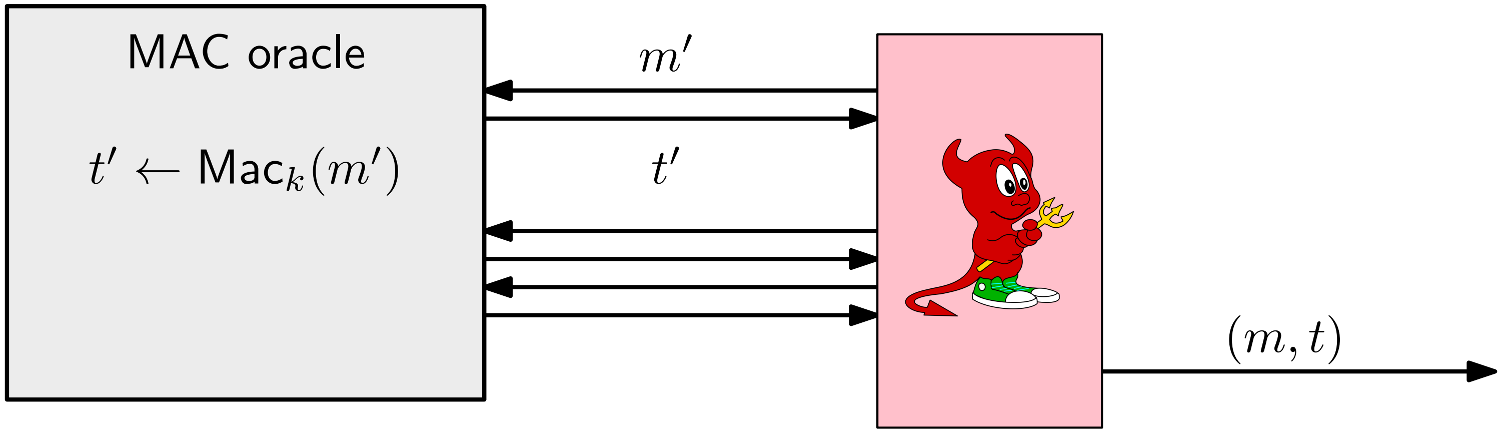
- The outcome of the experiment is 1 if (*) holds and $\mathsf{Vrfy}_k(m, t) = 1$. Otherwise the outcome is 0.



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Good news:

All deterministic secure MACs that use canonical verification are also strongly secure.