## Information Systems and Network Security

**Docente:** Stefano Leucci



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### Basic Info

### **Course length:** 48 hours (6 CFU)

• 24 lectures

When/where:



- Tuesday 16:30 18:30 **Room:** C1.16
- Thursday 11:30 13:30 **Room:** C1.16

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- Thursday 11:30 13:30 **Room:** C1.16

**Office hours:** 

- Thursday 14:30 16:30
- Please send an email to stefano.leucci@gmail.com or ask before/after class

### **Course material:**

https://people.disim.univaq.it/~stefanoleucci/isns25/



### Cryptography



(Discrete) Math: Basic algebra, Modular arithmetic, some concepts from group theory and number theory



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**Probability theory**: Events, random variables, expectation, independence, conditional probability, Bayes' theorem, ...





(Discrete) Math: Basic algebra, Modular arithmetic, some concepts from group theory and number theory

**Probability theory**: Events, random variables, expectation, independence, conditional probability, Bayes' theorem, ...

**Theoretical computer science:** Algorithms, complexity, asymptotic notation, reductions

Cryptography







Two entities wish to communicate





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Two entities wish to communicate

Alice can send a message to Bob through a communication channel







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The communication channel is "insecure"







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### The communication channel is "insecure"

• Anybody can view a message in transit







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### Can Alice and Bob communicate "securely"?



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What does "secure" mean?



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- Secrecy/Confidentiality: The contents messages sent over the channel will be known only to Alice and Bob
- Authentication: any message received by Bob can be confirmed to have originated from Alice (we can detect any message injected by the adversary)
- **Integrity:** it is not possible to alter the contents of a message sent across the channel (without the tampering being detected)





Intuitively, we established a virtual "secure" communication channel on top of an underlying insecure channel





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• Introduction (why cryptography?)

YOU ARE HERE

- Introduction (why cryptography?)
- Cryptography in the past: historic ciphers



- Introduction (why cryptography?)
- Cryptography in the past: historic ciphers
- Cryptography today: formalizing security



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opics

#### **Private-key** cryptography



- Hash functions
- Key distribution

At the "boundary" between private-key and public-key cryptography

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At the "boundary" between private-key and public-key cryptography

• Public-key cryptography, Hybrid cryptography

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Public-key cryptography

- Hash functions
- Key distribution

At the "boundary" between private-key and public-key cryptography

- Public-key cryptography, Hybrid cryptography
- Digital signatures
- Digital certificates, SSL/TLS
- Advanced applications: secret sharing, multiparty computation, zero-knowledge proofs



Public-key cryptography

### **Books**





Introduction to Modern Cryptography

Jonathan Katz, Yehuda Lindell

ISBN: 978-0815354369

### Mike Rosulek https://joyofcryptography.com/

#### The Joy Of Cryptography



Draft of January 3, 2021

Mike Rosulek (mike@joyofcryptography.com) School of Electrical Engineering & Computer Science Oregon State University, Corvallis, Oregon, USA

THE JOY OF CRYPTOGRAPHY



### Books



Almost all course material can be found in this book

We will use the same notation





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### Exams

Written exam:

- Questions on the theoretical concepts (e.g., security definitions)
- Exercises (e.g., prove security, carry out an attack, etc...)



### Some advanced applications

Imagine some sensitive information that is kept by a single agent

- A master encryption key
- Your bitcoin wallet
- Nuclear codes

. . .







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Single point of failure!





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An attacker can compromise one machine and steal the sensitive information





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### Single point of failure!

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**Idea:** Share the information across several agents



"Magic box"



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- Share 1
- Share 2
- Share 3
- Share 4

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#### Idea:

• The shares of all agents can be used to reconstruct the secret



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What if the adversary destroys a share?



What if the adversary destroys a share?

#### Idea:

• If there are n agents, any subset of at least t agents must be able to recover the secret.

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#### **Example:** n = 4, t = 3



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### *t*-out-of-*n* threshold secret-sharing scheme

• Agents can be different servers connected over the Internet









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- The "magic boxes" can be distributed algorithms



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• The system remains secure if < t servers are compromised
# Secret Sharing

- Agents can be different servers connected over the Internet
- The "magic boxes" can be distributed algorithms



- The system remains secure if < t servers are compromised
- The system remains operational (the secret can be recovered) if  $\leq n t$  servers are unavailable

The Domain Name System is the system responsible for converting human-readable domain names into IP addresses





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To prevent this, DNSSEC is used to authenticate DNS mappings

Who can be trusted with the master cryptographic keys to the system?

DNSSEC is managed by the Internet Corporation for Assigned Names and Numbers (ICANN)

The master key is split into 7 pieces and distributed on smart cards to 7 geographically diverse people

At least five key-holding members of this fellowship would have to meet at a secure data center in the United States to reboot [DNSSEC] in case of a very unlikely system collapse.

"If you round up five of these guys, they can decrypt [the root key] should the West Coast fall in the water and the East Coast get hit by a nuclear bomb"

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Alice and Bob want to decide on a movie to watch











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Each of Alice and Bob has their own preferences...

However, they don't want to reveal that they like a movie unless the other person also likes it





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However, they don't want to reveal that they like a movie unless the other person also likes it

- They vote on one movie at a time until they find a movie that they both like
- Voting is done using a secure **2-party computation protocol**





# person also likes it th like

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Alice and Bob are **honest** (they follow the protocol) but **curious** 





# person also likes it th like

They will use some cards with two different faces, say hearts and spades





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They will place 5 cards in a row on a table









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The middle card is face up and is always a hearts card









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They will place 5 cards in a row on a table

The middle card is face up and is always a hearts card

Each of Alice and Bob has a hearts card and a spades card











If Alice likes the movie, she will place her two cards face down in the order

Otherwise she will place her two cards **face down** in the order







# erences



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# 





### **Possible configurations:**





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There are three hearts in a row if and only if both Alice and Bob like the movie



### **Possible configurations:**



There are three hearts in a row if and only if both Alice and Bob like the movie This holds (in a modular sense) even if any rotation of the cards is considered!





### Alice and Bob pile up the cards, all face down, ensuring that their order is preserved





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Alice and Bob take turns cutting the deck (i.e., they perform a rotation)





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Alice and Bob reveal the cards and watch the movie if and only if there are 3 consecutive hearts (in a modular fashion), otherwise they "pass"





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**Observation:** If they end up not watching the movie, all possible "pass" configurations are equiprobable





The previous scenario is a secure multiparty computation problem:

• There are n agents (parties)  $1, 2, \ldots, n$ 

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In the movie selection problem:

• 
$$n=2$$

•  $s_1, s_2 \in \{\mathsf{pass}, \mathsf{watch}\}$ 

• 
$$f(s_1, s_2) = \begin{cases} \text{watch} & \text{if } s_1 = s_2 = \text{watch} \\ \text{pass} & \text{otherwise} \end{cases}$$

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$$f(s_1, s_2) = \begin{cases} 1 & \text{if } s_1 = s_2 = 1 \\ 0 & \text{otherwise} \end{cases}$$

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 $\begin{array}{c|c} s_1 & \ \\ s_2 & \ \end{array} \longrightarrow f(s_1, s_2)$ 

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What about arbitrary circuits?

В



 $- f(s_1, s_2)$ 

5	3			7				
6			1	9	5			
	9	8					6	
8				6				3
4			8		3			1
7				2				6
	6					2	8	
			4	1	9			5
				8			7	9

5	3	4	6	7	8	9	1	2
6	7	2	1	9	5	3	4	8
1	9	8	3	4	2	5	6	7
8	5	9	7	6	1	4	2	3
4	2	6	8	5	3	7	9	1
7	1	3	9	2	4	8	5	6
9	6	1	5	3	7	2	8	4
2	8	7	4	1	9	6	3	5
3	4	5	2	8	6	1	7	9

5	3			7				
6			1	9	5			
	9	8					6	
8				6				3
4			8		3			1
7				2				6
	6					2	8	
			4	1	9			5
				8			7	9



5	3	4	6	7	8	9	1	2
6	7	2	1	9	5	3	4	8
1	9	8	3	4	2	5	6	7
8	5	9	7	6	1	4	2	3
4	2	6	8	5	3	7	9	1
7	1	3	9	2	4	8	5	6
9	6	1	5	3	7	2	8	4
2	8	7	4	1	9	6	3	5
3	4	5	2	8	6	1	7	9

5	3			7				
6			1	9	5			
	9	8					6	
8				6				3
4			8		3			1
7				2				6
	6					2	8	
			4	1	9			5
				8			7	9

I know a solution to this Sudoku instance





5	3	4	6	7	8	9	1	2
6	7	2	1	9	5	3	4	8
1	9	8	3	4	2	5	6	7
8	5	9	7	6	1	4	2	3
4	2	6	8	5	3	7	9	1
7	1	3	9	2	4	8	5	6
9	6	1	5	3	7	2	8	4
2	8	7	4	1	9	6	3	5
3	4	5	2	8	6	1	7	9

5	3			7				
6			1	9	5			
	9	8					6	
8				6				3
4			8		3			1
7				2				6
	6					2	8	
			4	1	9			5
				8			7	9

Really? Show it to me!





5	3	4	6	7	8	9	1	2
6	7	2	1	9	5	3	4	8
1	9	8	3	4	2	5	6	7
8	5	9	7	6	1	4	2	3
4	2	6	8	5	3	7	9	1
7	1	3	9	2	4	8	5	6
9	6	1	5	3	7	2	8	4
2	8	7	4	1	9	6	3	5
3	4	5	2	8	6	1	7	9

5	3			7				
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8				6				3
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7				2				6
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5	3	4	6	7	8	9	1	2
6	7	2	1	9	5	3	4	8
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9	6	1	5	3	7	2	8	4
2	8	7	4	1	9	6	3	5
3	4	5	2	8	6	1	7	9

5	3			7				
6			1	9	5			
	9	8					6	
8				6				3
4			8		3			1
7				2				6
	6					2	8	
			4	1	9			5
				8			7	9

Then I don't believe you really have a solution





5	3	4	6	7	8	9	1	2
6	7	2	1	9	5	3	4	8
1	9	8	3	4	2	5	6	7
8	5	9	7	6	1	4	2	3
4	2	6	8	5	3	7	9	1
7	1	3	9	2	4	8	5	6
9	6	1	5	3	7	2	8	4
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5	3			7				
6			1	9	5			
	9	8					6	
8				6				3
4			8		3			1
$\overline{7}$				2				6
	6					2	8	
			4	1	9			5
				8			7	9

I can prove to you that I have a solution without revealing anything about it





5	3	4	6	7	8	9	1	2
6	7	2	1	9	5	3	4	8
1	9	8	3	4	2	5	6	7
8	5	9	7	6	1	4	2	3
4	2	6	8	5	3	7	9	1
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Zero Knowledge protocol



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I am now convinced you have the solution



Zero Knowledge protocol





Hey, Charlie! Alice has a solution to this Sudoku instance











5	3			7				
6			1	9	5			
	9	8					6	
8				6				3
4			8		3			1
7				2				6
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Even if I definitely know she has a solution, somehow I have no way of proving that







#### Graph isomorphism problem

 $G_1$  is isomorphic to  $G_2$  iff  $\exists$  bijection  $\pi: V_1 \to V_2$  s.t.  $(u, v) \in E_1 \iff (\pi(u), \pi(v)) \in E_2$ .



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Alice can use a Zero Knowledge protocol to convince bob that  $G_1$  and  $G_2$  are isomorphic without revealing  $\pi$ 

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#### Basic definitions

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For now, we will only be concerned with private-key cryptography

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**Perfect correctness:**  $\forall k \in \mathcal{K}, m \in \mathcal{M}$  if c can be output by  $Enc_k(m)$  then  $Dec_k(c) = m$ 







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If the key is leaked, it is easy to replace it



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- Parties only need to share a key (and not the exact details of the secret scheme).
- No need for Bob to implement Alice's weird scheme on his own (Bob can use public, vetted, implementations of well-known schemes)

