

Caesar cipher

An example of a simple symmetric encryption scheme is the Caesar cipher

“If he had anything confidential to say, he wrote it in cipher, that is, by so changing the order of the letters of the alphabet, that not a word could be made out. If anyone wishes to decipher these, and get at their meaning, he must substitute the fourth letter of the alphabet, namely D, for A, and so with the others.”

– Suetonius, Life of Julius Caesar



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Each character of the plaintext is replaced with the character 3 positions down the alphabet, in a modular fashion

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓
D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C



Caesar cipher: example



$m =$ A T T A C K A T D A W N

↓ $\text{Enc}(m)$

Caesar cipher: example



$m =$ A T T A C K A T D A W N

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$c =$ D W W D F N D W G D Z Q

Caesar cipher: example



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↓ $\text{Enc}(m)$

$c =$ D W W D F N D W G D Z Q

$c =$ U H W U H D W Q R Z

↓ $\text{Dec}(c)$

$m =$

Caesar cipher: example



$m =$ A T T A C K A T D A W N

↓ $\text{Enc}(m)$

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$c =$ U H W U H D W Q R Z

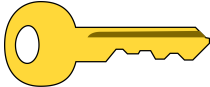
↓ $\text{Dec}(c)$

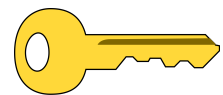
$m =$ R E T R E A T N O W

Shift ciphers

The Caesar cipher is a special type of *shift cipher*

In a shift cipher, each character is replaced with the character k positions down the alphabet (in a modular fashion)

The *key* of the cipher is the integer k 
(the key is also called the *shift* of the cipher)



$$k = 5$$

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓
F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E

Shift ciphers

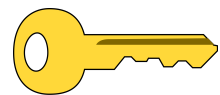
$m = \text{F L A N K T H E E N E M Y}$

↓ $\text{Enc}_5(m)$

$c = \text{K Q F S P Y M J J S J R D}$

$c = \text{X J S I M J Q U}$

↓ $\text{Dec}_5(c)$



$k = 5$

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓
F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E

Shift ciphers

$m = \text{F L A N K T H E E N E M Y}$

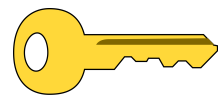
↓ $\text{Enc}_5(m)$

$c = \text{K Q F S P Y M J J S J R D}$

$c = \text{X J S I M J Q U}$

↓ $\text{Dec}_5(c)$

$m = \text{S E N D H E L P}$



$k = 5$

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓
F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E

Shift ciphers

Message space: $\mathcal{M} = \{A, \dots, Z\}^*$

Shift ciphers

Message space: $\mathcal{M} = \{0, 1, \dots, 25\}^*$

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Ciphertext space:

Shift ciphers

Message space: $\mathcal{M} = \{0, 1, \dots, 25\}^*$

Ciphertext space: $\mathcal{C} = \{0, \dots, 25\}^*$

Shift ciphers

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Key space:

Shift ciphers

Message space: $\mathcal{M} = \{0, 1, \dots, 25\}^*$

Ciphertext space: $\mathcal{C} = \{0, \dots, 25\}^*$

Key space: $\mathcal{K} = \{0, \dots, 25\}$

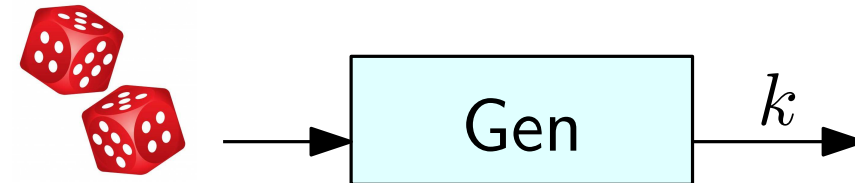
Shift ciphers

Message space: $\mathcal{M} = \{0, 1, \dots, 25\}^*$

Ciphertext space: $\mathcal{C} = \{0, \dots, 25\}^*$

Key space: $\mathcal{K} = \{0, \dots, 25\}$

Key generation:



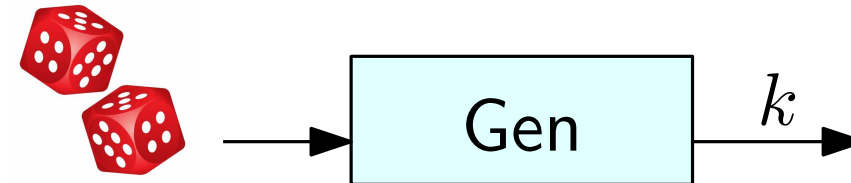
Shift ciphers

Message space: $\mathcal{M} = \{0, 1, \dots, 25\}^*$

Ciphertext space: $\mathcal{C} = \{0, \dots, 25\}^*$

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Key generation: return k chosen u.a.r. from \mathcal{K}



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Message space: $\mathcal{M} = \{0, 1, \dots, 25\}^*$

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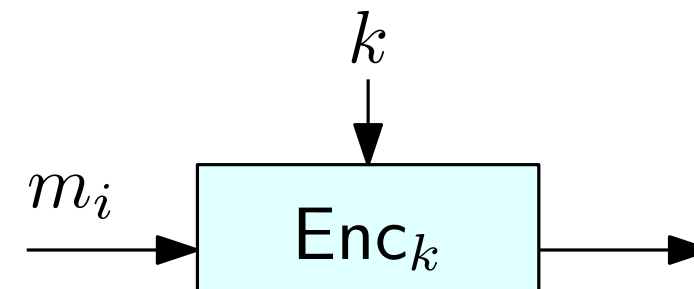
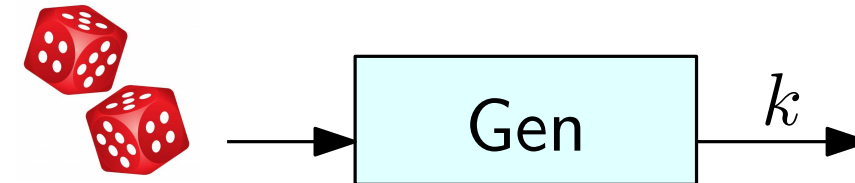
Key space: $\mathcal{K} = \{0, \dots, 25\}$

Key generation: return k chosen u.a.r. from \mathcal{K}

Encryption function:

$$\text{Enc}_k(m) = \text{Enc}_k(m_1) \parallel \text{Enc}_k(m_2) \parallel \dots \parallel \text{Enc}_k(m_\ell)$$

$$m = m_1 m_2 \dots m_\ell$$



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Message space: $\mathcal{M} = \{0, 1, \dots, 25\}^*$

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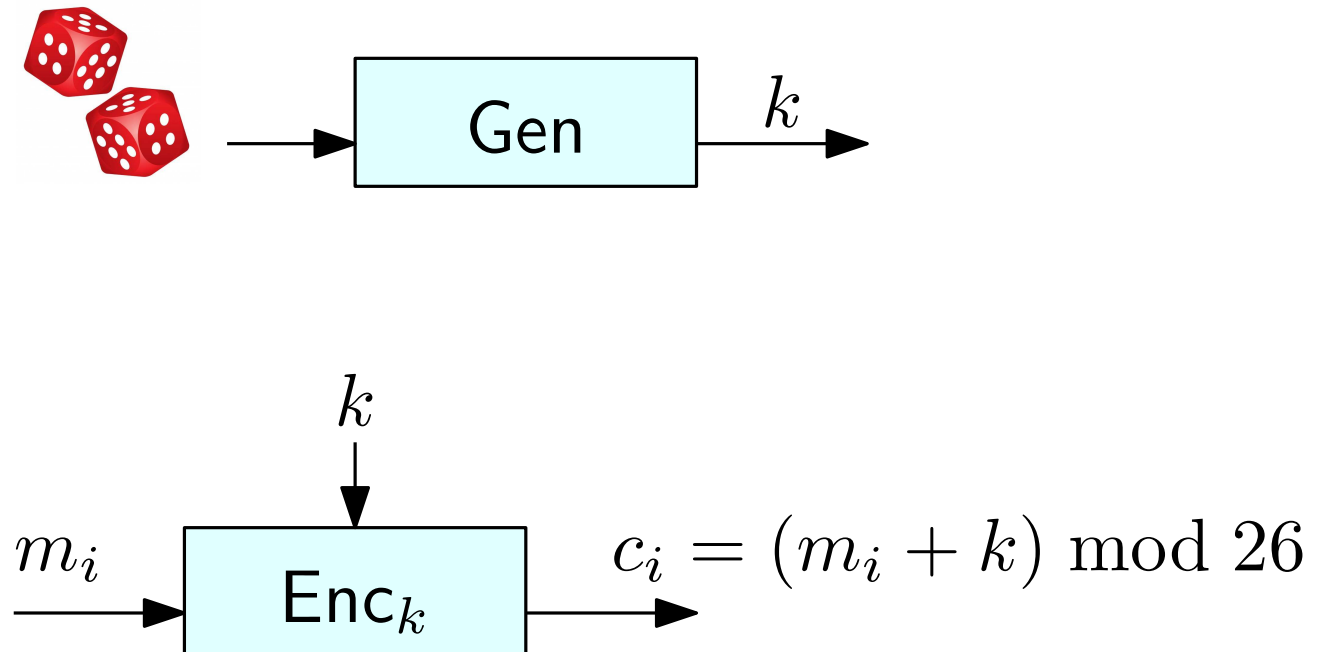
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$$\text{Enc}_k(m) = \text{Enc}_k(m_1) \parallel \text{Enc}_k(m_2) \parallel \dots \parallel \text{Enc}_k(m_\ell)$$

$$\text{Enc}_k(m_i) = (m_i + k) \bmod 26$$

$$m = m_1 m_2 \dots m_\ell$$



Shift ciphers

Message space: $\mathcal{M} = \{0, 1, \dots, 25\}^*$

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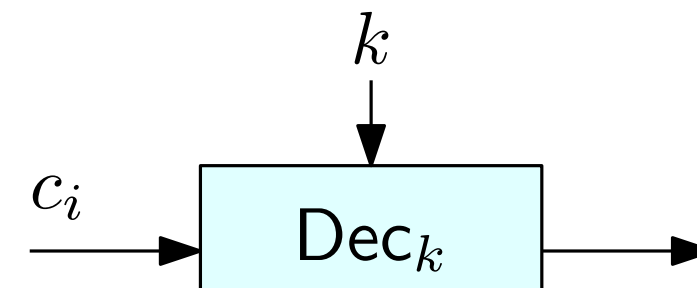
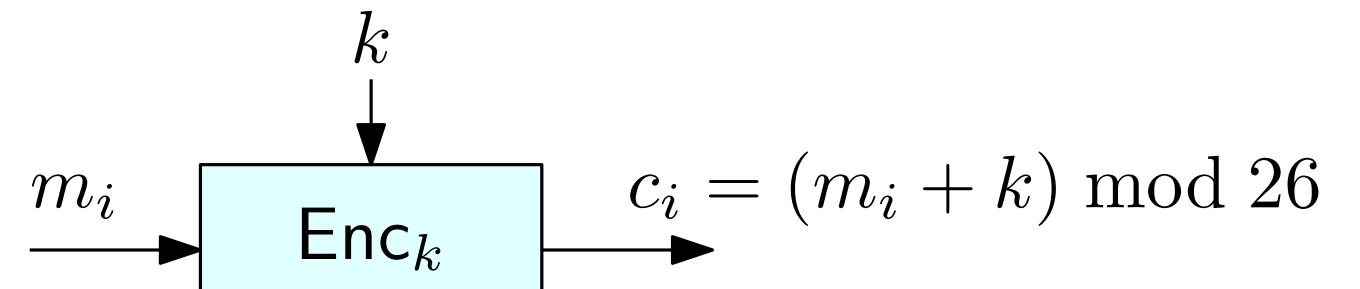
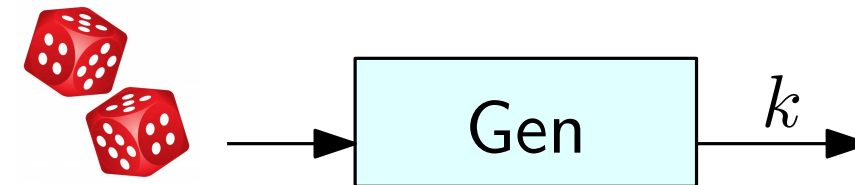
$$\text{Enc}_k(m_i) = (m_i + k) \bmod 26$$

Decryption function:

$$\text{Dec}_k(c) = \text{Dec}_k(c_1) \parallel \text{Dec}_k(c_2) \parallel \dots \parallel \text{Dec}_k(c_\ell)$$

$$m = m_1 m_2 \dots m_\ell$$

$$c = c_1 c_2 \dots c_\ell$$



Shift ciphers

Message space: $\mathcal{M} = \{0, 1, \dots, 25\}^*$

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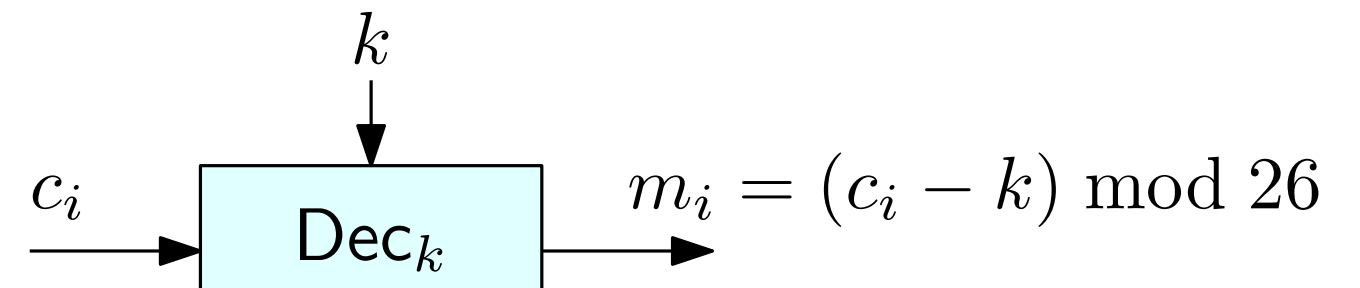
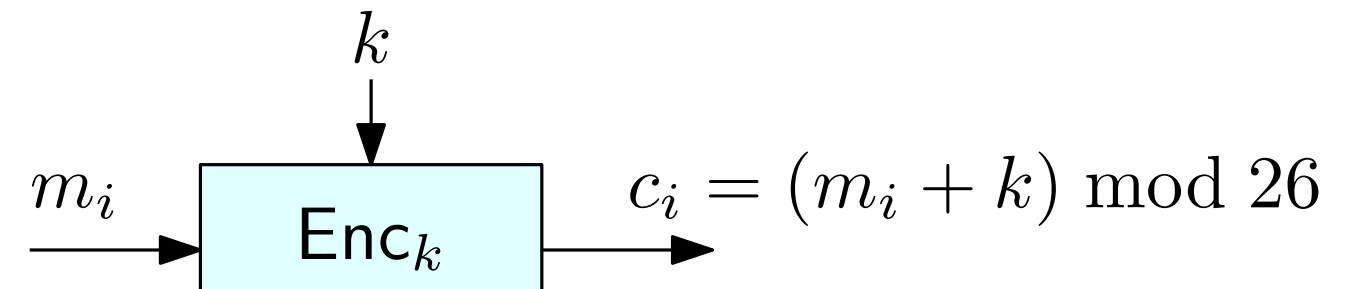
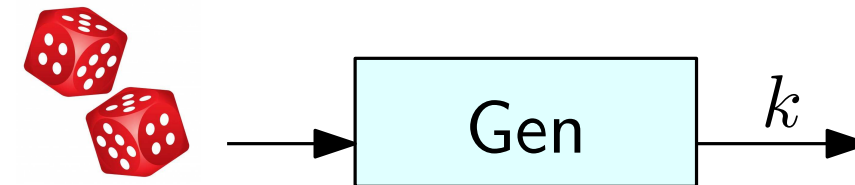
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$$\text{Dec}_k(c) = \text{Dec}_k(c_1) \parallel \text{Dec}_k(c_2) \parallel \dots \parallel \text{Dec}_k(c_\ell)$$

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$$m = m_1 m_2 \dots m_\ell$$

$$c = c_1 c_2 \dots c_\ell$$



Shift ciphers

Correctness:

We need to prove that $\text{Dec}_k(\text{Enc}_k(m)) = m$

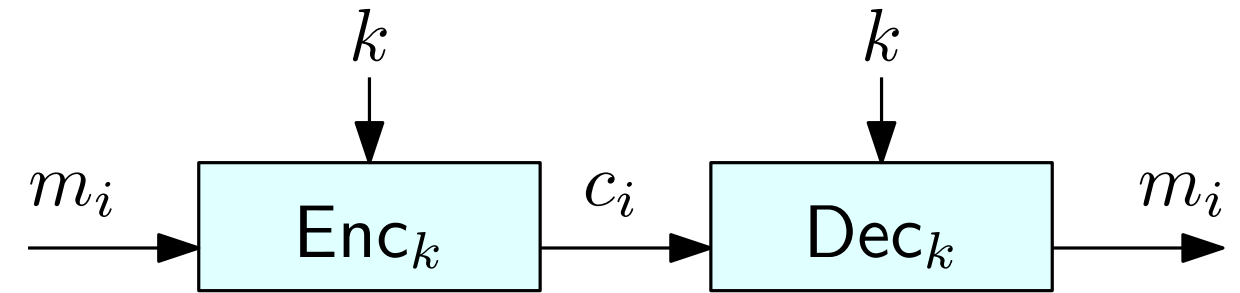
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$\text{Dec}_k(\text{Enc}_k(m_i))$



Shift ciphers

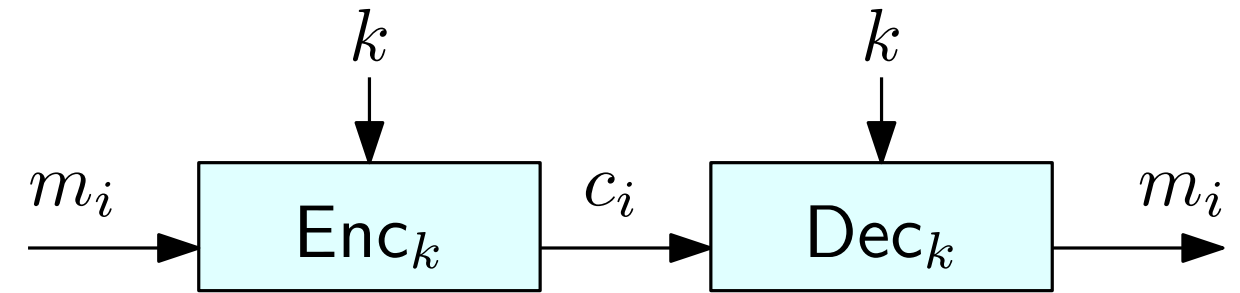
Correctness:

We need to prove that $\text{Dec}_k(\text{Enc}_k(m)) = m$

It suffices to show that $\text{Dec}_k(\text{Enc}_k(m_i)) = m_i$

$$\text{Dec}_k(\text{Enc}_k(m_i)) = \text{Dec}_k((m_i + k) \bmod 26)$$

(definition of Enc_k)

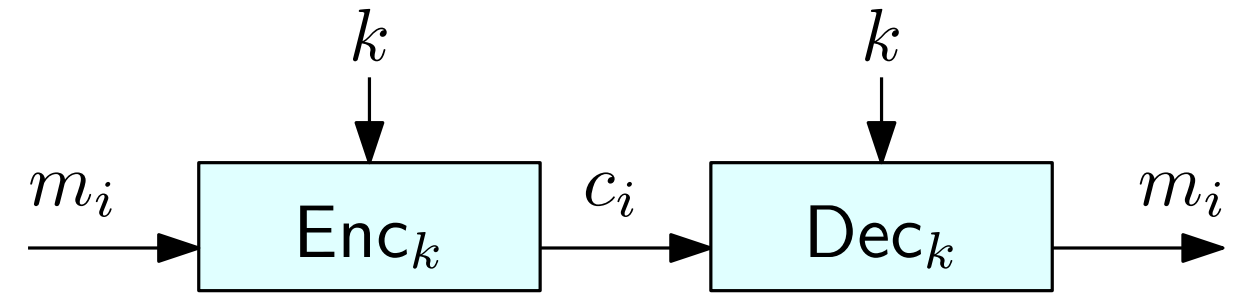


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$$\text{Dec}_k(\text{Enc}_k(m_i)) = \text{Dec}_k((m_i + k) \bmod 26) \quad (\text{definition of } \text{Enc}_k)$$

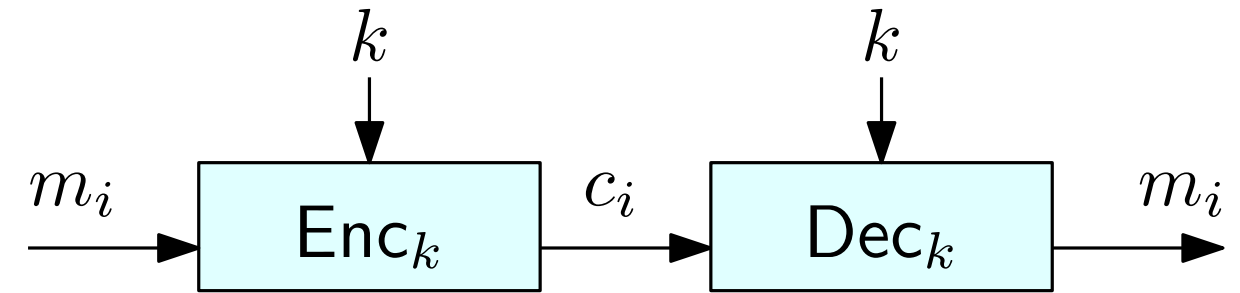
$$= (((m_i + k) \bmod 26) - k) \bmod 26 \quad (\text{definition of } \text{Dec}_k)$$

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$$\text{Dec}_k(\text{Enc}_k(m_i)) = \text{Dec}_k((m_i + k) \bmod 26) \quad \text{(definition of } \text{Enc}_k \text{)}$$

$$= (((m_i + k) \bmod 26) - k) \bmod 26 \quad \text{(definition of } \text{Dec}_k \text{)}$$

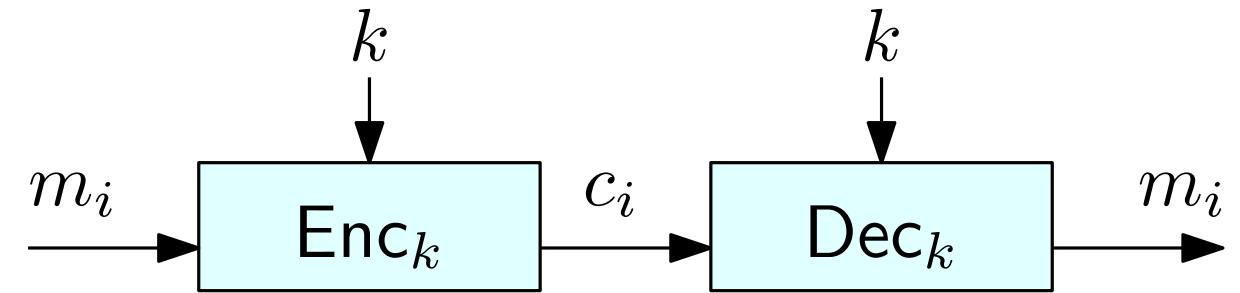
$$= (m_i + k - k) \bmod 26 \quad \text{(properties of mod)}$$

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It suffices to show that $\text{Dec}_k(\text{Enc}_k(m_i)) = m_i$



$$\text{Dec}_k(\text{Enc}_k(m_i)) = \text{Dec}_k((m_i + k) \bmod 26) \quad \text{(definition of Enc}_k\text{)}$$

$$= (((m_i + k) \bmod 26) - k) \bmod 26 \quad \text{(definition of Dec}_k\text{)}$$

$$= (m_i + k - k) \bmod 26 \quad \text{(properties of mod)}$$

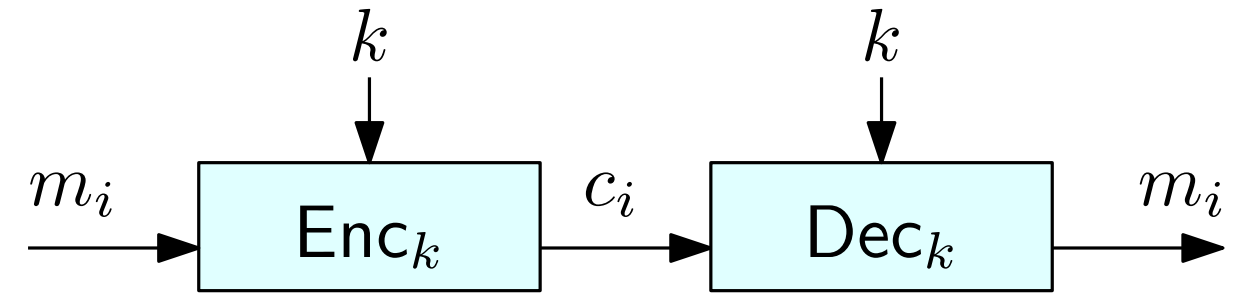
$$= m_i \bmod 26$$

Shift ciphers

Correctness:

We need to prove that $\text{Dec}_k(\text{Enc}_k(m)) = m$

It suffices to show that $\text{Dec}_k(\text{Enc}_k(m_i)) = m_i$



$$\text{Dec}_k(\text{Enc}_k(m_i)) = \text{Dec}_k((m_i + k) \bmod 26) \quad (\text{definition of } \text{Enc}_k)$$

$$= (((m_i + k) \bmod 26) - k) \bmod 26 \quad (\text{definition of } \text{Dec}_k)$$

$$= (m_i + k - k) \bmod 26 \quad (\text{properties of mod})$$

$$= m_i \bmod 26$$

$$= m_i \quad (m_i < 26)$$

Shift ciphers

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How many keys are there?

$$|\mathcal{K}| = 26$$

We can use a **brute-force** (or **exhaustive search**) attack

In a brute-force attack, the adversary systematically tries all possible keys until the correct one is found.



Shift ciphers

Brute-force attack:

$\text{Dec}_0(c) =$ X J S I M J Q U

$\text{Dec}_1(c) =$ W I R H L I P T

$\text{Dec}_2(c) =$ V H Q G K H O S

$\text{Dec}_3(c) =$ U G P F J G N R

$\text{Dec}_4(c) =$ T F O E I F M Q

$\text{Dec}_5(c) =$ S E N D H E L P

$\text{Dec}_6(c) =$ R D M C G D K O

\vdots

$\text{Dec}_{24}(c) =$ Z L U K O L S W

$\text{Dec}_{25}(c) =$ Y K T J N K R V

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\vdots

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Sufficient key-space principle: Any cipher should use a “large enough” key space to prevent brute-force attacks

(Monoalphabetic) Substitution ciphers

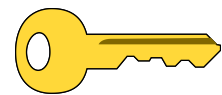
The key is now a permutation π of the alphabet $\Sigma = \{A, B, \dots, Z\}$

$$\mathcal{K} = \{\pi : \Sigma \rightarrow \Sigma \mid \pi \text{ is a permutation}\}$$

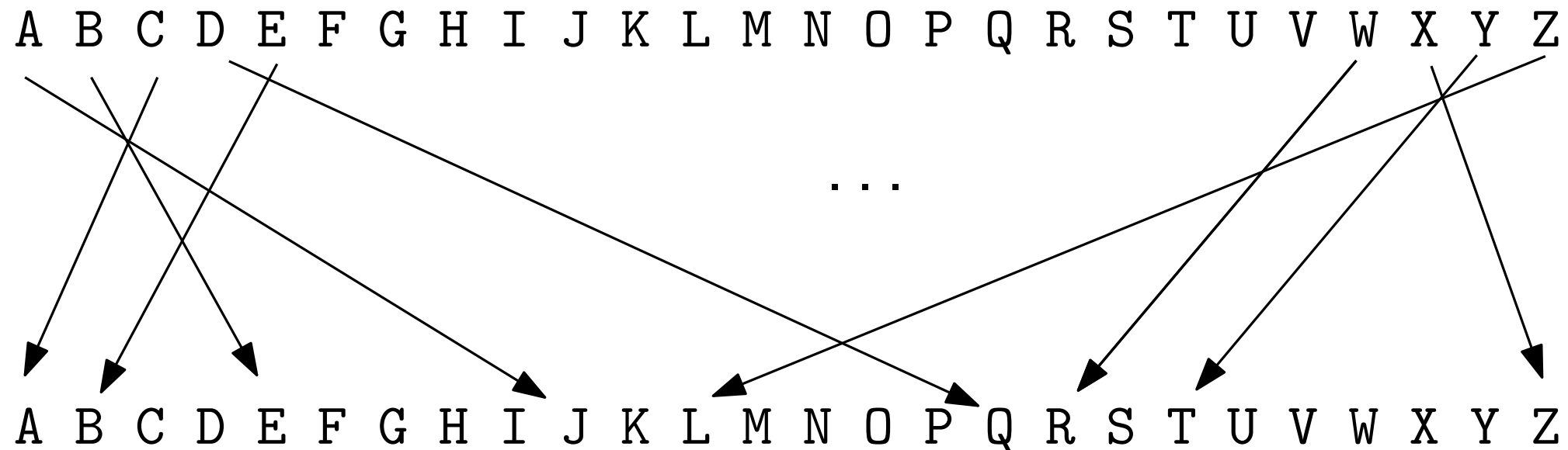
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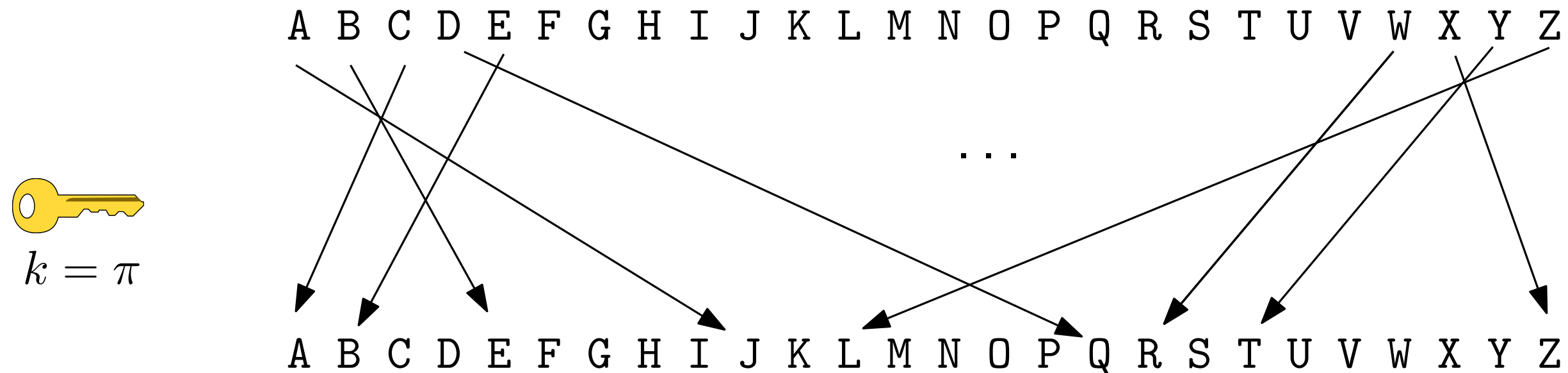
$$k = \pi$$



(Monoalphabetic) Substitution ciphers

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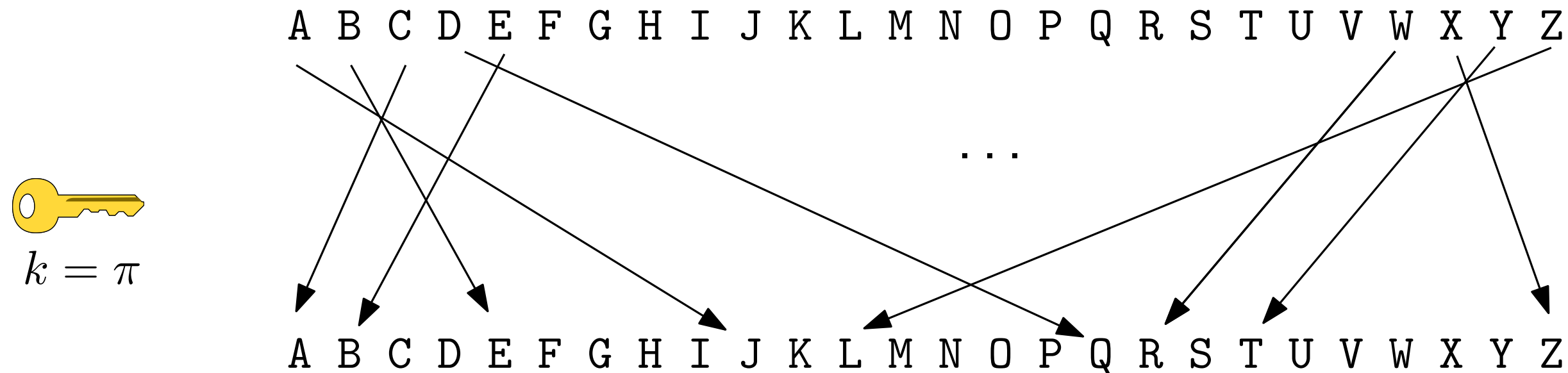
To encrypt a message, replace each character m_i in the plaintext with $k(m_i) = \pi(m_i)$

$$\text{Enc}_k(m) = k(m_1) \| k(m_2) \| \dots \| k(m_\ell)$$

(Monoalphabetic) Substitution ciphers

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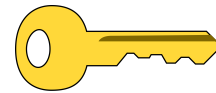
To decrypt a message, replace each character c_i of the ciphertext with $k^{-1}(c_i) = \pi^{-1}(c_i)$

$$\text{Dec}_k(m) = k^{-1}(c_1) \| k^{-1}(c_2) \| \dots \| k^{-1}(c_\ell)$$

(Monoalphabetic) Substitution ciphers

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
J	E	A	Q	B	Y	D	P	V	F	K	I	N	H	M	X	U	S	W	C	O	G	R	Z	T	L

k



$m = A W A I T O R D E R S$



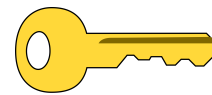
$\text{Enc}_k(m)$

(Monoalphabetic) Substitution ciphers

A B C D E F G H I J K L M N O P Q R S T U V W X Y Z

J E A Q B Y D P V F K I N H M X U S W C O G R Z T L

k



$m =$ A W A I T O R D E R S



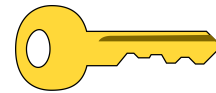
$\text{Enc}_k(m)$

$c =$ J R J V C M S Q B S W

(Monoalphabetic) Substitution ciphers

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
J	E	A	Q	B	Y	D	P	V	F	K	I	N	H	M	X	U	S	W	C	O	G	R	Z	T	L

k



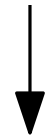
$m = \text{A W A I T O R D E R S}$



$\text{Enc}_k(m)$

$c = \text{J R J V C M S Q B S W}$

$c = \text{B H B N T Q M R H}$

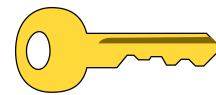


$\text{Dec}_k(c)$

(Monoalphabetic) Substitution ciphers

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
J	E	A	Q	B	Y	D	P	V	F	K	I	N	H	M	X	U	S	W	C	O	G	R	Z	T	L

k



$m = \text{A W A I T O R D E R S}$



$\text{Enc}_k(m)$

$c = \text{J R J V C M S Q B S W}$

$c = \text{B H B N T Q M R H}$



$\text{Dec}_k(c)$

$m = \text{E N E M Y D O W N}$

(Monoalphabetic) Substitution ciphers

How many keys are there?

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$$|\mathcal{K}| = |\Sigma|! = 26! \approx 2^{88}$$

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- ... but they might be susceptible to more sophisticated attack techniques

Observation (informal): A large key space is not a sufficient condition for a cipher to be secure



Substitution ciphers

Suppose that we somehow have deciphered a small portion of the ciphertext

We can replace each known ciphertext symbol x with its plaintext $k^{-1}(x)$ and then use the partially decrypted message to make further guesses about k

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A similar example: codebreaker word puzzle

1	12	1	12	19		21	2	13	9	26	20	19	16	17
19		14		13		19		9		4		4		16
5	16	19	1	5	12	16		22	12	12	21	12	26	25
19		16				24				15		5		19
21	2	15	10	11		19	18	19	3	9		14	12	17
25		9		19				13				9		
11	19	17		10	21	20	13	13		7	20	16	12	16
2				21		21		2		19				9
11	2	9	24	9		5	12	8	2	1		18	19	11
		15				16				3		14		6
7	19	10		23	20	19	21	15		11	14	12	24	9
20		19		20				19				12		1
21	19	16	2	19	5	11		7	19	1	3	6	12	5
9		26		3		2		12		20		9		9
6	16	12	7	9	1	5	12	16		10	21	9	4	17

Substitution ciphers

1	12	1	12	19		21	2	13	9	26	20	19	16	17	
19		14		13		19		9		4		4		16	
5	T	16	19	1	5	12	16		22	12	12	21	12	26	25
19		16				24				15		5		19	
21	2	15	10	11		19	18	19	3	9		14	12	17	
25		9		19				13				9			
11	19	17		10	21	20	13	13		7	20	16	12	16	
2				21		21		2		19				9	
11	2	9	24	9		5	12	8	2	1		18	19	11	
		15				16				3		14		6	
7	19	10		23	20	19	21	15		11	14	12	24	9	
20		19		20				19				12		1	
21	19	16	2	19	5	11		7	19	1	3	6	12	5	
9		26		3		2		12		20		9		9	
6	16	12	7	9	1	5	12	16		10	21	9	4	17	

Substitution ciphers

1C	12O	1C	12O	19A		21	2	13	9	26	20	19A	16	R	17
19A		14		13		19A		9		4		4			16
5T	16R	19A	1C	5T	12O	16R		22	12O	12O	21	12O	26	25	
19A		16R				24				15		5T		19A	
21	2	15	10	11		19A	18	19A	3	9		14	12O	17	
25		9		19A				13				9			
11	19A	17		10	21	20	13	13		7	20	16R	12O	16R	
2				21		21		2		19A				9	
11	2	9	24	9		5T	12O	8	2	1C		18	19A	11	
		15				16R				3		14		6	
7	19A	10		23	20	19A	21	15		11	14	12O	24	9	
20		19A		20				19A				12O		1C	
21	19A	16R	2	19A	5T	11		7	19A	1C	3	6	12O	5T	
9		26		3		2		12O		20		9		9	
6	16R	12O	7	9	1C	5T	12O	16R		10	21	9	4	17	

Substitution ciphers

1	C	12	O	1	C	12	O	19	A		21	2	13	9	26	20	19	A	16	R	17		
19	A			14				13			19	A		9		4		4			16	R	
5	T	16	R	19	A	1	C	5	T	12	O	R		22	O	O	21	12	O	26	25		
19	A			16	R						24					15		5	T		19	A	
21	2	15	10	11						19	A	18	19	A	3	9		14	12	O	17		
25			9			19	A							13				9					
11	19	A	17			10	21	20	13	13				7	20	16	R	12	O	16	R		
2						21			21		2			19	A						9		
11	2	9	24	9				5	T	12	O	8	2	1	C		18	19	A	11			
			15						16	R				3			14			6			
7	19	A	10			23	20	19	A	21	15			11	14	12	O	24	9				
20			19	A		20					19	A					12	O		1	C		
21	19	A	16	R	2	19	A	5	T	11			7	19	A	1	C	3	6	12	O	5	T
9			26			3			2				12	O		20		9			9		
6	16	R	O	7	9	1	C	5	T	12	O	16	R		10	21	9	4	17				

Substitution ciphers

1C	12O	1C	12O	19A		21	2	13	9E	26	20	19A	16R	17
19A		14		13		19A		9E		4		4		16R
5T	16R	19A	1C	5T	12O	16R		22	12O	12O	21	12O	26	25
19A		16R				24				15		5T		19A
21	2	15	10	11		19A	18	19A	3	9E		14	12O	17
25		9E		19A				13				9E		
11	19A	17		10	21	20	13	13		7J	20	16R	12O	16R
2				21		21		2		19A				9E
11	2	9E	24	9E		5T	12O	8	2	1C		18	19A	11
		15				16R				3		14		6P
7J	19A	10		23	20	19A	21	15		11	14	12O	24	9E
20		19A		20				19A				12O		1C
21	19A	16R	2	19A	5T	11		7J	19A	1C	3	6P	12O	5T
9E		26		3		2		12O		20		9E		9E
6P	16R	12O	7J	9E	1C	5T	12O	16R		10	21	9E	4	17

Substitution ciphers

1C	12O	1C	12O	19A		21	2	13	9E	26	20	19A	16R	17
19A		14		13		19A		9E		4		4		16R
5T	16R	19A	1C	5T	12O	16R		22	12O	12O	21	12O	26	25
19A		16R				24				15		5T		19A
21	2	15	10	11		19A	18	19A	3	9E		14	12O	17
25		9E		19A				13				9E		
11	19A	17		10	21	20	13	13		7J	20	16R	12O	16R
2				21		21		2		19A				9E
11	2	9E	24	9E		5T	12O	8	2	1C		18	19A	11
		15				16R				3		14		6P
7J	19A	10		23	20	19A	21	15		11	14	12O	24	9E
20		19A		20				19A				12O		1C
21	19A	16R	2	19A	5T	11		7J	19A	1C	3	6P	12O	5T
9E		26		3		2		12O		20		9E		9E
6P	16R	12O	7J	9E	1C	5T	12O	16R		10	21	9E	4	17

Substitution ciphers

1C	12O	1C	12O	19A		21	2	13	9E	26	20	19A	16R	17D
19A		14		13		19A		9E		4		4		16R
5T	16R	19A	1C	5T	12O	16R		22	12O	12O	21	12O	26	25
19A		16R				24				15		5T		19A
21	2	15	10	11S		19A	18	19A	3	9E		14	12O	17D
25		9E		19A				13				9E		
11S	19A	17		10	21	20	13	13		7J	20	16R	12O	16R
2				21		21		2		19A				9E
11S	2	9E	24	9E		5T	12O	8	2	1C		18	19A	11S
		15				16R				3		14		6P
7J	19A	10		23	20	19A	21	15		11S	14	12O	24	9E
20		19A		20				19A				12O		1C
21	19A	16R	2	19A	5T	11S		7J	19A	1C	3	6P	12O	5T
9E		26		3		2		12O		20		9E		9E
6P	16R	12O	7J	9E	1C	5T	12O	16R		10	21	9E	4	17D

Substitution ciphers

1C	12O	1C	12O	19A		21	2	13	9E	26	20	19A	16R	17D
19A		14		13		19A		9E		4		4		16R
5T	16R	19A	1C	5T	12O	16R		22	12O	12O	21	12O	26	25
19A		16R				24				15		5T		19A
21	2	15	10	11S		19A	18	19A	3	9E		14	12O	17D
25		9E		19A				13				9E		
11S	19A	17		10	21	20	13	13		7J	20	16R	12O	16R
2				21		21		2		19A				9E
11S	2	9E	24	9E		5T	12O	8	2	1C		18	19A	11S
		15				16R				3		14		6P
7J	19A	10		23	20	19A	21	15		11S	14	12O	24	9E
20		19A		20				19A				12O		1C
21	19A	16R	2	19A	5T	11S		7J	19A	1C	3	6P	12O	5T
9E		26		3		2		12O		20		9E		9E
6P	16R	12O	7J	9E	1C	5T	12O	16R		10	21	9E	4	17D

Substitution ciphers

1C	12O	1C	12O	19A		21	2	13	9E	26	20U	19A	16R	17D
19A		14		13		19A		9E		4		4		16R
5T	16R	19A	1C	5T	12O	16R		22	12O	12O	21	12O	26	25
19A		16R				24				15		5T		19A
21	2	15	10	11S		19A	18	19A	3	9E		14	12O	17D
25		9E		19A				13				9E		
11S	19A	17		10	21	20U	13	13		7J	20U	16R	12O	16R
2				21		21		2		19A				9E
11S	2	9E	24	9E		5T	12O	8	2	1C		18	19A	11S
		15				16R				3		14		6P
7J	19A	10		23	20U	19A	21	15		11S	14	12O	24	9E
20U		19A		20U				19A				12O		1C
21	19A	16R	2	19A	5T	11S		7J	19A	1C	3	6P	12O	5T
9E		26		3		2		12O		20U		9E		9E
6P	16R	12O	7J	9E	1C	5T	12O	16R		10	21	9E	4	17D

Substitution ciphers

1C	12O	1C	12O	19A		21	2	13	9E	26	20U	19A	16R	17D
19A		14		13		19A		9E		4		4		16R
5T	16R	19A	1C	5T	12O	16R		22	12O	12O	21	12O	26	25
19A		16R				24				15		5T		19A
21	2	15	10	11S		19A	18	19A	3	9E		14	12O	17D
25		9E		19A				13				9E		
11S	19A	17		10	21	20U	13	13		7J	20U	16R	12O	16R
2				21		21		2		19A				9E
11S	2	9E	24	9E		5T	12O	8	2	1C		18	19A	11S
		15				16R				3		14		6P
7J	19A	10		23	20U	19A	21	15		11S	14	12O	24	9E
20U		19A		20U				19A				12O		1C
21	19A	16R	2	19A	5T	11S		7J	19A	1C	3	6P	12O	5T
9E		26		3		2		12O		20U		9E		9E
6P	16R	12O	7J	9E	1C	5T	12O	16R		10	21	9E	4	17D

Substitution ciphers

1C	12O	1C	12O	19A		21L	2I	12F	9E	26G	20U	19A	16R	17D
19A		14		12F		19A		9E		4		4		16R
5T	16R	19A	1C	5T	12O	16R		22	12O	12O	21L	12O	26G	25
19A		16R				24				15		5T		19A
21L	2I	15	10	11S		19A	18	19A	3	9E		14	12O	17D
25		9E		19A				12F				9E		
11S	19A	17		10	21L	20U	12F	12F		7J	20U	16R	12O	16R
2I				21L		21L		2I		19A				9E
11S	2I	9E	24	9E		5T	12O	8	2I	1C		18	19A	11S
		15				16R				3		14		6P
7J	19A	10		23	20U	19A	21L	15		11S	14	12O	24	9E
20U		19A		20U				19A				12O		1C
21L	19A	16R	2I	19A	5T	11S		7J	19A	1C	3	6P	12O	5T
9E		26G		3		2I		12O		20U		9E		9E
6P	16R	12O	7J	9E	1C	5T	12O	16R		10	21L	9E	4	17D

Substitution ciphers



Substitution ciphers

1C	12O	1C	12O	19A		21L	2I	12F	9E	26G	20U	19A	16R	17D
19A		14		12F		19A		9E		4		4		16R
5T	16R	19A	1C	5T	12O	16R		22	12O	12O	21L	12O	26G	25
19A		16R				24				15		5T		19A
21L	2I	15	19B	11S		19A	18W	19A	3K	9E		14	12O	17D
25		9E		19A				12F				9E		
11S	19A	17		19B	21L	20U	12F	12F		7J	20U	16R	12O	16R
2I				21L		21L		2I		19A				9E
11S	2I	9E	24V	9E		5T	12O	8X	2I	1C		19W	19A	11S
		15				16R				3K		14		6P
7J	19A	19B		23	20U	19A	21L	15		11S	14	12O	24V	9E
20U		19A		20U				19A				12O		1C
21L	19A	16R	2I	19A	5T	11S		7J	19A	1C	3K	6P	12O	5T
9E		26G		3K		2I		12O		20U		9E		9E
6P	16R	12O	7J	9E	1C	5T	12O	16R		19B	21L	9E	4	17D

Substitution ciphers



Substitution ciphers

1C	12O	1C	12O	19A		21L	2I	12F	9E	26G	20U	19A	16R	17D
19A		14		12F		19A		9E		4N		4N		16R
5T	16R	19A	1C	5T	12O	16R		22O	12O	21L	12O	26G	25	
19A		16R				24				15G		5T		19A
21L	2I	15M	18B	11S		19A	18W	19A	3K	9E		14O	12D	17
25		9E		19A				12F				9E		
11S	19A	17		18B	21L	20U	12F	12F		7J	20U	16R	12O	16R
2I				21L		21L		2I		19A				9E
11S	2I	9E	24V	9E		5T	12O	8X	2I	1C		18W	19A	11S
		15M				16R				3K		14H		6P
7J	19A	18B		23Q	20U	19A	21L	15M		11S	14H	12O	24V	9E
20U		19A		20U				19A				12O		1C
21L	19A	16R	2I	19A	5T	11S		7J	19A	1C	3K	6P	12O	5T
9E		26G		3K		2I		12O		20U		9E		9E
6P	16R	12O	7J	9E	1C	5T	12O	16R		18B	21L	9E	4N	17D

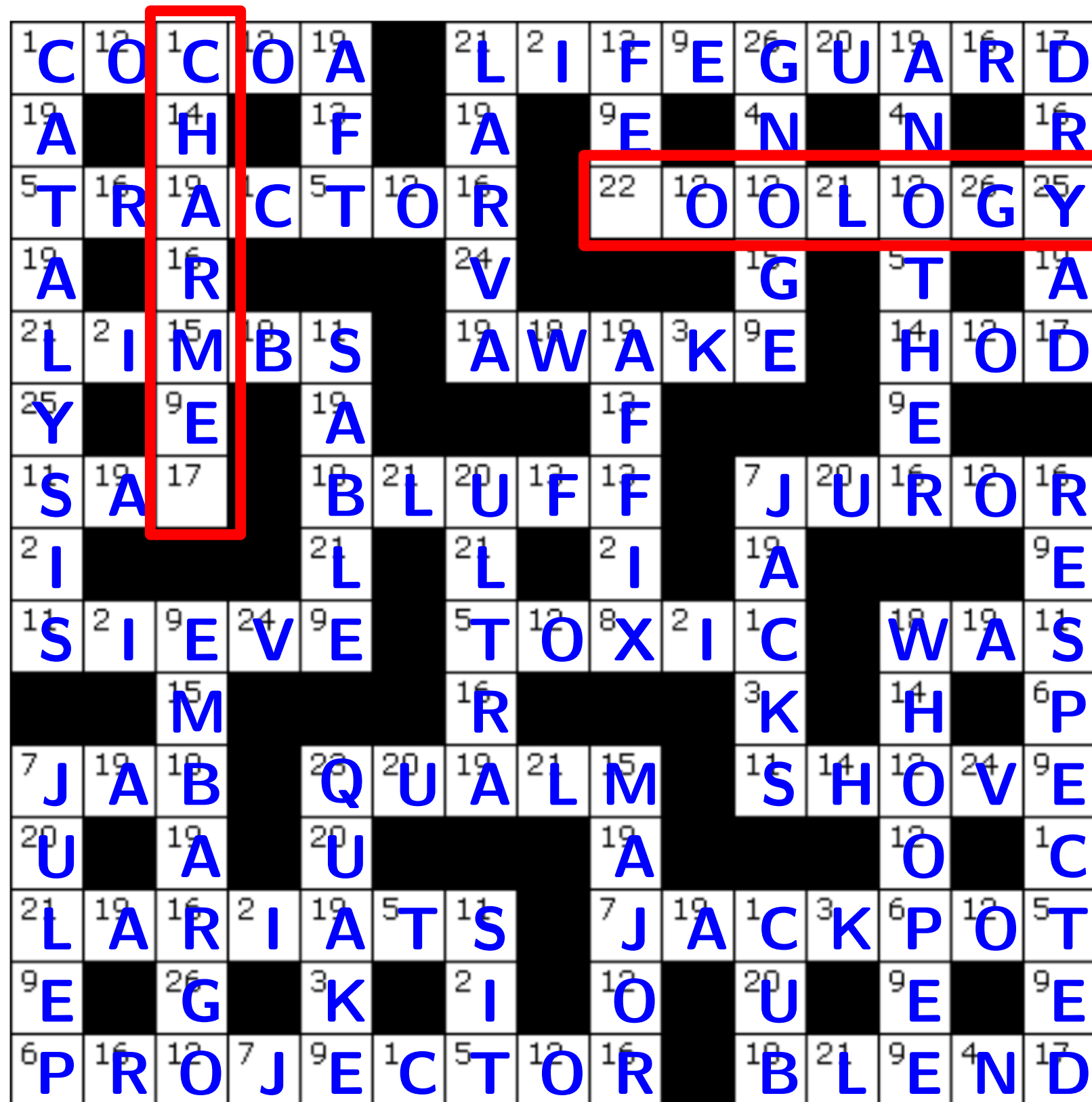
Substitution ciphers

1C	12O	1C	12O	19A		21L	2I	12F	9E	26G	20U	19A	16R	17D
19A		14		12F		19A		9E		4N		4N		16R
5T	16R	19A	1C	5T	12O	16R		22O	12O	21L	12O	26G	25	
19A		16R				24				15G		5T		19A
21L	2I	15M	18B	11S		19A	18W	19A	3K	9E		14O	12D	17
25		9E		19A				12F				9E		
11S	19A	17		18B	21L	20U	12F	12F		7J	20U	16R	12O	16R
2I				21L		21L		2I		19A				9E
11S	2I	9E	24V	9E		5T	12O	8X	2I	1C		18W	19A	11S
		15M				16R				3K		14H		6P
7J	19A	18B		23Q	20U	19A	21L	15M		11S	14H	12O	24V	9E
20U		19A		20U				19A				12O		1C
21L	19A	16R	2I	19A	5T	11S		7J	19A	1C	3K	6P	12O	5T
9E		26G		3K		2I		12O		20U		9E		9E
6P	16R	12O	7J	9E	1C	5T	12O	16R		18B	21L	9E	4N	17D

Substitution ciphers

1C	12O	1C	12O	19A		21L	2I	12F	9E	26G	20U	19A	16R	17D
19A		14H		12F		19A		9E		4N		4N		16R
5T	16R	19A	1C	5T	12O	16R		22O	12O	12L	21O	26G	25Y	
19A		16R				24V				15G		5T		19A
21L	2I	15M	18B	11S		19A	18W	19A	3K	9E		14H	12O	17D
25Y		9E		19A				12F				9E		
11S	19A	17		18B	21L	20U	12F	12F		7J	20U	16R	12O	16R
2I				21L		21L		2I		19A				9E
11S	2I	9E	24V	9E		5T	12O	8X	2I	1C		18W	19A	11S
		15M				16R				3K		14H		6P
7J	19A	18B		23Q	20U	19A	21L	15M		11S	14H	12O	24V	9E
20U		19A		20U				19A				12O		1C
21L	19A	16R	2I	19A	5T	11S		7J	19A	1C	3K	6P	12O	5T
9E		26G		3K		2I		12O		20U		9E		9E
6P	16R	12O	7J	9E	1C	5T	12O	16R		18B	21L	9E	4N	17D

Substitution ciphers



Substitution ciphers

1C	12O	1C	12O	19A		21L	2I	12F	9E	26G	20U	19A	16R	17D
19A		14H		12F		19A		9E		4N		4N		16R
5T	16R	19A	1C	5T	12O	16R		22Z	12O	12O	21L	12O	26G	25Y
19A		16R				24V				15G		5T		19A
21L	2I	15M	18B	11S		19A	18W	19A	3K	9E		14H	12O	17D
25Y		9E		19A				12F				9E		
11S	19A	17D		18B	21L	20U	12F	12F		7J	20U	16R	12O	16R
2I				21L		21L		2I		19A				9E
11S	2I	9E	24V	9E		5T	12O	8X	2I	1C		18W	19A	11S
		15M				16R				3K		14H		6P
7J	19A	18B		23Q	20U	19A	21L	15M		11S	14H	12O	24V	9E
20U		19A		20U				19A				12O		1C
21L	19A	16R	2I	19A	5T	11S		7J	19A	1C	3K	6P	12O	5T
9E		26G		3K		2I		12O		20U		9E		9E
6P	16R	12O	7J	9E	1C	5T	12O	16R		18B	21L	9E	4N	17D

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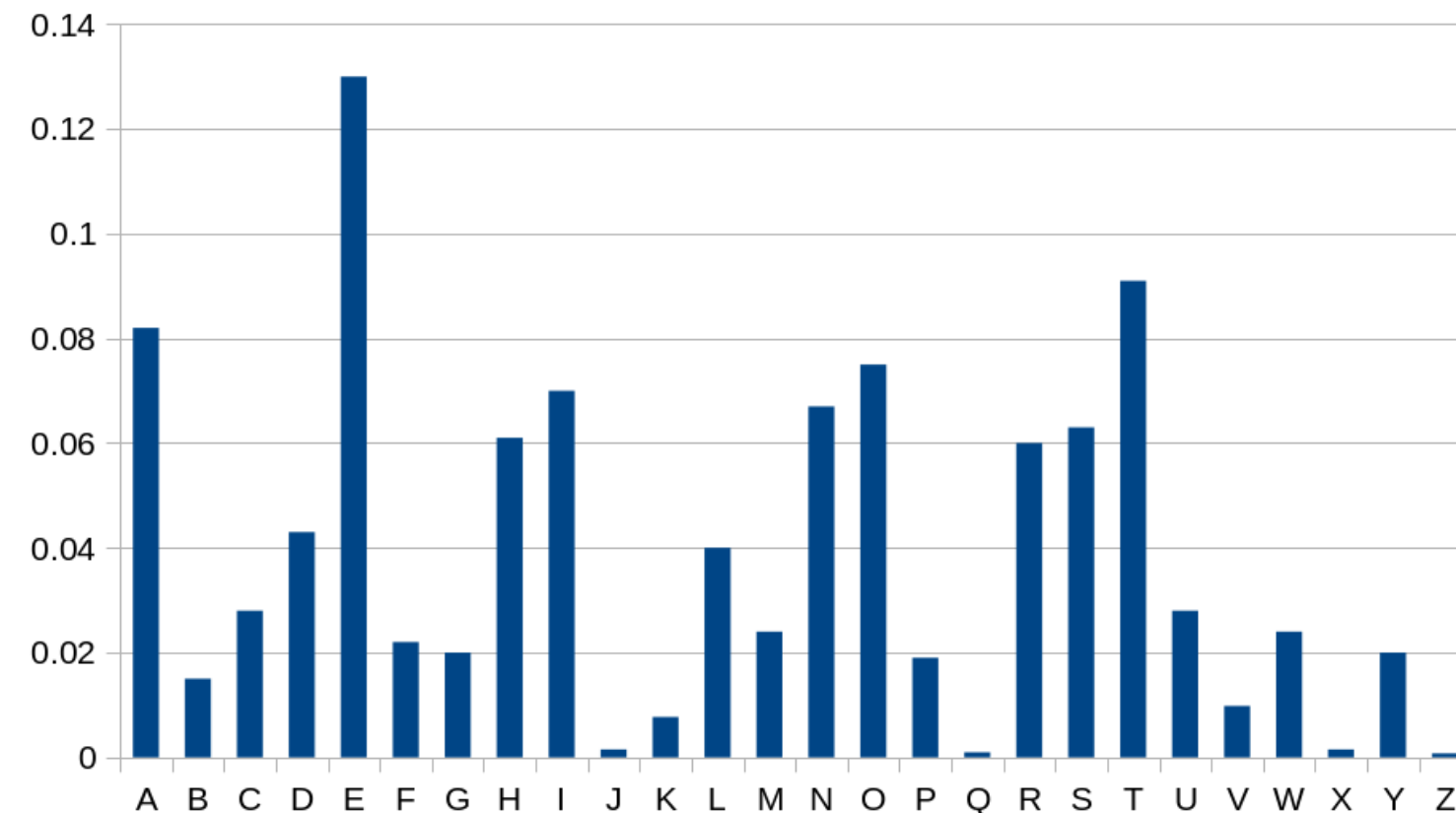
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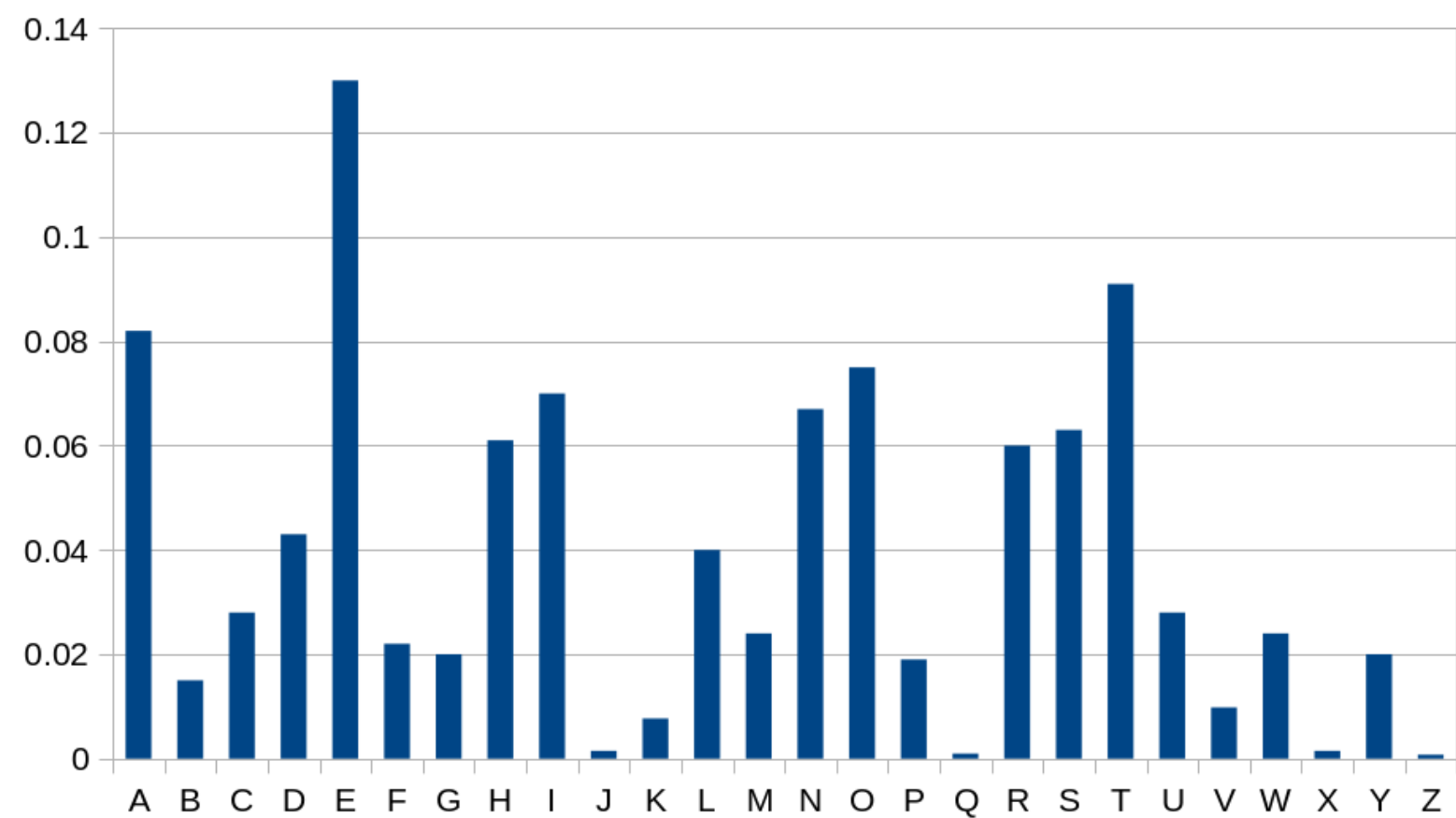
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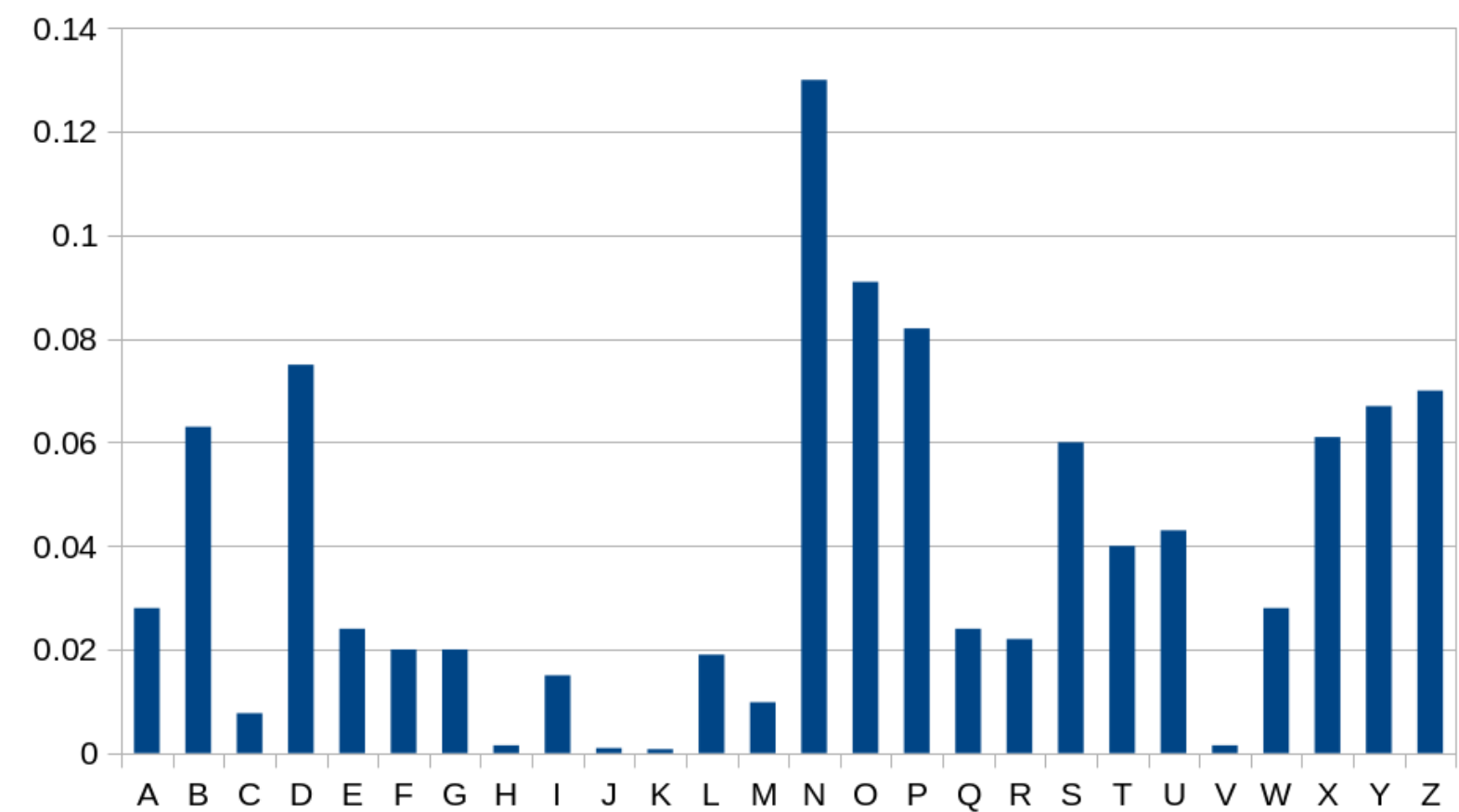
Substitution ciphers

Compare the expected frequencies in the message language with the observed frequencies in the ciphertext

Expected



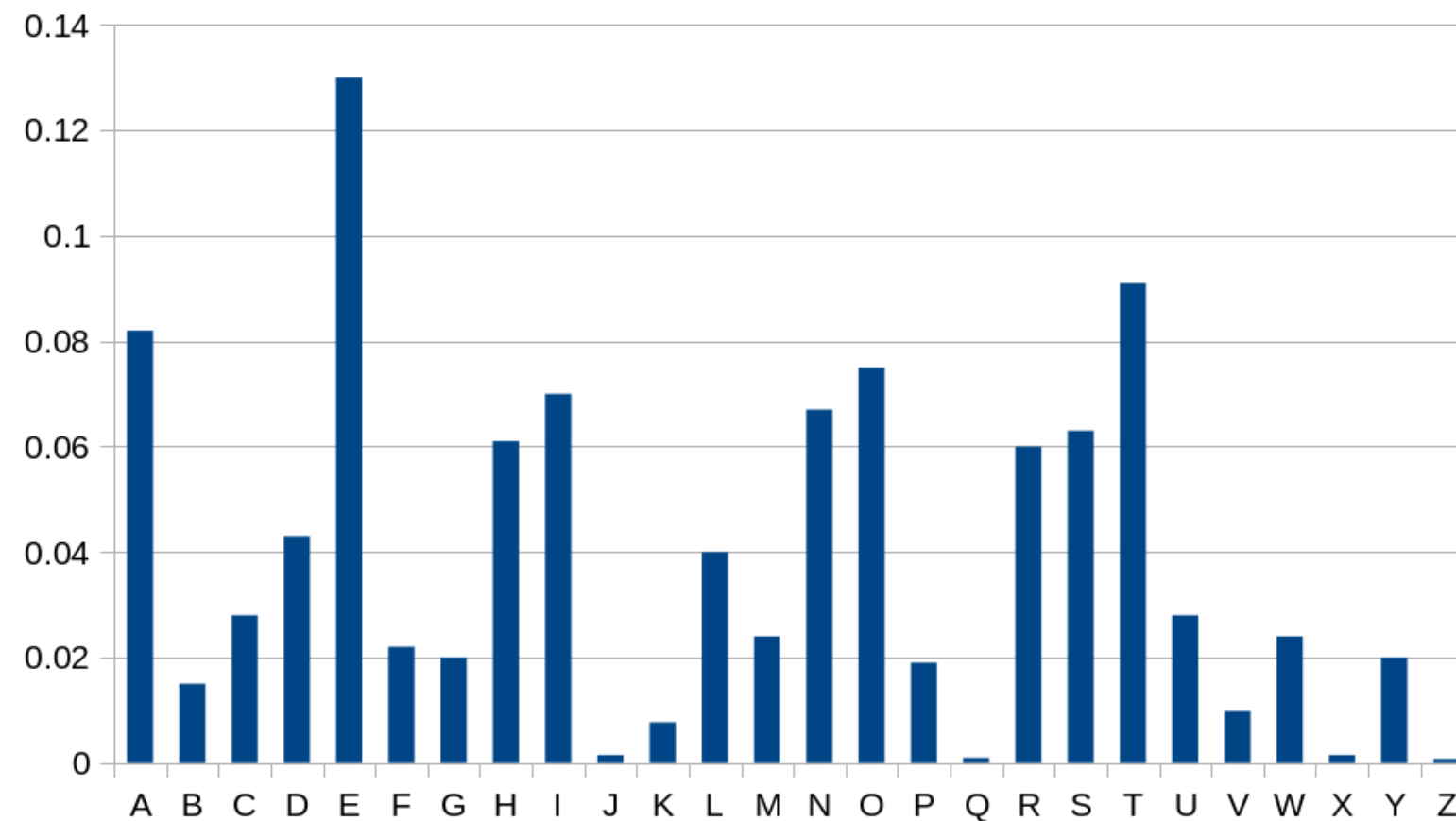
Observed (in the ciphertext)



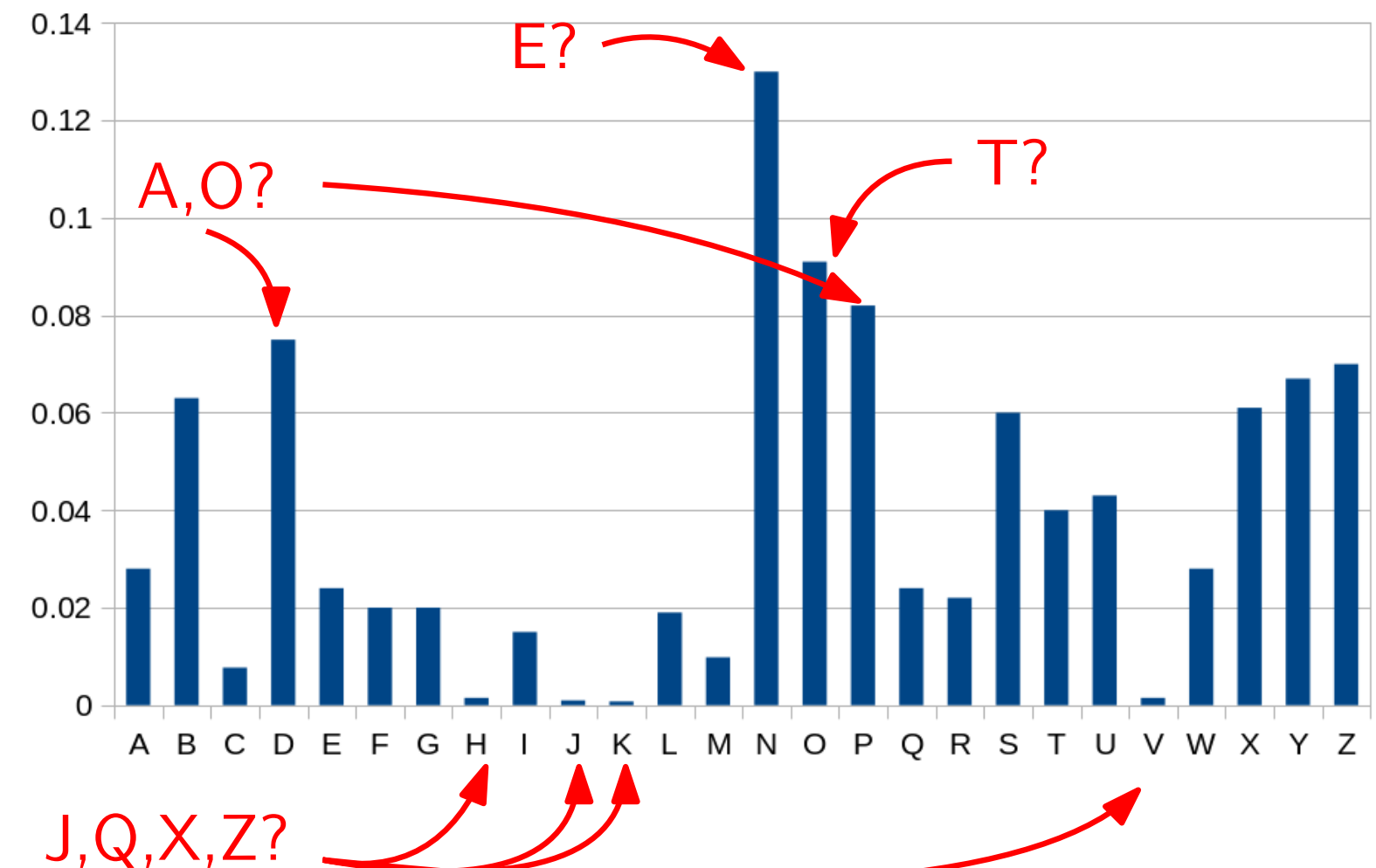
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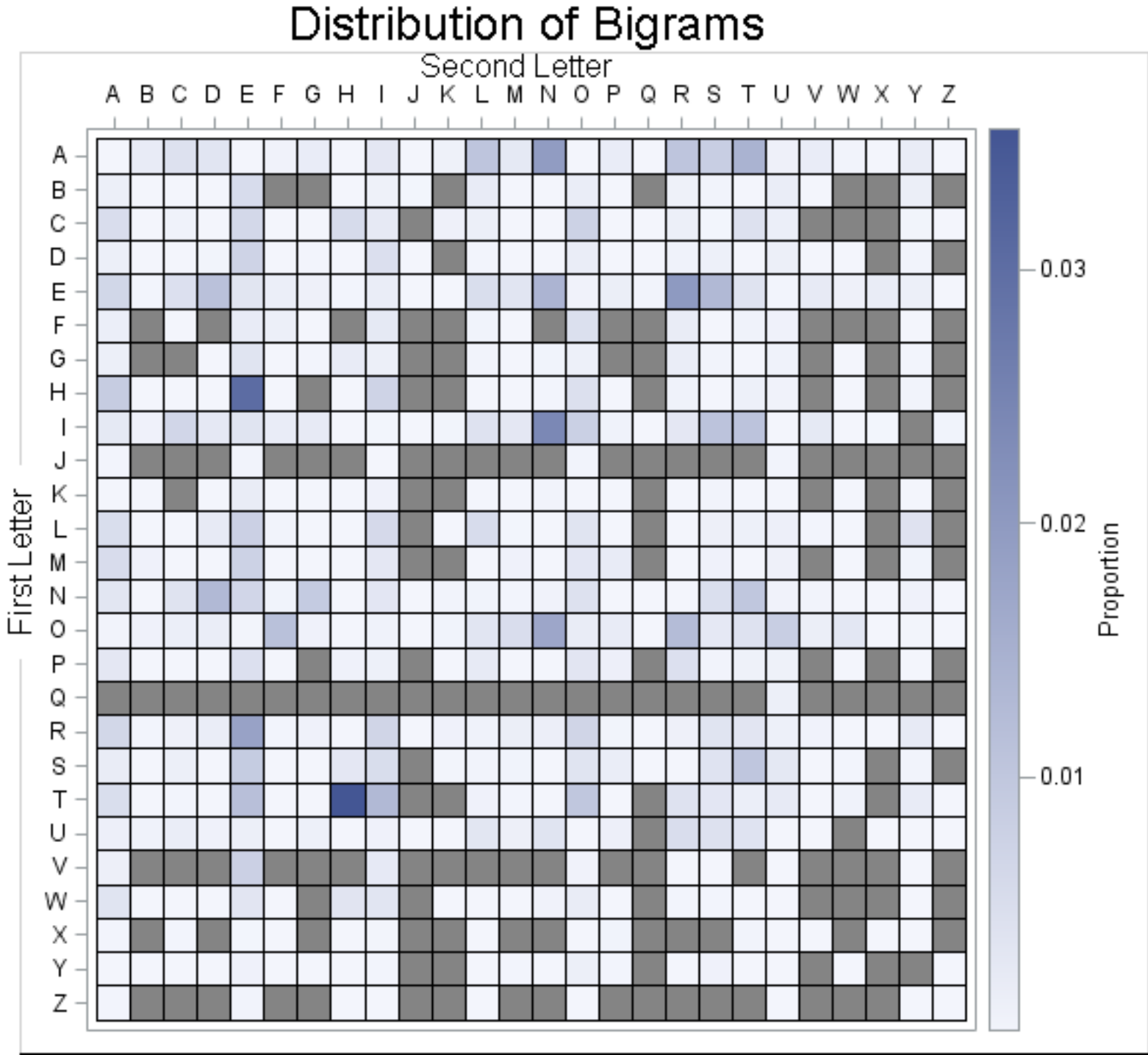
Observed (in the ciphertext)



Guess part of the key and use the guesses to break the cipher (as shown before)

Substitution ciphers

The same analysis can be repeated for bigrams, trigrams, etc



Vigenère cipher

Monoalphabetic substitution ciphers are vulnerable to frequency analysis



Blaise de Vigenère
(1523 - 1596)

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The generic i -th character m_i of the message $m = m_0 m_1 \dots m_{\ell-1}$ is encrypted using a shift cipher with shift $s_i \bmod t$



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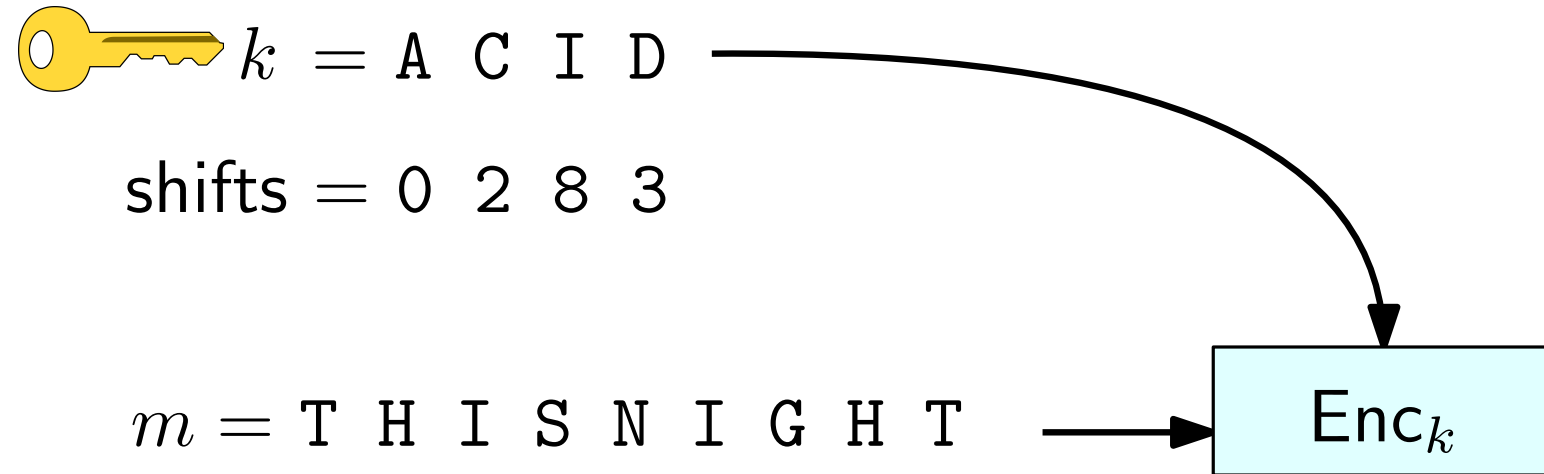


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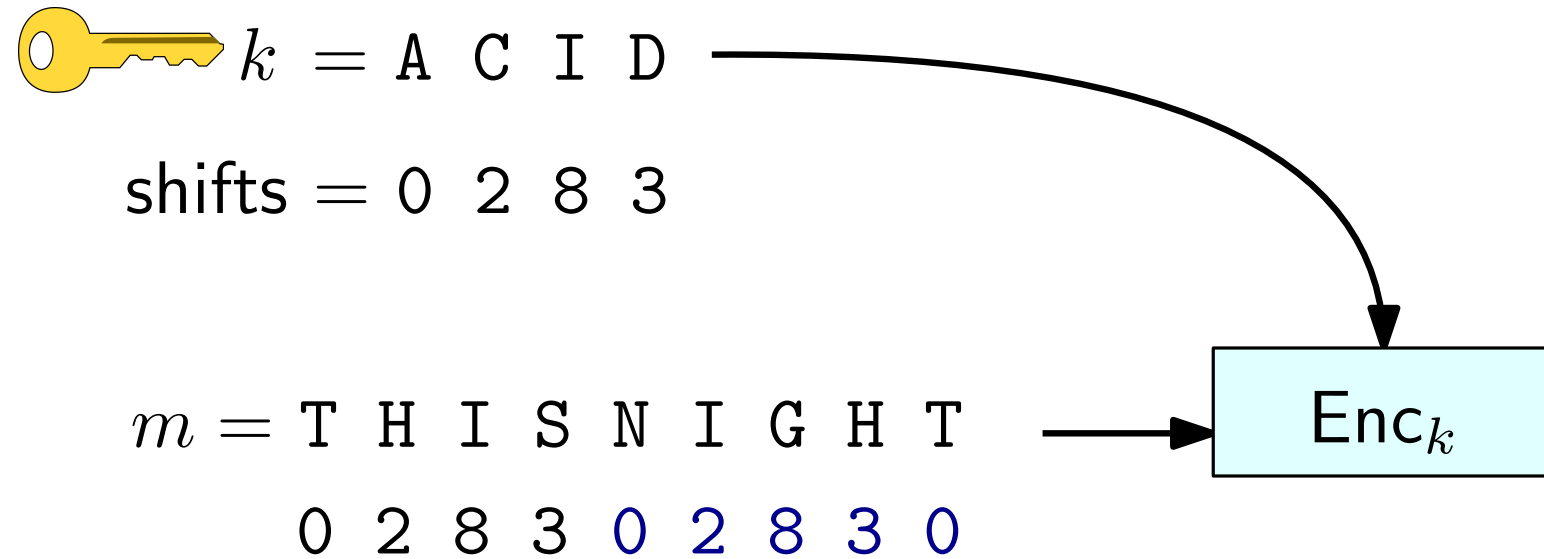


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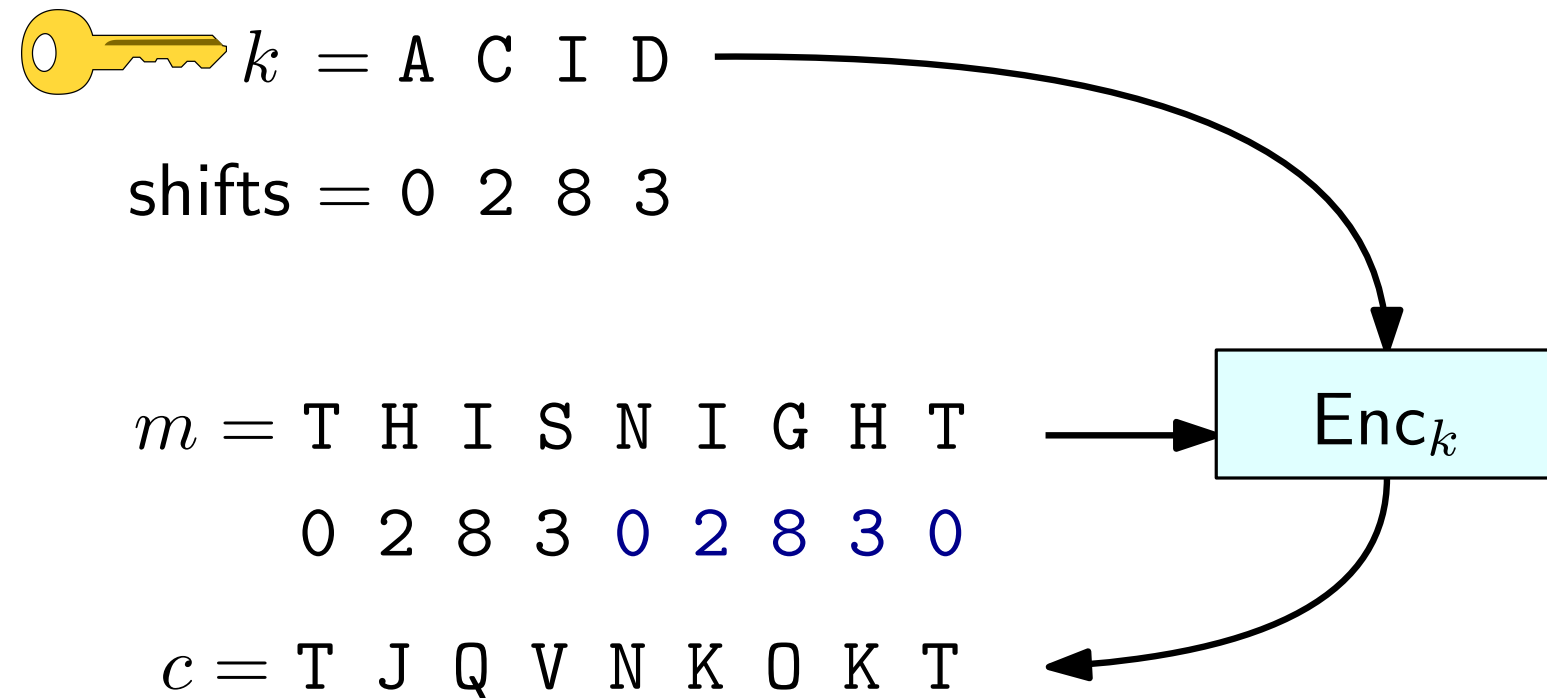


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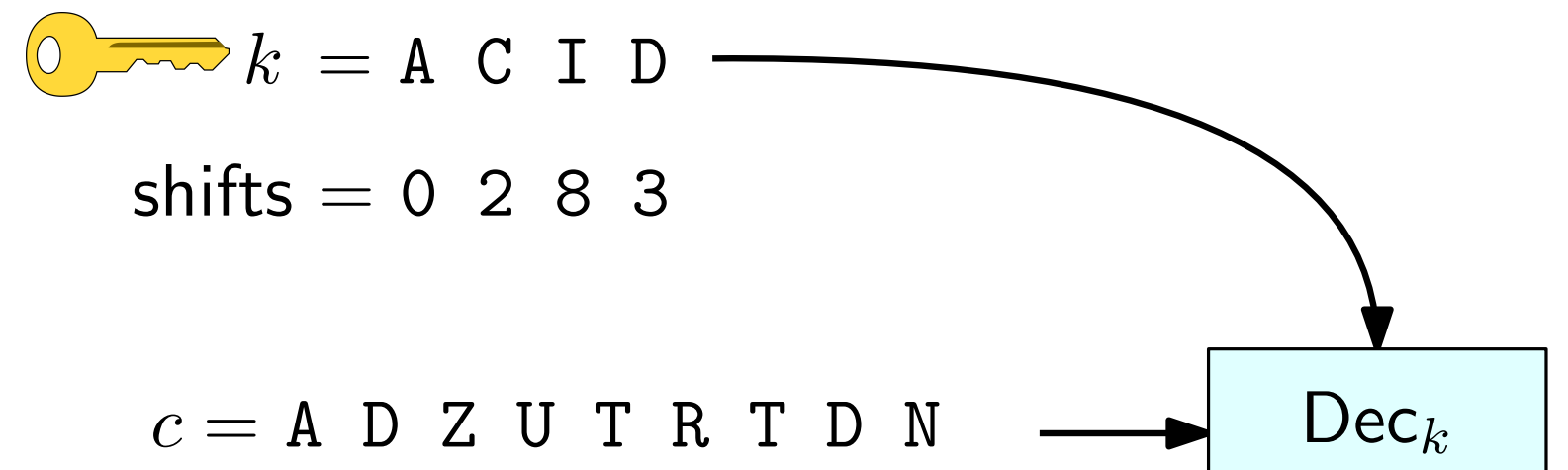
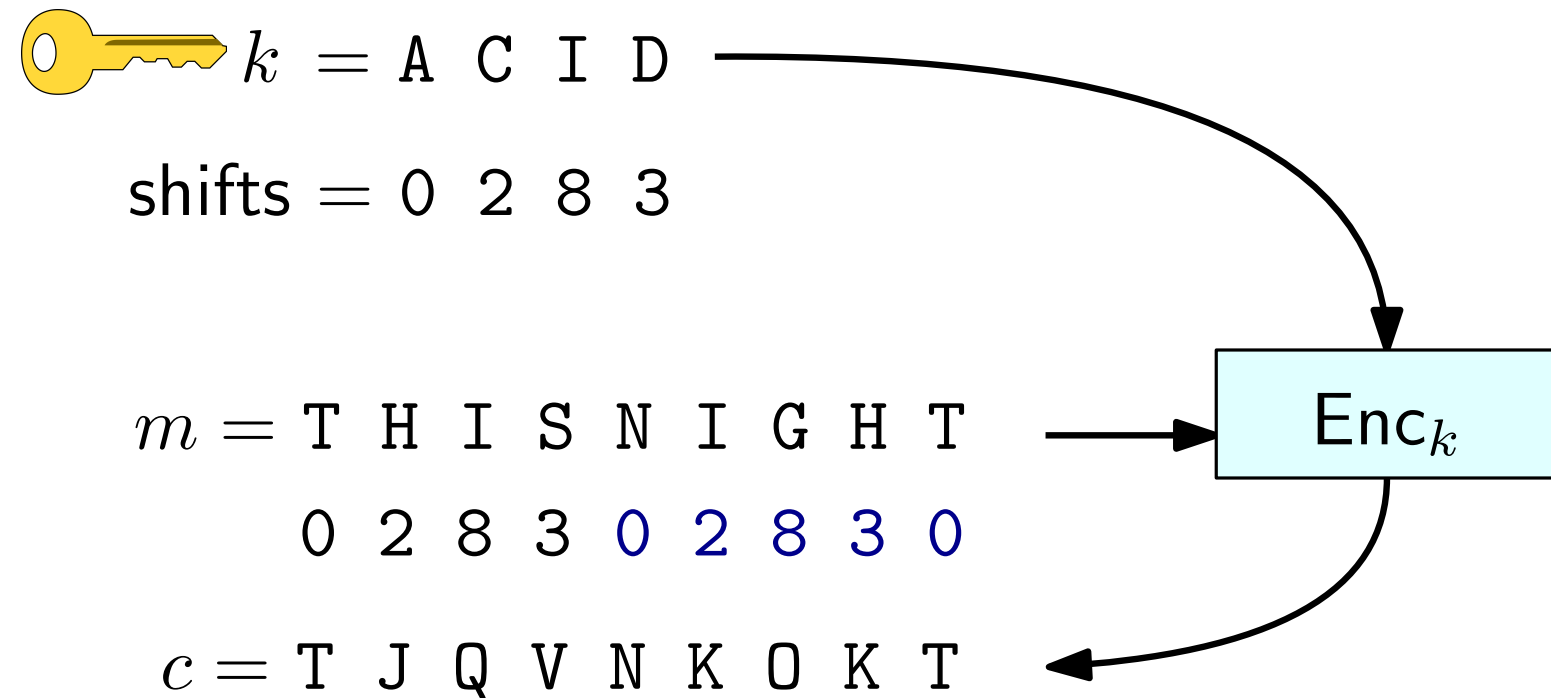


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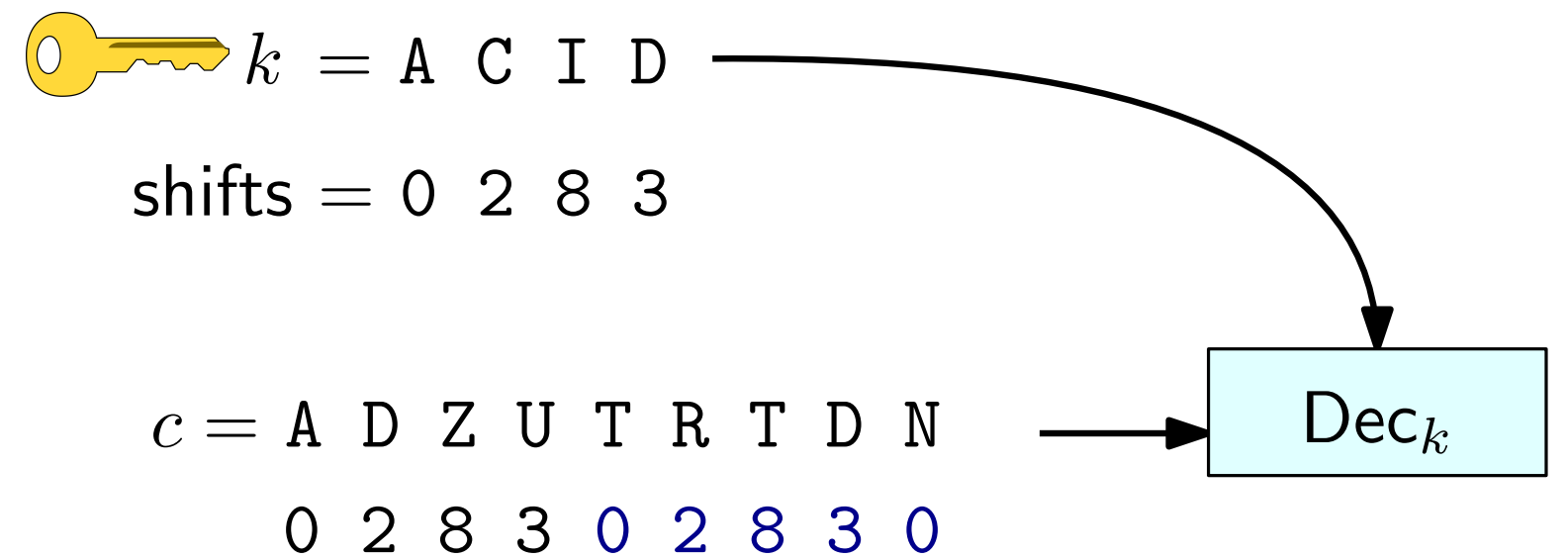
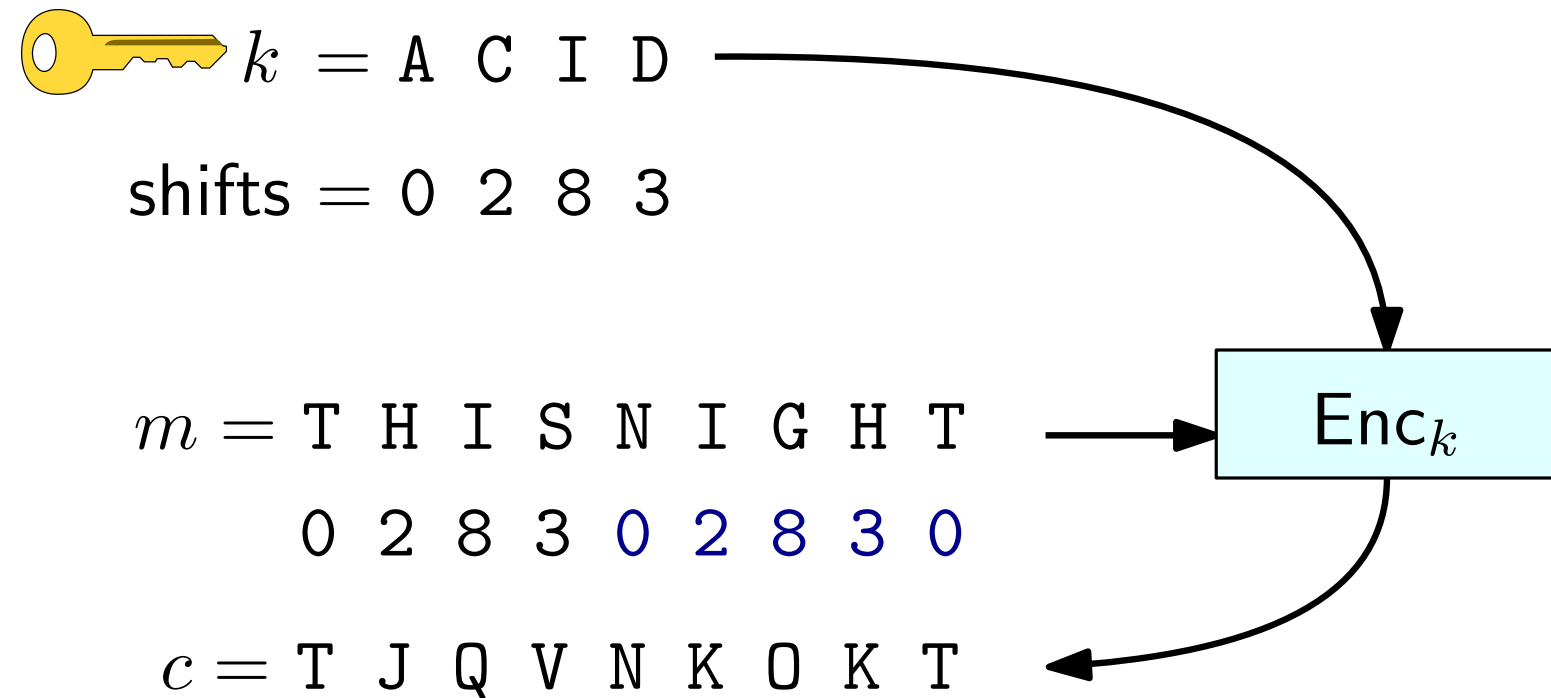


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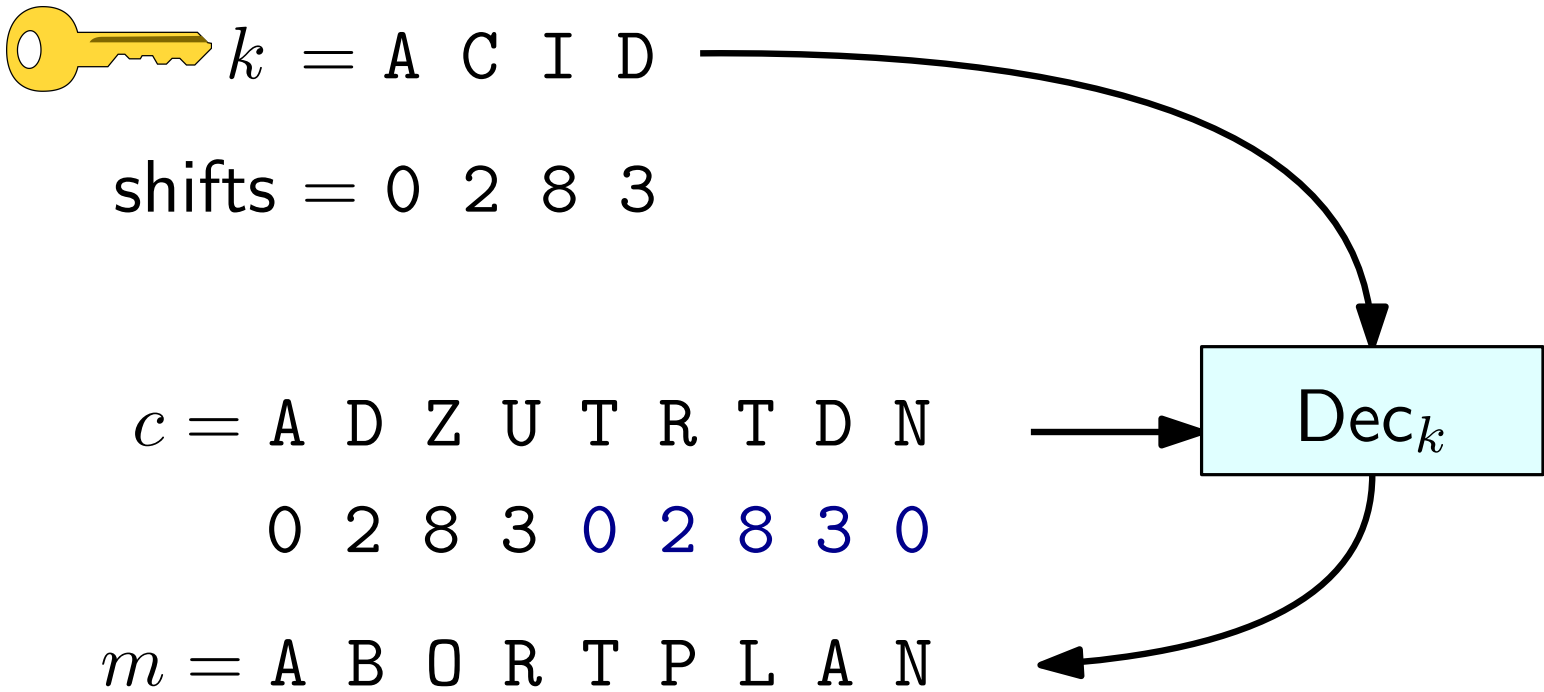
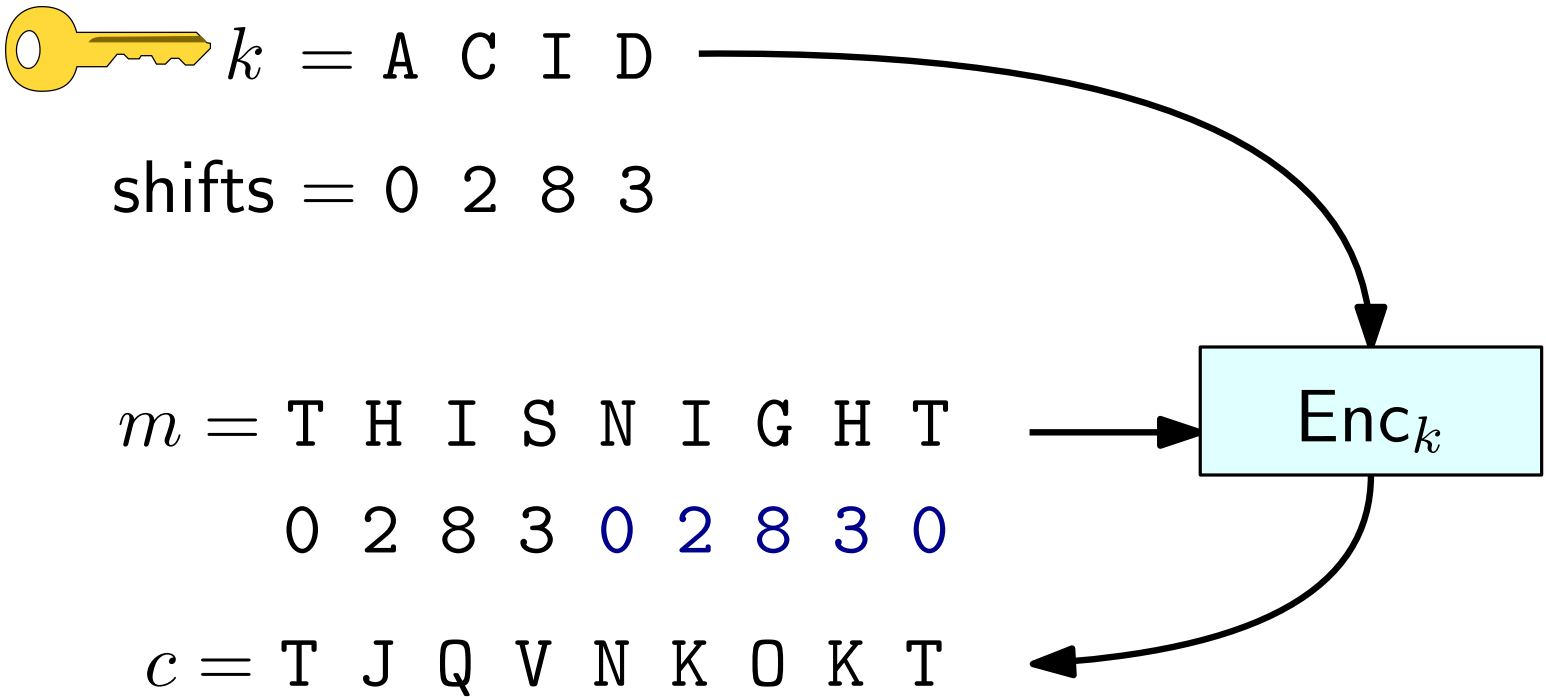


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A	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
B	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A
C	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B
D	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C
E	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D
F	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E
G	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F
H	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G
I	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H
J	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I
K	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J
L	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K
M	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L
N	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M
O	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N
P	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
Q	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
R	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q
S	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R
T	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S
U	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T
V	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U
W	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V
X	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W
Y	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X
Z	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y

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B	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A
C	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B
D	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C
E	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D
F	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E
G	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F
H	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G
I	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H
J	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I
K	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J
L	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K
M	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L
N	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M
O	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N
P	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
Q	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
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T	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S
U	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T
V	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U
W	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V
X	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W
Y	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X
Z	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y

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B	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A
C	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B
D	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C
E	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D
F	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E
G	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F
H	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G
I	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H
J	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I
K	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J
L	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K
M	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L
N	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M
O	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N
P	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
Q	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
R	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q
S	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R
T	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S
U	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T
V	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U
W	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V
X	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W
Y	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X
Z	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y

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	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
A	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
B	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A
C	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B
D	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C
E	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D
F	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E
G	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F
H	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G
I	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H
J	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I
K	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J
L	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K
M	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L
N	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M
O	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N
P	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
Q	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
R	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q
S	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R
T	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S
U	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T
V	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U
W	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V
X	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W
Y	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X
Z	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y

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A	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
B	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A
C	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B
D	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C
E	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D
F	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E
G	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F
H	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G
I	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H
J	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I
K	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J
L	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K
M	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L
N	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M
O	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N
P	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
Q	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
R	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q
S	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R
T	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S
U	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T
V	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U
W	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V
X	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W
Y	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X
Z	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y

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A	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
B	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A
C	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B
D	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C
E	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D
F	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E
G	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F
H	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G
I	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H
J	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I
K	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J
L	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K
M	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L
N	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M
O	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N
P	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
Q	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
R	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q
S	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R
T	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S
U	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T
V	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U
W	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V
X	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W
Y	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X
Z	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y

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A	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
B	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A
C	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B
D	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C
E	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D
F	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E
G	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F
H	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G
I	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H
J	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I
K	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J
L	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K
M	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L
N	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M
O	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N
P	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
Q	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
R	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q
S	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R
T	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S
U	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T
V	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U
W	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V
X	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W
Y	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X
Z	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y

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Suppose that the adversary is somehow able to figure out what the length t of the key is

E.g.: $t = 4$

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$c =$

A	M	A	P	A	A	U	H	K	G	O	O	T	W	F	I	O	G	G	G	T	B	T
Q	I	N	N	A	V	S	M	B	T	K	Q	O	M	O	I	W	C	P	C	T	W	T
U	O	I	F	A	G	O	G	T	I	M	O	U	C	F	P	B	T	W	T	B	N	P
W	C	P	C	Q	B	S	J	D	G	F	A	U	O	W	B	O	E	E	K	D	A	E
R	K	R	E	M	L	K	B	F	P	R	O	O	T	J	C	C	S	U	O	O	F	S
I	Q	I	W	U	R	B	N	F	W	M	B	T	G	A	A	U	I	E	W	D	F	L
Z	L	S	F	C	Q	Z	O															

The ciphertext can be decomposed into n ciphertext $c^{(1)}, c^{(2)}, \dots, c^{(t)}$.

Each $c^{(i)}$ is encrypted using the same shift

Each ciphertext can be attacked separately (but we cannot simply bruteforce them)

Breaking the Vigenère cipher

How do we determine the key length?

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Kasiski's method

- Consider some (unknown) sequence of characters that appears frequently in the plaintext (for example the word “the”)

T H E M A N A N D T H E W O M A N R E T R I E V E D T H E L E T T E R F R O M T H E P O S T B O X
B E A D S B E A D S B E A D S B E A D S B E A D S B E A D S B E A D S B E A D S B E A D S B E A D

U L E P S O E N G L I I W R E B R R H L S M E Y W E X H H D F X T H J G V O P L I I P R K U F O A

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T H E M A N A N D T H E W O M A N R E T R I E V E D T H E L E T T E R F R O M T H E P O S T B O X
B E A D S B E A D S B E A D S B E A D S B E A D S B E A D S B E A D S B E A D S B E A D S B E A D S B E A D
U L E P S O E N G L I I W R E B R R H L S M E Y W E X H H D F X T H J G V O P L I I P R K U F O A

Kasiski's method

- Consider some (unknown) sequence of characters that appears frequently in the plaintext (for example the word “the”)
- In general, distinct occurrences of the word will be encrypted using different portions of the key and the ciphertext characters will differ

T	H	E	M	A	N	A	N	D	T	H	E	W	O	M	A	N	R	E	T	R	I	E	V	E	D	T	H	E	L	E	T	T	E	R	F	R	O	M	T	H	E	P	O	S	T	B	O	X
B	E	A	D	S	B	E	A	D	S	B	E	A	D	S	B	E	A	D	S	B	E	A	D	S	B	E	A	D	S	B	E	A	D	S	B	E	A	D	S	B	E	A	D	S	B	E	A	D
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U	L	E	P	S	O	E	N	G	L	I	I	W	R	E	B	R	R	H	L	S	M	E	Y	W	E	X	H	H	D	F	X	T	H	J	G	V	O	P	L	I	I	P	R	K	U	F	O	A

Kasiski's method

- Consider some (unknown) sequence of characters that appears frequently in the plaintext (for example the word “the”)
- In general, distinct occurrences of the word will be encrypted using different portions of the key and the ciphertext characters will differ
- However, some occurrences will happen to *line up* (i.e., be encrypted with the same portion of the key)

T	H	E	M	A	N	A	N	D	T	H	E	W	O	M	A	N	R	E	T	R	I	E	V	E	D	T	H	E	L	E	T	T	E	R	F	R	O	M	T	H	E	P	O	S	T	B	O	X
B	E	A	D	S	B	E	A	D	S	B	E	A	D	S	B	E	A	D	S	B	E	A	D	S	B	E	A	D	S	B	E	A	D	S	B	E	A	D	S	B	E	A	D	S	B	E	A	D
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U	L	E	P	S	O	E	N	G	L	I	I	W	R	E	B	R	R	H	L	S	M	E	Y	W	E	X	H	H	D	F	X	T	H	J	G	V	O	P	L	I	I	P	R	K	U	F	O	A

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- When this happens, the corresponding portions of ciphertext will be equal

T H E	M A N A N D	T H E	W O M A N R E T R I E V E D	T H E	L E T T E R F R O M	T H E	P O S T B O X
B E A D S	B E A D S	B E A D S	B E A D S	B E A D S	B E A D S	B E A D S	B E A D S
<hr/>							
U L E	P S O E N G	L I I	W R E B R R H L S M E Y W E	X H H	D F X T H J G V O P	L I I	P R K U F O A

Kasiski's method

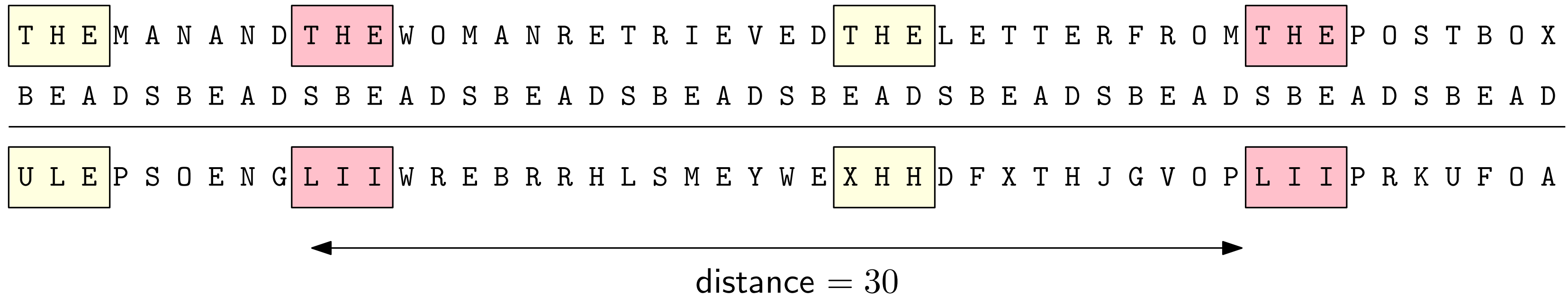
T H E M A N A N D T H E W O M A N R E T R I E V E D T H E L E T T E R F R O M T H E P O S T B O X
B E A D S B E A D S B E A D S B E A D S B E A D S B E A D S B E A D S B E A D S B E A D S B E A D S B E A D

U L E P S O E N G L I I W R E B R R H L S M E Y W E X H H D F X T H J G V O P L I I P R K U F O A

← distance = 30 →

Obs: The distance between repeated patterns in the ciphertext is likely to be a multiple of the key length

Kasiski's method



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- Find some repeated patterns of small length (e.g., 2 or 3) in the ciphertext

Kasiski's method

THE MAN AND THE WOMAN RETRIEVED THE LETTER FROM THE POST BOX
BEADS BEADS BEADS BEADS BEADS BEADS BEADS BEADS BEADS BEADS

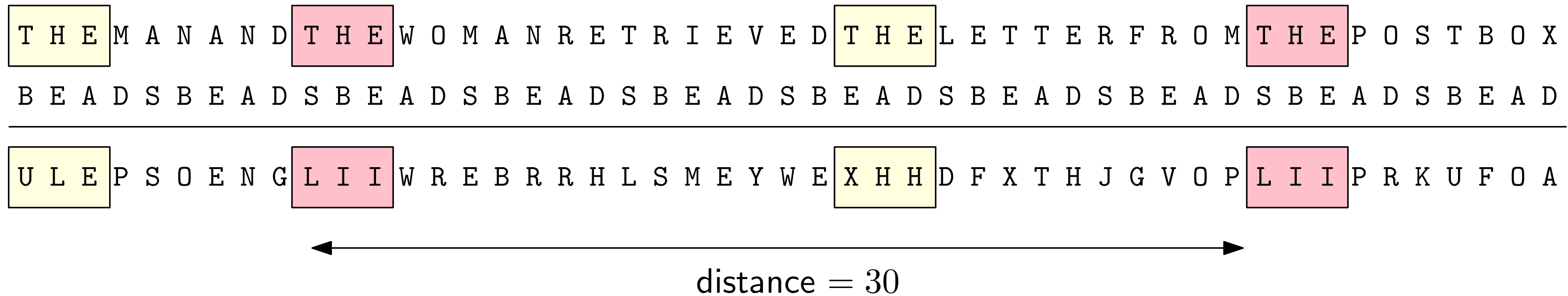
ULEP SOENG LIIWREBRRHLSMEYWE XHH DFXTHJGVOP LIIPRKUFOA

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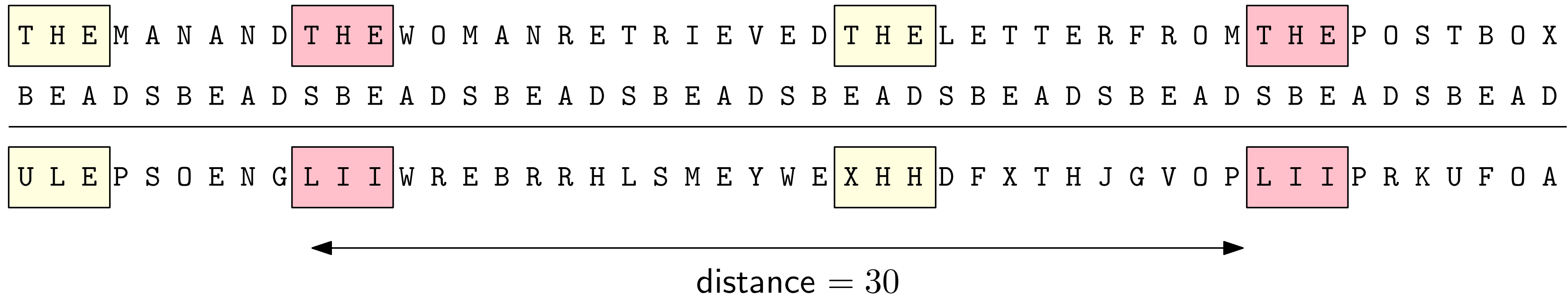
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In the example the key length t is 5 and the distance between patterns is 30

Breaking the Vigenère cipher

How do we determine the key length?

- **Option 1:** brute-force (guess n and try decrypting the n shift ciphers)
- **Option 2:** Kasiski's method
- **Option 3:** Index of coincidence method

Index of coincidence method

Let p_j be the expected frequency of the j -th letter ($j = 0 \dots, 25$) in the language of the plaintext

Using the frequencies in the English language:

$$\sum_{j=0}^{25} p_j^2 \approx 0.065$$

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Compute $S_\tau = \sum_{i=0}^{25} q_j^2$

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$\implies S_\tau \approx \sum_{j=0}^{25} \left(\frac{1}{26}\right)^2 \approx 0.038$

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The smallest value of τ such that $S_\tau \approx 0.065$ is probably the length of the key

This can be validated by repeating the check for other values of i

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Compute $I_j = \sum_{i=0}^{25} p_i q_{(i+j) \bmod 26}$ for all possible shifts j and choose the one for which I_j is closest to 0.065.

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Marian Adam Rejewski



Alan Mathison Turing

Scytale cipher

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What is the key of this cipher? The diameter of the rod!

To decrypt the ciphertext, simply wind it around a rod of the same diameter



Breaking the scytale cipher



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Wind the parchment around a cone

Look for the portion of the cone where letters start to line up and produce sensible words

The corresponding diameter is the key!



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The scytale cipher is a (specific type of) transposition cipher!

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The plaintext is arranged in a matrix with n columns (and the appropriate number of rows)

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T	H	E	M	E	E
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O	W	A	T	T	H
E	D	O	C	K	

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The message might or might not be padded with random characters (X in this case) to fill the last row.



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O W A T T H

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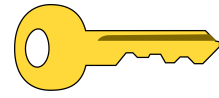
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E.g., if the permutation is 4, 2, 1, 6, 5, 3, then the ciphertext is:

$c =$ M G O T C H I O W D T T T O E E S R H X E I R T K E N M A O

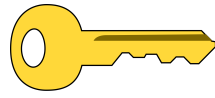
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What is the key?



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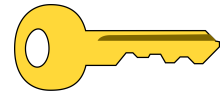
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The pair (n, π)

(Columnar) Transposition ciphers

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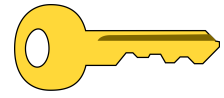


The pair (n, π)

How do we decrypt the ciphertext?

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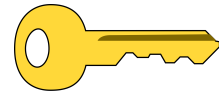
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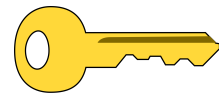
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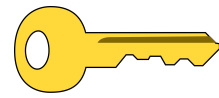
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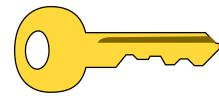
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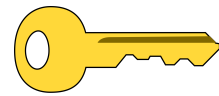
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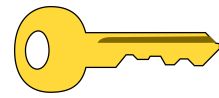
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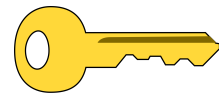
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The plaintext can be found by reading the rows in order (left to right, top to bottom)

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Are columnar transposition ciphers secure?

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- Look for anagrams (that simultaneously yield intelligible text on multiple rows)



Other transposition ciphers

To make cryptanalysis harder, a double (irregular) transposition cipher is often used:

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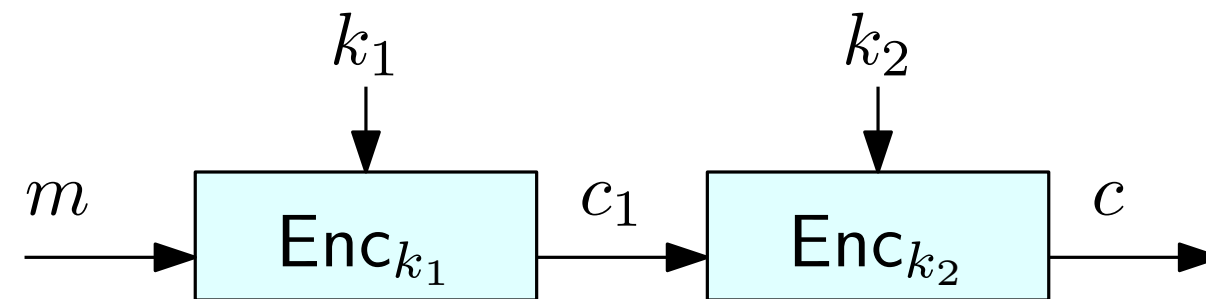
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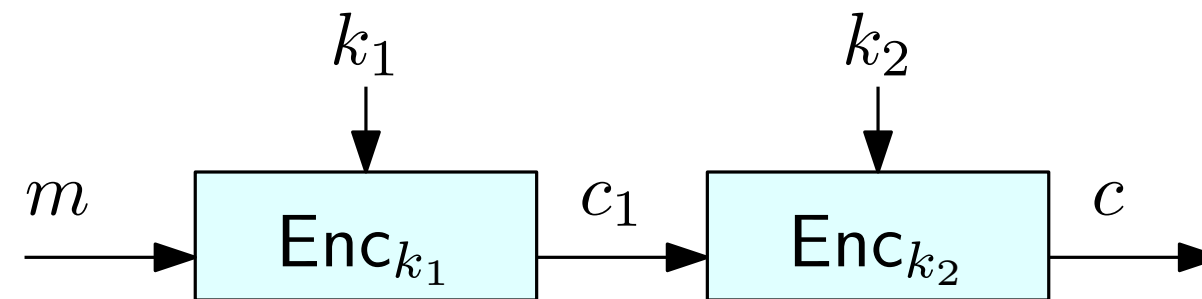
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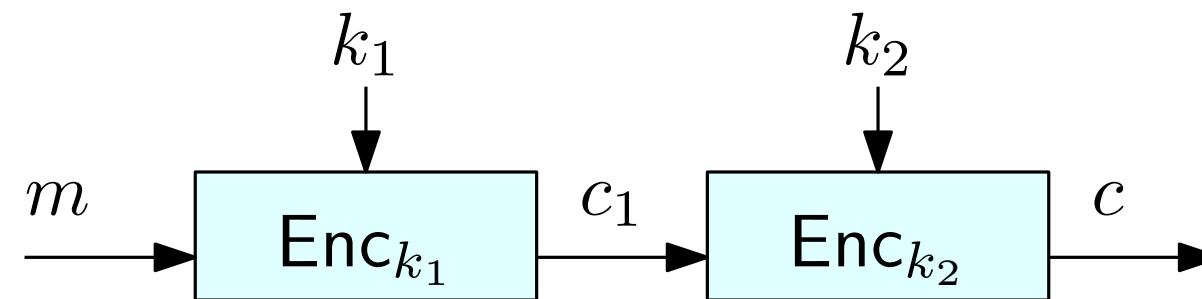


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- Many other (more complex) transposition ciphers have been used

Other transposition ciphers

The Zodiac Z-340 cipher remained unsolved for 51 years!



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