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The historic ciphers from the previous lectures are intuitively "insecure". Can we prove that formally?

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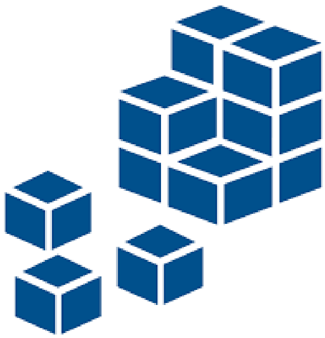
Another benefit of formal definitions is *modularity*:

- A designer can replace an encryption scheme with another (that satisfies the same security definition)
- The security of the overall application is unaffected



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Threat models

One can define several different threat models depending on the environment in which the encryption scheme is going to be used

- A threat model only specifies **what** the abilities of the adversary are

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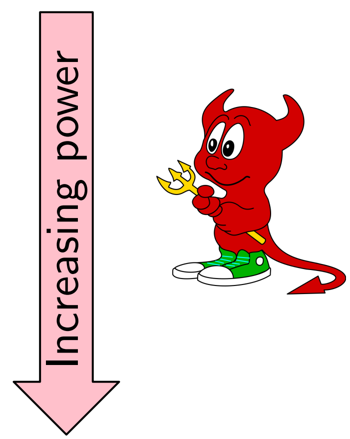

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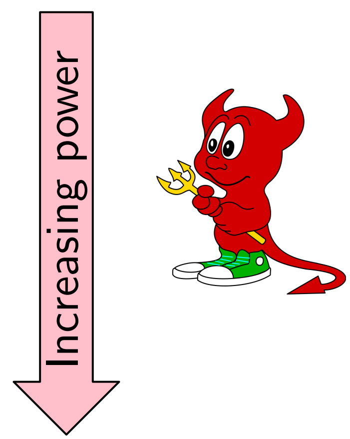

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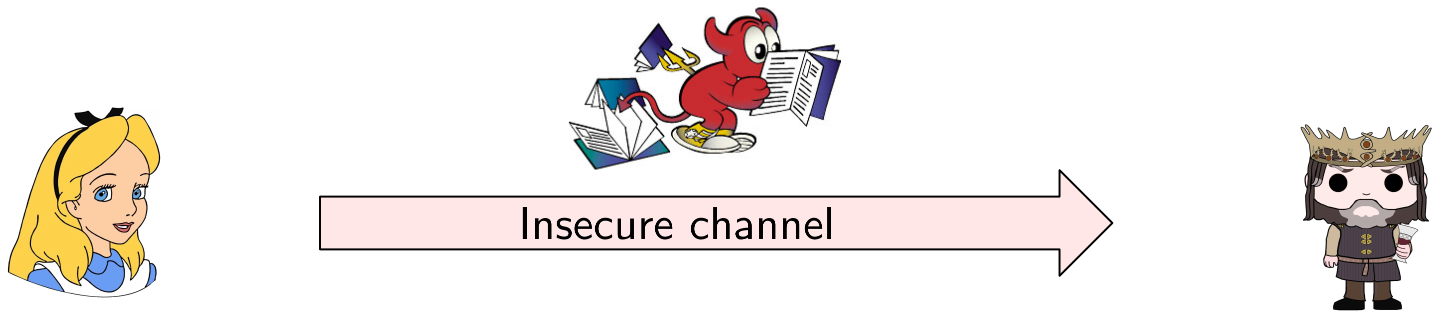

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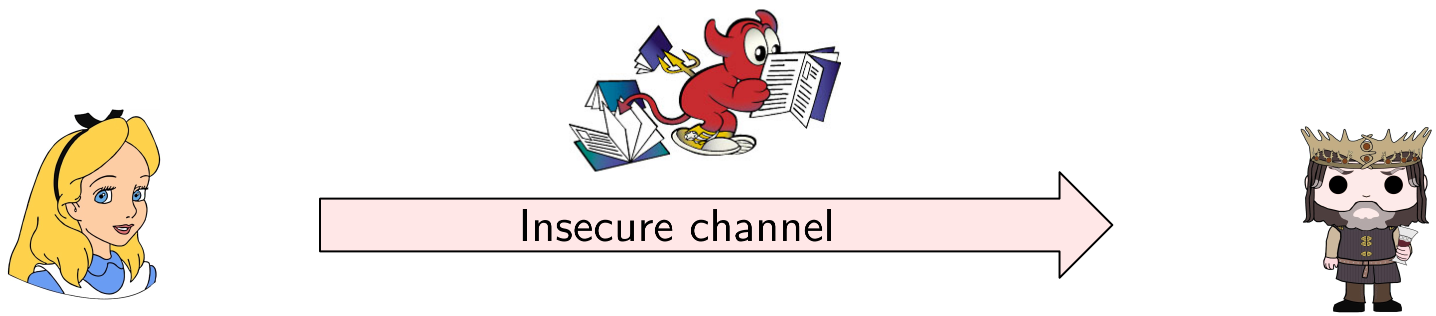

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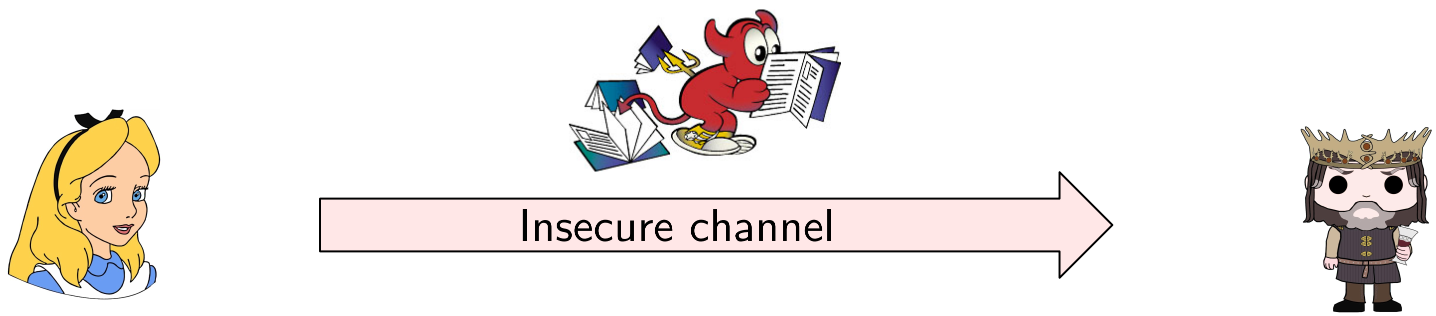

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It is the attack type that we have been implicitly considering in our discussion about historic ciphers

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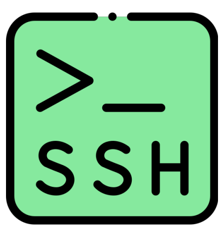

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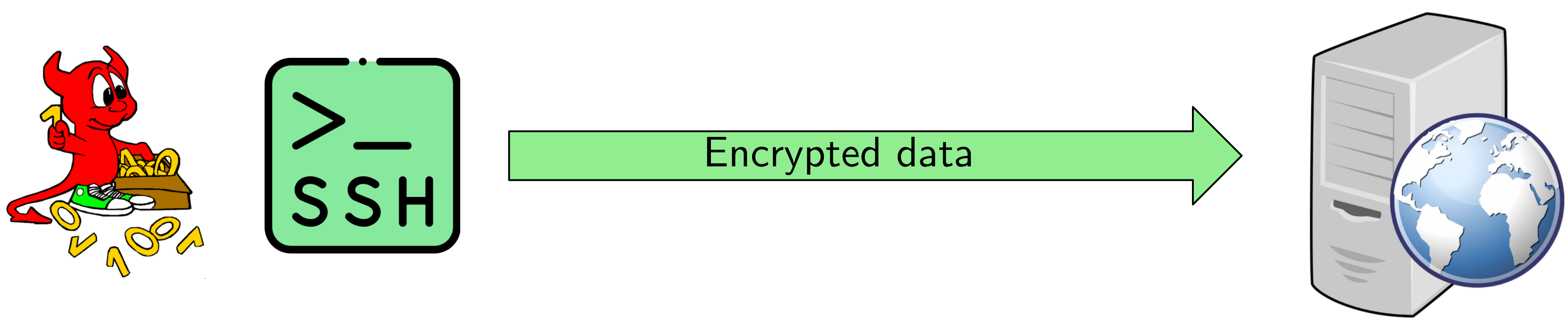

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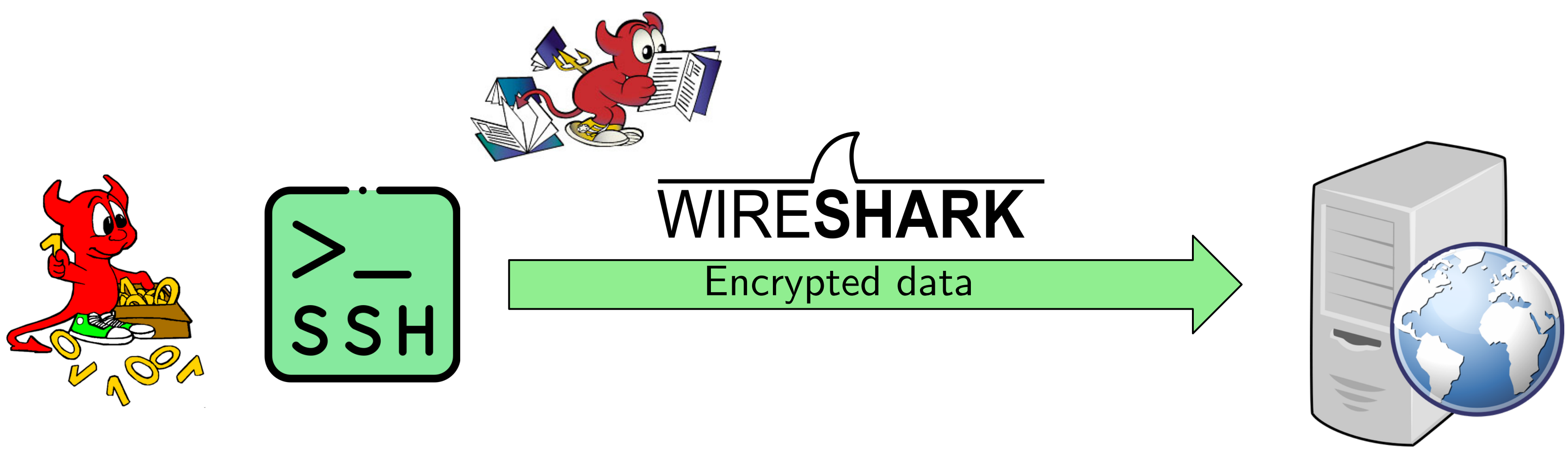

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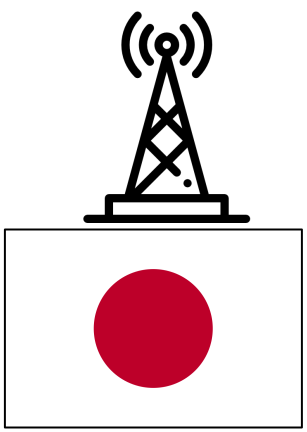

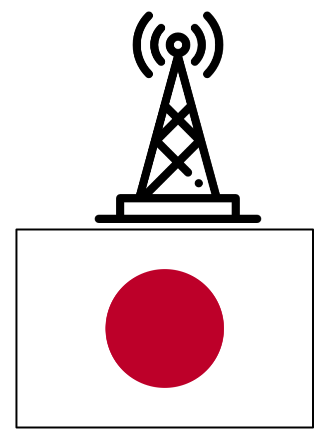

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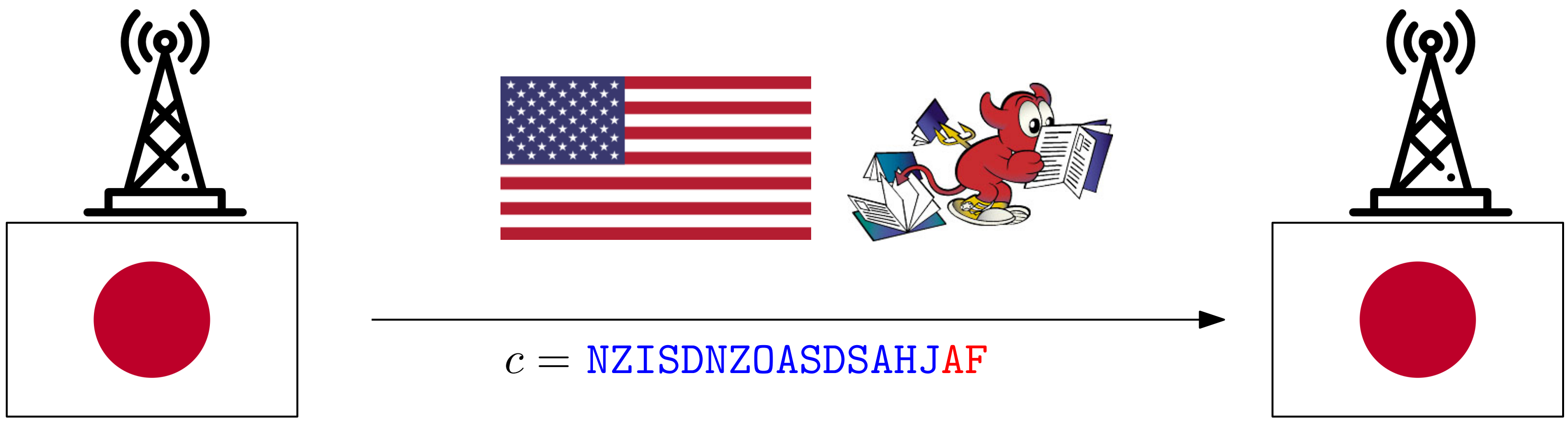

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The U.S. cryptanalysts believed that `AF` meant Midway Island, but they were not 100% sure

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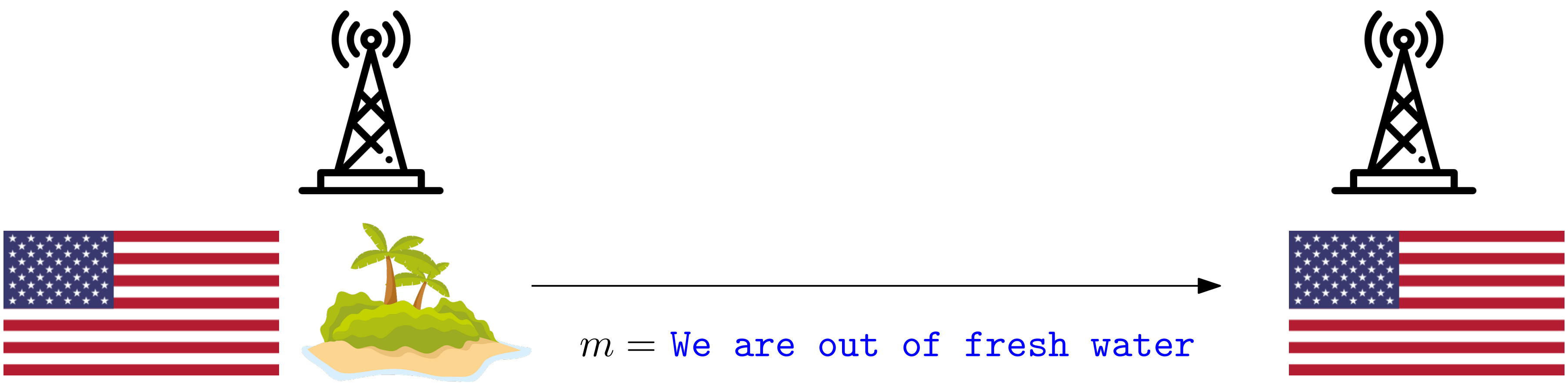

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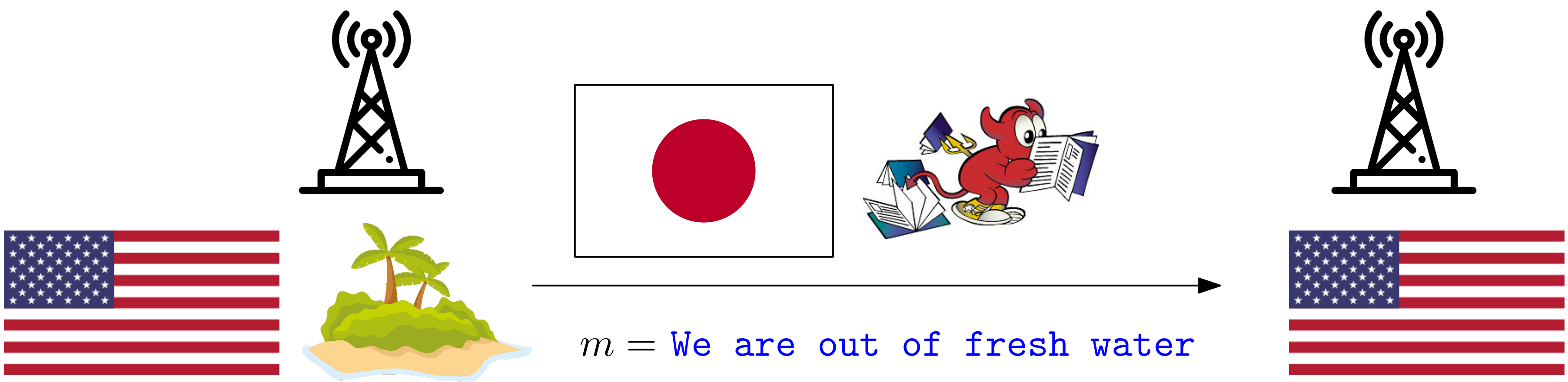

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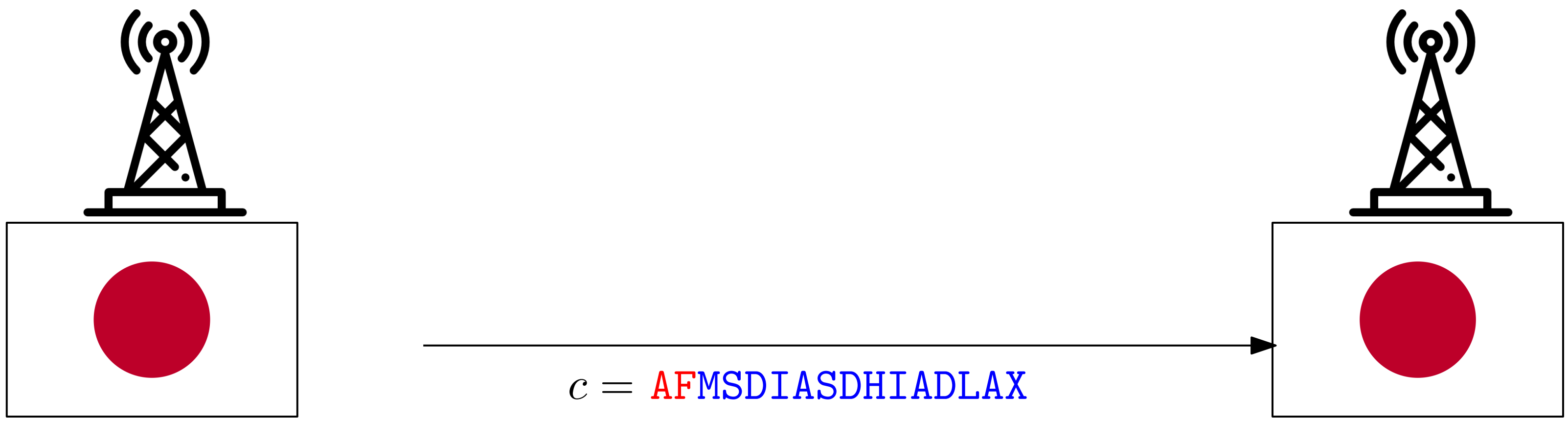

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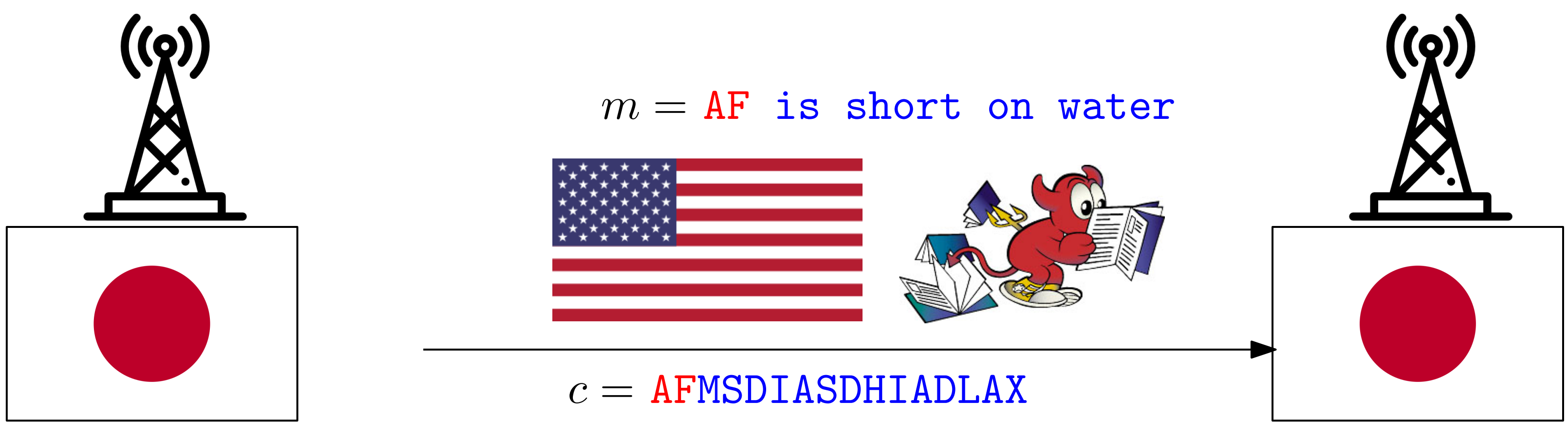

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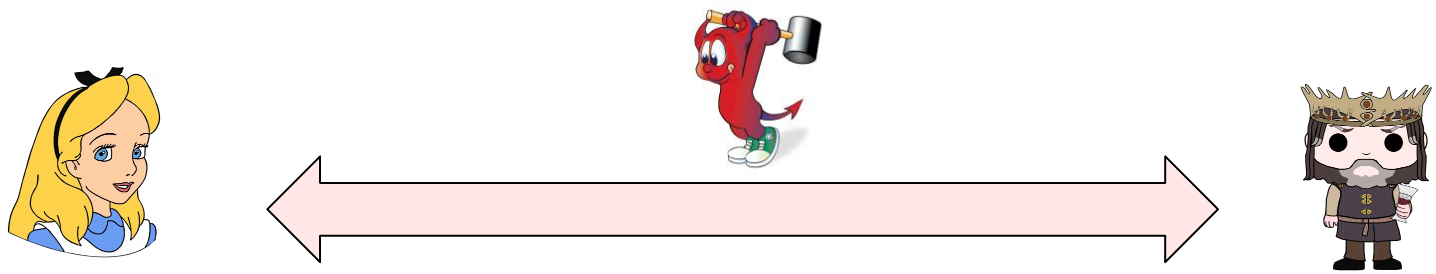

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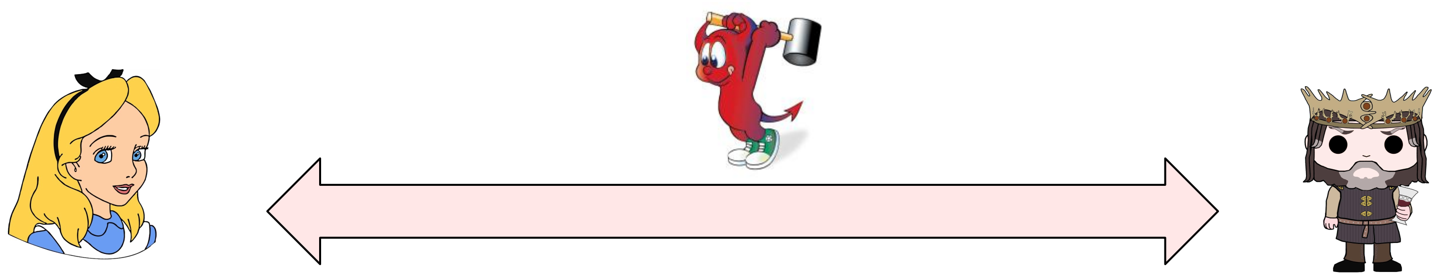

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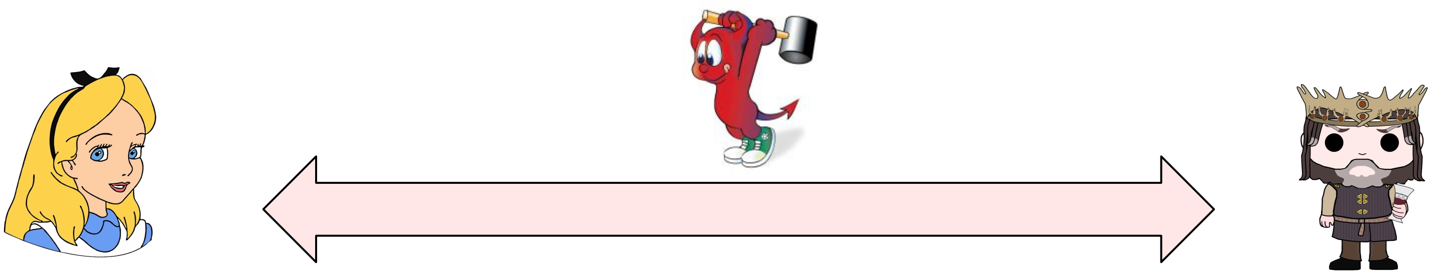

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Many protocols close a connection or request a retransmission when a bad message is received

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Being able to know whether a ciphertext is valid enables “Padding oracle” attacks:

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What about the following private-key encryption scheme?

- Gen returns a random key
- $\mathsf{Enc}_k(m) = m$
- $\mathsf{Dec}_k(c) = c$

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- $\mathsf{Enc}_k(m) = \begin{cases} \mathtt{A} \| f_k(m) & \text{if } m \geq 100 \\ \mathtt{B} \| f_k(m) & \text{if } m < 100 \end{cases}$, for some $f_k(\cdot)$?

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Is it a "good" definition?

What about $f(m) = |m|$?

What about $f(m) = 42$?

Security guarantees

What should a secure encryption scheme guarantee?

Candidate definition 5 (inf.): *Regardless of any information an attacker already has, a ciphertext should leak <u>no additional information</u> about the underlying plaintext.*

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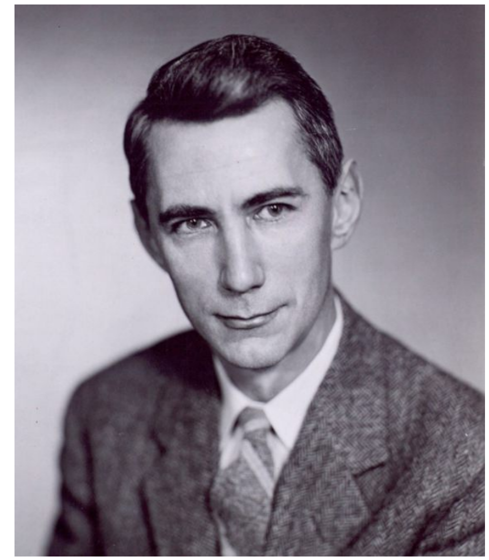
Shannon's Treatment

Messages come from a probability distribution over the message space $\mathcal{M}$



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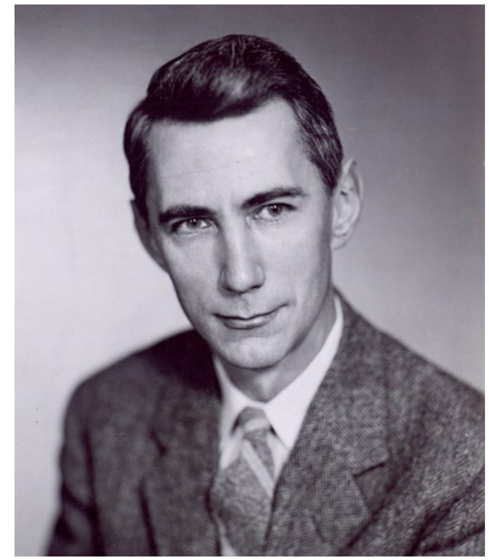

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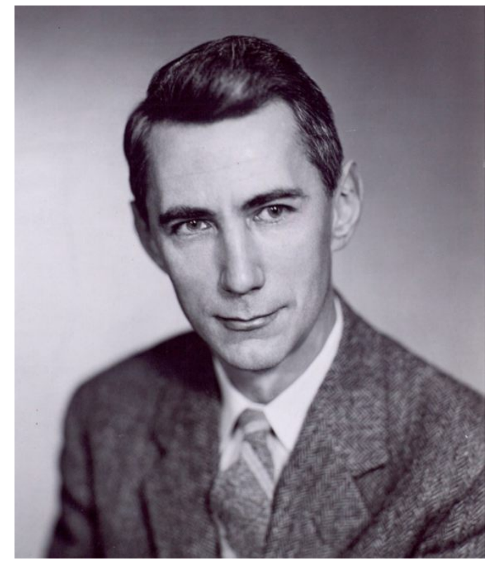

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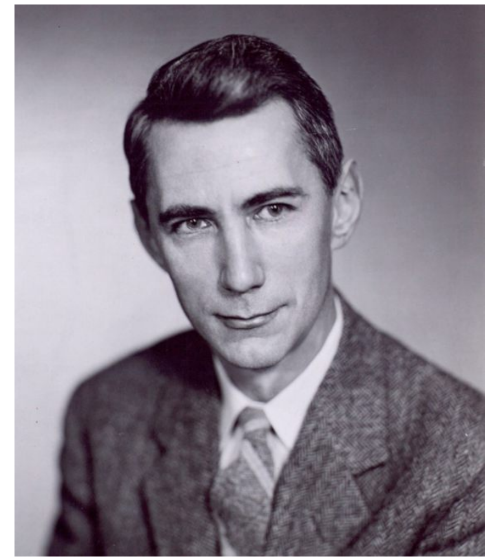

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$\Pr[M = m]$ ← probability that the plaintext is m

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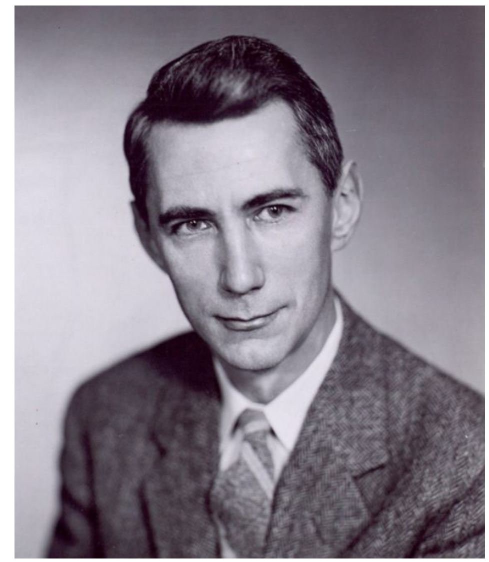

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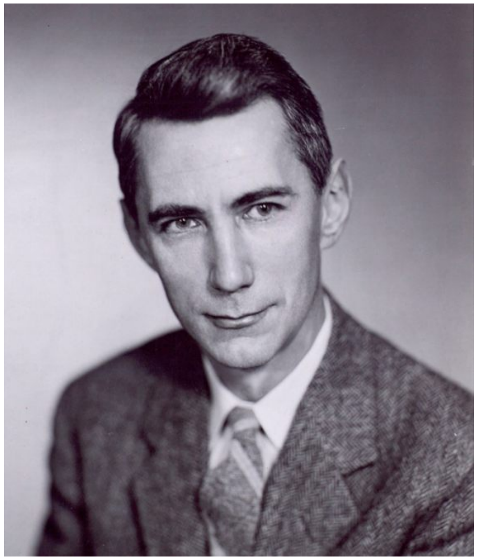

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C is a random variable (over $\mathcal{C}$) denoting the resulting ciphertext.

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The adversary knows that the message is going to be either `ATTACK` or `RETREAT`

Moreover, he believes that the probability of attack is 70%

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a priori probability

Gen outputs a binary string of length 3 chosen uniformly at random (u.a.r.):

$\Pr[K = \texttt{011}] = \frac{1}{8}$

Example 1

Consider a shift cipher:

$\mathcal{M} = \{\mathtt{a}, \ldots, \mathtt{z}\}^*$ $\quad$ $\mathcal{C} = \{\mathtt{A}, \ldots, \mathtt{Z}\}^*$ $\quad$ $\mathcal{K} = \{0, \ldots, 25\}$

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Lower-case for plaintexts $\qquad$ Upper-case for ciphertexts

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What is the probability that the ciphertext is `B`?

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$\Pr[C = \texttt{B}]$

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$$\Pr[C = \texttt{B}] \;\; = \sum_{m \in \mathcal{M}} \Pr[C = \texttt{B} \wedge M = m]$$

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What is the probability that the ciphertext is B?

$$\Pr[C = \mathtt{B}] = \sum_{m \in \mathcal{M}} \Pr[C = \mathtt{B} \wedge M = m] = \Pr[C = \mathtt{B} \wedge M = \mathtt{a}] + \Pr[C = \mathtt{B} \wedge M = \mathtt{b}]$$

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$$= \Pr[C = \mathtt{B} \mid M = \mathtt{a}] \cdot \Pr[M = \mathtt{a}] + \Pr[C = \mathtt{B} \mid M = \mathtt{b}] \cdot \Pr[M = \mathtt{b}]$$

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\Pr[C = \mathtt{B}] &= \textstyle\sum_{m \in \mathcal{M}} \Pr[C = \mathtt{B} \wedge M = m] \;= \Pr[C = \mathtt{B} \wedge M = \mathtt{a}] + \Pr[C = \mathtt{B} \wedge M = \mathtt{b}] \\
&= \Pr[C = \mathtt{B} \mid M = \mathtt{a}] \cdot \Pr[M = \mathtt{a}] \;+\; \Pr[C = \mathtt{B} \mid M = \mathtt{b}] \cdot \Pr[M = \mathtt{b}] \\
&= \Pr[K = 1] \cdot \Pr[M = \mathtt{a}] \;+\; \Pr[K = 0] \cdot \Pr[M = \mathtt{b}]
\end{aligned}$$

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\Pr[C = \mathtt{B}] &= \textstyle\sum_{m \in \mathcal{M}} \Pr[C = \mathtt{B} \wedge M = m] \ = \Pr[C = \mathtt{B} \wedge M = \mathtt{a}] + \Pr[C = \mathtt{B} \wedge M = \mathtt{b}] \\
&= \Pr[C = \mathtt{B} \mid M = \mathtt{a}] \cdot \Pr[M = \mathtt{a}] \ + \ \Pr[C = \mathtt{B} \mid M = \mathtt{b}] \cdot \Pr[M = \mathtt{b}] \\
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What is the probability that the plaintext is `a` if the adversary has observed the ciphertext `B`?

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$\Pr[M = \mathtt{a} \mid C = \mathtt{B}]$

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What is the probability that the plaintext is $\texttt{a}$ if the adversary has observed the ciphertext $\texttt{B}$?

$$\Pr[M = \texttt{a} \mid C = \texttt{B}] \; = \Pr[C = \texttt{B} \mid M = \texttt{a}] \cdot \frac{\Pr[M = \texttt{a}]}{\Pr[C = \texttt{B}]}$$

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$$\begin{aligned}\Pr[M = \texttt{a} \mid C = \texttt{B}] &= \Pr[C = \texttt{B} \mid M = \texttt{a}] \cdot \tfrac{\Pr[M=\texttt{a}]}{\Pr[C=\texttt{B}]} \\ &= \Pr[C = \texttt{B} \mid M = \texttt{a}] \cdot \tfrac{7/10}{1/26}\end{aligned}$$

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&= \Pr[C = \mathtt{B} \mid M = \mathtt{a}] \cdot \tfrac{7/10}{1/26} \\
&= \Pr[K = 1] \cdot \tfrac{7/10}{1/26}
\end{aligned}$$

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\end{aligned}$$

a posteriori probability

Example 2

Consider a shift cipher:

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The adversary has the following a priori distribution over $\mathcal{M}$:

$$\Pr[M = \mathtt{kim}] = 0.5 \qquad \Pr[M = \mathtt{ann}] = 0.2 \qquad \Pr[M = \mathtt{boo}] = 0.3$$

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What is the probability that the ciphertext is DQQ?

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$$\begin{aligned}\Pr[C = \texttt{DQQ}] = \quad & \Pr[C = \texttt{DQQ} \mid M = \texttt{kim}] \Pr[M = \texttt{kim}] \\ & + \Pr[C = \texttt{DQQ} \mid M = \texttt{ann}] \Pr[M = \texttt{ann}] \\ & + \Pr[C = \texttt{DQQ} \mid M = \texttt{boo}] \Pr[M = \texttt{boo}]\end{aligned}$$

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$$\Pr[C = \mathtt{DQQ}] = \Pr[C = \mathtt{DQQ} \mid M = \mathtt{ann}] \Pr[M = \mathtt{ann}] + \Pr[C = \mathtt{DQQ} \mid M = \mathtt{boo}] \Pr[M = \mathtt{boo}]$$

Example 2

Consider a shift cipher:

$\mathcal{M} = \{\mathtt{a}, \ldots, \mathtt{z}\}^*$ $\qquad$ $\mathcal{C} = \{\mathtt{A}, \ldots, \mathtt{Z}\}^*$ $\qquad$ $\mathcal{K} = \{0, \ldots, 25\}$

K is distributed uniformly over $\mathcal{K}$

The adversary has the following a priori distribution over $\mathcal{M}$:

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&= \tfrac{1}{26} \cdot 0.2 + \tfrac{1}{26} \cdot 0.3 = \tfrac{1}{52}
\end{aligned}$$

Perfect secrecy

Candidate definition 5 (inf.): *Regardless of any information an attacker already has, a ciphertext should leak* <u>*no additional information*</u> *about the underlying plaintext.*

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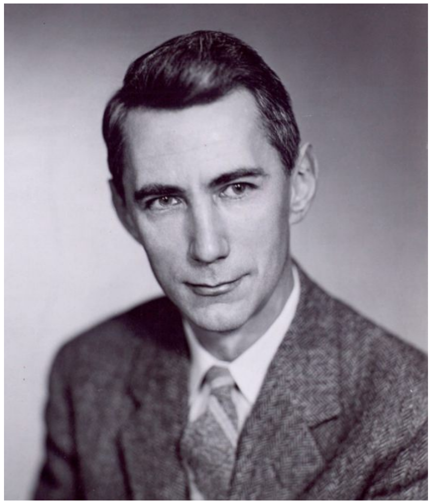

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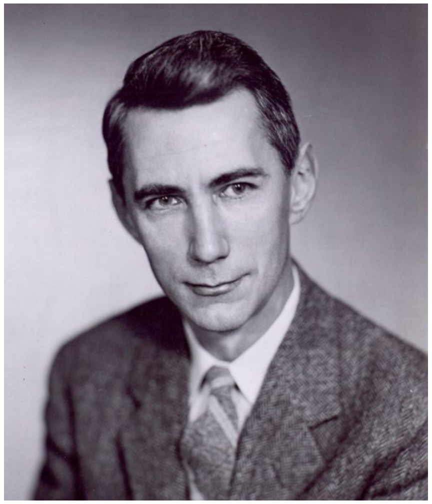

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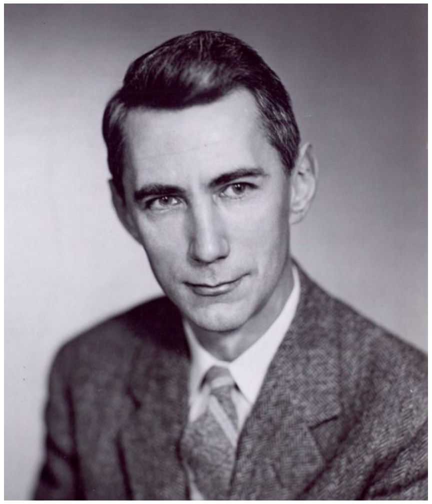

A posteriori probability

The knowledge the adversary has about m after observing c

All the a priori information known by the adversary about the plaintexts

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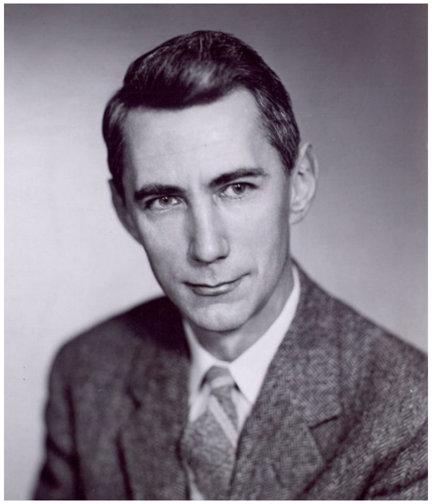

The adversary learns nothing **new**

Example

Are shift ciphers perfectly secret?

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Idea: Two occurrences of the same characters in the plaintext must produce the same characters in the ciphertext

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Ciphertext: $c = \texttt{XX}$

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This is a valid choice since:

$$\begin{aligned}\Pr[C = \texttt{XX}] &\geq \Pr[C = \texttt{XX} \wedge M = \texttt{aa}] \\ &= \Pr[C = \texttt{XX} \mid M = \texttt{aa}] \Pr[M = \texttt{aa}] \\ &= \Pr[K = 23] \Pr[M = \texttt{aa}] > 0\end{aligned}$$

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Intuition: the distribution of the ciphertexts does not depend on the plaintext

- If the distribution of the ciphertexts obtained when m is encrypted is identical to the distribution obtained when m' is encrypted, then it is impossible to tell m and m' apart when observing c

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$$\Pr[\mathsf{Enc}_K(\mathtt{aa}) = \mathtt{CC}] = \Pr[K = 2] = \tfrac{1}{26}$$

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$$\not\Updownarrow$$

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$\square$

Relating the two definitions

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They are equivalent!

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Proof of equivalence

$\forall$ *probability distribution over* $\mathcal{M}$, $\forall m \in \mathcal{M}, c \in \mathcal{C}$ *with* $\Pr[C = c] \neq 0$*:*

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$$\Pr[M = m] = \Pr[M = m \mid C = c] = \Pr[C = c \mid M = m] \cdot \frac{\Pr[M=m]}{\Pr[C=c]} = \Pr[\mathsf{Enc}_K(m) = c] \cdot \frac{\Pr[M=m]}{\Pr[C=c]}$$

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$\forall$ *probability distribution over* $\mathcal{M}$*,* $\forall m \in \mathcal{M}, c \in \mathcal{C}$ *with* $\Pr[C = c] \neq 0$*:*

$$\Pr[M = m \mid C = c] = \Pr[M = m]$$

$$\Downarrow$$

$\forall m, m' \in \mathcal{M}, c \in \mathcal{C}$:

$$\Pr[\mathit{Enc}_K(m) = c] = \Pr[\mathit{Enc}_K(m') = c]$$

Pick the uniform distribution over $\mathcal{M}$ and any c s.t. $\Pr[C = c] \neq 0$. For an arbitrary m:

$$\cancel{\Pr[M = m]} = \Pr[M = m \mid C = c] = \Pr[C = c \mid M = m] \cdot \frac{\Pr[M=m]}{\Pr[C=c]} = \Pr[\mathsf{Enc}_K(m) = c] \cdot \frac{\cancel{\Pr[M=m]}}{\Pr[C=c]}$$

$$\Pr[\mathsf{Enc}_K(m) = c] = \Pr[C = c]$$

This does not depend on the choice of m!

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$$\Pr[\mathsf{Enc}_K(m) = c] = \Pr[C = c] = \Pr[\mathsf{Enc}_K(m') = c]$$ (repeating the same argument for m')

This does not depend on the choice of m!

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$$\forall m, m' \in \mathcal{M}, c \in \mathcal{C}\text{:}$$
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Proof of equivalence

$\forall m, m' \in \mathcal{M}, c \in \mathcal{C}$:

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$\square$

Perfect indistinguishability

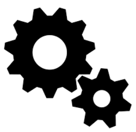
Adversary $\mathcal{A}$ (deterministic, computationally unbounded algorithm)

Verifier

Perfect indistinguishability

Adversary $\mathcal{A}$ (deterministic, computationally unbounded algorithm)

Verifier



$m_0, m_1 \in \mathcal{M}$

Perfect indistinguishability

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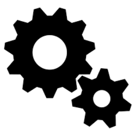

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$k \leftarrow \boxed{\mathsf{Gen}} \leftarrow$

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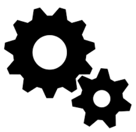

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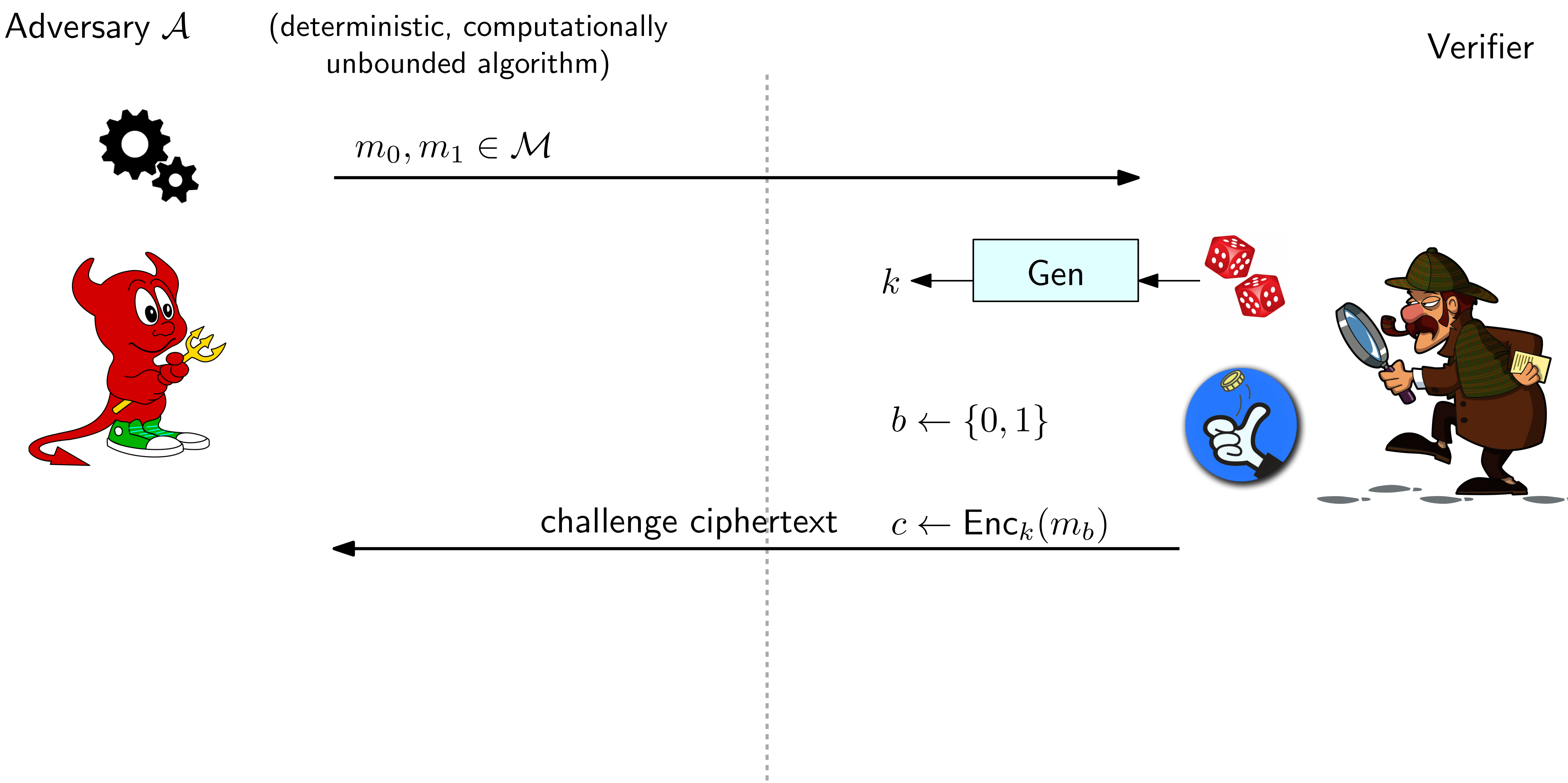
$b \leftarrow \{0,1\}$

Perfect indistinguishability



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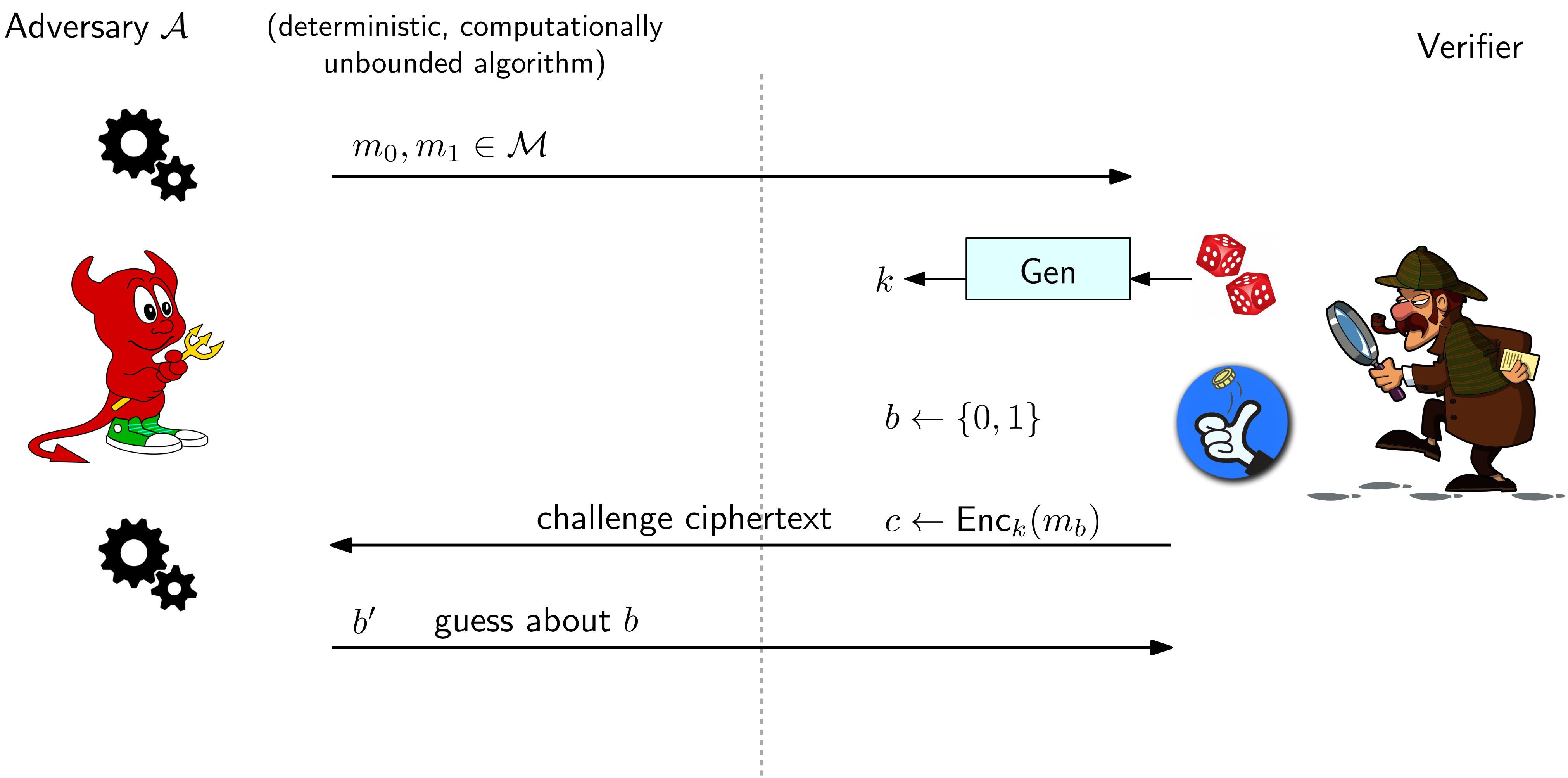

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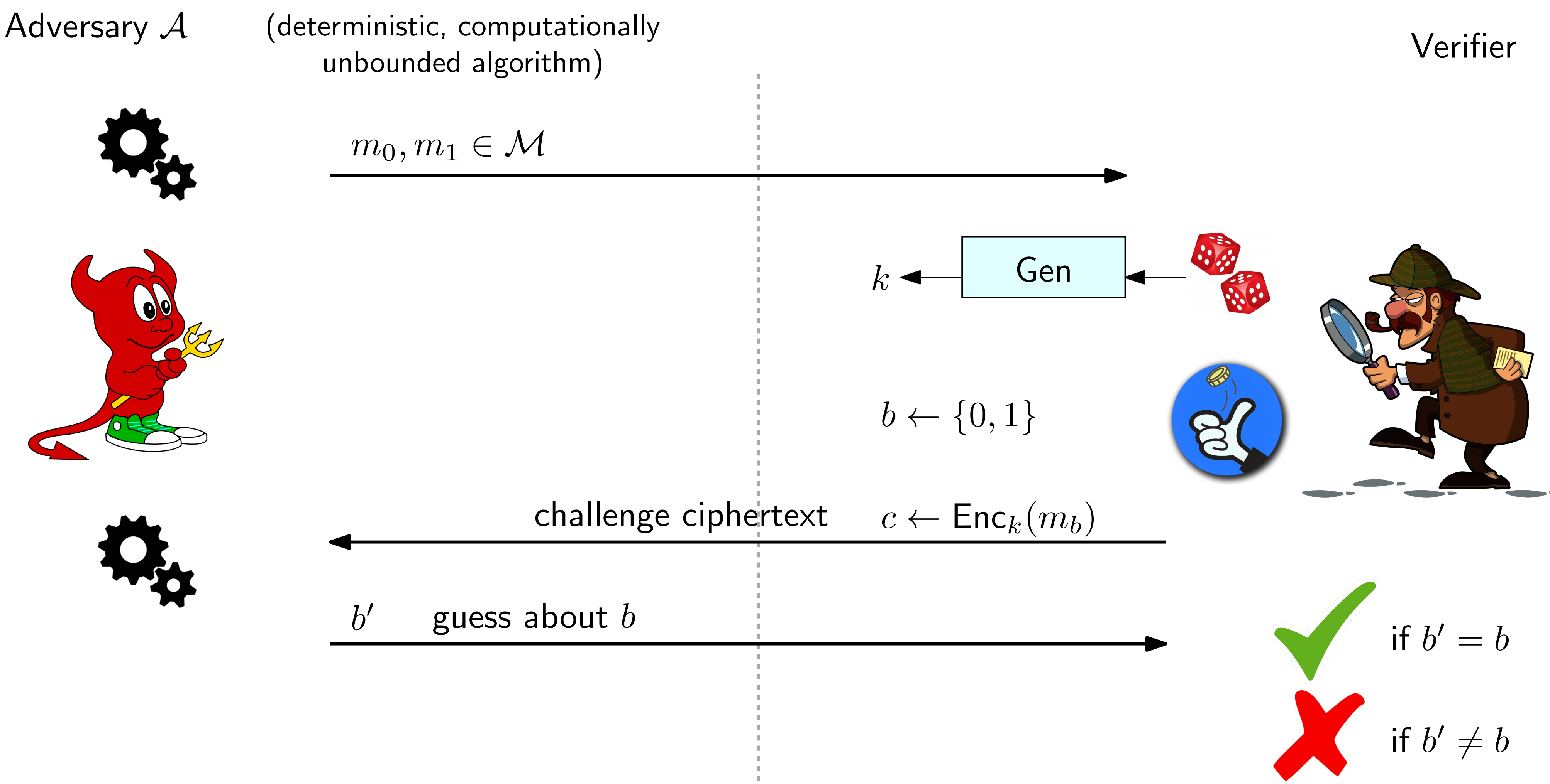

Perfect indistinguishability

Formally, if $\Pi = (\mathsf{Gen}, \mathsf{Enc}, \mathsf{Dec})$ is a private key encryption scheme with message space $\mathcal{M}$, we denote the previous experiment by $\mathsf{PrivK}^{\mathsf{eav}}_{\mathcal{A},\Pi}$

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The algorithm $\mathcal{A}$ used by the adversary

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The considered encryption scheme

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- $\mathcal{A}$ chooses two messages $m_0, m_1 \in \mathcal{M}$
- A random key k is generated (by running Gen)
- A uniform random bit $b \in \{0, 1\}$ is generated
- The *challenge ciphertext* c is computed by running $\mathsf{Enc}_k(m_b)$, and it is given to $\mathcal{A}$
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- The *output of the experiment* is defined to be 1 if $b' = b$, and 0 otherwise

We write $\mathsf{PrivK}^{\mathsf{eav}}_{\mathcal{A},\Pi} = 1$ (resp. $\mathsf{PrivK}^{\mathsf{eav}}_{\mathcal{A},\Pi} = 0$) to denote that the output of the experiment is 1 (resp. 0)

Perfect indistinguishability

Definition: *A private key encryption scheme* $\Pi = (\mathit{Gen}, \mathit{Enc}, \mathit{Dec})$ *with message space* $\mathcal{M}$ *is* **perfectly indistinguishable** *if for every* $\mathcal{A}$ *it holds:*

$$\Pr[\mathit{PrivK}^{\mathit{eav}}_{\mathcal{A},\Pi} = 1] = \frac{1}{2}$$

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(even if the two candidate plaintexts are chosen by the adversary)

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Advantage of $\mathcal{A}$

Perfect indistinguishability: Example

Consider the Vigenère cipher Π with:

$\mathcal{M} = \{a, b, \ldots, z\}^2$ $\mathcal{K} = \{A, \ldots, Z\} \cup \{A, \ldots, Z\}^2$ $\mathcal{C} = \{A, B, \ldots, Z\}^2$

Where the key is selected as follows:

- Pick a key length ℓ uniformly at random in $\{1, 2\}$
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Is Π perfectly indistinguishable?

We need to devise a "distinguisher", i.e., an algorithm $\mathcal{A}$ that wins the $\mathsf{PrivK}^{\mathsf{eav}}_{\mathcal{A},\Pi}$ experiment with probability greater than $\frac{1}{2}$

Perfect indistinguishability: Example

Algorithm $\mathcal{A}$:

- Output $m_0 = \texttt{aa}$, $m_1 = \texttt{ab}$
- Upon receiving the challenge ciphertext $c = c^{(1)}c^{(2)}$:
  - If $c^{(1)} = c^{(2)}$ output $b' = 0$
  - Otherwise (i..e, $c^{(1)} \neq c^{(2)}$) output $b' = 1$

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$\Pr[\mathsf{PrivK}^{\mathsf{eav}}_{\mathcal{A},\Pi} = 1]$

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- When $b = 0$, $\mathsf{PrivK}^{\mathsf{eav}}_{\mathcal{A},\Pi} = 1 \iff \ell = 1$ or $\ell = 2$ and the two characters of the key are equal

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- When $b = 0$, $\mathsf{PrivK}^{\mathsf{eav}}_{\mathcal{A},\Pi} = 1 \iff \ell = 1$ or $\ell = 2$ and the two characters of the key are equal

$$\Pr[\mathsf{PrivK}^{\mathsf{eav}}_{\mathcal{A},\Pi} = 1 \mid b = 0] = \tfrac{1}{2} + \tfrac{1}{2} \cdot \tfrac{1}{26}$$

Perfect indistinguishability: Example

Algorithm $\mathcal{A}$:

- Output $m_0 = \texttt{aa}$, $m_1 = \texttt{ab}$
- Upon receiving the challenge ciphertext $c = c^{(1)}c^{(2)}$:
  - If $c^{(1)} = c^{(2)}$ output $b' = 0$
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  $$\Pr[\mathsf{PrivK}^{\mathsf{eav}}_{\mathcal{A},\Pi} = 1 \mid b = 1] = \tfrac{1}{2} + \tfrac{1}{2} \cdot \tfrac{25}{26}$$

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Algorithm $\mathcal{A}$:

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$$\begin{aligned}\Pr[\mathsf{PrivK}^{\mathsf{eav}}_{\mathcal{A},\Pi} = 1] &= \tfrac{1}{2}\Pr[\mathsf{PrivK}^{\mathsf{eav}}_{\mathcal{A},\Pi} = 1 \mid b = 0] + \tfrac{1}{2}\Pr[\mathsf{PrivK}^{\mathsf{eav}}_{\mathcal{A},\Pi} = 1 \mid b = 1] \\ &= \tfrac{1}{2} \cdot (\tfrac{1}{2} + \tfrac{1}{2} \cdot \tfrac{1}{26}) + \tfrac{1}{2} \cdot (\tfrac{1}{2} + \tfrac{1}{2} \cdot \tfrac{25}{26})\end{aligned}$$

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$$= \tfrac{1}{2} \cdot (\tfrac{1}{2} + \tfrac{1}{2} \cdot \tfrac{1}{26}) + \tfrac{1}{2} \cdot (\tfrac{1}{2} + \tfrac{1}{2} \cdot \tfrac{25}{26}) \quad = \tfrac{3}{4} = \tfrac{1}{2} + \tfrac{1}{4} > \tfrac{1}{2}$$

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Advantage of $\mathcal{A}$

Perfect secrecy & perfect indistinguishability

A private key encryption scheme is **perfectly secret** if and only if it is **perfectly indistinguishable**.

$\forall$ *probability distribution over* $\mathcal{M}$, $\forall m \in \mathcal{M}, c \in \mathcal{C}$ *with* $\Pr[C = c] \neq 0$*:*

$$\Pr[M = m \mid C = c] = \Pr[M = m]$$

$$\Updownarrow$$

$\forall m, m' \in \mathcal{M}, c \in \mathcal{C}$*:*

$$\Pr[Enc_K(m) = c] = \Pr[Enc_K(m') = c]$$

$$\Updownarrow$$

$$\Pr[\mathsf{PrivK}^{\mathsf{eav}}_{\mathcal{A},\Pi} = 1] = \frac{1}{2} \quad \forall \mathcal{A}$$

Proof of equivalence

$$\forall m, m' \in \mathcal{M}, c \in \mathcal{C}\text{:}$$

$$\Pr[\mathit{Enc}_K(m) = c] = \Pr[\mathit{Enc}_K(m') = c]$$

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Fix any algorithm $\mathcal{A}$, and let m_0, m_1 be the messages output by $\mathcal{A}$

Partition $\mathcal{C}$ into $\mathcal{C}_0, \mathcal{C}_1$, where C_i is the set of ciphertexts for which $\mathcal{A}$ guesses $b' = i$

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$$\begin{aligned}\Pr[\mathsf{PrivK}^{\mathsf{eav}}_{\mathcal{A},\Pi} = 1] &= \tfrac{1}{2} \cdot \Pr[b' = 0 \mid b = 0] + \tfrac{1}{2} \cdot \Pr[b' = 1 \mid b = 1] \\ &= \tfrac{1}{2} \cdot \Pr[\mathsf{Enc}_K(m_0) \in \mathcal{C}_0] + \tfrac{1}{2} \cdot \Pr[\mathsf{Enc}_K(m_1) \in \mathcal{C}_1]\end{aligned}$$

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$\forall m, m' \in \mathcal{M}, c \in \mathcal{C}$:

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&= \tfrac{1}{2} \cdot \textstyle\sum_{c \in \mathcal{C}_0} \Pr[\mathsf{Enc}_K(m_0) = c] + \tfrac{1}{2} \cdot \textstyle\sum_{c \in \mathcal{C}_1} \Pr[\mathsf{Enc}_K(m_1) = c]
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$\forall m, m' \in \mathcal{M}, c \in \mathcal{C}$:

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&= \tfrac{1}{2} \cdot \textstyle\sum_{c \in \mathcal{C}_0} \Pr[\mathsf{Enc}_K(m_0) = c] + \tfrac{1}{2} \cdot \textstyle\sum_{c \in \mathcal{C}_1} \Pr[\mathsf{Enc}_K(\color{red}{m_0}) = c] \\
&= \tfrac{1}{2} \cdot \textstyle\sum_{c \in \mathcal{C}} \Pr[\mathsf{Enc}_K(m_0) = c]
\end{aligned}$$

Proof of equivalence

$$\forall m, m' \in \mathcal{M}, c \in \mathcal{C}:$$

$$\Pr[\mathit{Enc}_K(m) = c] = \Pr[\mathit{Enc}_K(m') = c]$$

$$\Downarrow$$

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&= \tfrac{1}{2} \cdot \textstyle\sum_{c \in \mathcal{C}_0} \Pr[\mathsf{Enc}_K(m_0) = c] + \tfrac{1}{2} \cdot \textstyle\sum_{c \in \mathcal{C}_1} \Pr[\mathsf{Enc}_K({\color{red}m_0}) = c] \\
&= \tfrac{1}{2} \cdot \textstyle\sum_{c \in \mathcal{C}} \Pr[\mathsf{Enc}_K(m_0) = c] \quad = \tfrac{1}{2}
\end{aligned}$$

Proof of equivalence

NOT

$$\forall m, m' \in \mathcal{M}, c \in \mathcal{C}:$$

$$\Pr[\mathsf{Enc}_K(m) = c] = \Pr[\mathsf{Enc}_K(m') = c]$$

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- Output m_0, m_1
- Upon receiving the challenge ciphertext c
  - If $c = c^*$ output $b' = 0$
  - Otherwise output a b' chosen u.a.r. in $\{0, 1\}$

Proof of equivalence

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$$\forall m, m' \in \mathcal{M}, c \in \mathcal{C}\text{:}$$
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$$\forall m, m' \in \mathcal{M}, c \in \mathcal{C}\text{:}$$
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$\Pr[b' = 0 \mid b = 0] = \Pr[b' = 0 \wedge \mathsf{Enc}_K(m_0) = c^*] + \Pr[b' = 0 \wedge \mathsf{Enc}_K(m_0) \neq c^*]$

Proof of equivalence

NOT

$$\forall m, m' \in \mathcal{M}, c \in \mathcal{C}:$$
$$\Pr[\mathit{Enc}_K(m) = c] = \Pr[\mathit{Enc}_K(m') = c]$$

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$$\begin{aligned}\Pr[b' = 0 \mid b = 0] &= \Pr[b' = 0 \wedge \mathsf{Enc}_K(m_0) = c^*] + \Pr[b' = 0 \wedge \mathsf{Enc}_K(m_0) \neq c^*] \\ &= \Pr[\mathsf{Enc}_K(m_0) = c^*] + \Pr[b' = 0 \mid \mathsf{Enc}_K(m_0) \neq c^*] \cdot \Pr[\mathsf{Enc}_K(m_0) \neq c^*]\end{aligned}$$

Proof of equivalence

NOT

$$\forall m, m' \in \mathcal{M}, c \in \mathcal{C}:$$
$$\Pr[\mathit{Enc}_K(m) = c] = \Pr[\mathit{Enc}_K(m') = c]$$

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$$\Pr[\mathsf{PrivK}^{\mathsf{eav}}_{\mathcal{A},\Pi} = 1] = \tfrac{1}{2} \cdot \Pr[b' = 0 \mid b = 0] + \tfrac{1}{2} \cdot \Pr[b' = 1 \mid b = 1]$$

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\Pr[b' = 0 \mid b = 0] &= \Pr[b' = 0 \wedge \mathsf{Enc}_K(m_0) = c^*] + \Pr[b' = 0 \wedge \mathsf{Enc}_K(m_0) \neq c^*] \\
&= \Pr[\mathsf{Enc}_K(m_0) = c^*] + \Pr[b' = 0 \mid \mathsf{Enc}_K(m_0) \neq c^*] \cdot \Pr[\mathsf{Enc}_K(m_0) \neq c^*] \\
&= \Pr[\mathsf{Enc}_K(m_0) = c^*] + \tfrac{1}{2} \cdot \Pr[\mathsf{Enc}_K(m_0) \neq c^*]
\end{aligned}$$

Proof of equivalence

NOT

$$\forall m, m' \in \mathcal{M}, c \in \mathcal{C}\text{:}$$

$$\Pr[\mathit{Enc}_K(m) = c] = \Pr[\mathit{Enc}_K(m') = c]$$

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Pick $m_0, m_1 \in \mathcal{M}, c^* \in \mathcal{C}$ s.t. $\Pr[\mathsf{Enc}_K(m_0) = c^*] \neq \Pr[\mathsf{Enc}_K(m_1) = c^*]$

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$$\begin{aligned}\Pr[b' = 0 \mid b = 0] &= \Pr[b' = 0 \wedge \mathsf{Enc}_K(m_0) = c^*] + \Pr[b' = 0 \wedge \mathsf{Enc}_K(m_0) \neq c^*] \\ &= \Pr[\mathsf{Enc}_K(m_0) = c^*] + \Pr[b' = 0 \mid \mathsf{Enc}_K(m_0) \neq c^*] \cdot \Pr[\mathsf{Enc}_K(m_0) \neq c^*] \\ &= \Pr[\mathsf{Enc}_K(m_0) = c^*] + \tfrac{1}{2} \cdot (1 - \Pr[\mathsf{Enc}_K(m_0) = c^*])\end{aligned}$$

Proof of equivalence

NOT

$$\forall m, m' \in \mathcal{M}, c \in \mathcal{C}:$$

$$\Pr[\mathit{Enc}_K(m) = c] = \Pr[\mathit{Enc}_K(m') = c]$$

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$$\begin{aligned}
\Pr[b' = 0 \mid b = 0] &= \Pr[b' = 0 \wedge \mathsf{Enc}_K(m_0) = c^*] + \Pr[b' = 0 \wedge \mathsf{Enc}_K(m_0) \neq c^*] \\
&= \Pr[\mathsf{Enc}_K(m_0) = c^*] + \Pr[b' = 0 \mid \mathsf{Enc}_K(m_0) \neq c^*] \cdot \Pr[\mathsf{Enc}_K(m_0) \neq c^*] \\
&= \Pr[\mathsf{Enc}_K(m_0) = c^*] + \tfrac{1}{2} \cdot (1 - \Pr[\mathsf{Enc}_K(m_0) = c^*]) \\
&= \tfrac{1}{2} + \tfrac{1}{2} \Pr[\mathsf{Enc}_K(m_0) = c^*]
\end{aligned}$$

Proof of equivalence

NOT

$$\forall m, m' \in \mathcal{M}, c \in \mathcal{C}:$$
$$\Pr[\textit{Enc}_K(m) = c] = \Pr[\textit{Enc}_K(m') = c]$$

$$\Downarrow$$

NOT

$$\Pr[\mathsf{PrivK}^{\mathsf{eav}}_{\mathcal{A},\Pi} = 1] = \frac{1}{2} \quad \forall \mathcal{A}$$

Pick $m_0, m_1 \in \mathcal{M}, c^* \in \mathcal{C}$ s.t. $\Pr[\mathsf{Enc}_K(m_0) = c^*] \neq \Pr[\mathsf{Enc}_K(m_1) = c^*]$

$\Pr[\mathsf{PrivK}^{\mathsf{eav}}_{\mathcal{A},\Pi} = 1] = \frac{1}{2} \cdot \Pr[b' = 0 \mid b = 0] + \frac{1}{2} \cdot \Pr[b' = 1 \mid b = 1]$

$\Pr[b' = 0 \mid b = 0] = \frac{1}{2} + \frac{1}{2} \cdot \Pr[\mathsf{Enc}_K(m_0) = c^*]$

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$\Pr[b' = 0 \mid b = 0] = \frac{1}{2} + \frac{1}{2} \cdot \Pr[\mathsf{Enc}_K(m_0) = c^*]$

$\Pr[b' = 1 \mid b = 1] = \Pr[b' = 1 \wedge \mathsf{Enc}_K(m_1) = c^*] + \Pr[b' = 1 \wedge \mathsf{Enc}_K(m_1) \neq c^*]$

Proof of equivalence

NOT	$\forall m, m' \in \mathcal{M}, c \in \mathcal{C}$: $\Pr[\mathit{Enc}_K(m) = c] = \Pr[\mathit{Enc}_K(m') = c]$

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Pick $m_0, m_1 \in \mathcal{M}, c^* \in \mathcal{C}$ s.t. $\Pr[\mathsf{Enc}_K(m_0) = c^*] \neq \Pr[\mathsf{Enc}_K(m_1) = c^*]$

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$$\Pr[b' = 0 \mid b = 0] = \tfrac{1}{2} + \tfrac{1}{2} \cdot \Pr[\mathsf{Enc}_K(m_0) = c^*]$$

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Proof of equivalence

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$$= \Pr[b' = 1 \mid \mathsf{Enc}_K(m_1) \neq c^*] \cdot \Pr[\mathsf{Enc}_K(m_1) \neq c^*]$$

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$$\forall m, m' \in \mathcal{M}, c \in \mathcal{C}\text{:}$$
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$$\begin{aligned}\Pr[b' = 1 \mid b = 1] &= \cancel{\Pr[b' = 1 \wedge \mathsf{Enc}_K(m_1) = c^*]} + \Pr[b' = 1 \wedge \mathsf{Enc}_K(m_1) \neq c^*] \\ &= \Pr[b' = 1 \mid \mathsf{Enc}_K(m_1) \neq c^*] \cdot \Pr[\mathsf{Enc}_K(m_1) \neq c^*] \\ &= \tfrac{1}{2} \cdot \Pr[\mathsf{Enc}_K(m_1) \neq c^*]\end{aligned}$$

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$\Pr[\mathsf{PrivK}^{\mathsf{eav}}_{\mathcal{A},\Pi} = 1] = \frac{1}{4} + \frac{1}{4} \cdot \Pr[\mathsf{Enc}_K(m_0) = c^*] + \frac{1}{4} \cdot \Pr[\mathsf{Enc}_K(m_1) \neq c^*]$

Proof of equivalence

NOT

$$\forall m, m' \in \mathcal{M}, c \in \mathcal{C}:$$
$$\Pr[\mathit{Enc}_K(m) = c] = \Pr[\mathit{Enc}_K(m') = c]$$

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$$\neq \tfrac{1}{4} + \tfrac{1}{4} \cdot \Pr[\mathsf{Enc}_K(m_1) = c^*] + \tfrac{1}{4} \cdot \Pr[\mathsf{Enc}_K(m_1) \neq c^*]$$

Proof of equivalence

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$$\forall m, m' \in \mathcal{M}, c \in \mathcal{C}\text{:}$$

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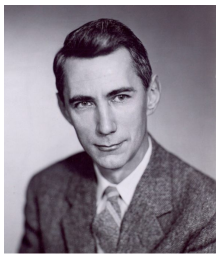
$$= \tfrac{1}{4} + \tfrac{1}{4} = \tfrac{1}{2}$$

$\square$

Recap: Equivalent definitions



Definition: *An encryption scheme* (Gen, Enc, Dec) *with message space* $\mathcal{M}$ *is* **perfectly secret** *if* **for every** *probability distribution over* $\mathcal{M}$*,* **every** *message* $m \in \mathcal{M}$*, and* **every** *ciphertext* $c \in \mathcal{C}$ *with* $\Pr[C = c] \neq 0$*:*

$$\Pr[M = m \mid C = c] = \Pr[M = m]$$

Definition: *An encryption scheme* (Gen, Enc, Dec) *with message space* $\mathcal{M}$ *is* **perfectly secret** *if* **for every** $m, m' \in \mathcal{M}$*, and* **every** $c \in \mathcal{C}$*:*

$$\Pr[Enc_K(m) = c] = \Pr[Enc_K(m') = c]$$

Definition: *A private key encryption scheme* $\Pi = (Gen, Enc, Dec)$ *with message space* $\mathcal{M}$ *is* **perfectly indistinguishable** *if for every* $\mathcal{A}$ *it holds:*

$$\Pr[\mathit{PrivK}^{eav}_{\mathcal{A},\Pi} = 1] = \frac{1}{2}$$