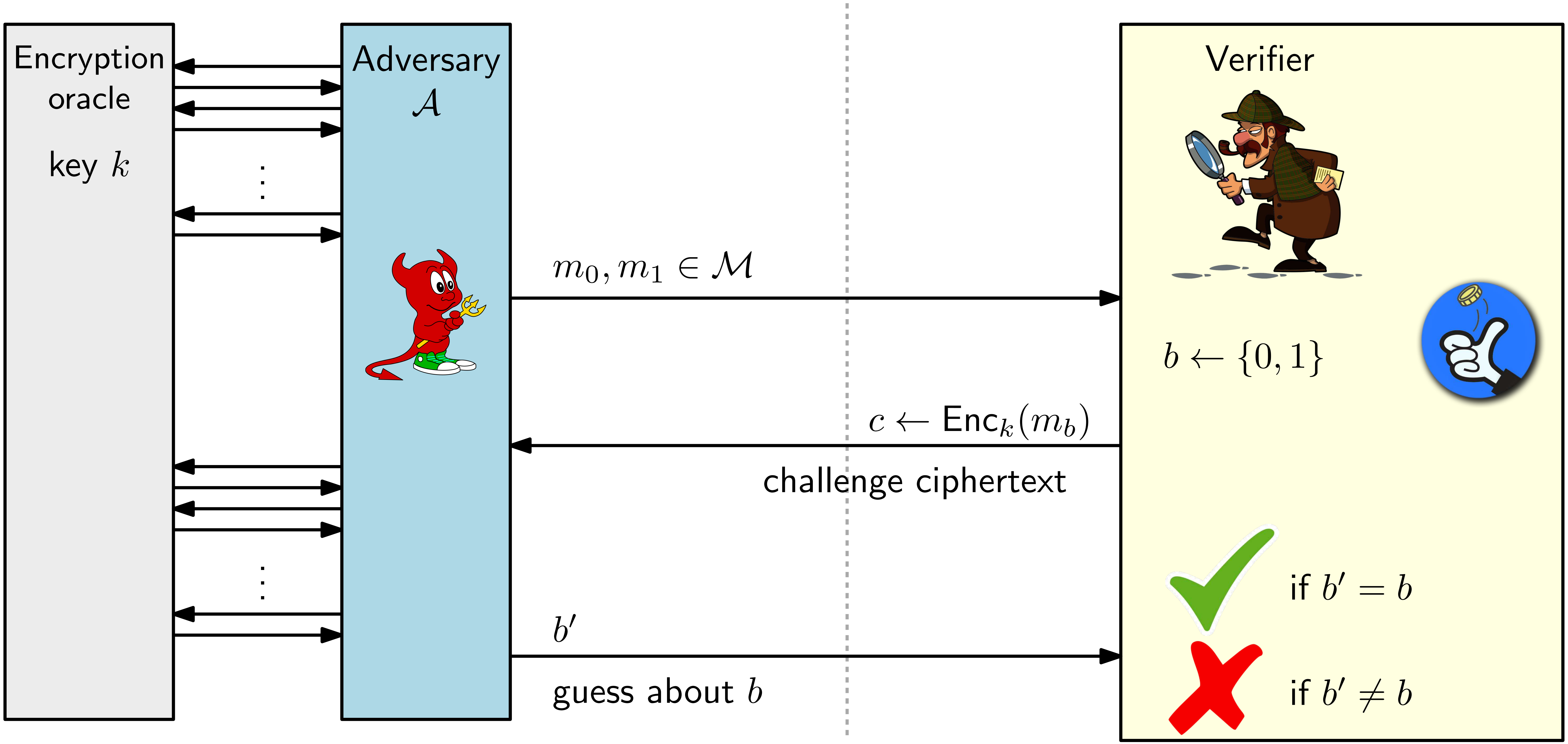
Recap: Modeling CPA security

A key $k \leftarrow \mathsf{Gen}(1^n)$ is generated



Encryption oracle

key k

Adversary $\mathcal{A}$

$m_0, m_1 \in \mathcal{M}$

Verifier

$b \leftarrow \{0,1\}$

$c \leftarrow \mathsf{Enc}_k(m_b)$

challenge ciphertext

b'

guess about b

if $b' = b$

if $b' \neq b$

Definition of CPA-security

Definition: *A private key encryption scheme* Π *has indistinguishable encryptions under a chosen-plaintext attack (is* **CPA-secure***) if, for every probabilistic polynomial-time adversary* $\mathcal{A}$*, there is a negligible function* ε *such that:*

$$\Pr[\mathit{PrivK}^{cpa}_{\mathcal{A},\Pi}(n) = 1] \leq \frac{1}{2} + \varepsilon(n)$$

Any private-key encryption scheme that is CPA-secure is also CPA-secure for multiple encryptions

$$\Downarrow$$

If Π is CPA-secure then Π has indistinguishable multiple encryptions in the presence of an eavesdropper (and hence it is also EAV-secure)

Achieving CPA security (a first attempt)

Let $F : \{0,1\}^n \times \{0,1\}^n \to \{0,1\}^n$ be a (length-preserving) pseudorandom permutation

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No stateless, deterministic encryption scheme can be CPA-secure

Actually achieving CPA security

Let $F : \{0,1\}^n \times \{0,1\}^n \to \{0,1\}^n$ be a (length-preserving) pseudorandom function

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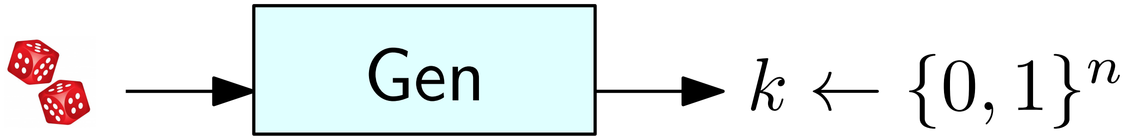

Gen

$k \leftarrow \{0,1\}^n$

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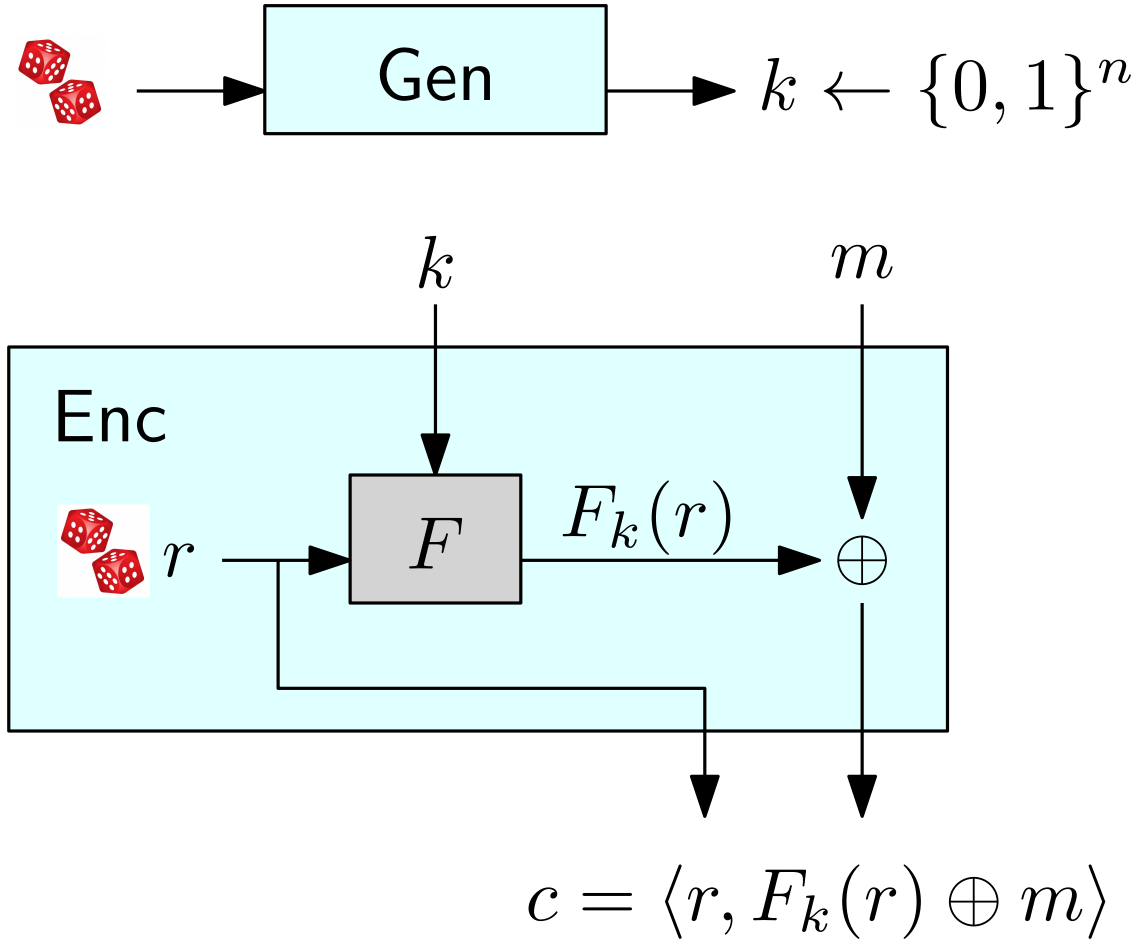

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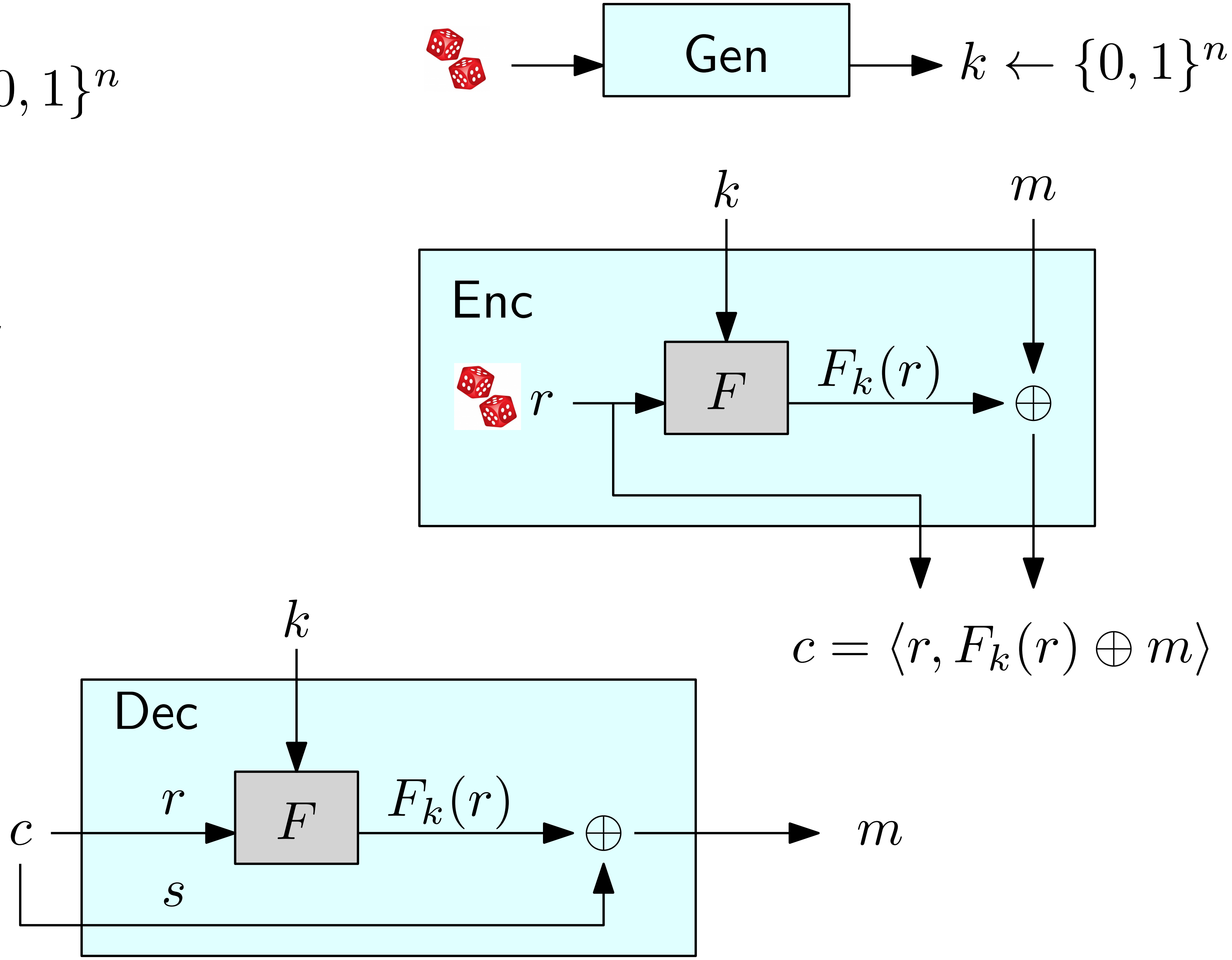

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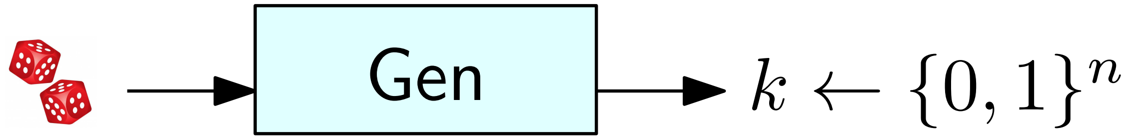
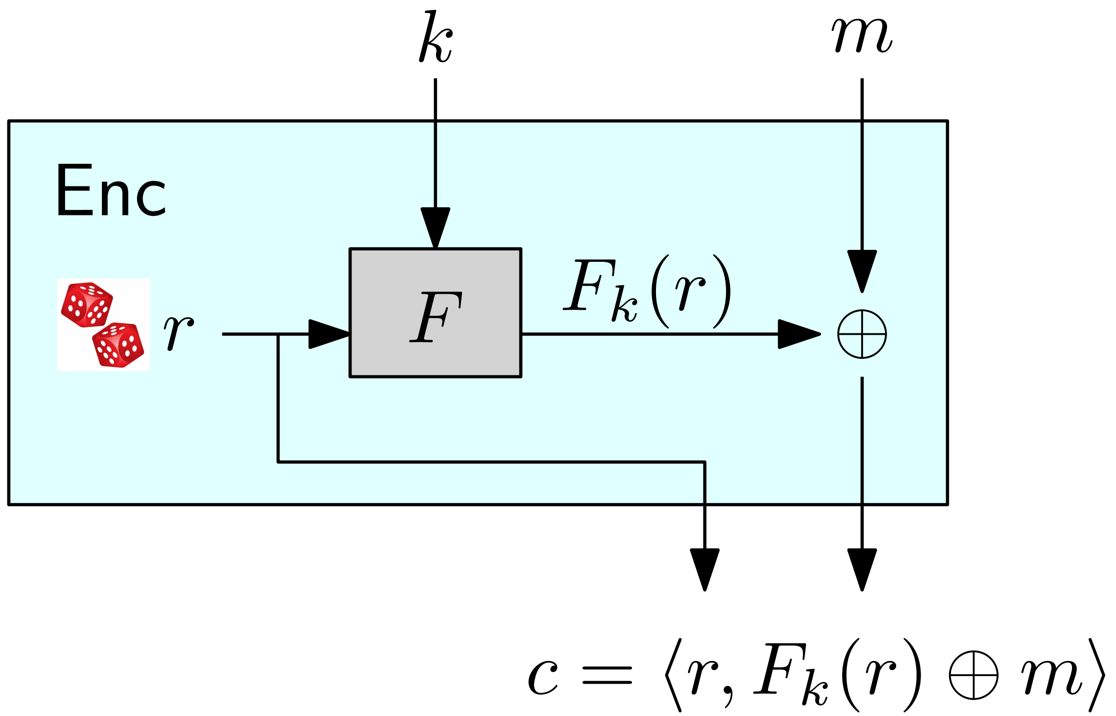
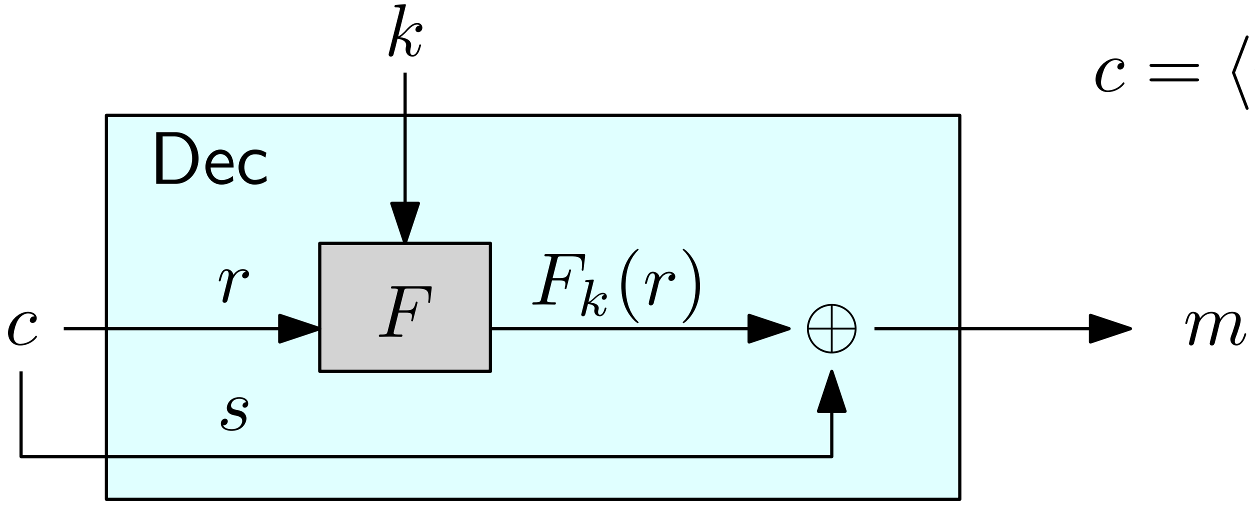
$\mathcal{M} = \{0,1\}^n$ $\mathcal{K} = \{0,1\}^n$ $\mathcal{C} = \{0,1\}^{2n}$







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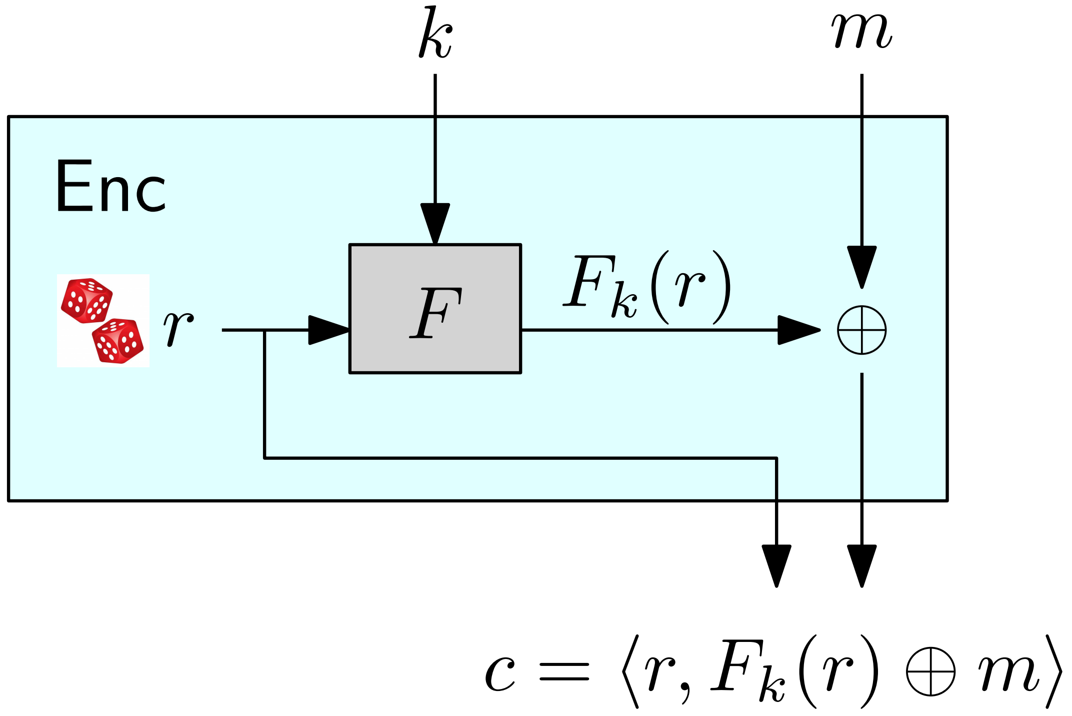

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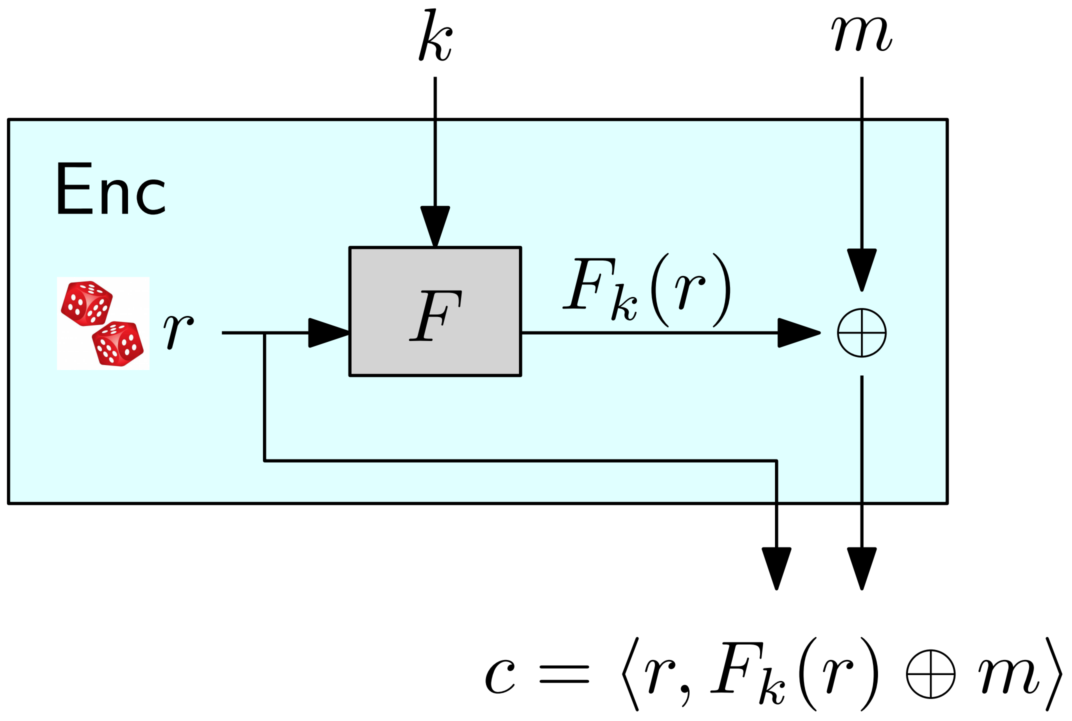

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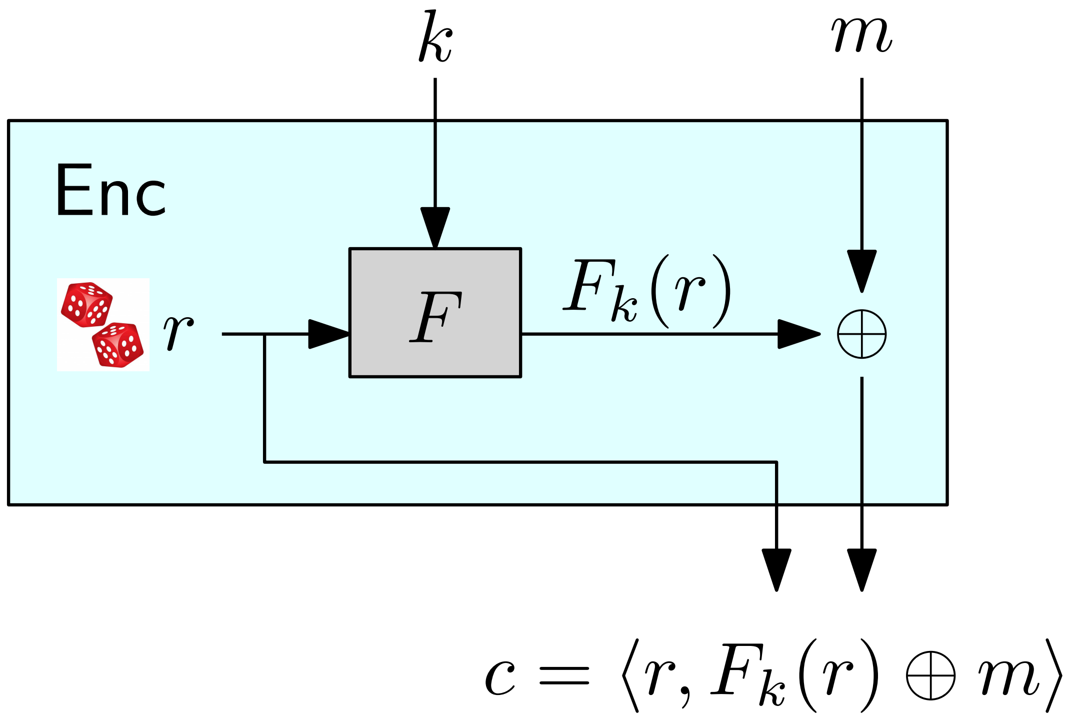

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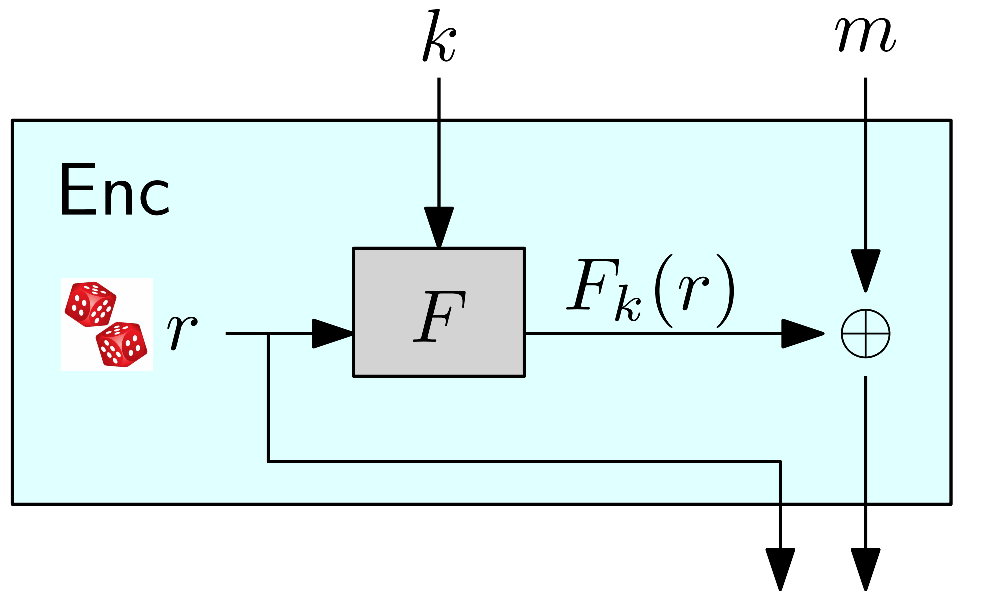

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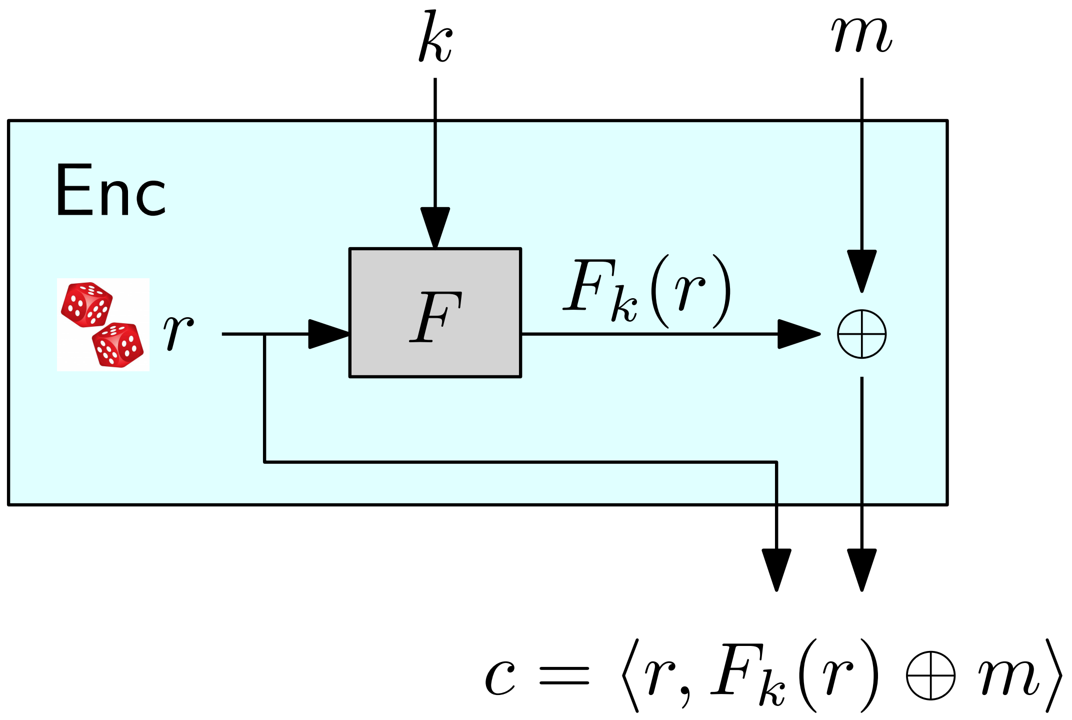
- The process behaves similarly to the "real" OTP if the parties were to"agree on a new key" after each message



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  - Return 1 if $b' = b$ and 0 otherwise

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Can we **prove** that this encryption scheme is secure?

High-level proof strategy:

- Consider a variant $\widetilde{\Pi}$ of Π in which a truly random function f is used instead of F
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- If n is too short, or it is not chosen from a uniform distribution then repeats might happen!

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Can we do better?

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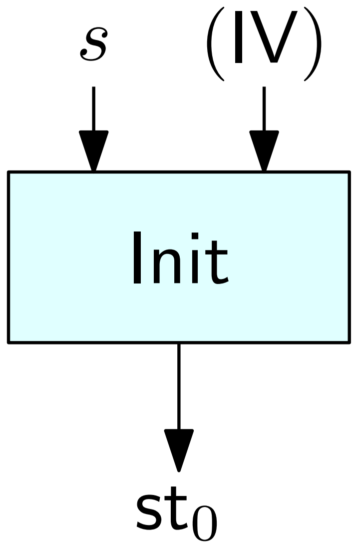
Warning: Sometimes the term "stream cipher" is used to refer to the encryption scheme built from the actual stream cipher (as defined here)

Stream ciphers

A stream cipher is a pair of deterministic polynomial-time algorithms

- **Init:** takes a n-bit seed s, and possibly a n-bit *initialization vector* (IV), and outputs a *state* st_0



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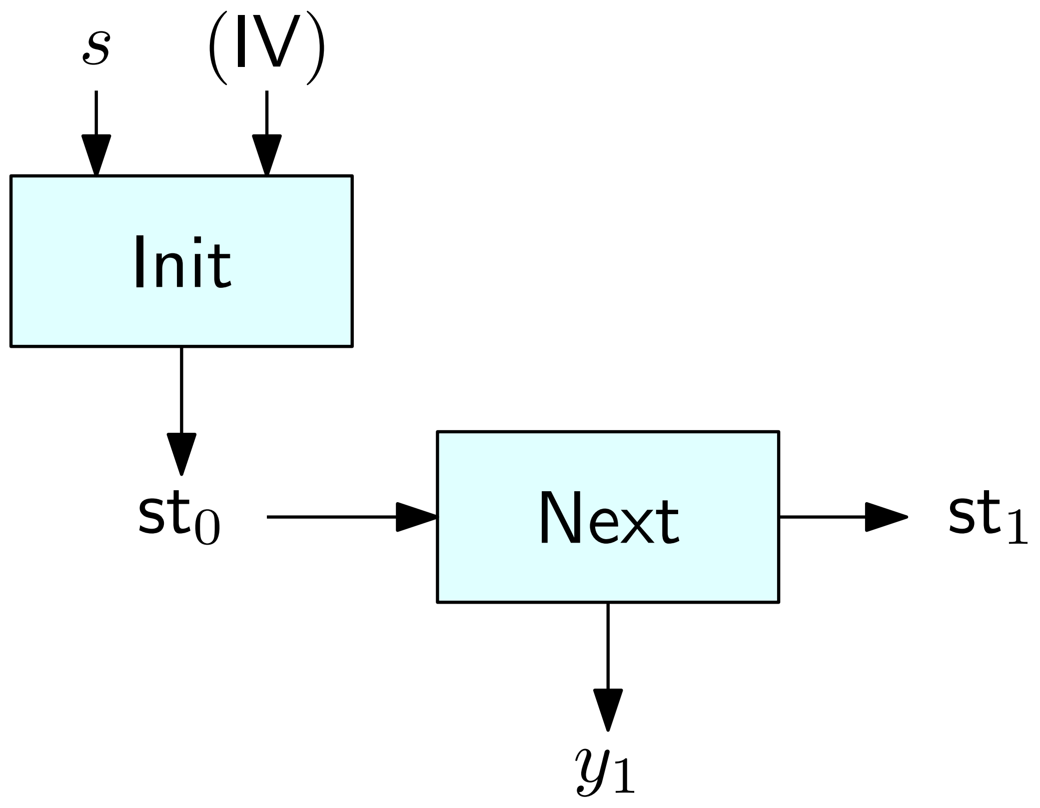

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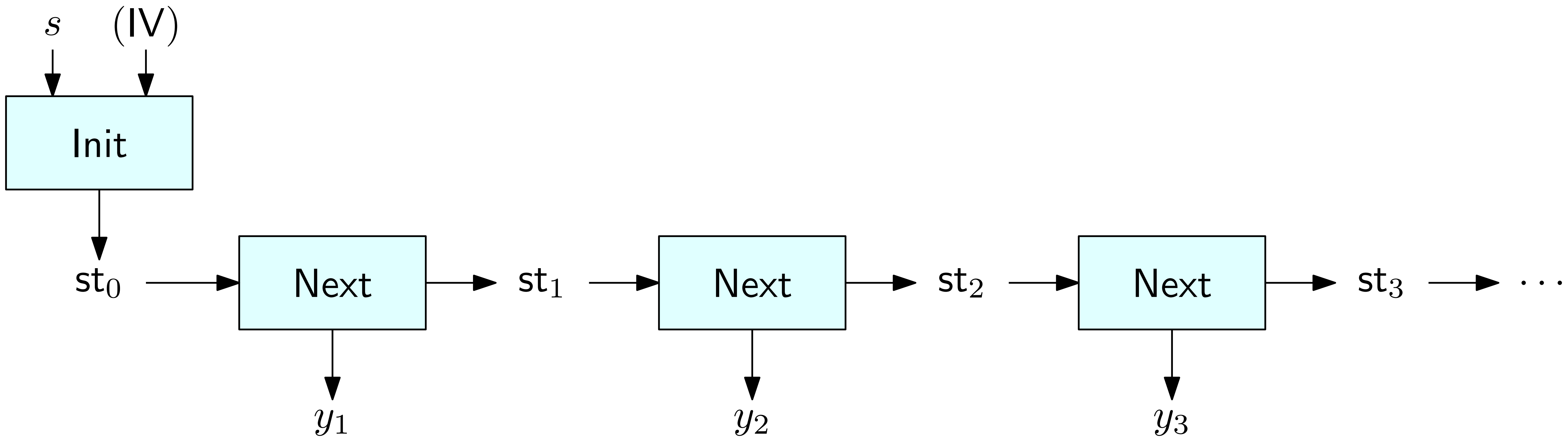

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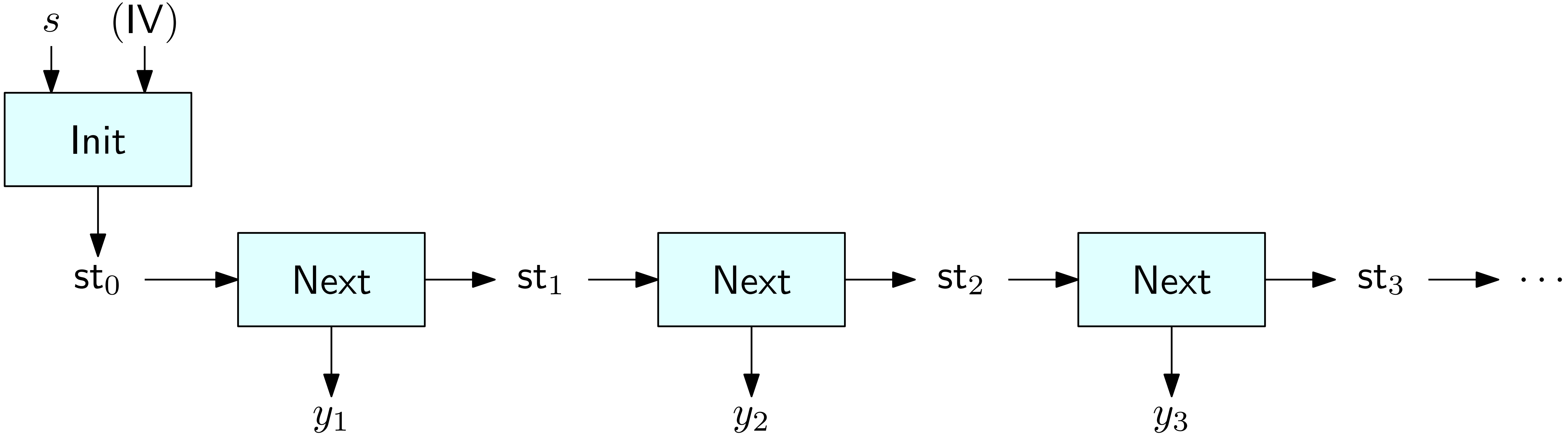

* In practice, **Next** can output multiple bits at once (e.g., a byte)

Secure Stream Ciphers

A stream cipher is secure if the output stream generated by starting from a seed chosen u.a.r. is pseudorandom

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If the stream cipher does not use IVs:

- Define the function $G^\ell(s)$ (from $\{0, 1\}^n$ to $\{0, 1\}^\ell$) as the string y of $\mathsf{GetBits}(\mathsf{Init}(s), 1^\ell)$

- The stream cipher is **secure** if $G^\ell(s)$ is a **pseudorandom generator** for any polynomial ℓ

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If the stream cipher uses IVs:

- Define the function $F_s^\ell(\mathsf{IV})$ (from $\{0, 1\}^n \times \{0, 1\}^n$ to $\{0, 1\}^\ell$) as the string y of $\mathsf{GetBits}(\mathsf{Init}(s, \mathsf{IV}), 1^\ell)$
- The stream cipher is **secure** if $F_s^\ell(\mathsf{IV})$ is a **pseudorandom function** for any polynomial ℓ

Secure Stream ciphers with IVs from Pseudorandom Functions

If we have a pseudorandom function $F : \{0,1\}^n \times \{0,1\}^n \to \{0,1\}^n$, we can use it to build a stream cipher that takes an initialization vector

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Next(st):
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$\langle i \rangle$ = Binary encoding of i using $n/4$ bits

Modes of operation of Stream Ciphers

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  - Useful for short communication sessions. Each message must be delivered exactly once and all messages must be received in order
  - Example: data exchanged over a TCP connection
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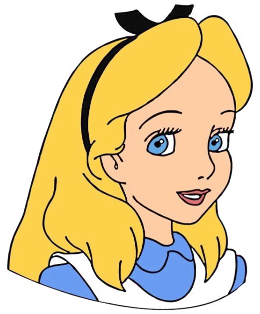
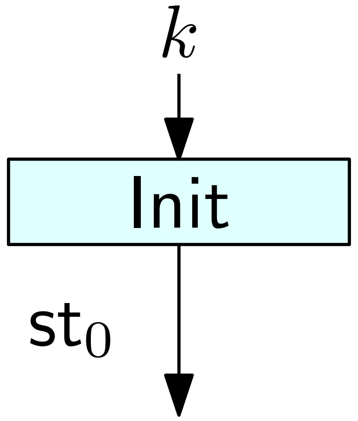
  - Example: data exchanged over a TCP connection
  - Does not need to use IVs, Ciphertext length = message length
- **Unsynchronized mode**: The sender and receiver do not need to store any information during the communication session (i.e., they are stateless)
  - Useful for long messages, and communication over a long period of time. Does not require messages to be delivered in order
  - Each message uses its own IV
  - Needs IVs, Ciphertext length = message length + IV length ($\approx$ message length for long messages)

Synchronized mode





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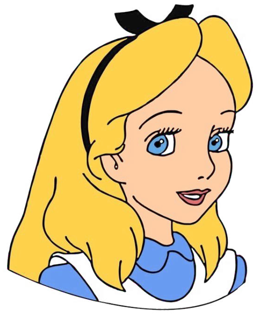
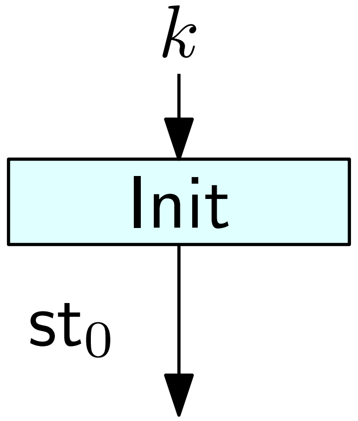
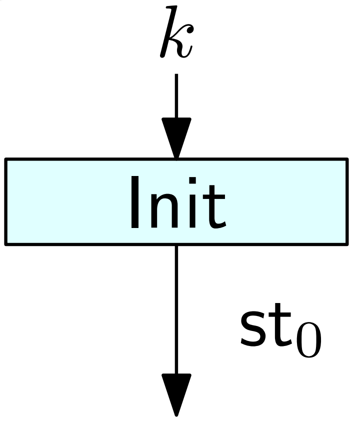




k

Init

st_0



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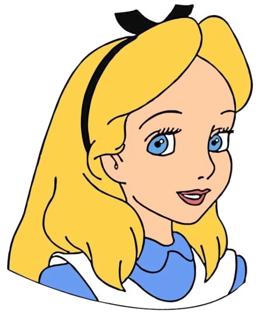

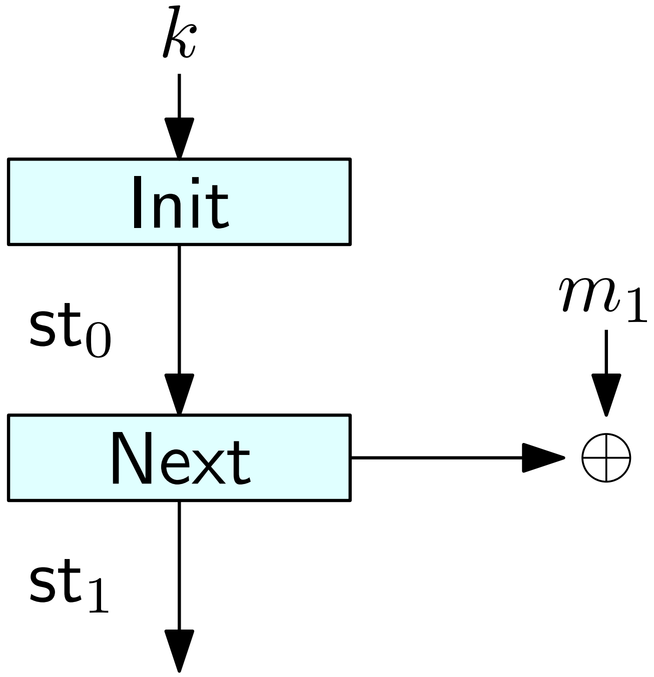

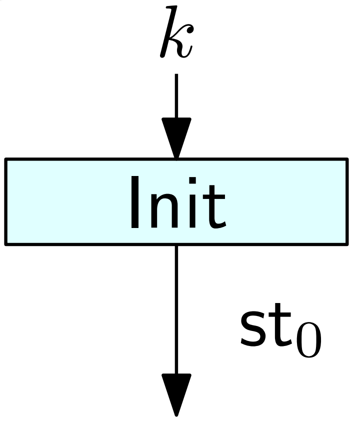

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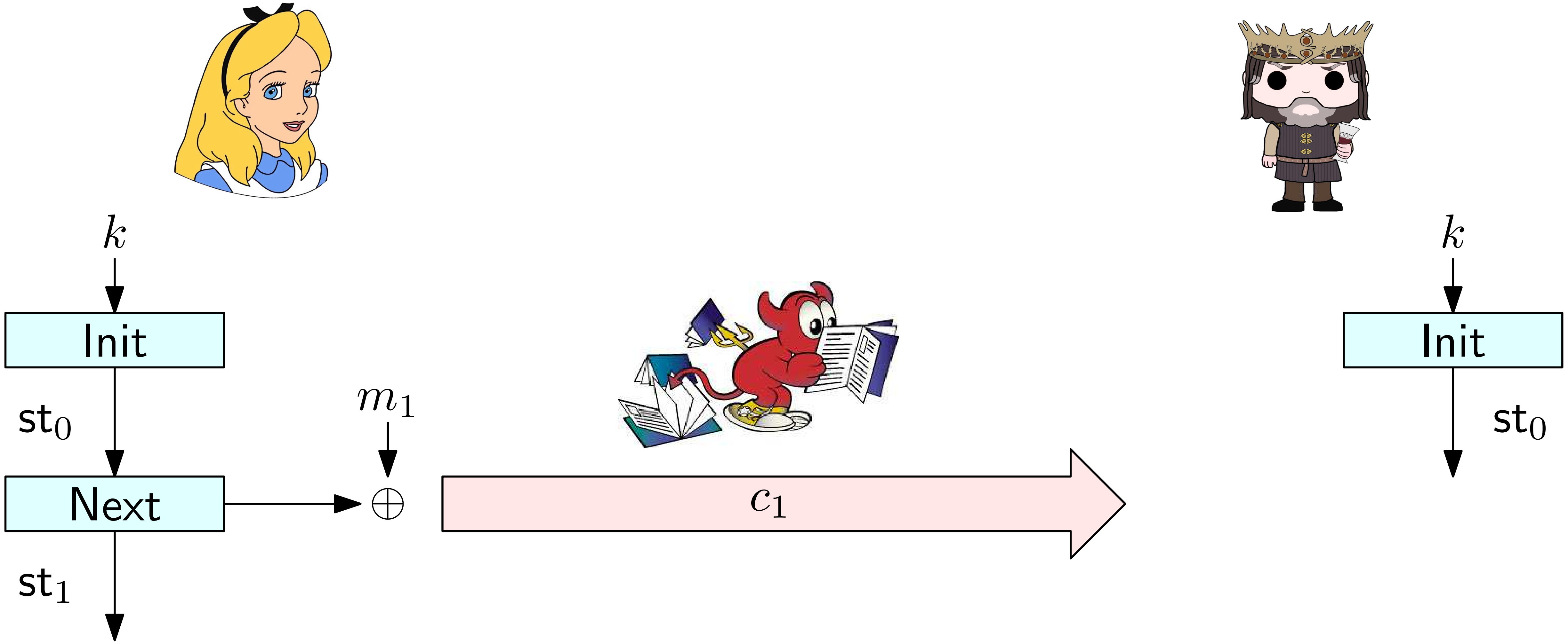


k

Init

st_0

m_1

Next

$\oplus$

c_1

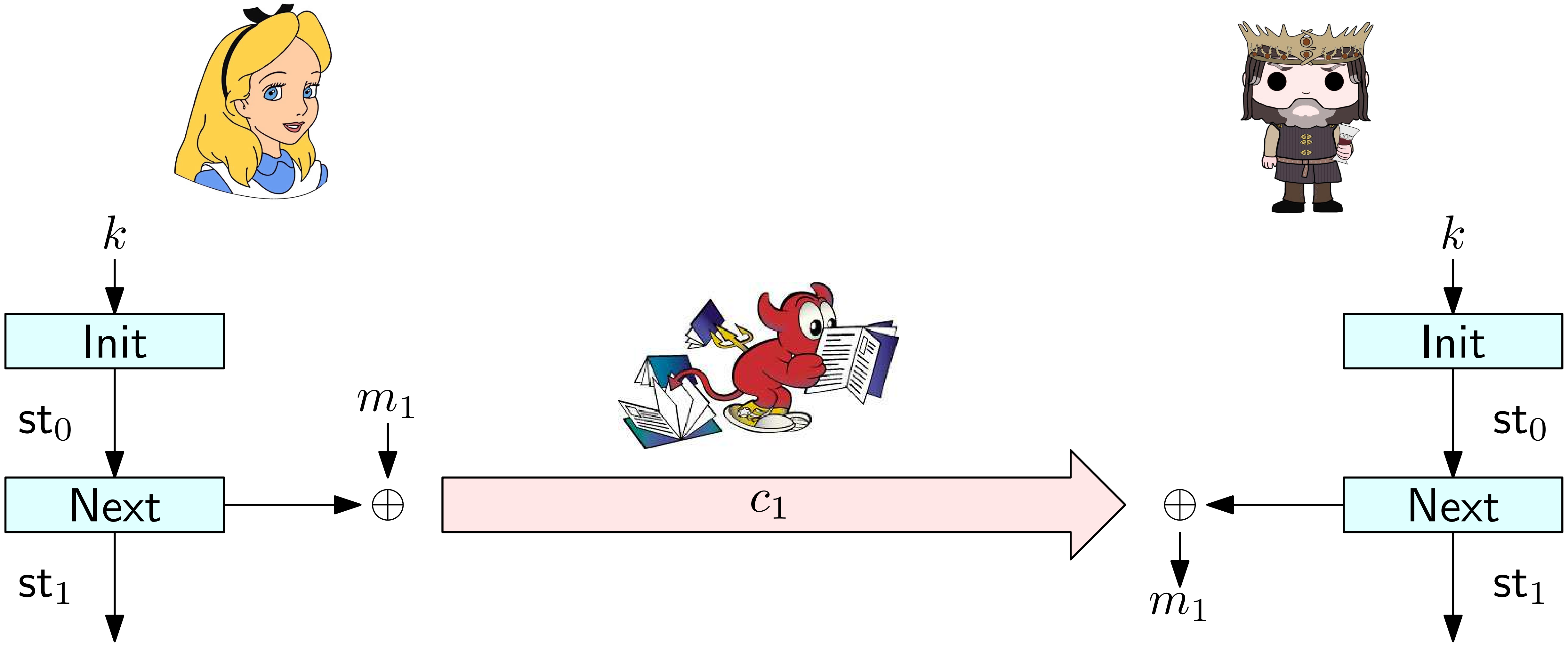
st_1

k

Init

st_0

Synchronized mode



k

Init

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m_1

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m_1

k

Init

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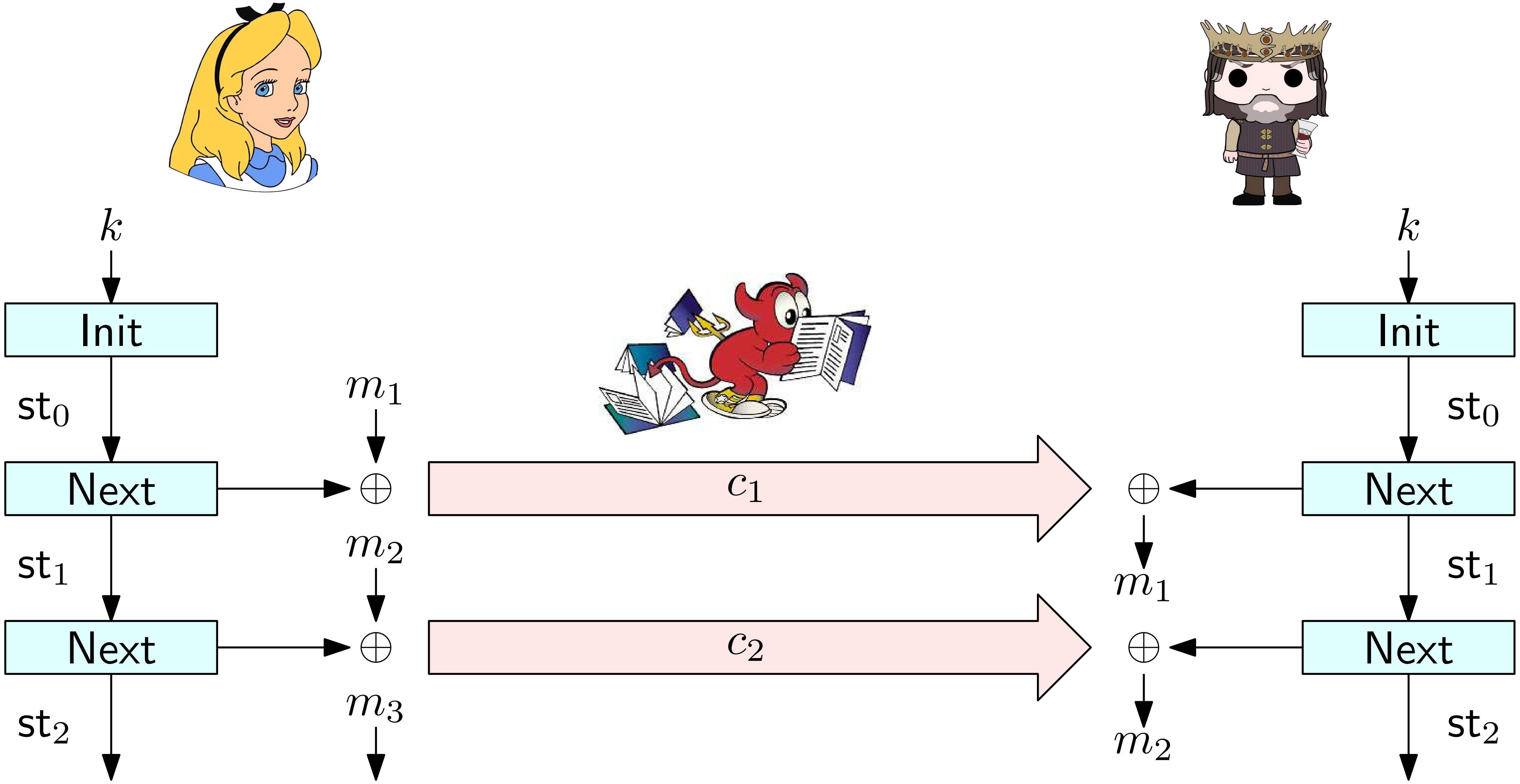
Next

st_1

Synchronized mode



k

Init

st_0

m_1

Next $\rightarrow \oplus$

c_1

$\oplus \leftarrow$ Next

st_1

m_2

Next $\rightarrow \oplus$

c_2

m_1

$\oplus \leftarrow$ Next

st_2

m_3

m_2

k

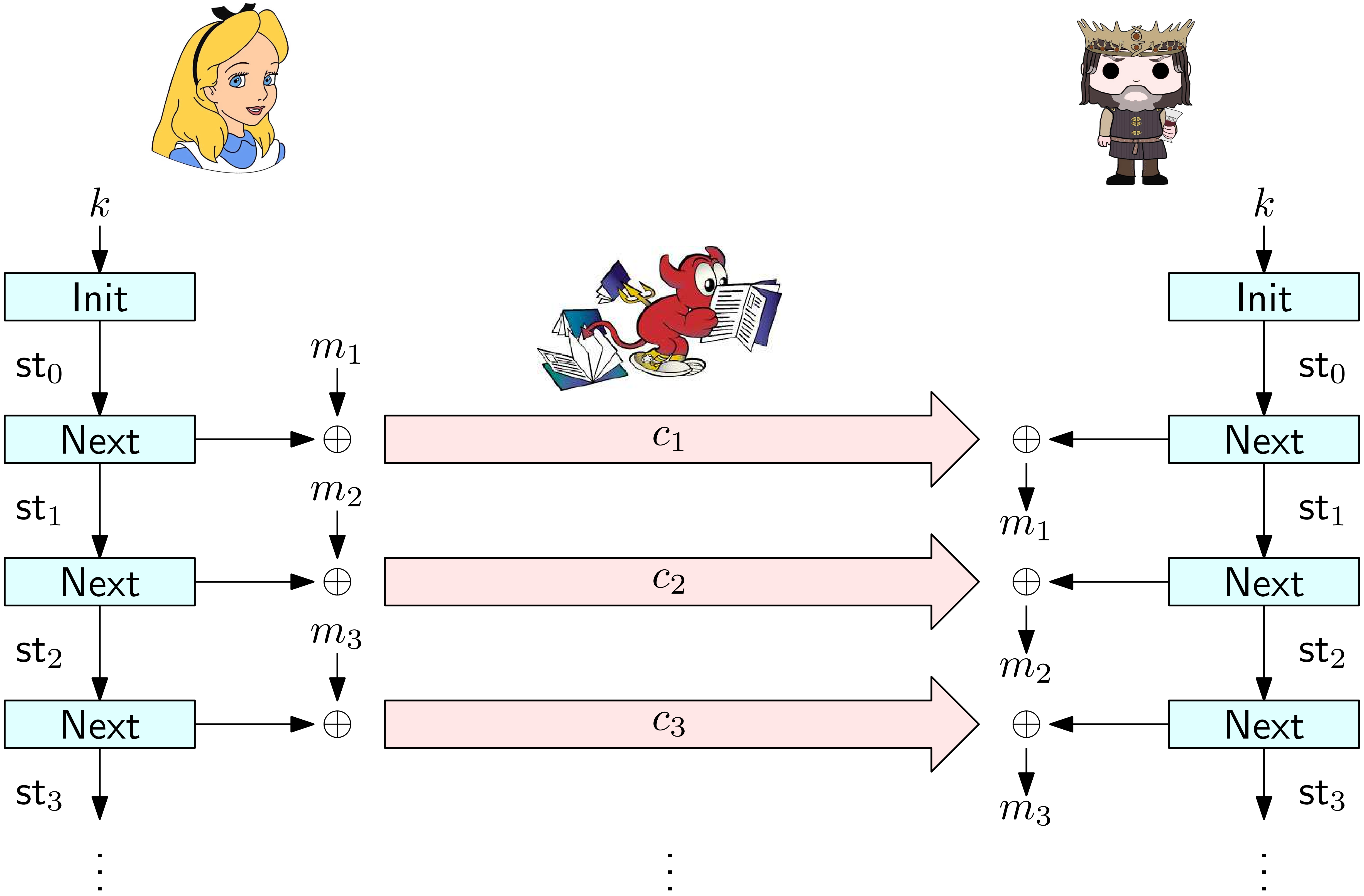
Init

st_0

st_1

st_2

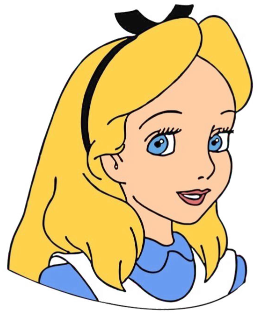
Synchronized mode

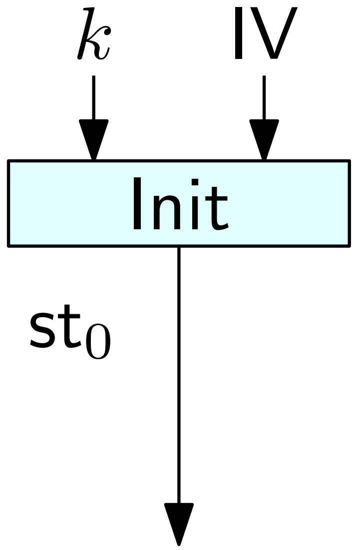


Alice & Bob need to keep track of the last state for as long as they wish to communicate

Unsynchronized mode

Alice picks a random IV





Unsynchronized mode

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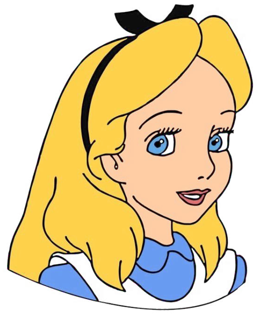
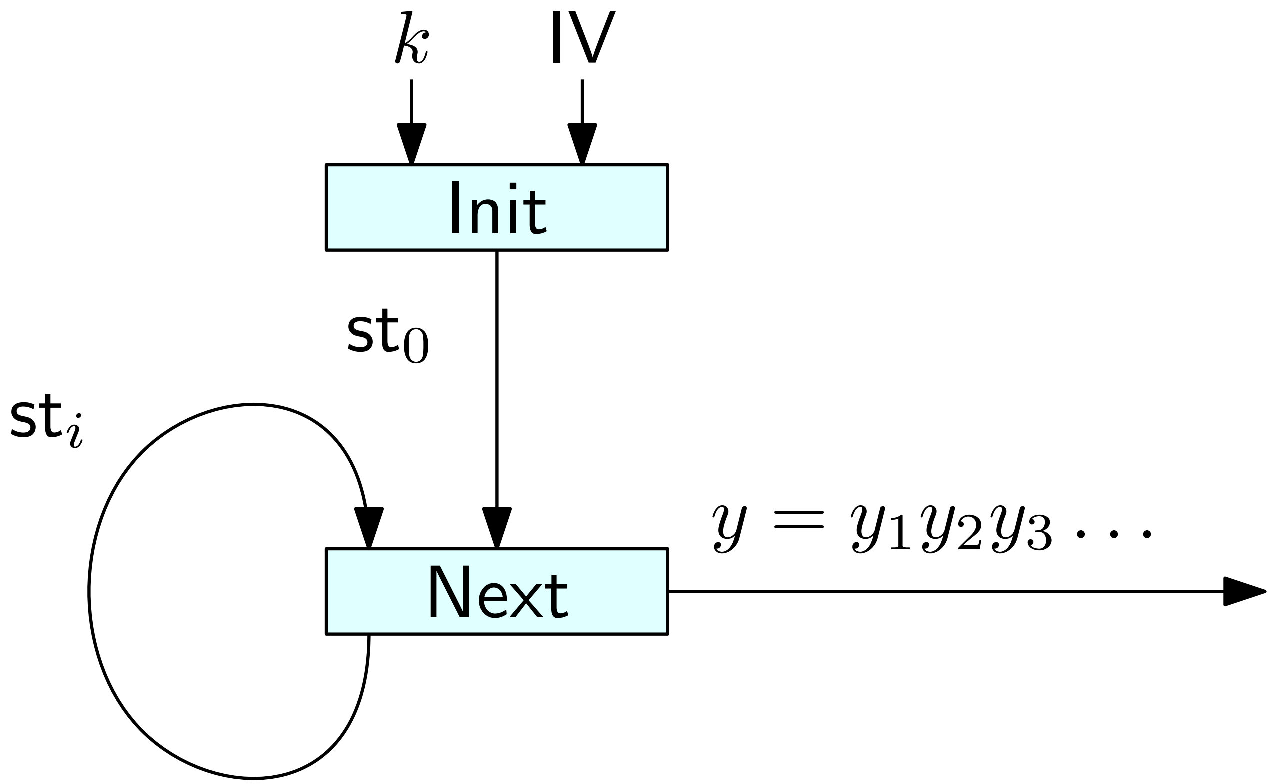




k IV

Init

st$_0$

st$_i$

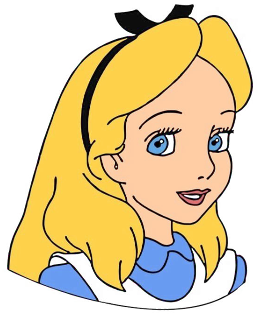
Next

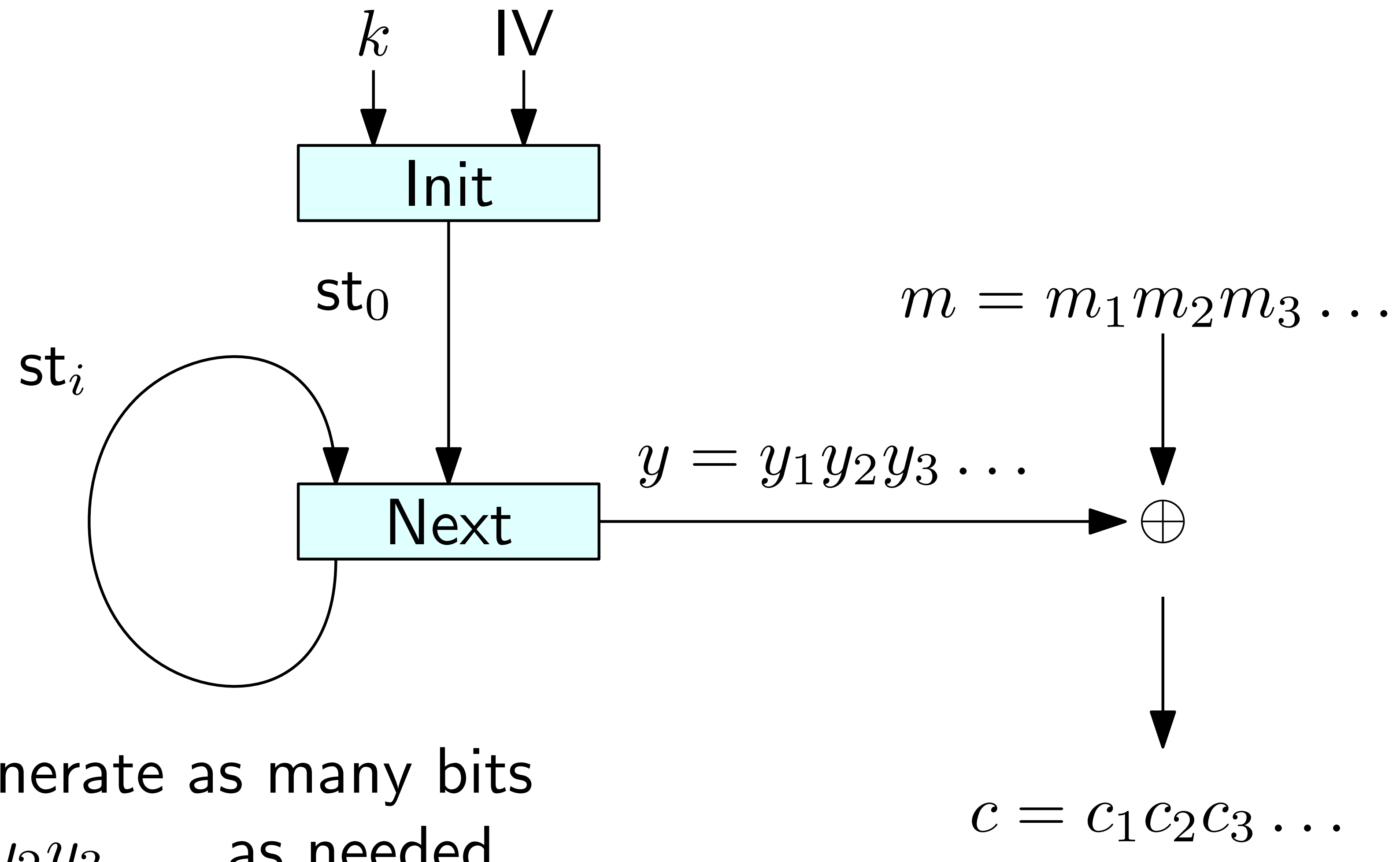
$y = y_1 y_2 y_3 \dots$

Generate as many bits
$y_1 y_2 y_3 \dots$ as needed

Unsynchronized mode

Alice picks a random IV





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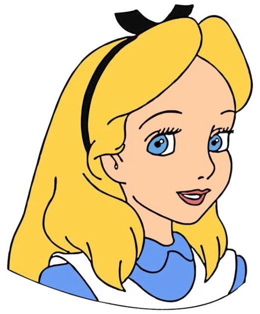

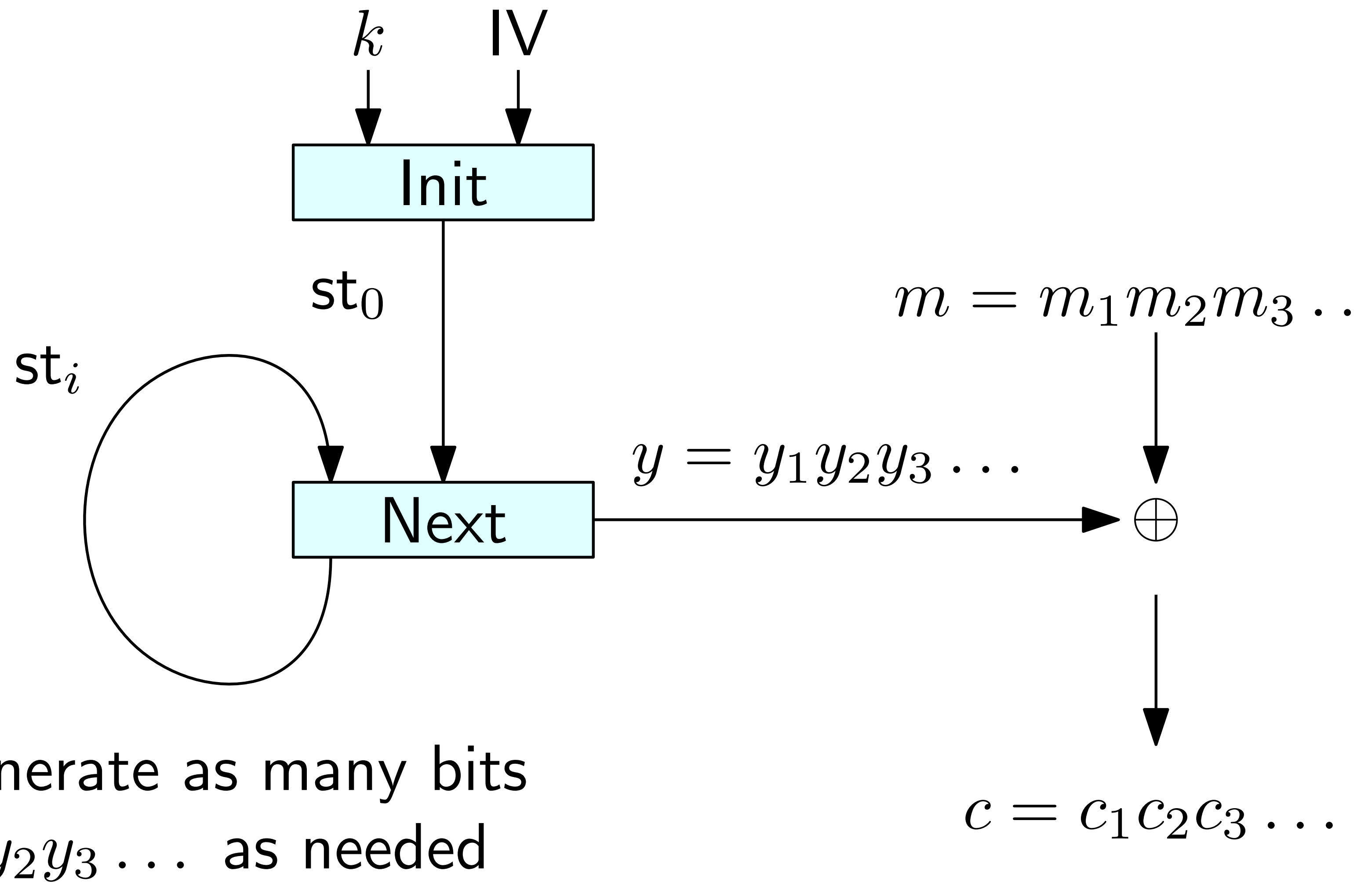

k IV

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st_i

Next

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$m = m_1 m_2 m_3 \ldots$

$\oplus$

$c = c_1 c_2 c_3 \ldots$

$\langle \mathsf{IV}, c \rangle$

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Unsynchronized mode

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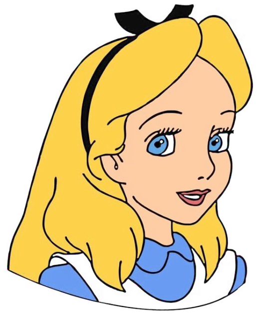

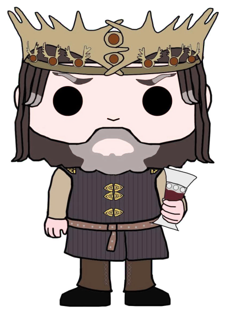

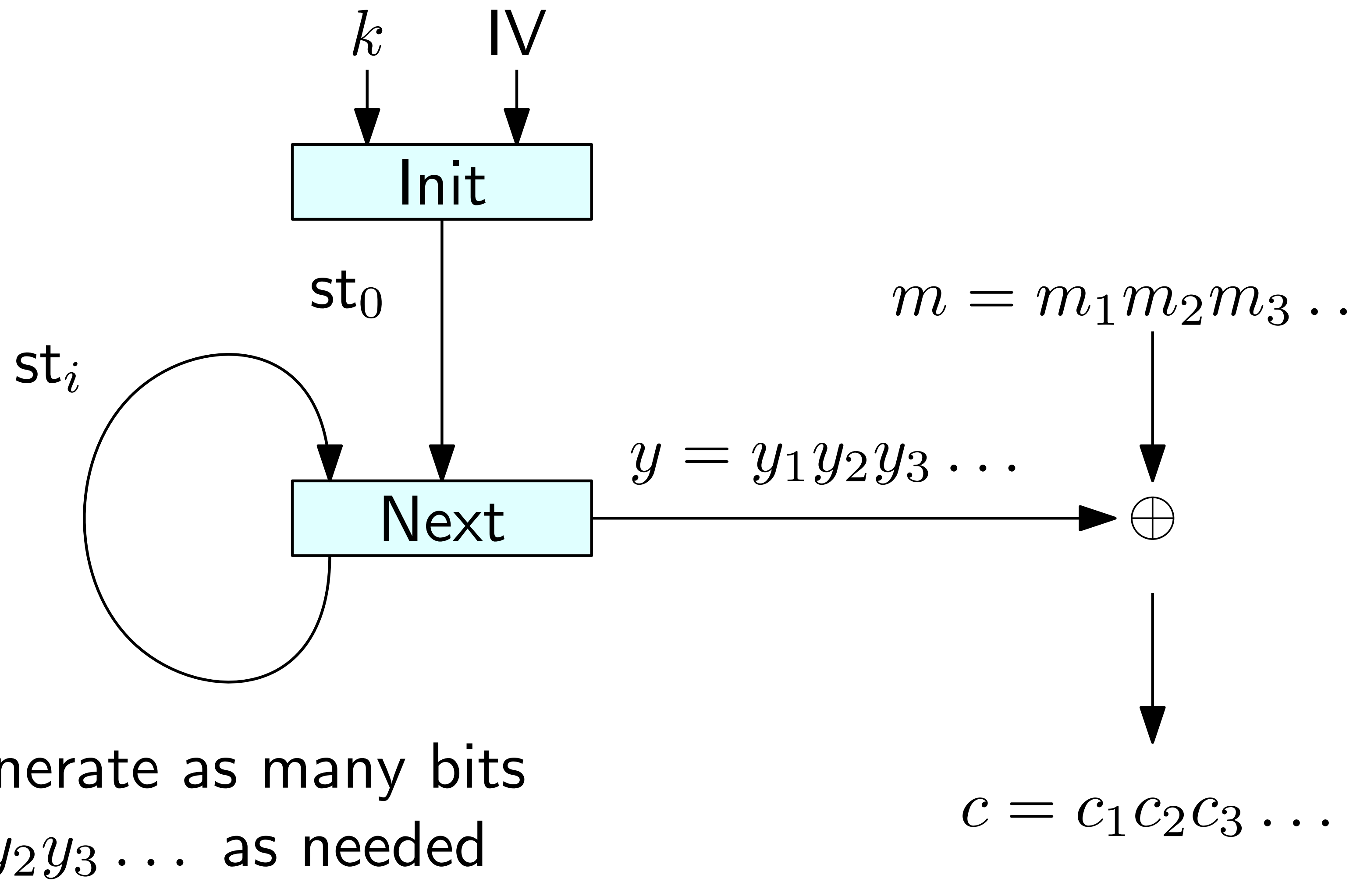

k IV

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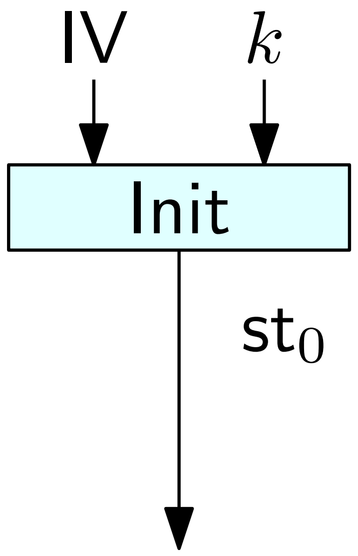

IV k

Init

st_0

Generate as many bits
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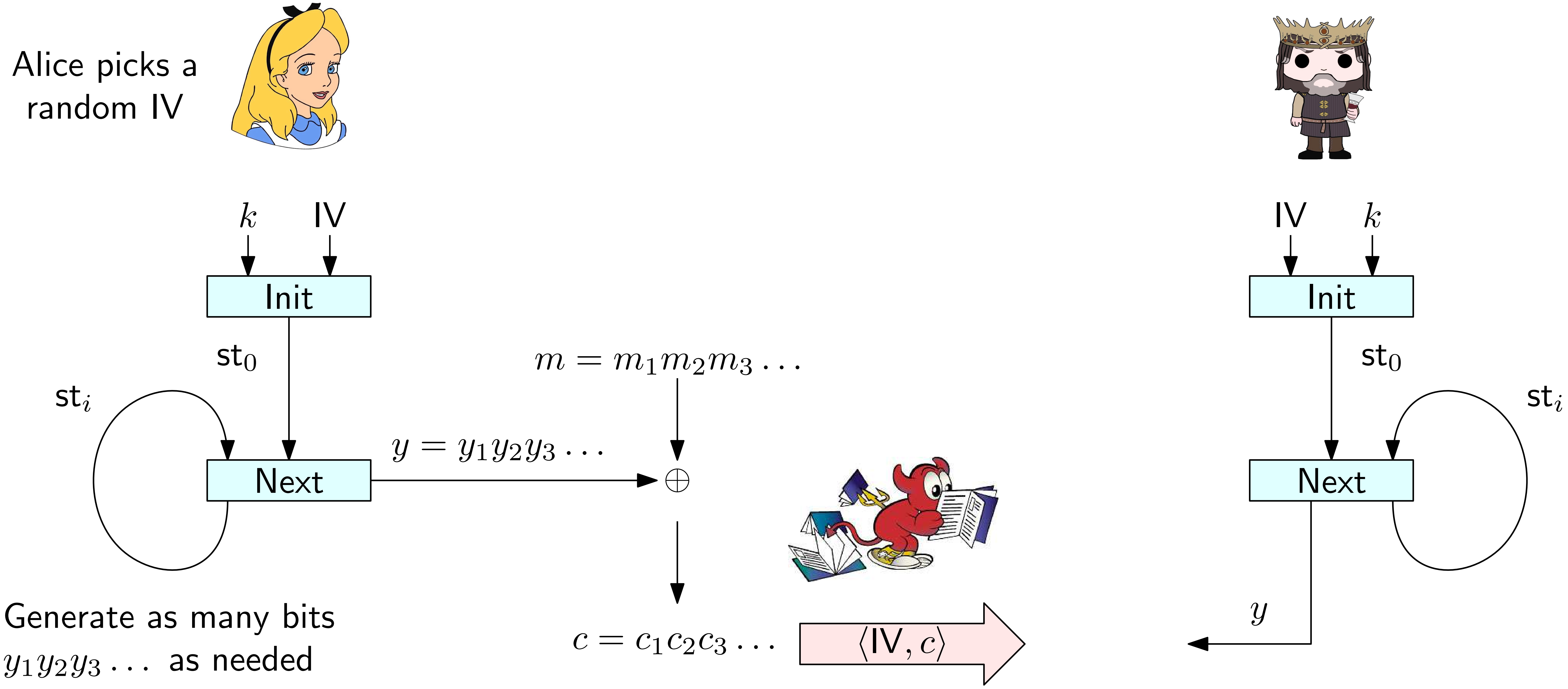
Unsynchronized mode



Alice picks a
random IV

k IV

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st_i

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IV k

Init

st_0

st_i

Next

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Unsynchronized mode

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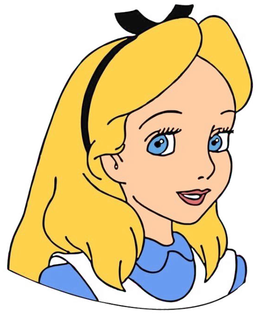

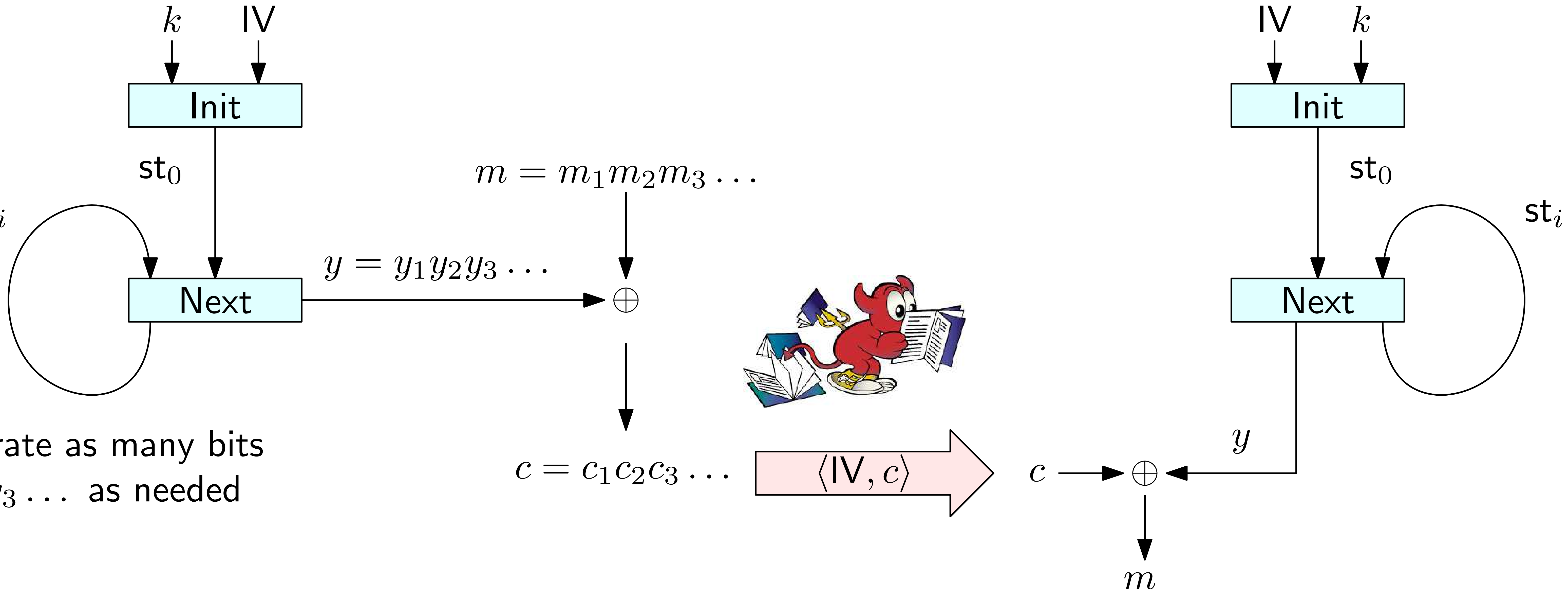

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IV k

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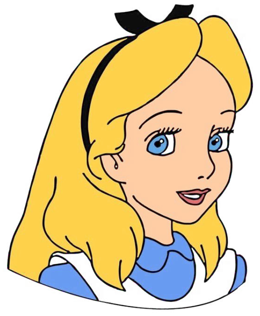

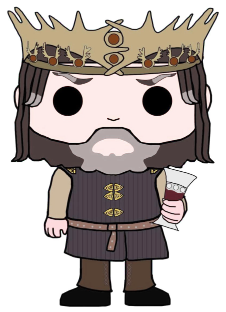

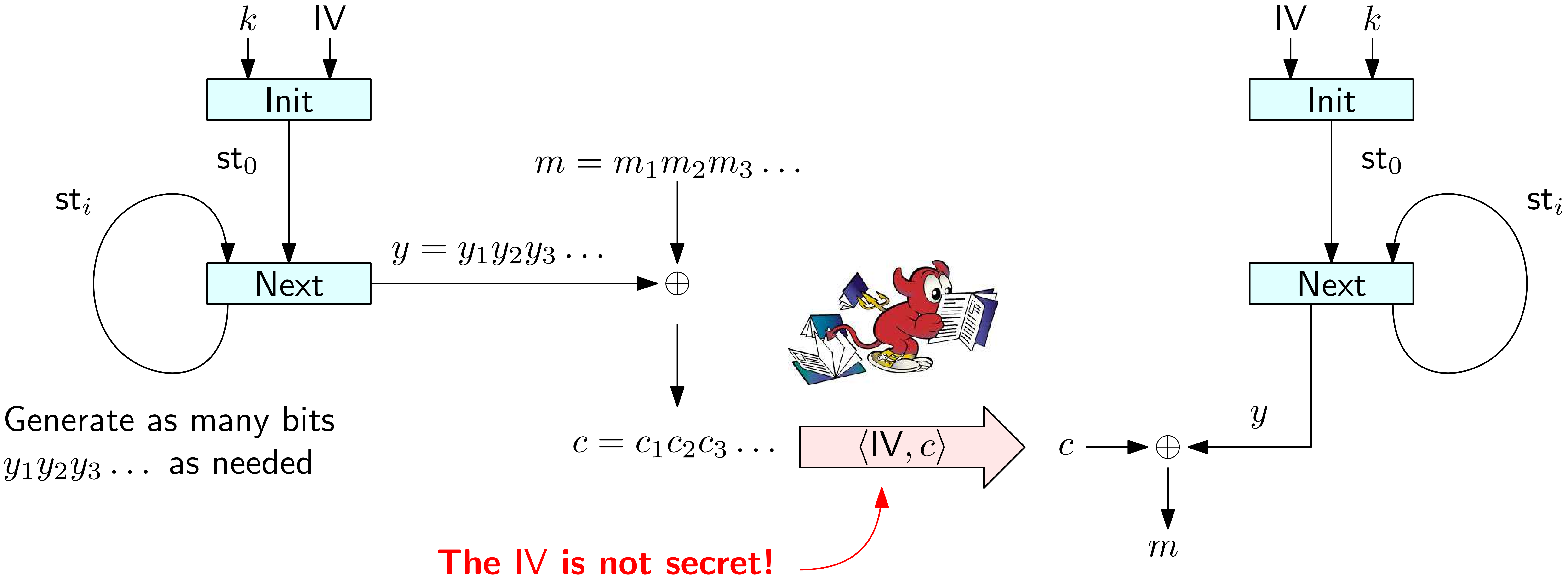

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$\oplus$

$c = c_1 c_2 c_3 \ldots$

$\langle \mathsf{IV}, c \rangle$

Generate as many bits $y_1 y_2 y_3 \ldots$ as needed

The IV is not secret!

IV k

Init

st$_0$

st$_i$

Next

y

$c \rightarrow \oplus$

m

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- Some popular practical constructions of stream ciphers:
    - Trivium: optimized for hardware
    - RC4 (insecure): optimized for software
    - ChaCha20: replacement of RC4

WEP Encryption