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- **Chosen ciphertext attack**: The attacker can query both F_k and F_k^{-1} (with values of its choice)

Designing Block Ciphers

- To design a block cipher, we want the computed function to be "indistinguishable" from a uniform permutation over $\{0,1\}^\ell$
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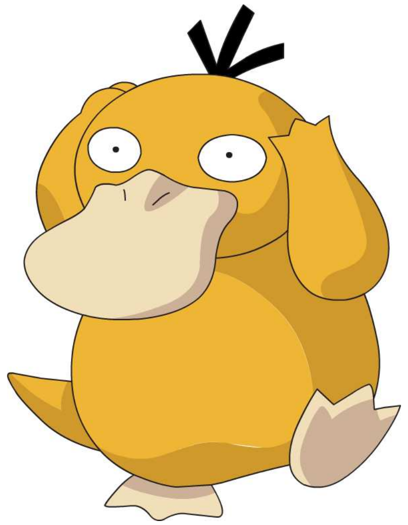

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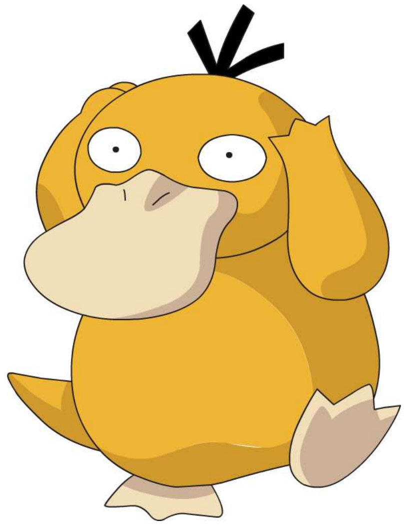

- **Confusion**: A small change in the input produces a small "random" change in the output

- **Diffusion:** The bits in the input are mixed so that a local change is spread throughout the block

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There are **many** random permutations

- Recall that $|\mathsf{Perm}_\ell| = (2^\ell)!$

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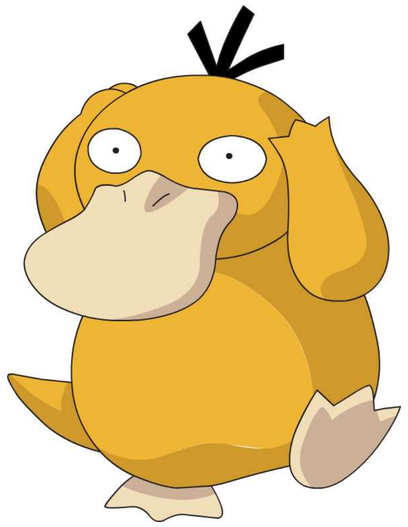

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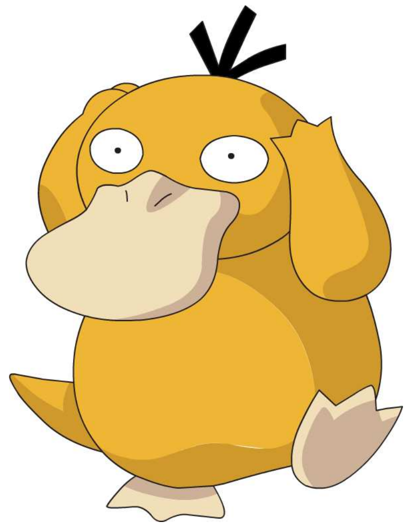

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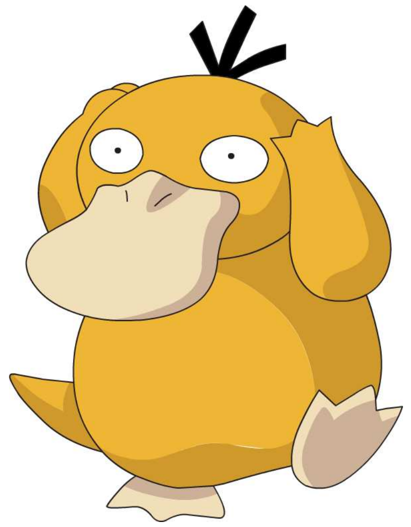

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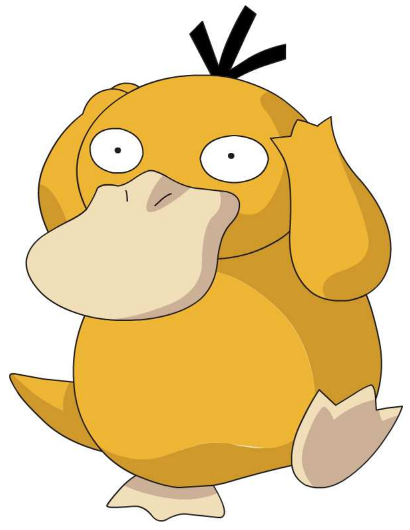

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Idea: Build a "random" permutation on **long** inputs by using many "random" permutations on **short** inputs

Example: To store 8 permutations over $\{0,1\}^8$ we need less than $8 \cdot (8 \cdot 2^8)$ b $= 2$ KB

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Consider a keyed PRP F_k with a block length 64 bits defined as follows: (the length is just an example)

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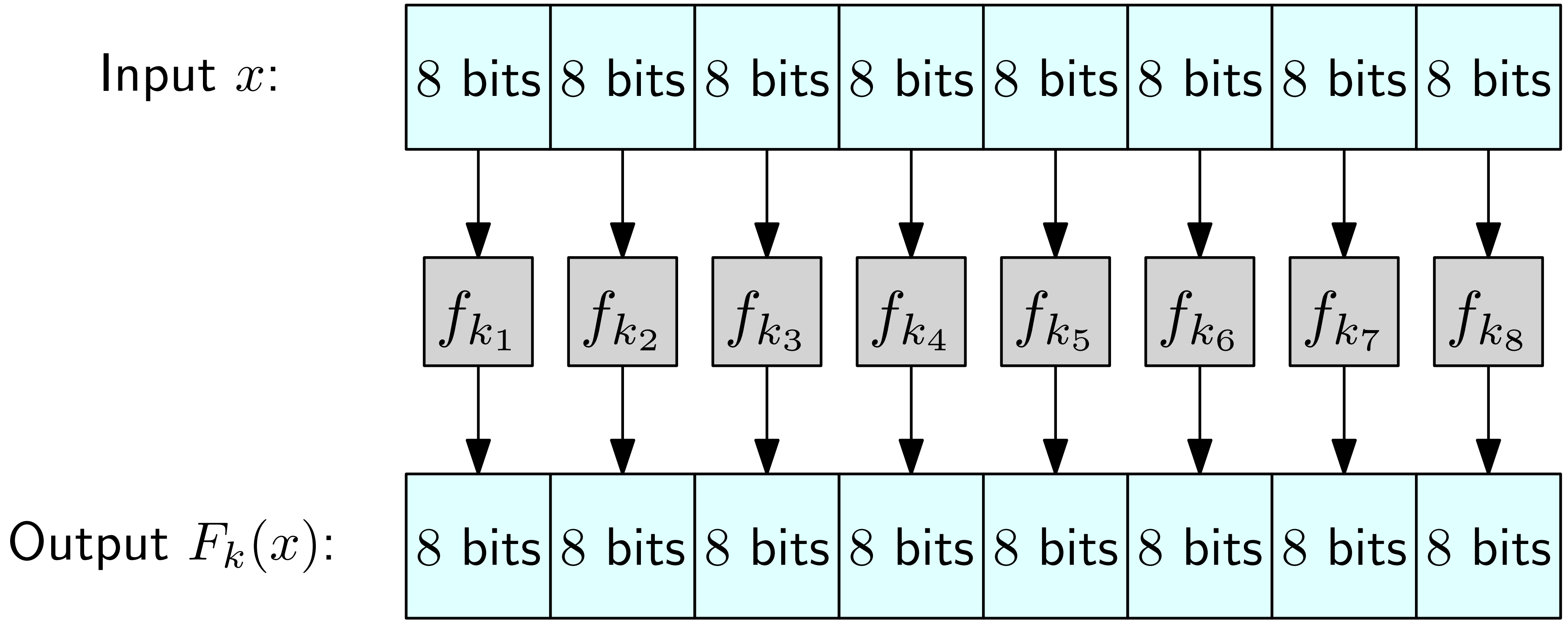

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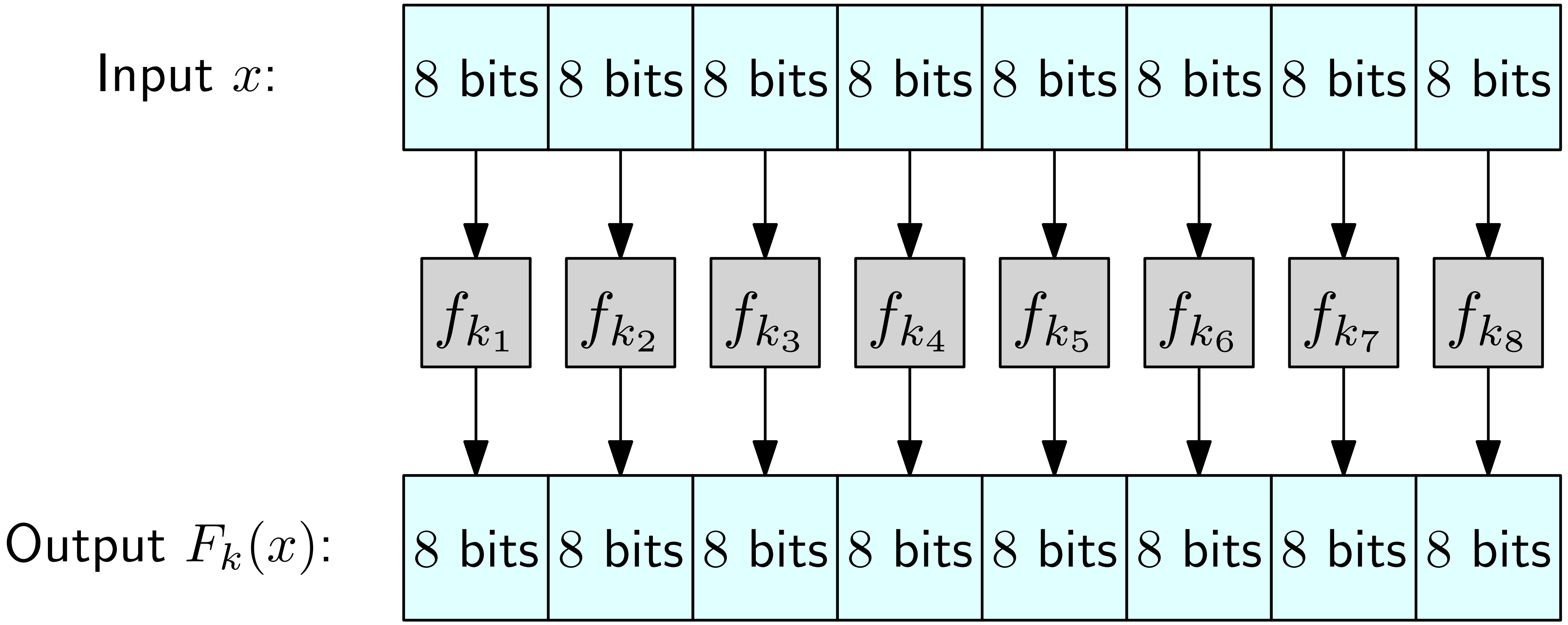

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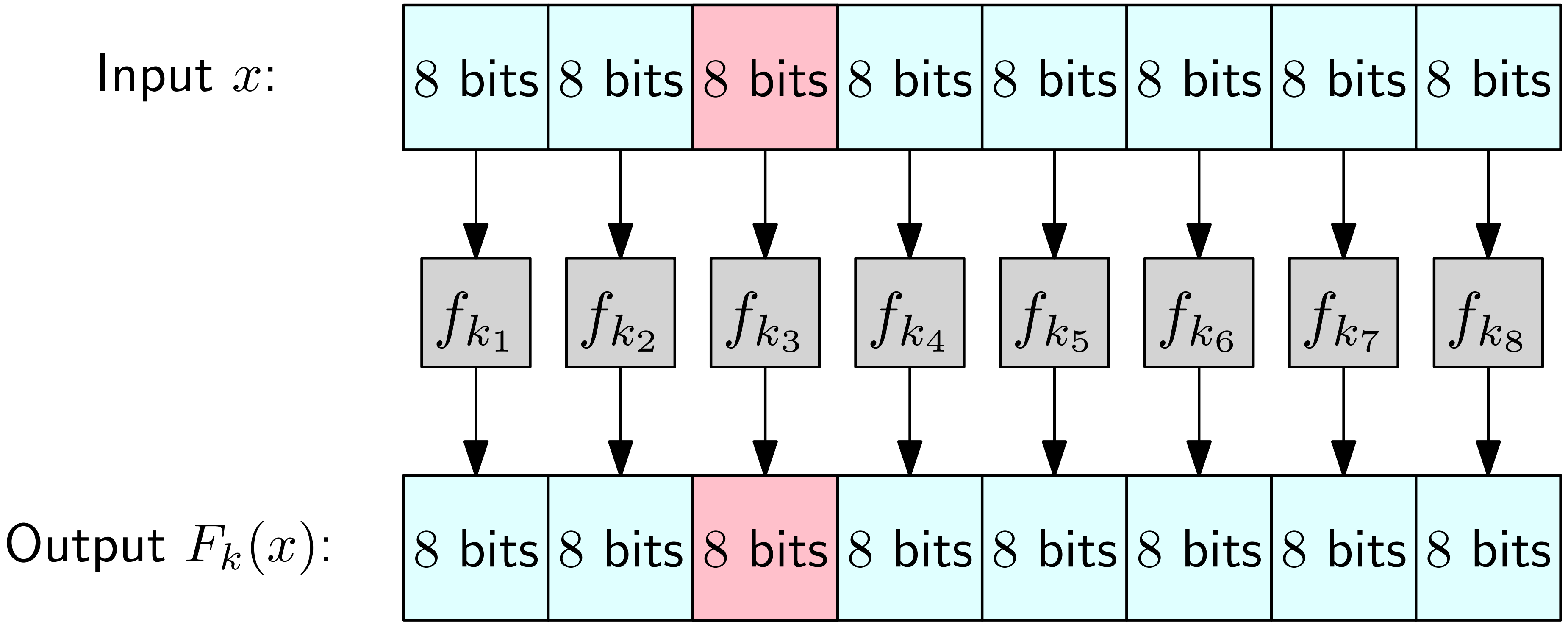

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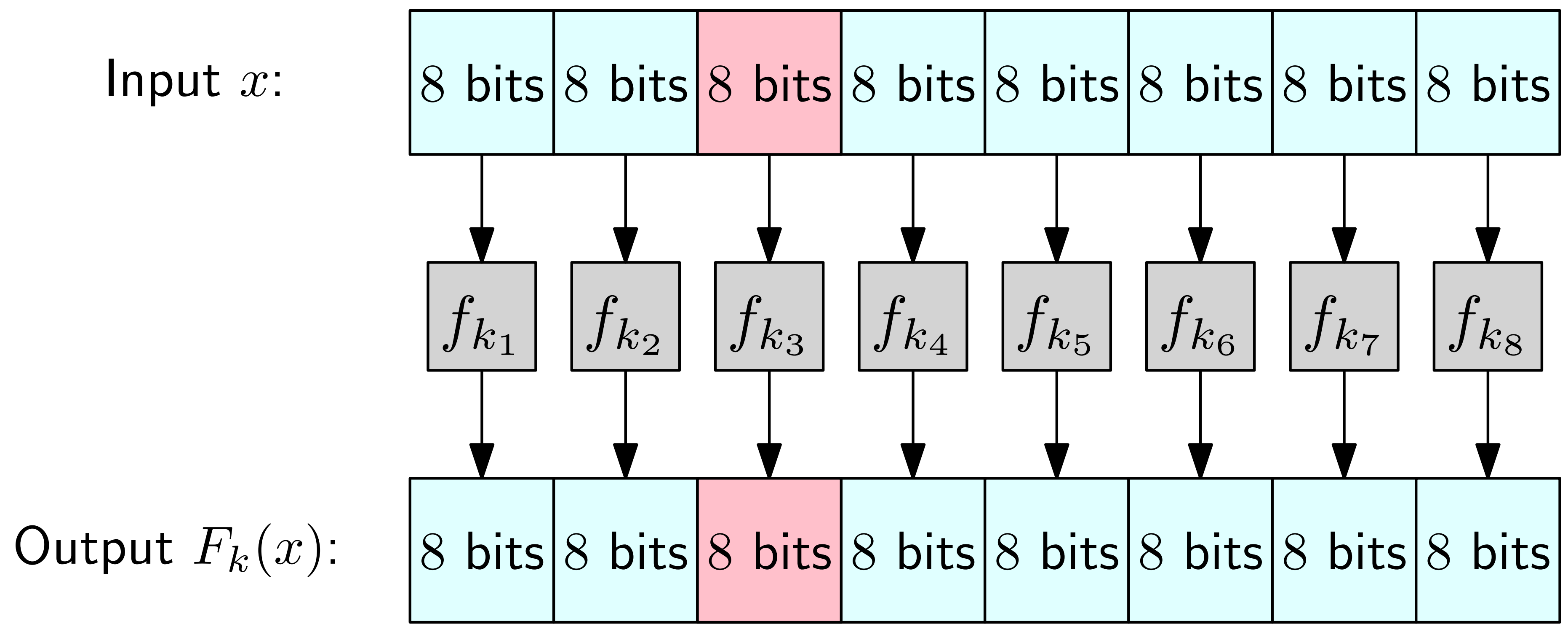

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Adding diffusion

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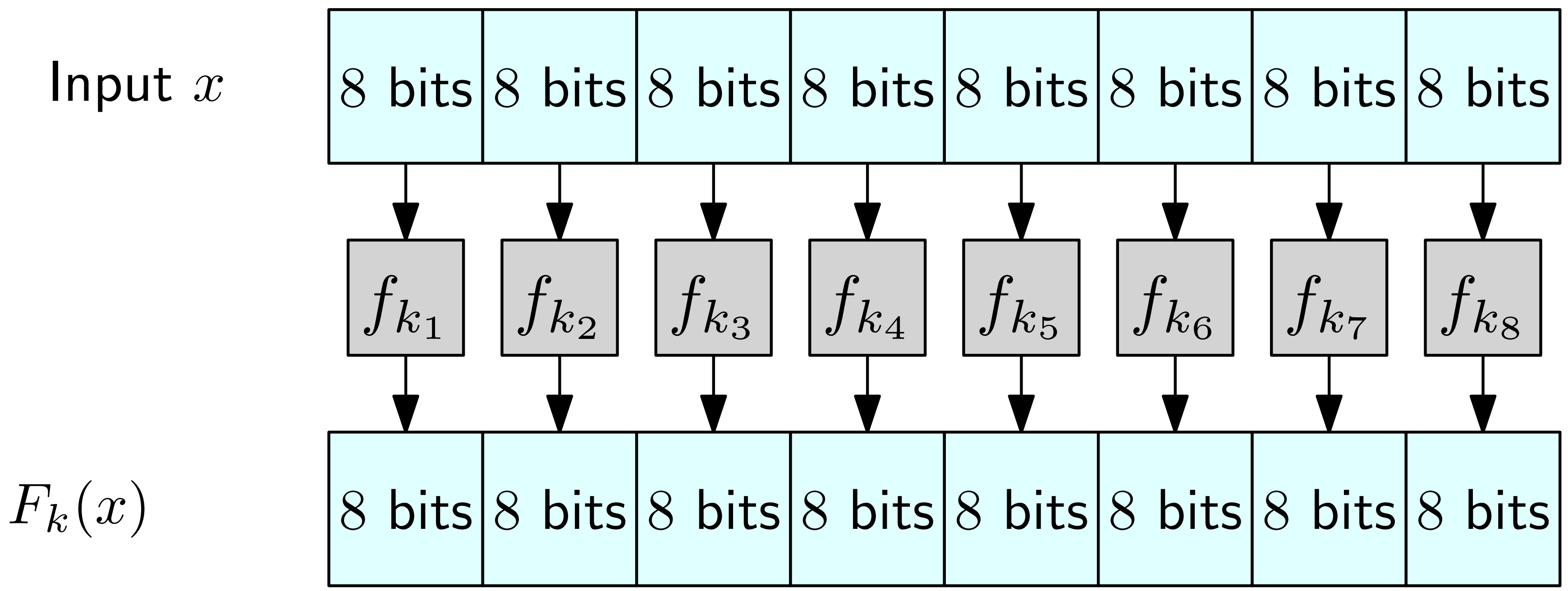
We move a generic bit in the i-th position of the input to the $\pi(i)$-th position of the output



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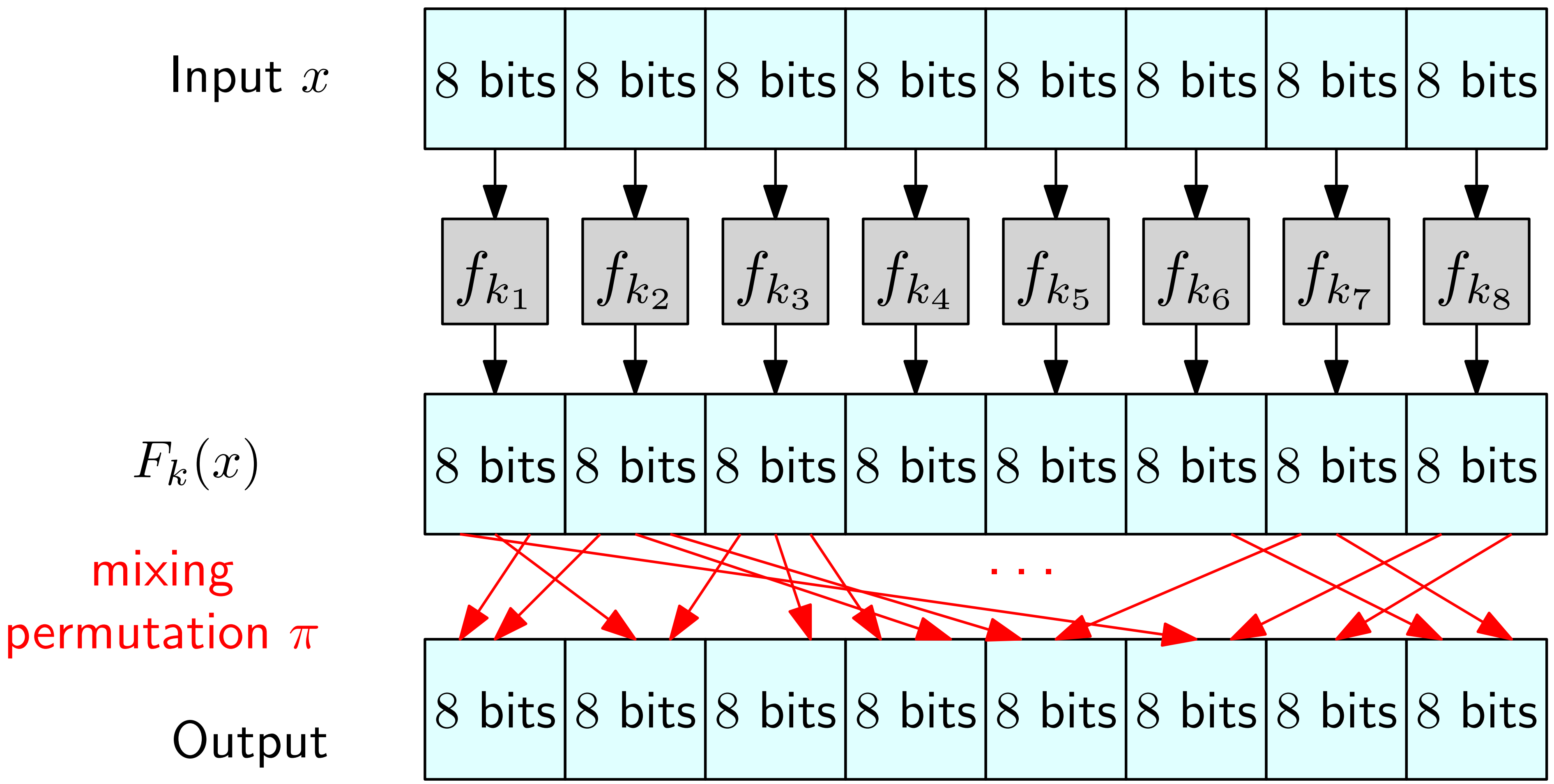

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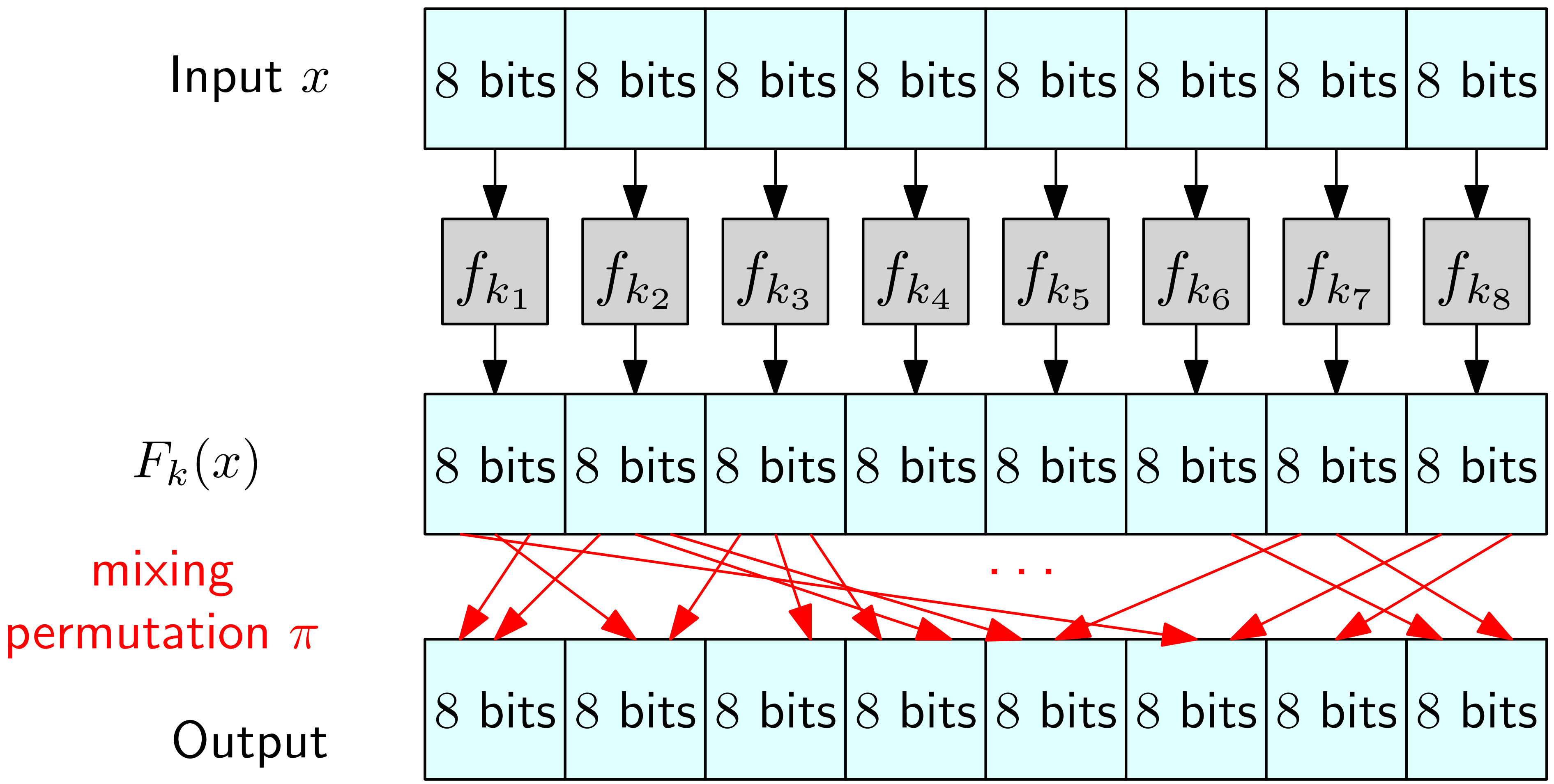

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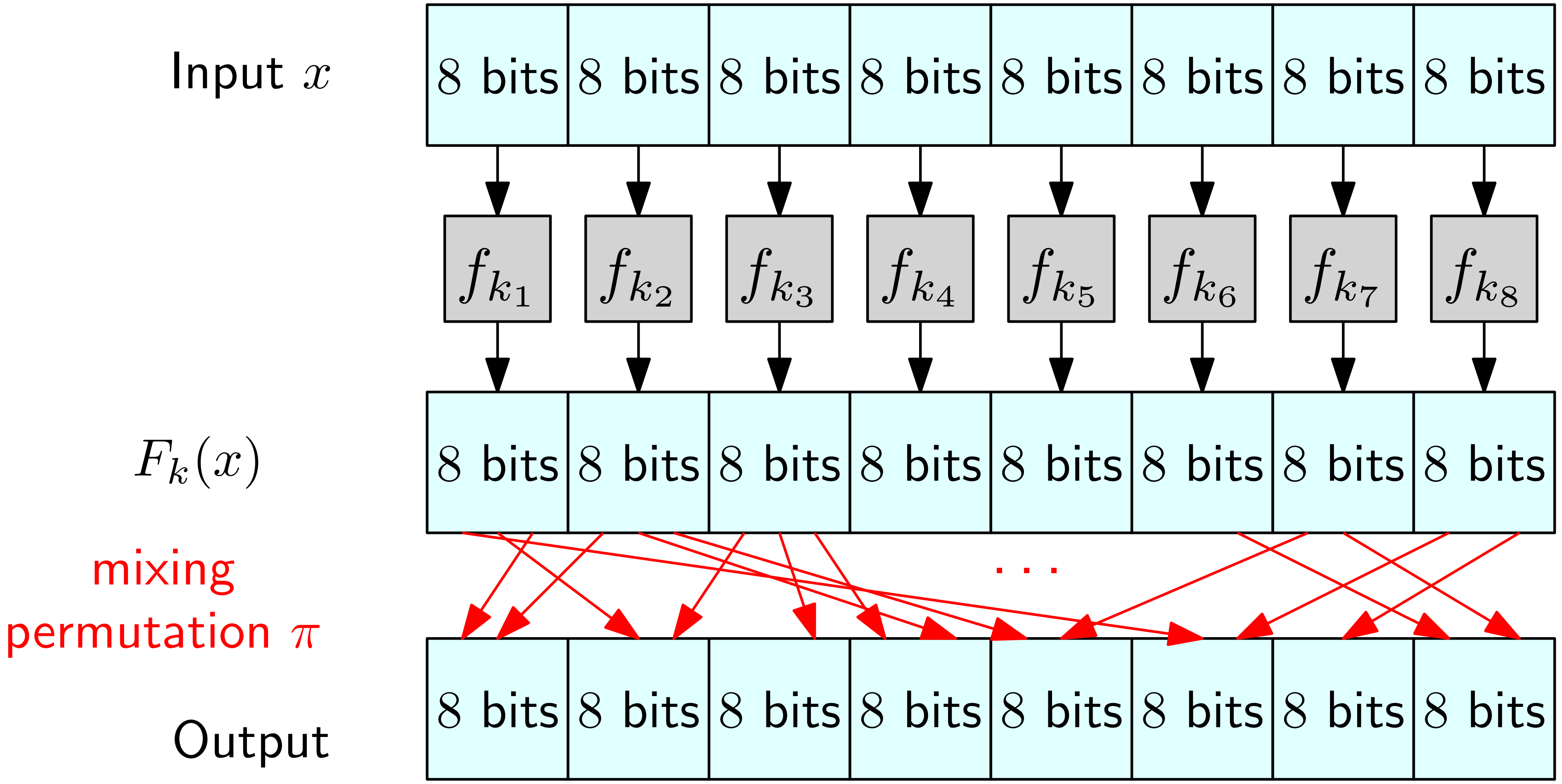

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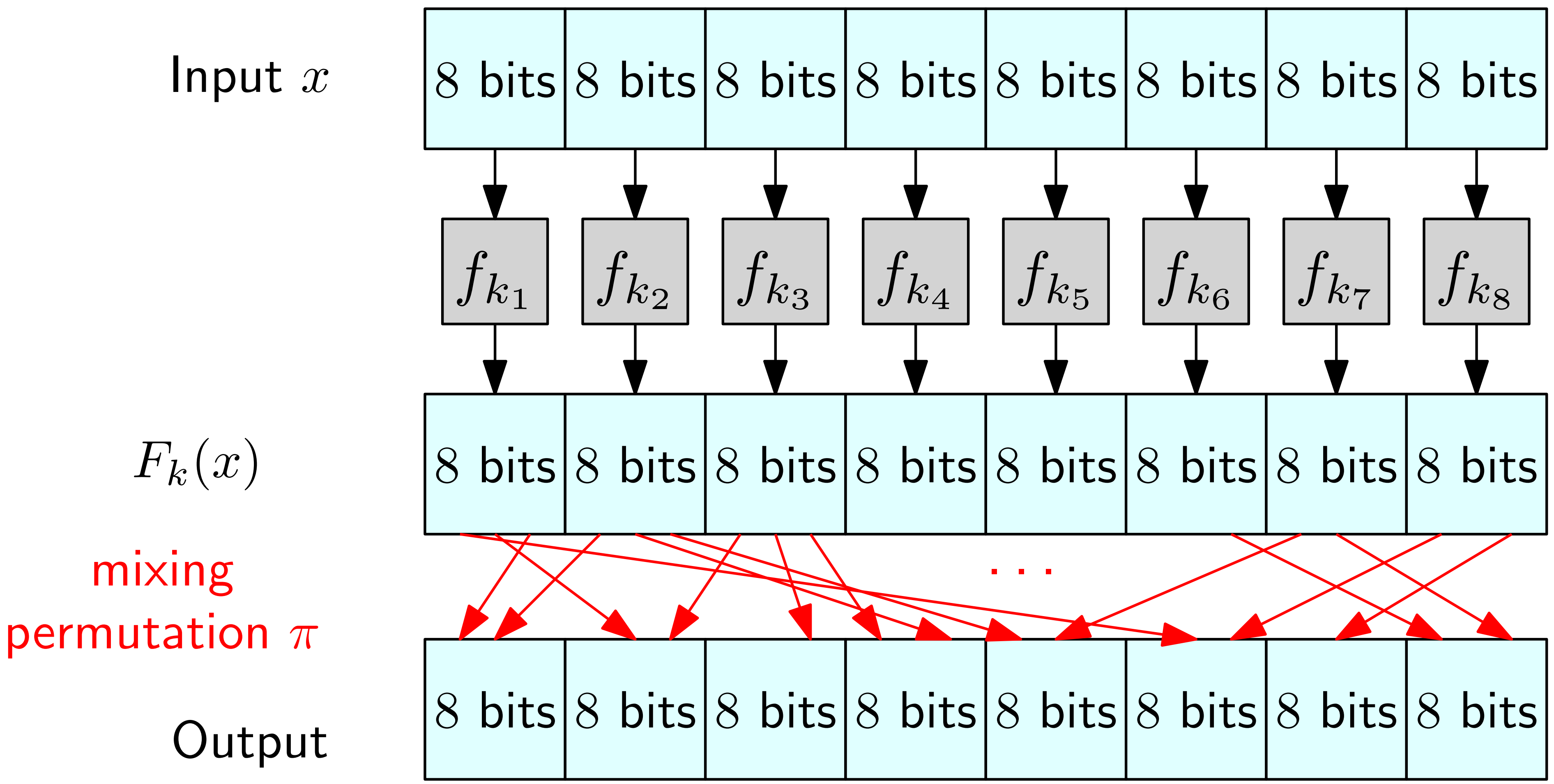
In practice the mixing permutation does not depend on the key and is carefully designed and **fixed**



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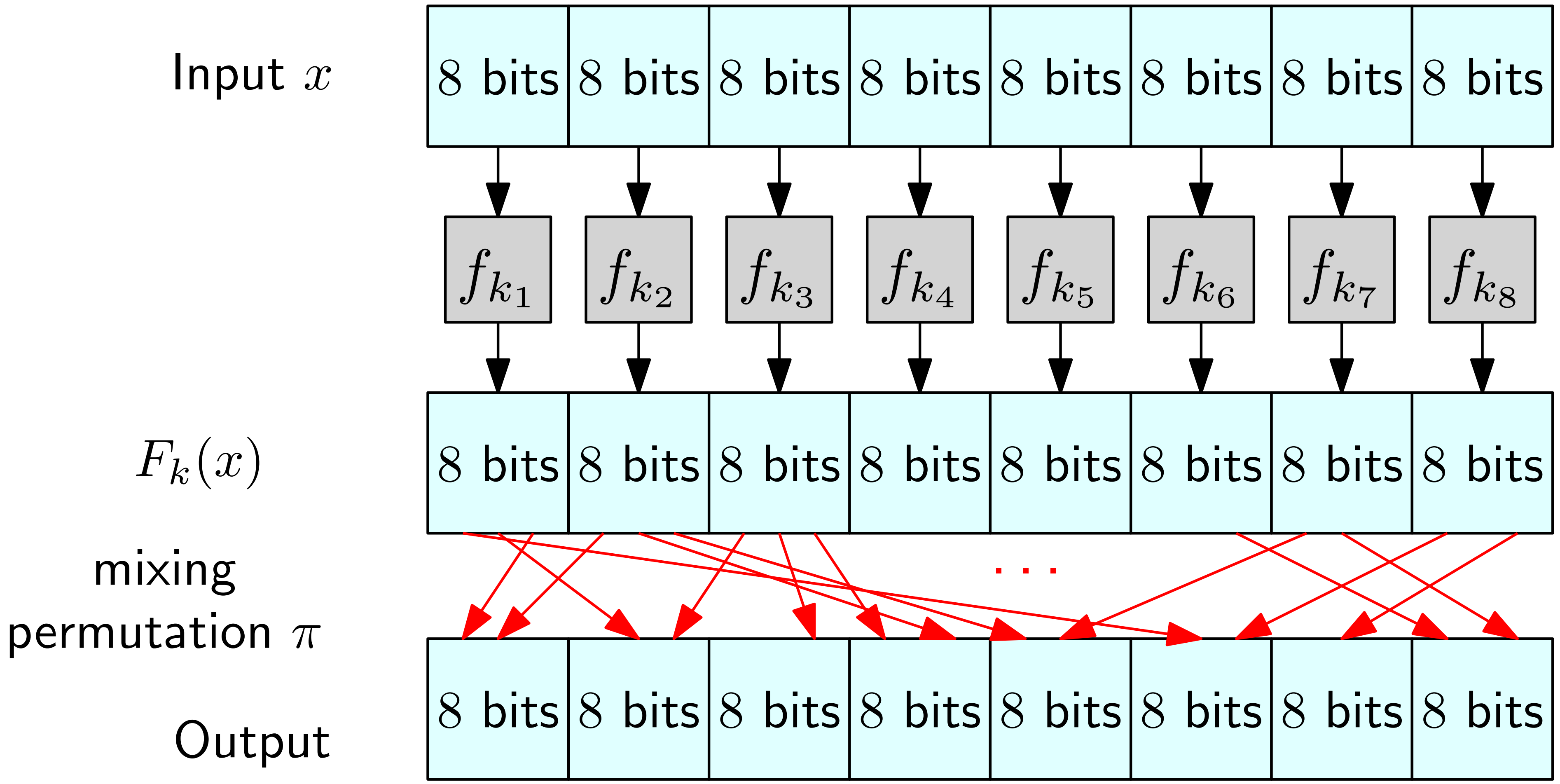

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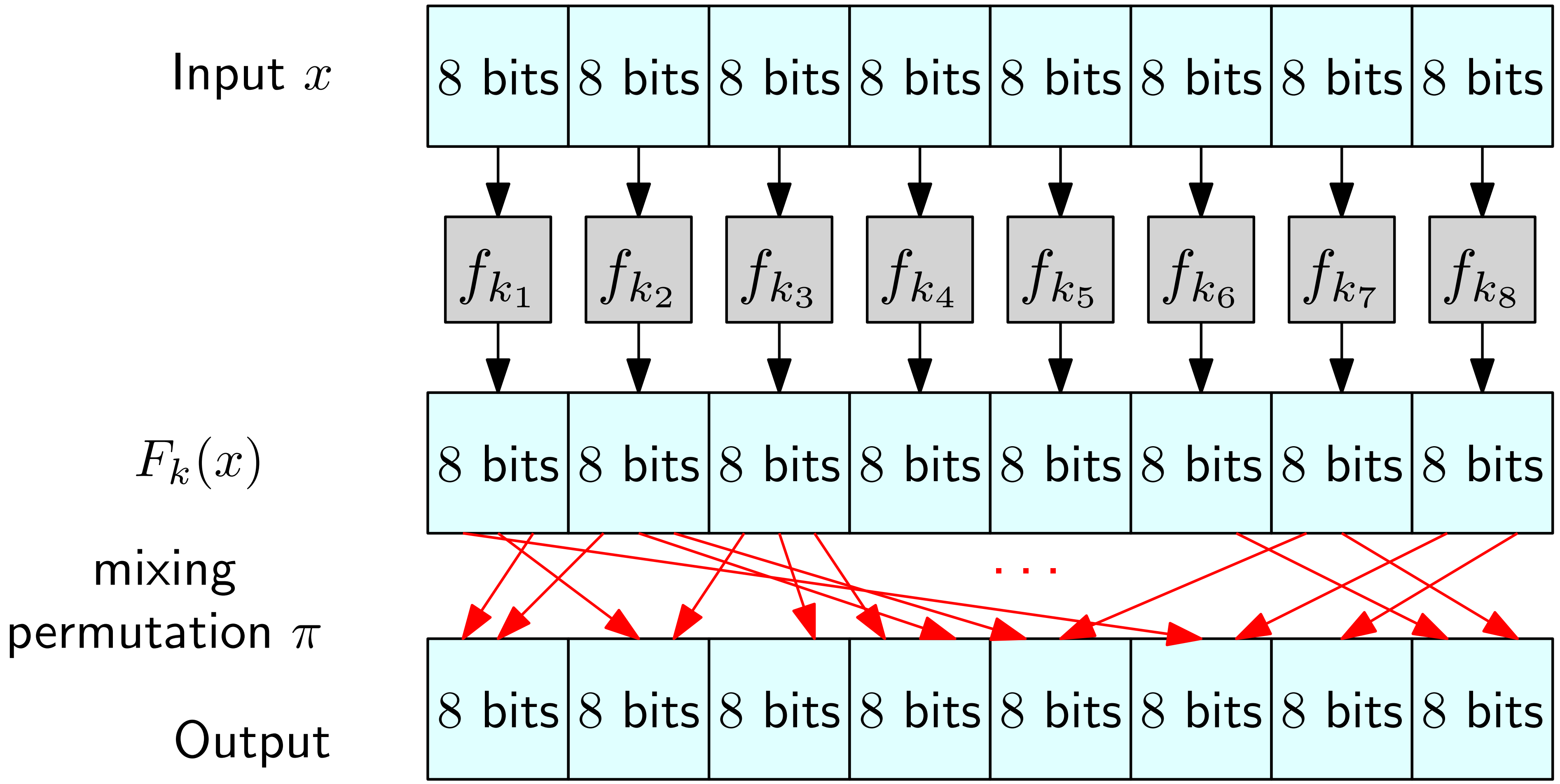

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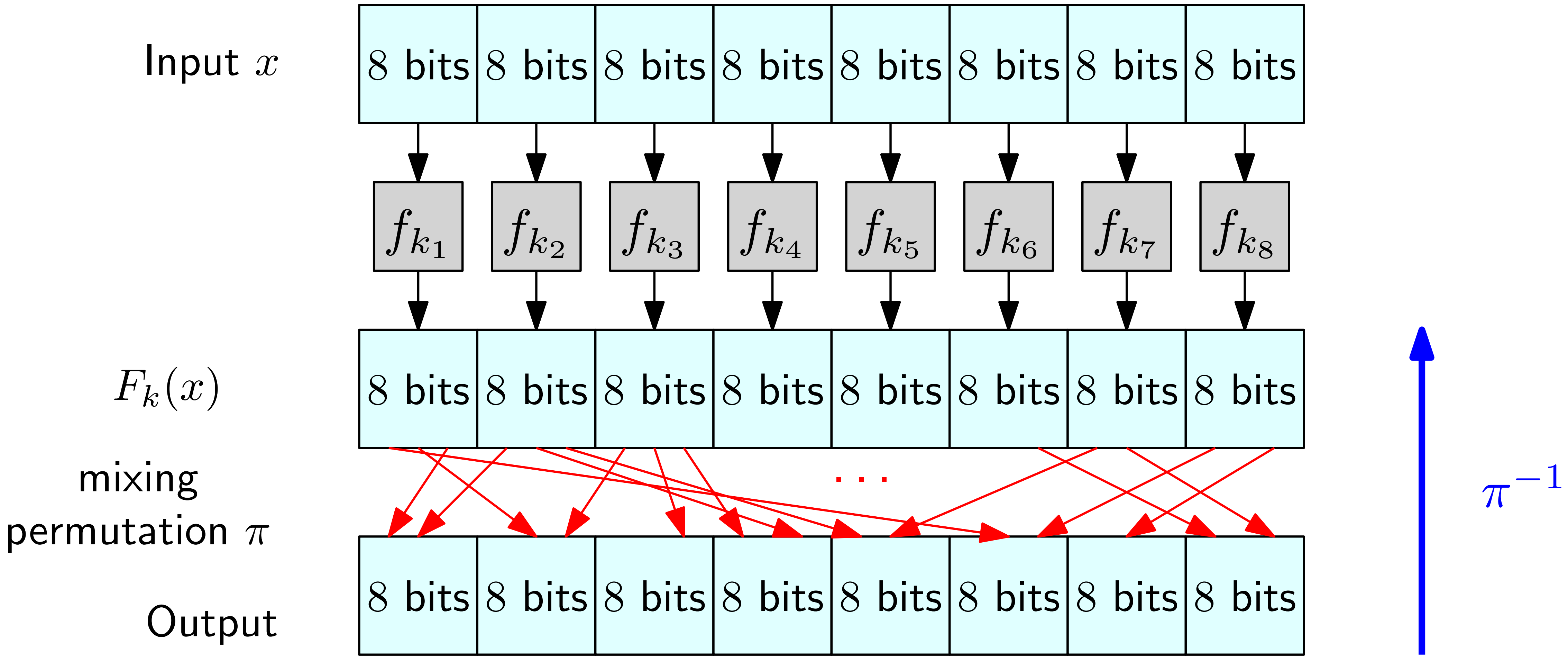

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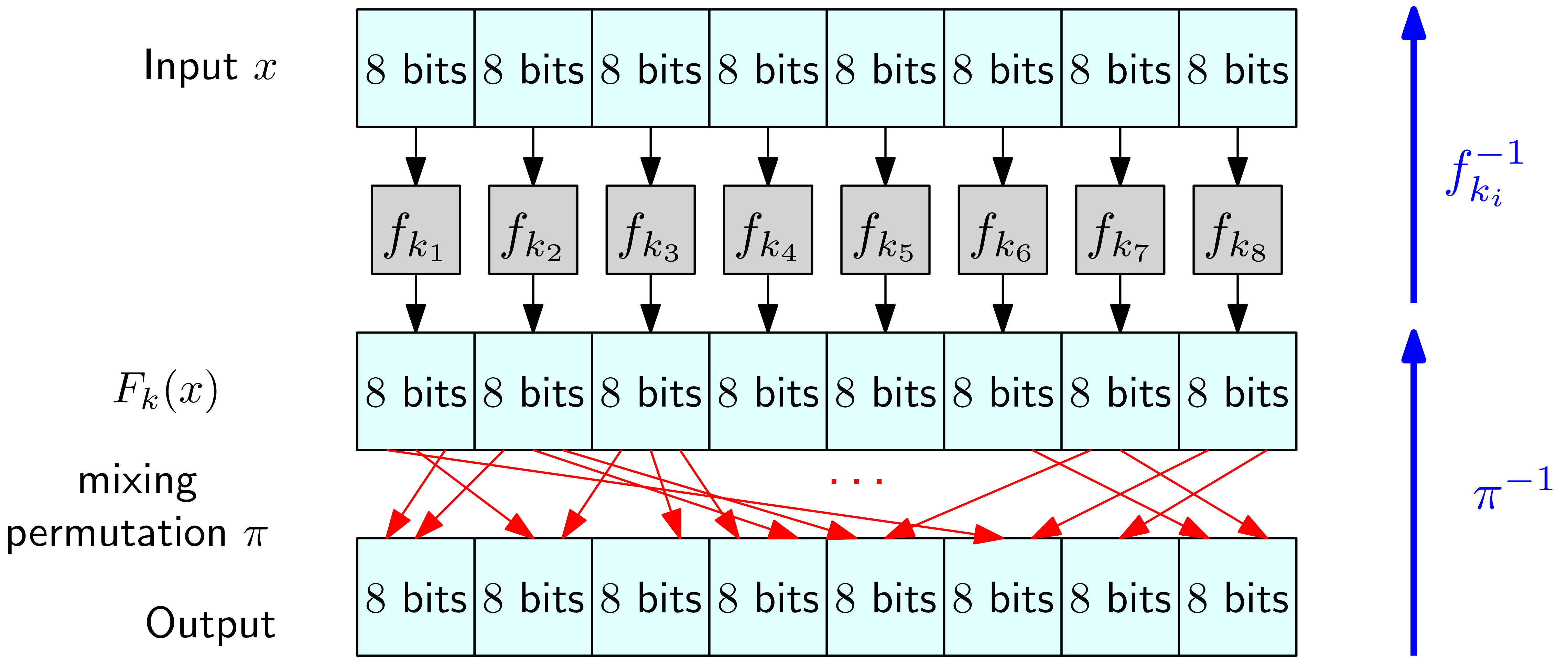

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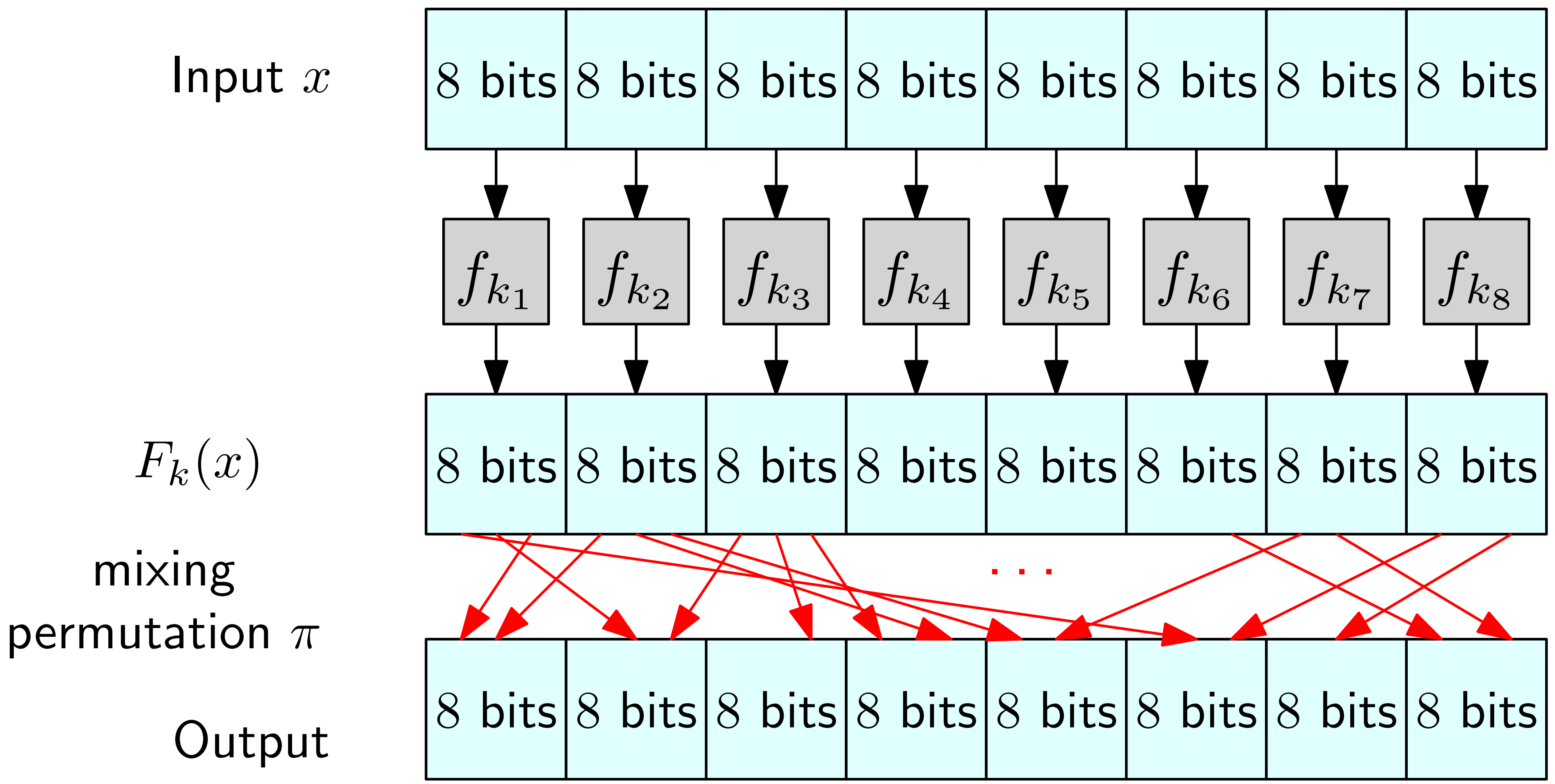
Substitution Permutation Networks



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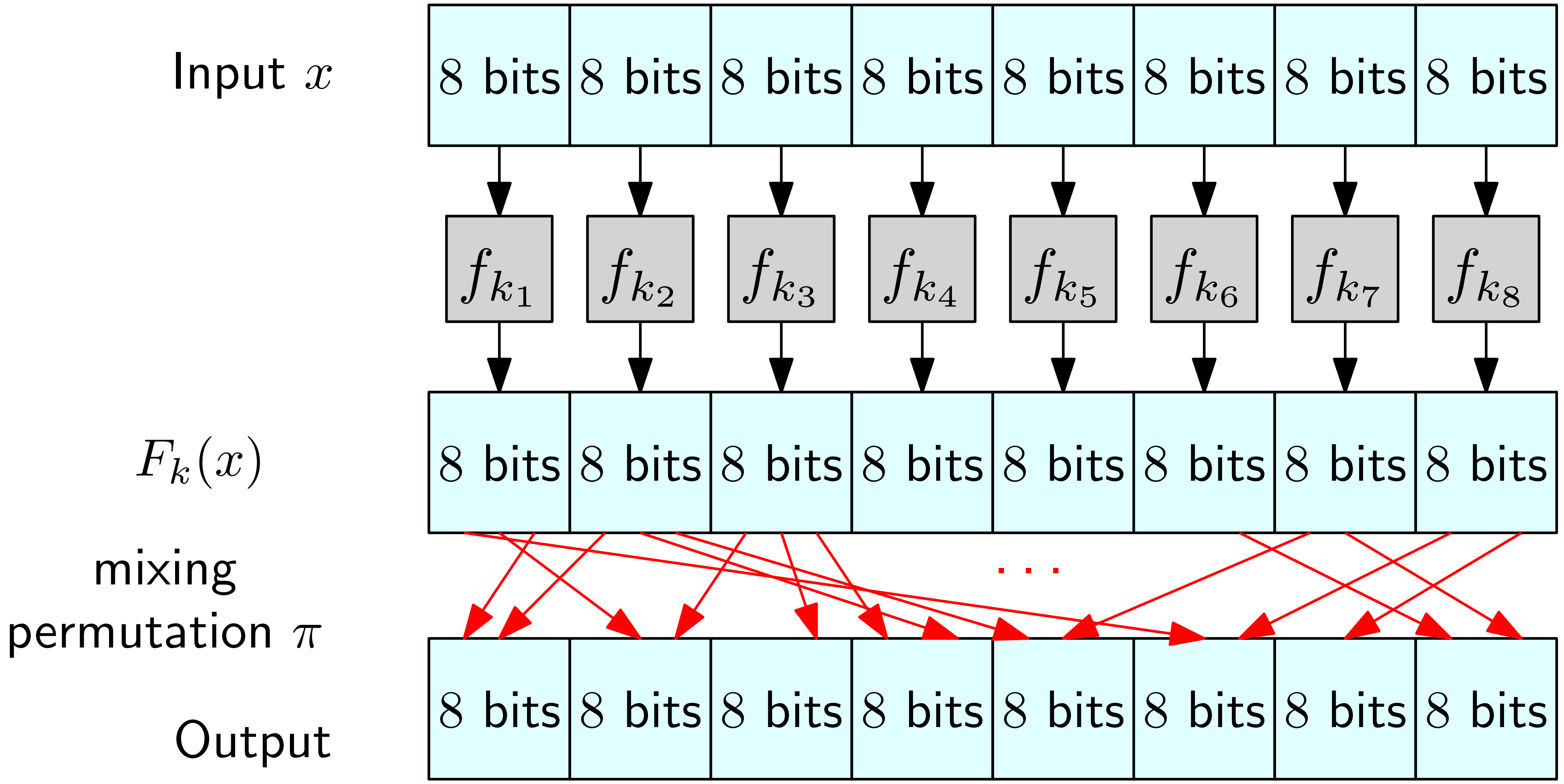

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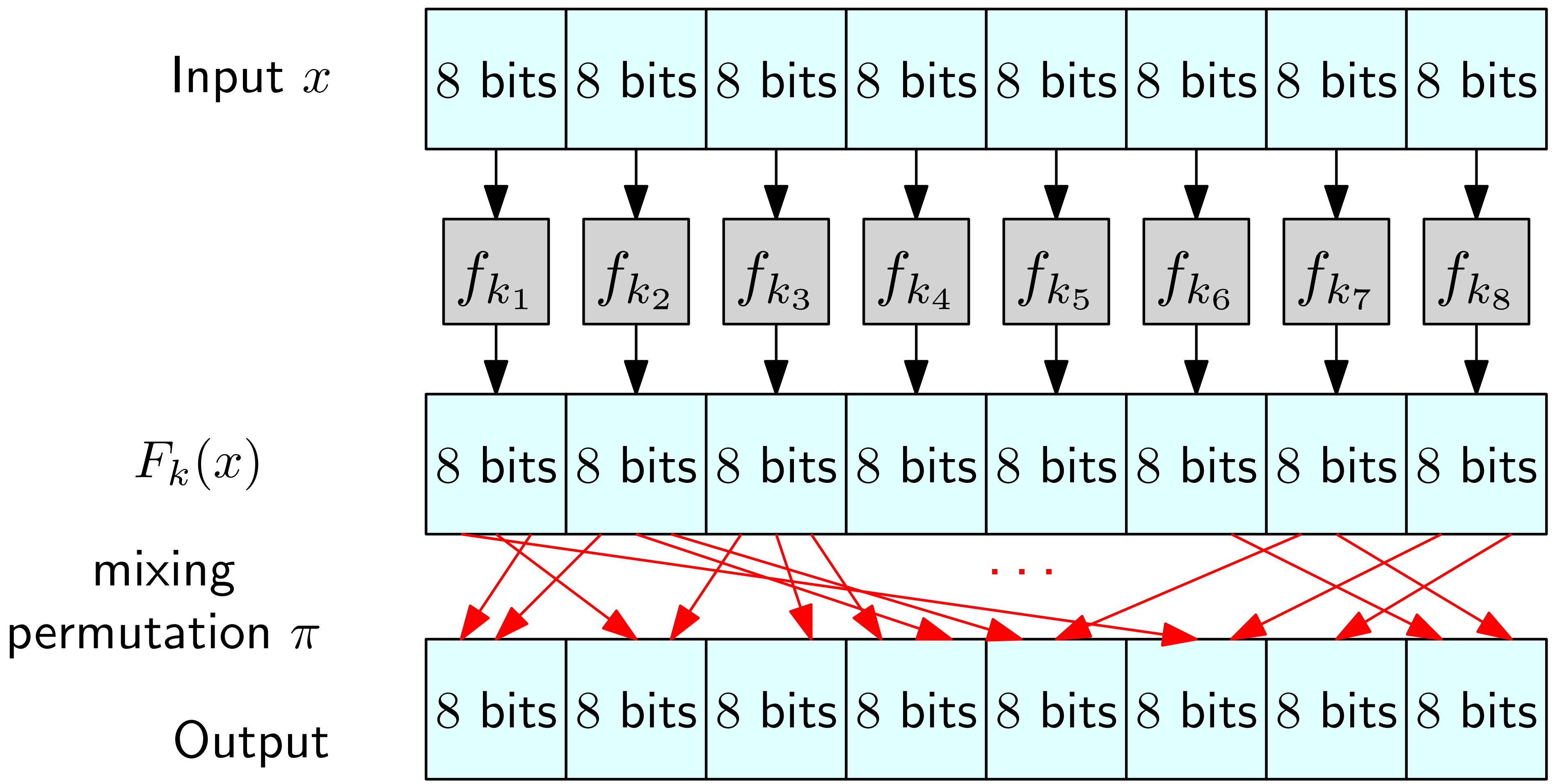

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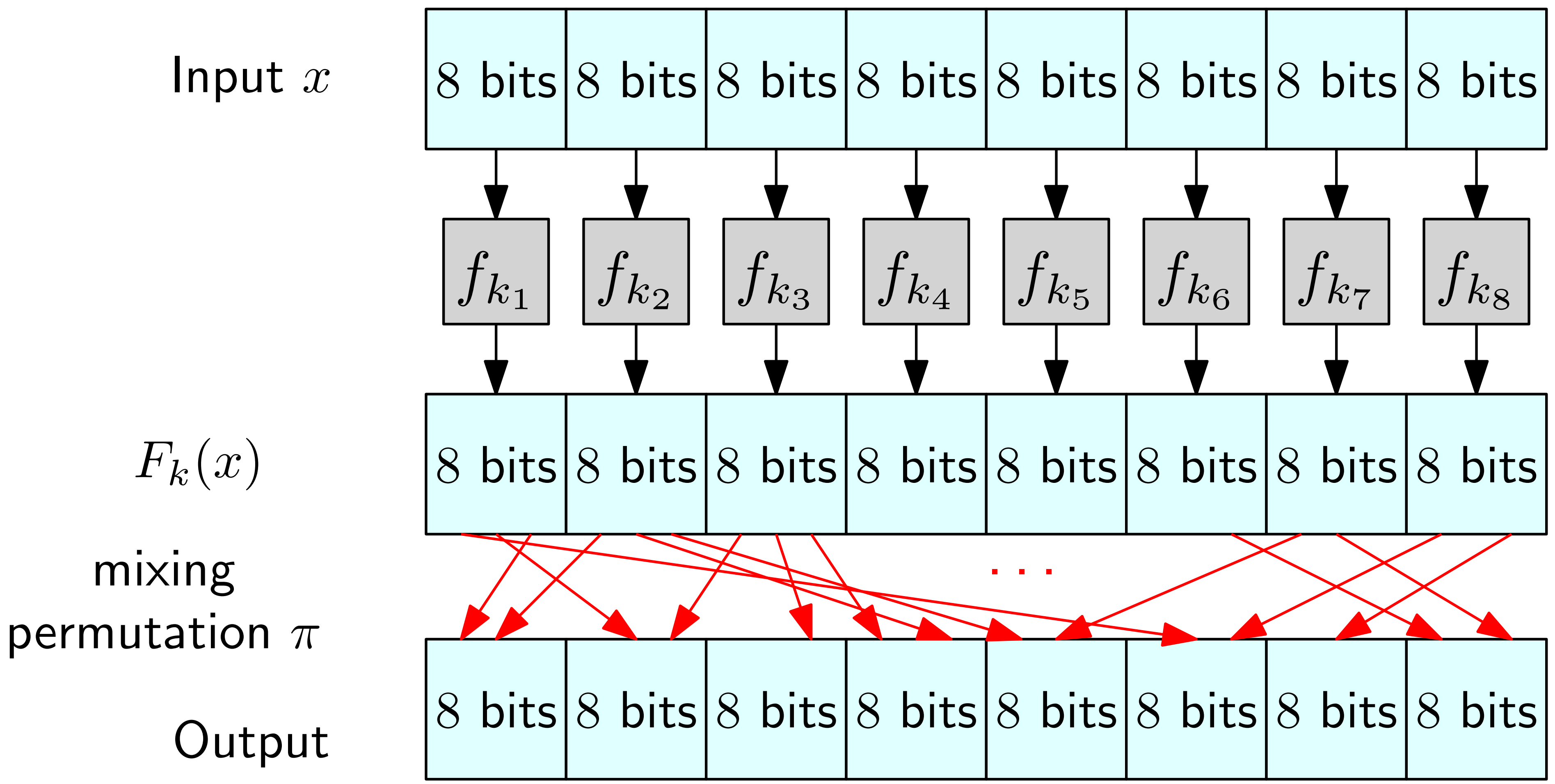

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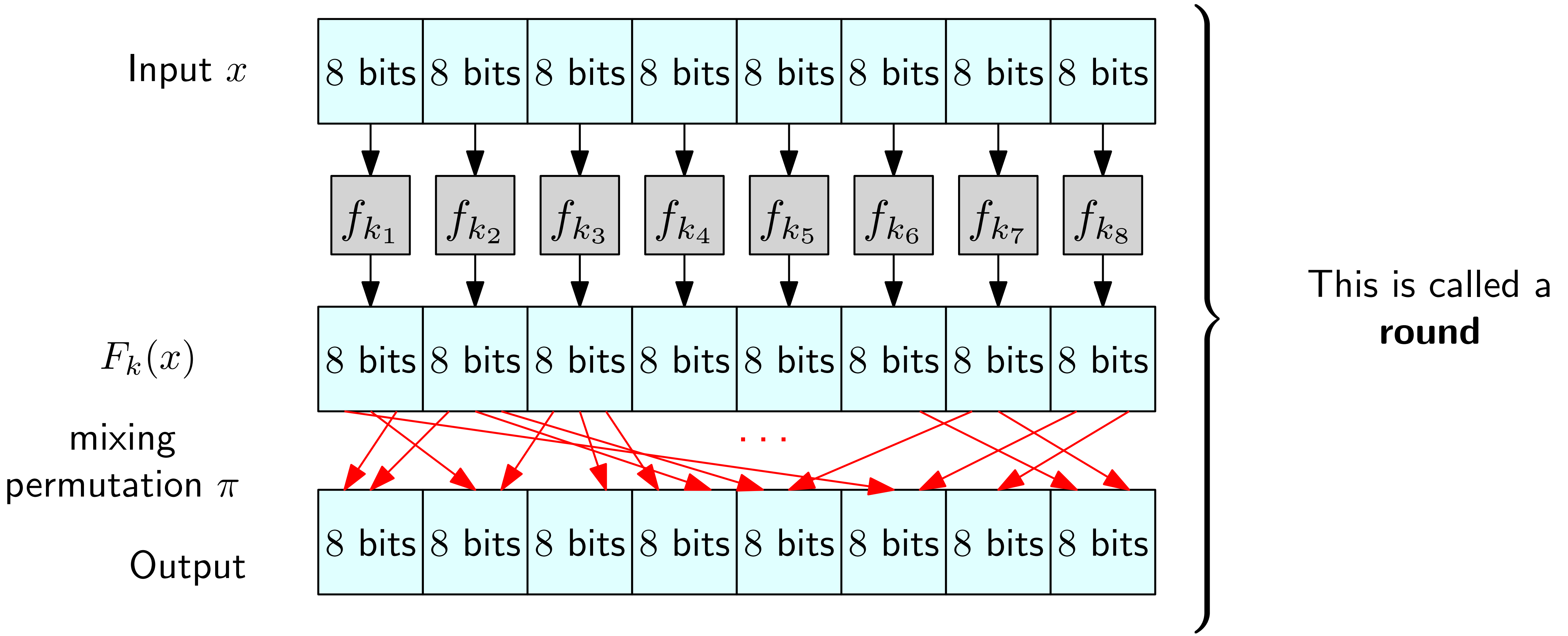

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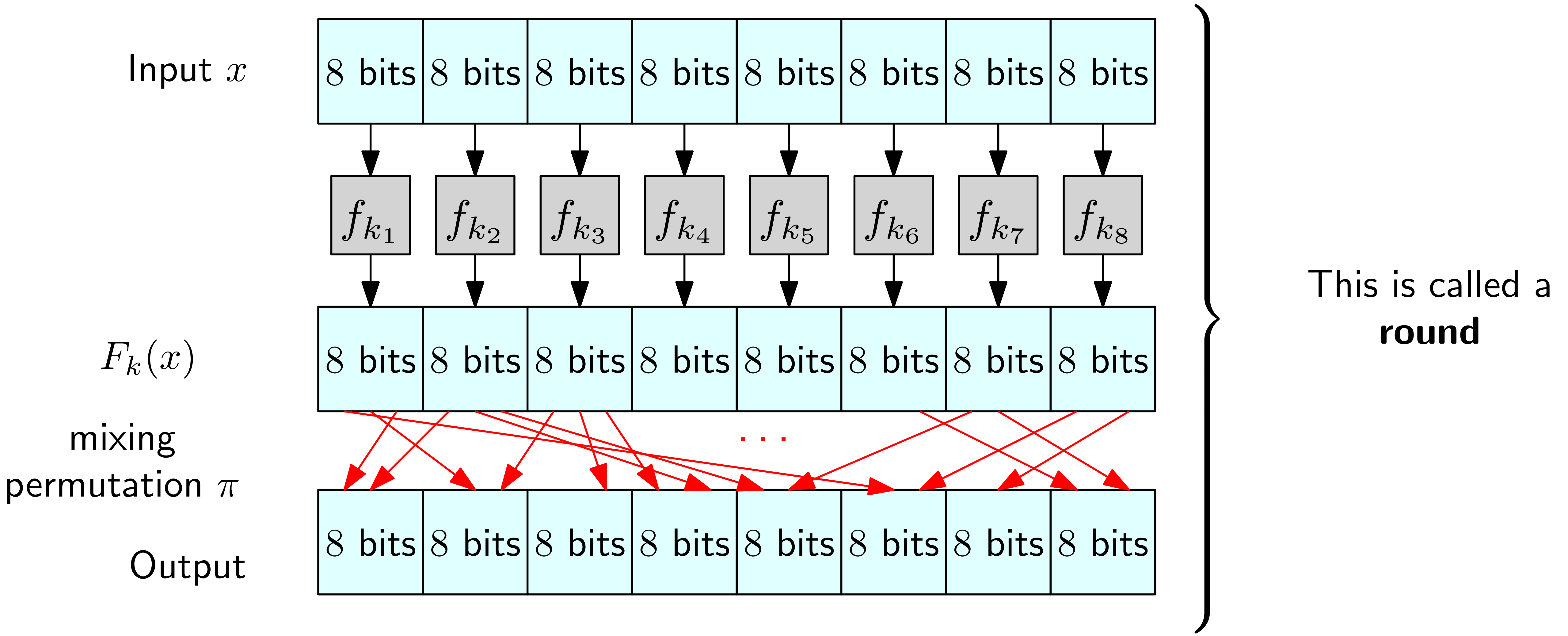

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What if we do another <u>round</u> with fresh functions f_{k_i}?

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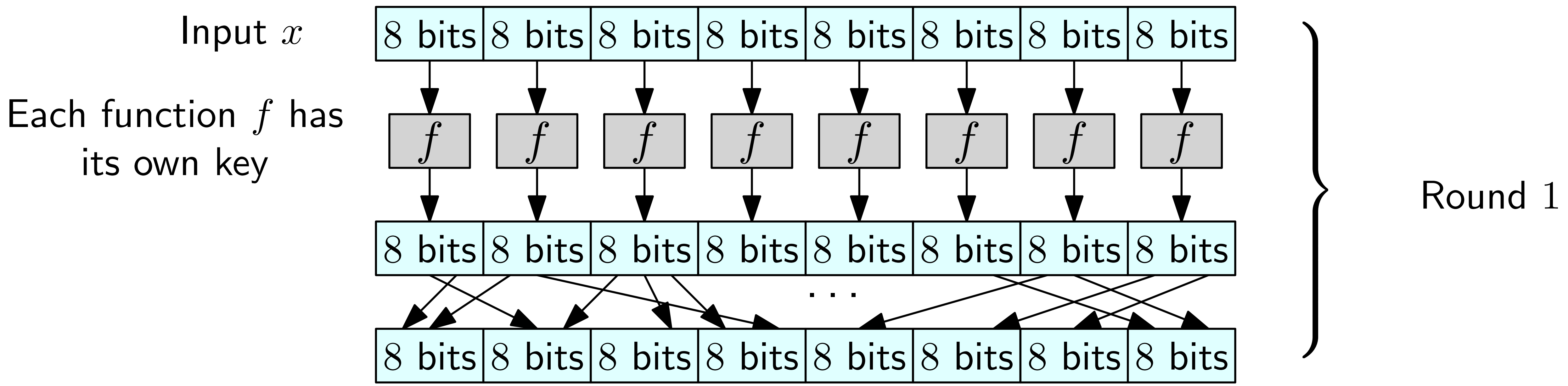

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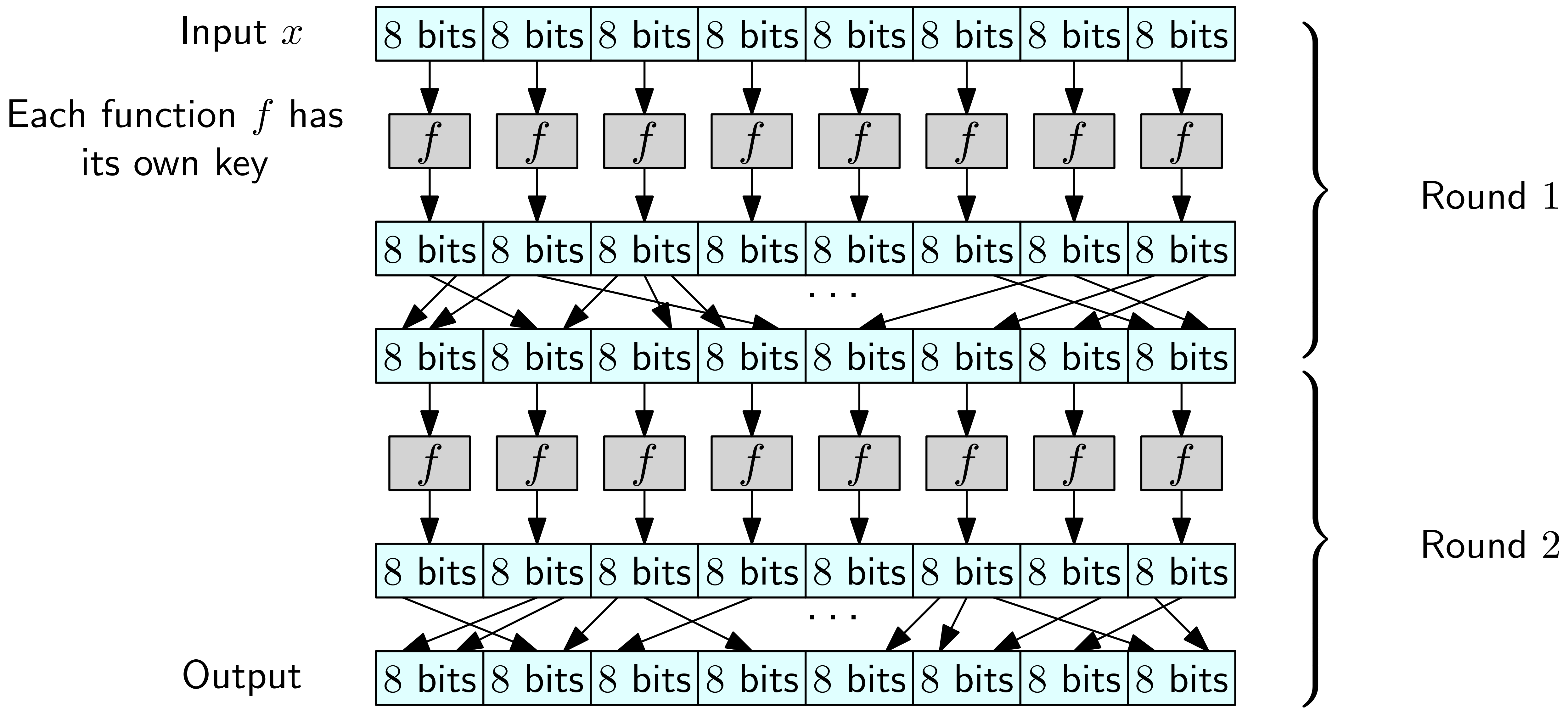

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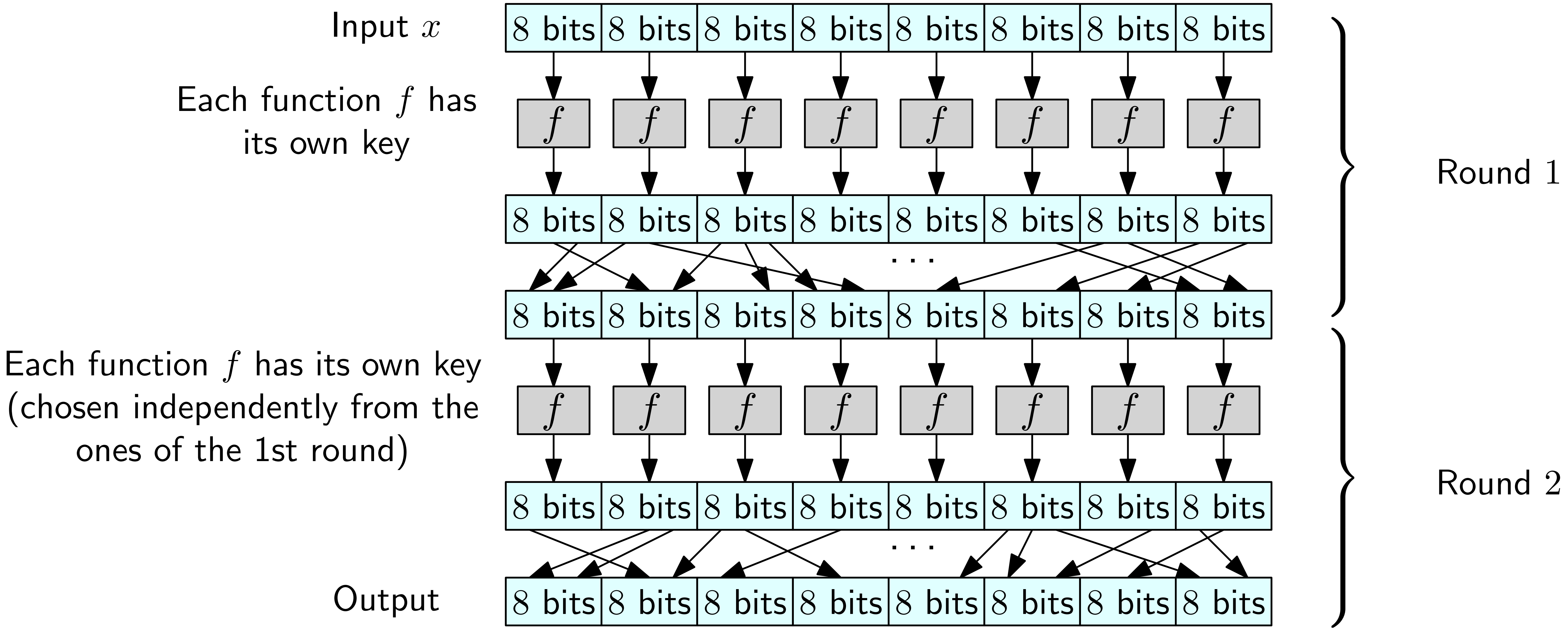

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Observation: the overall permutation remains invertible regardless of the number of rounds

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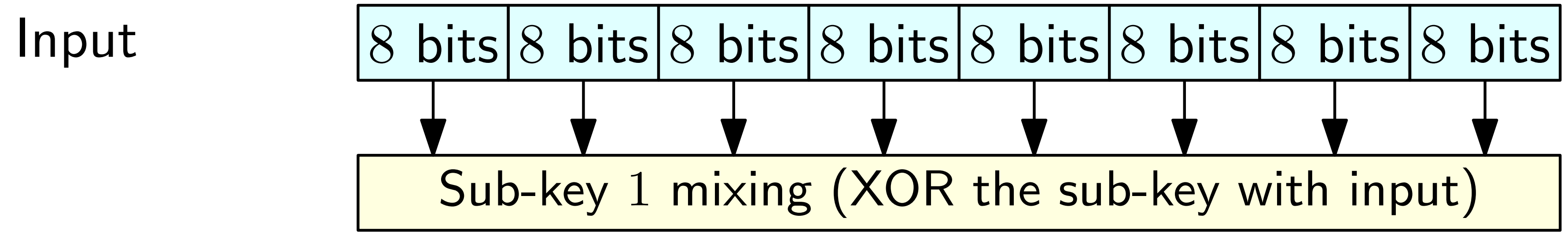
- The round keys are derived from the master key according to a **key schedule**

Input	8 bits	8 bits	8 bits	8 bits	8 bits	8 bits	8 bits	8 bits

Sample structure of a 2-round block cipher



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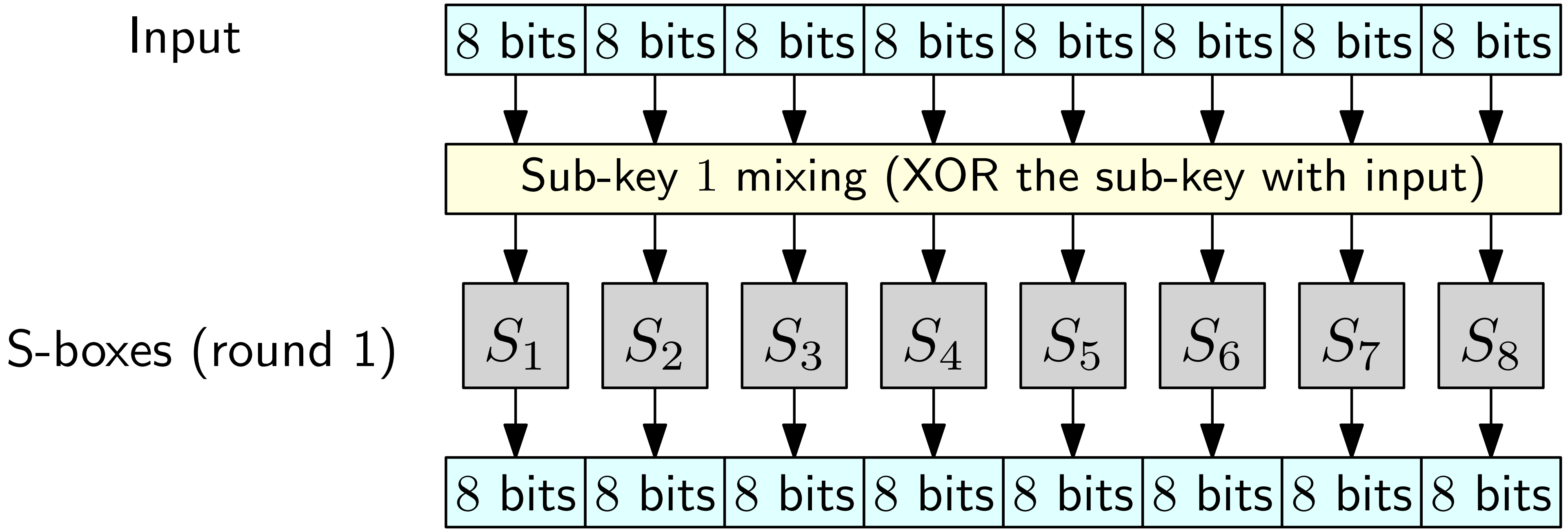

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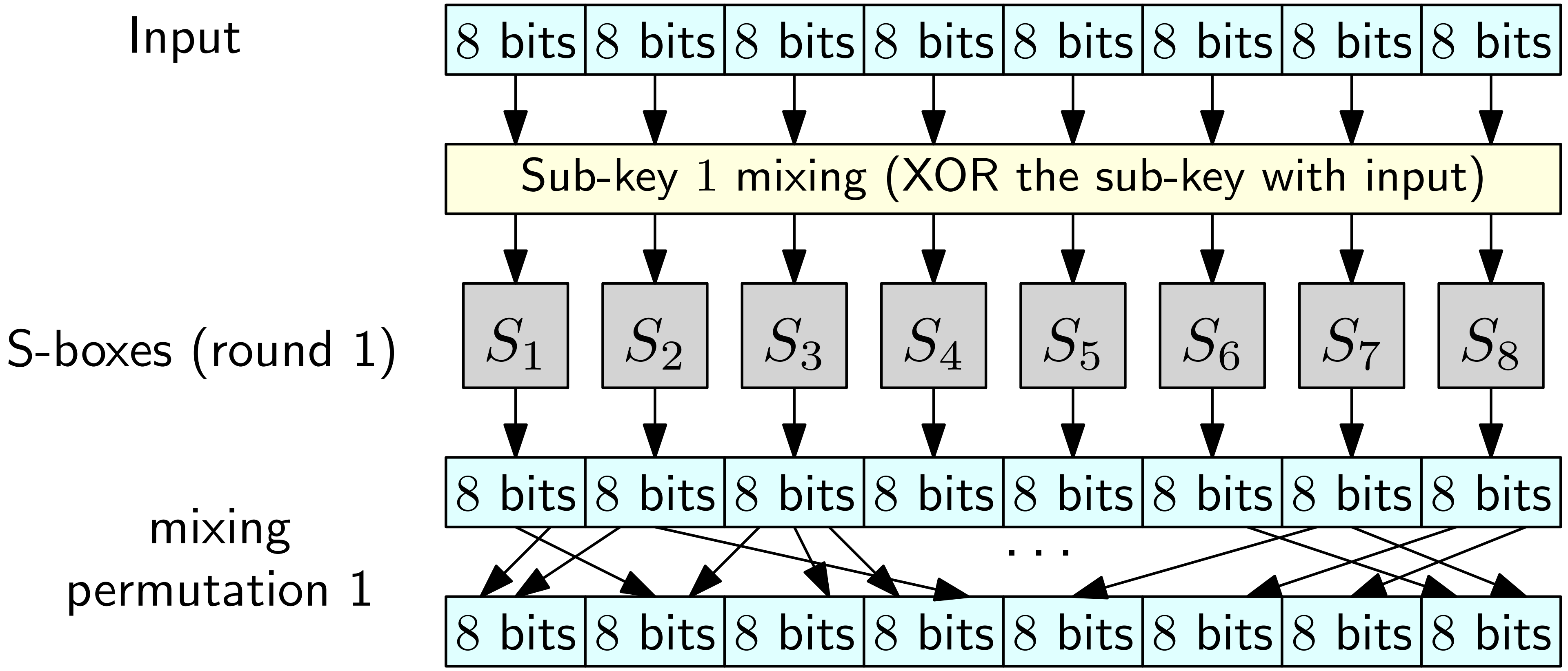

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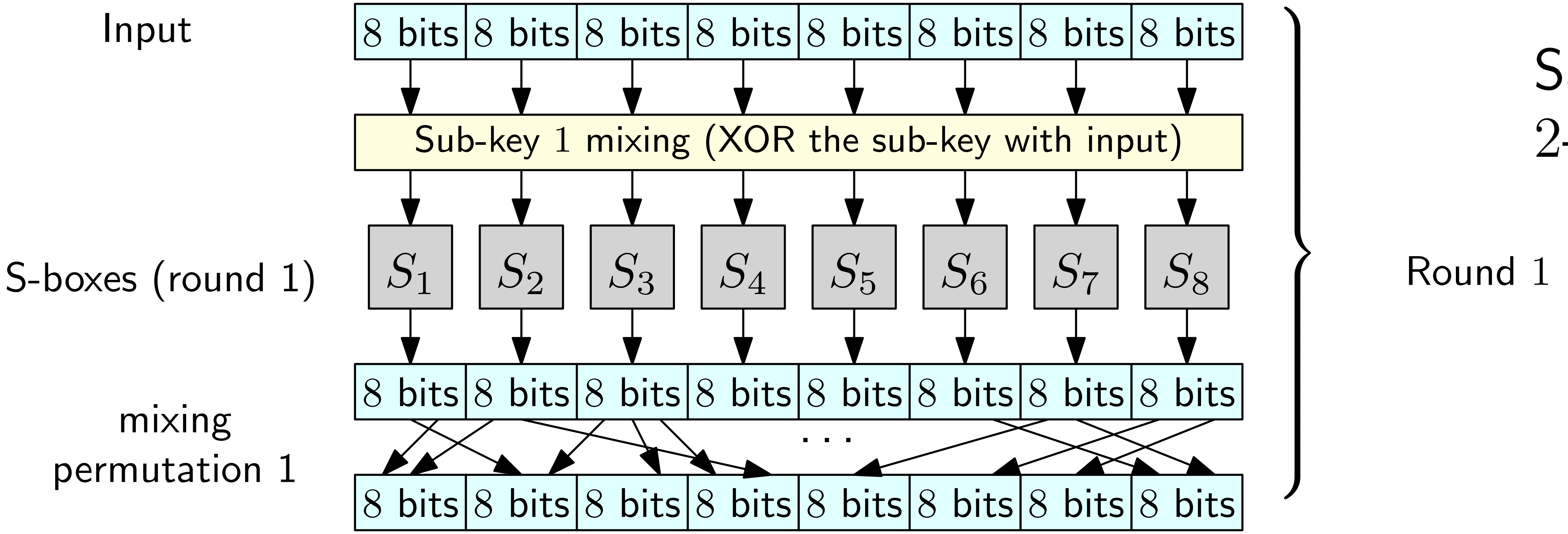

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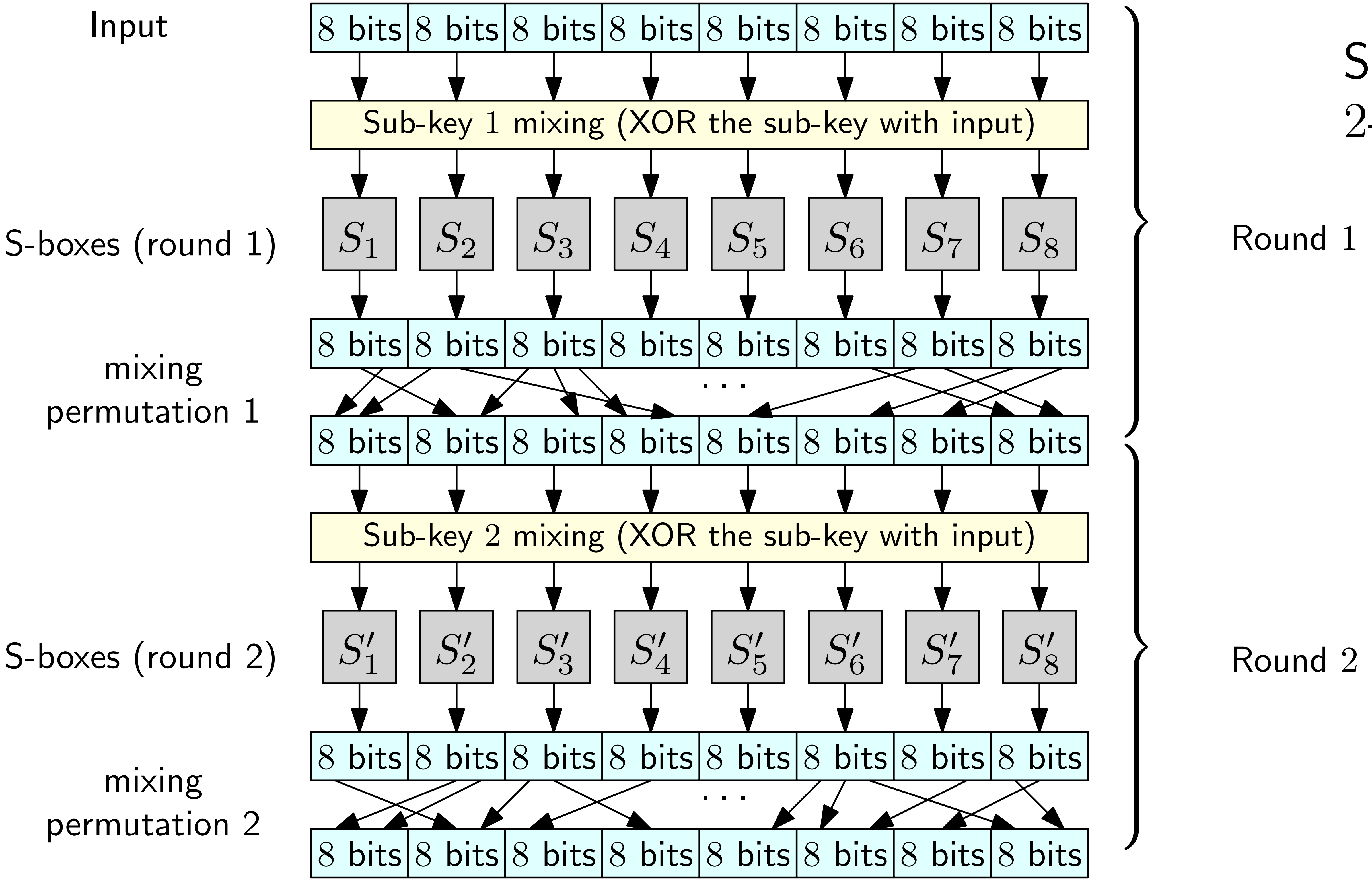

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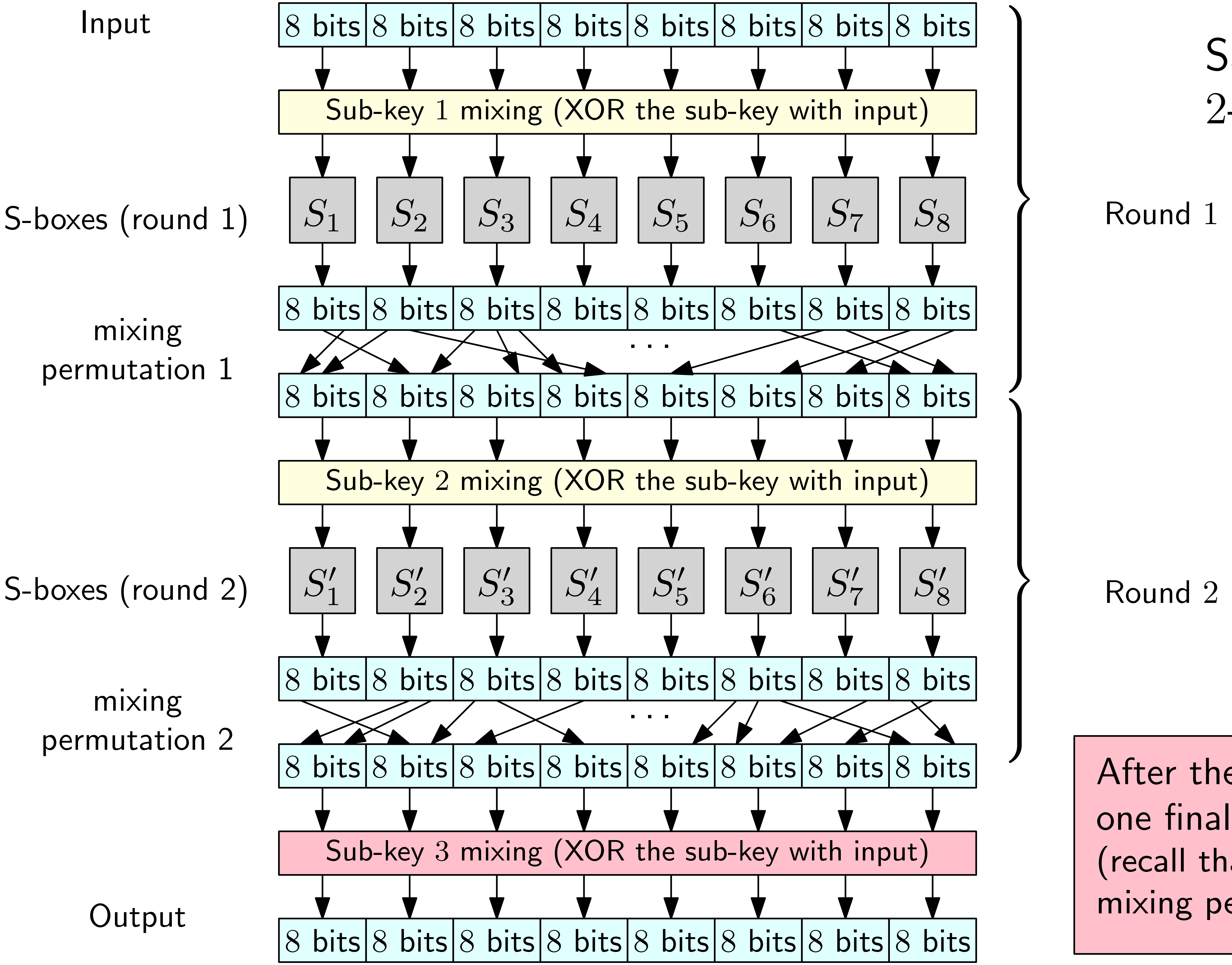

After the last round, we perform one final key mixing step (recall that it is useless to apply a mixing permutation as the last step)

The Avalanche Effect

We want to design the S-boxes and the mixing permutation to achieve the **avalanche effect**

- Even a small difference in the input should eventually (over multiple rounds) propagate to the entire output

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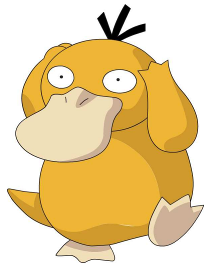

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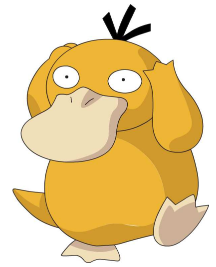
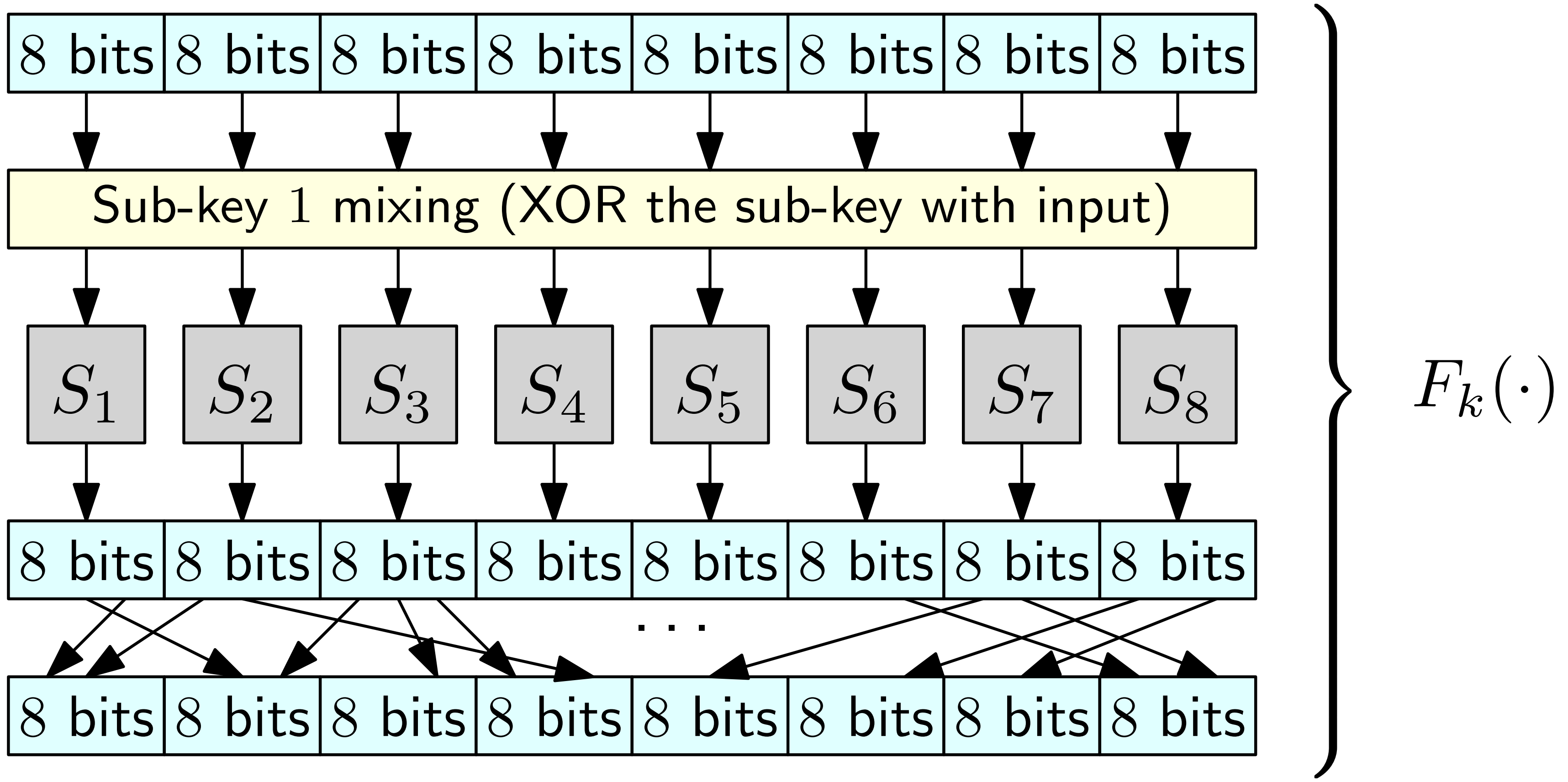


For the mixing permutation:

- A bit output from a S-box should be fed into a different S-box into the next round
- This adds diffusion

Key recovery attack against a simplified 1-round SPN

Simple case: 1-round SPN and no final key mixing step



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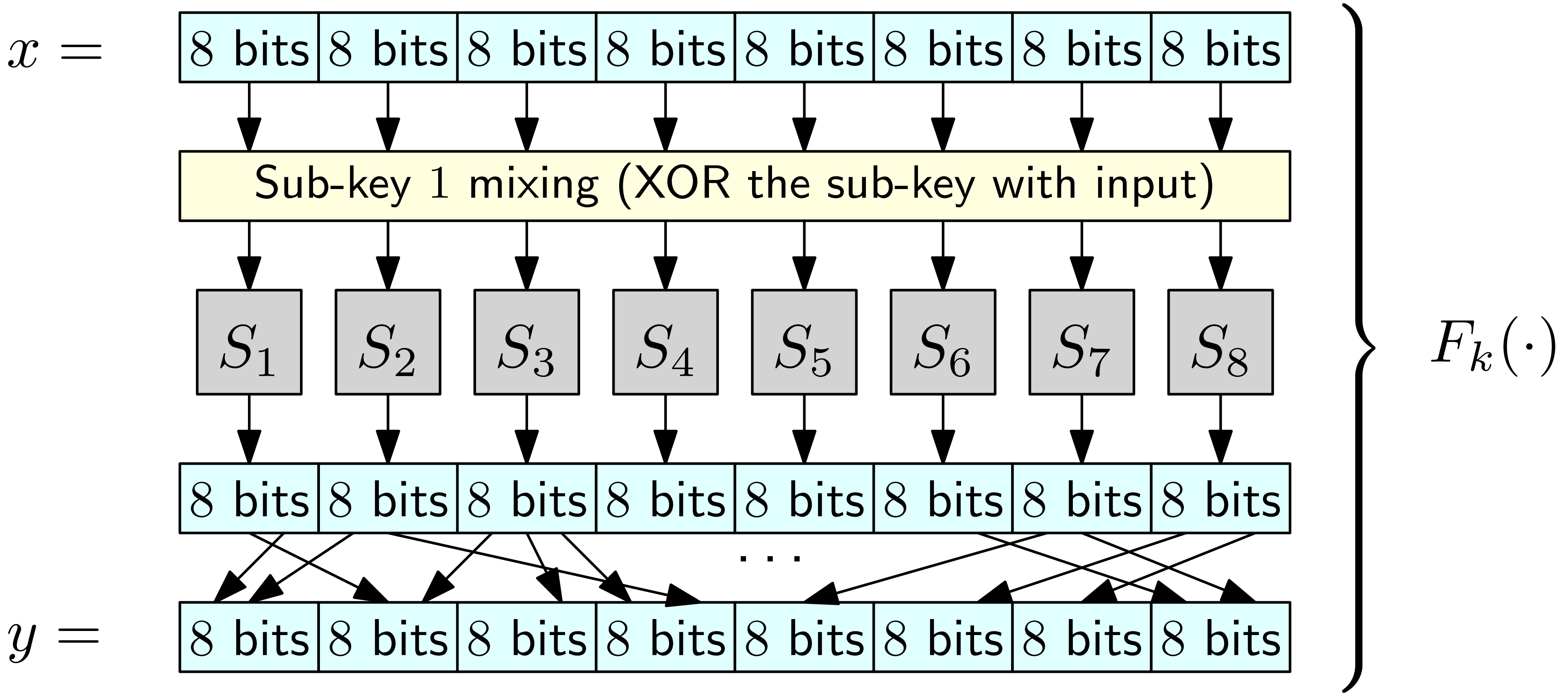

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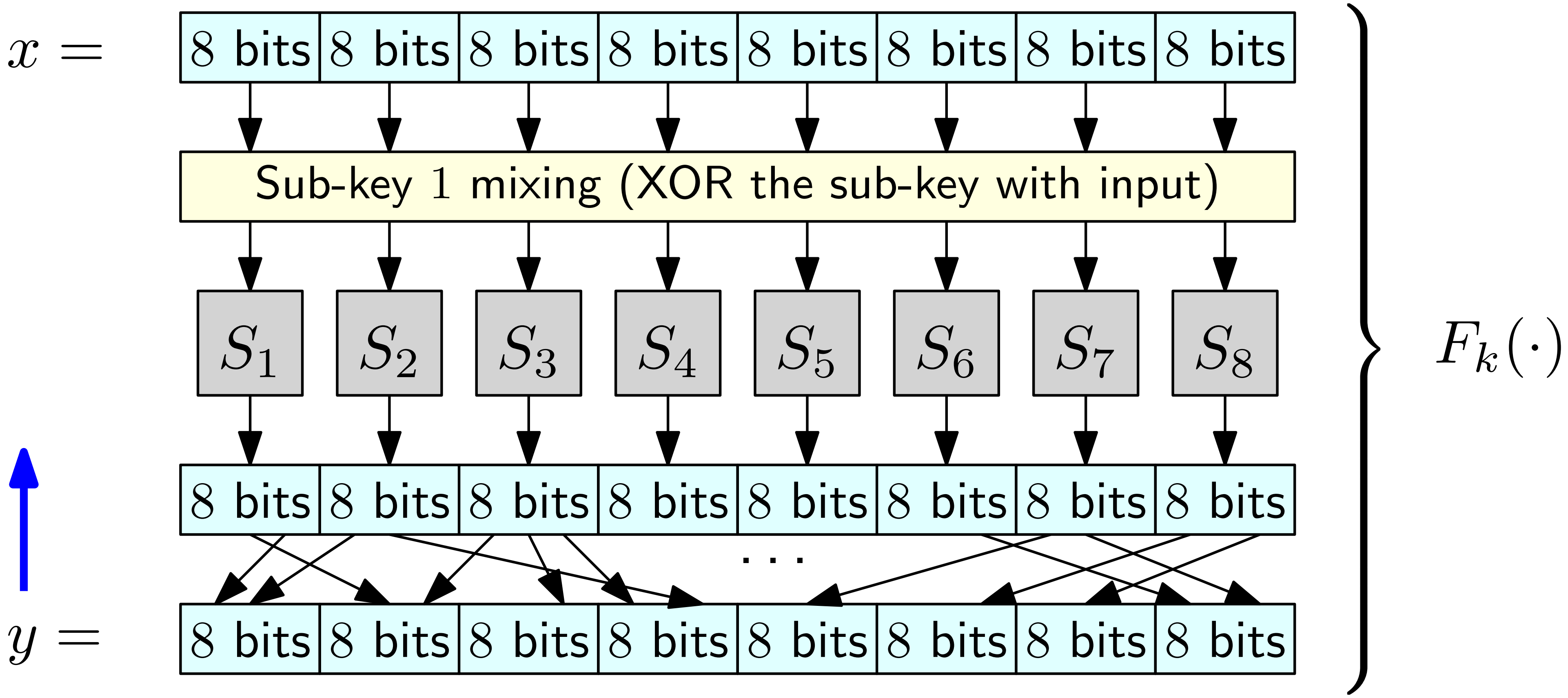

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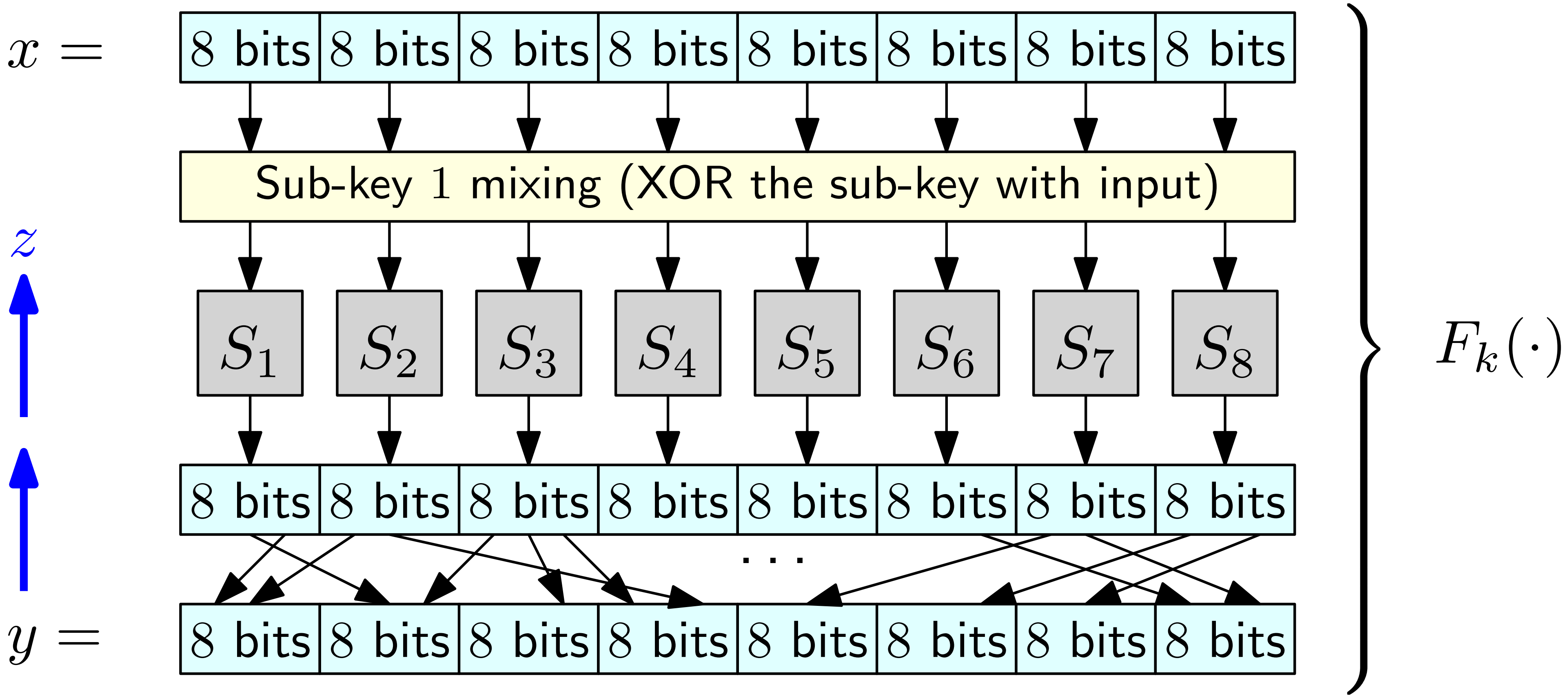

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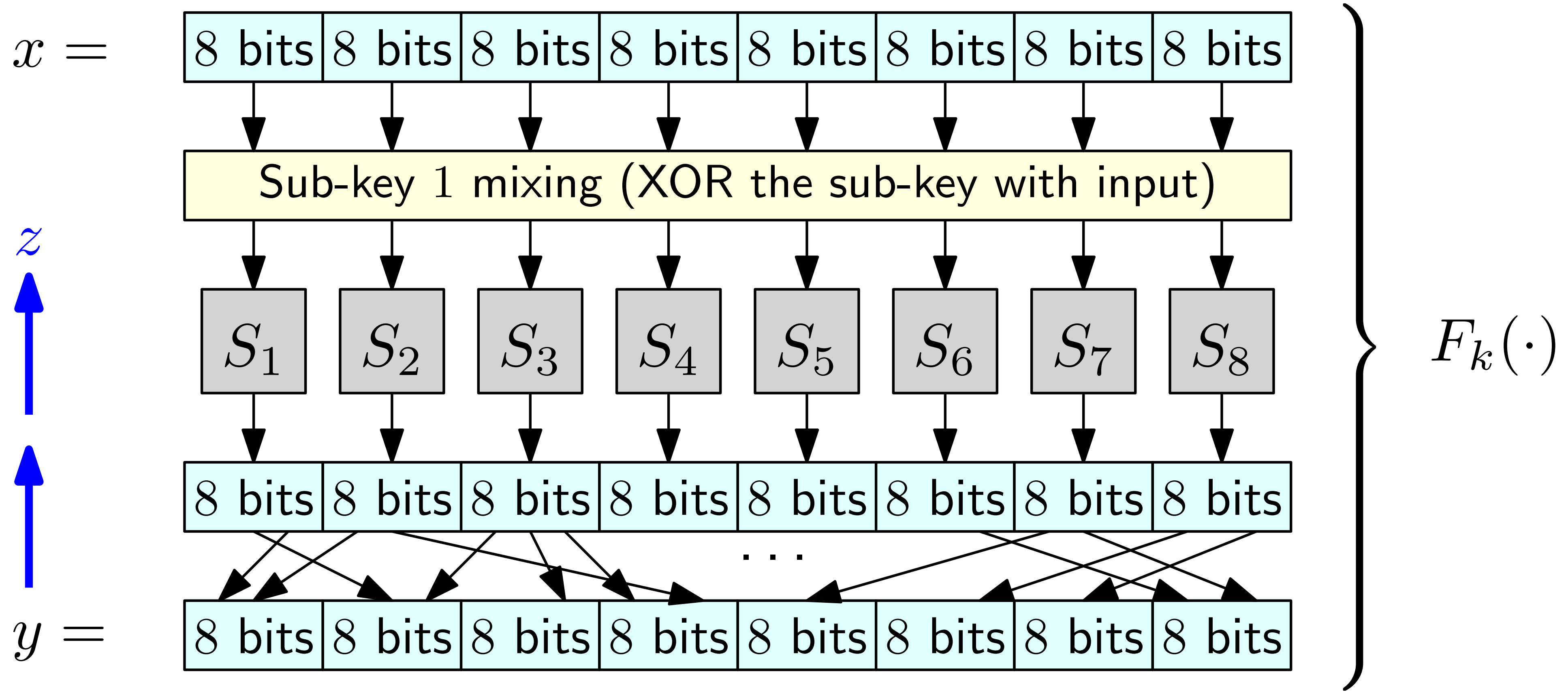

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- The (round and master) key is $k = z \oplus x = (x \oplus k) \oplus x$

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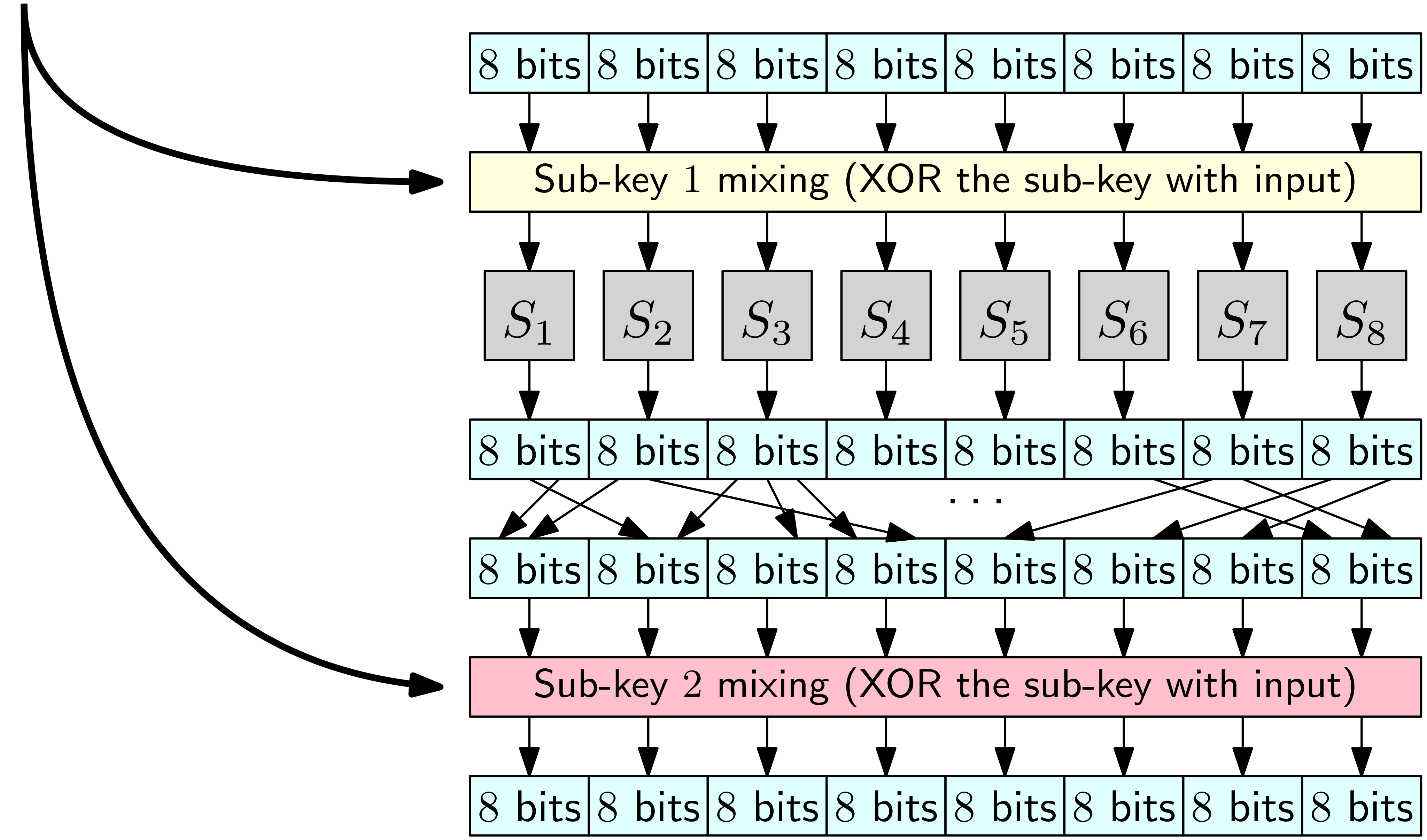

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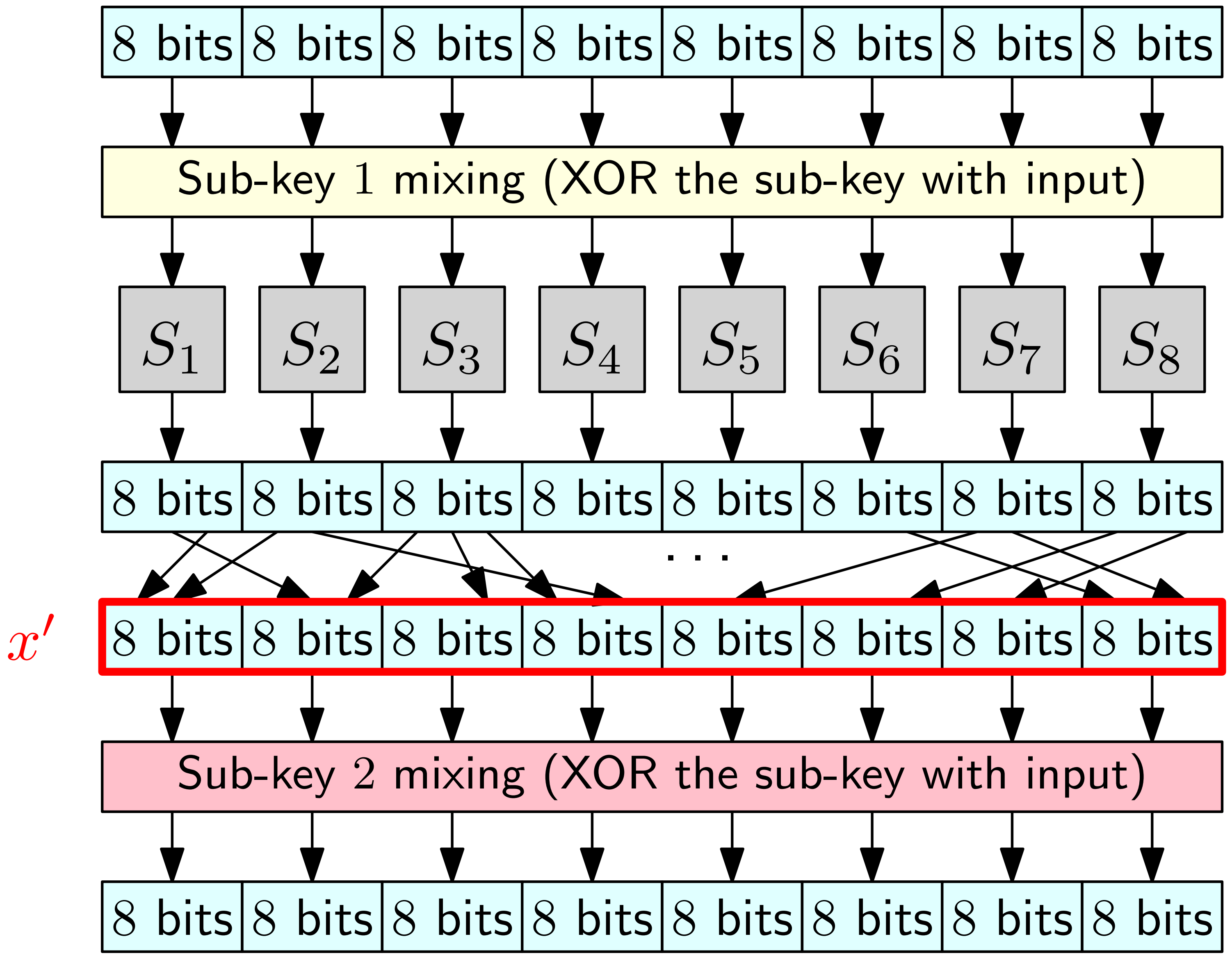

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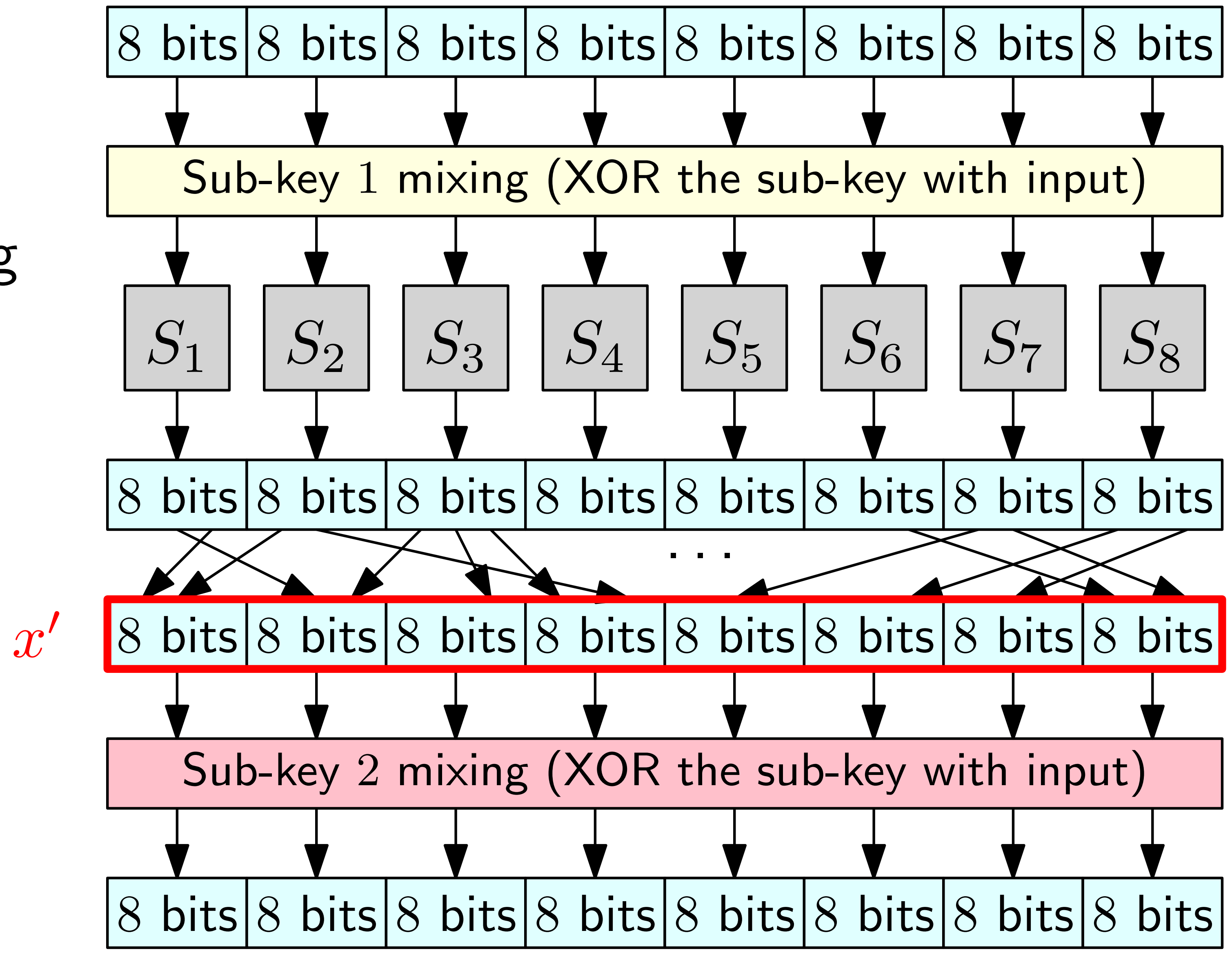

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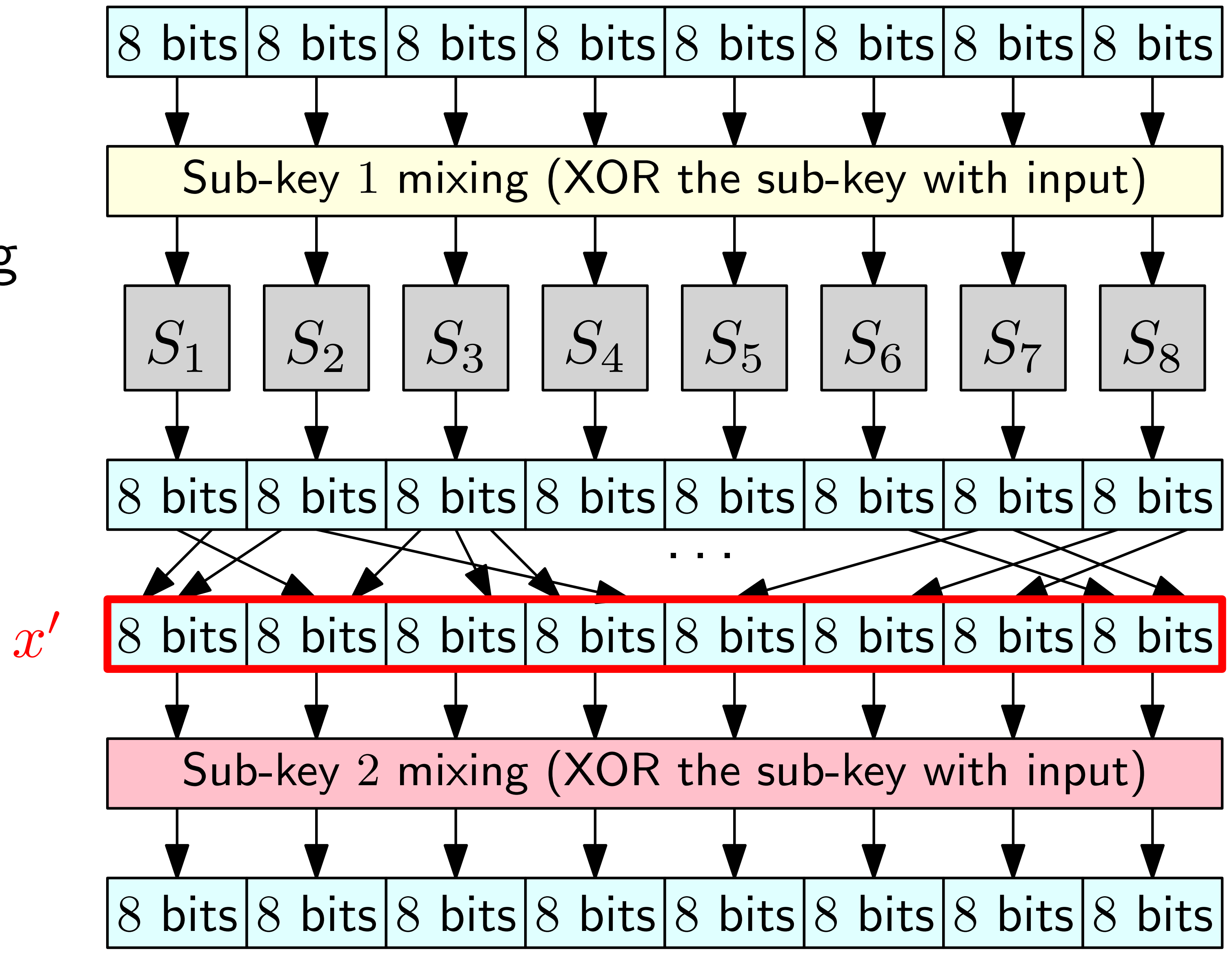

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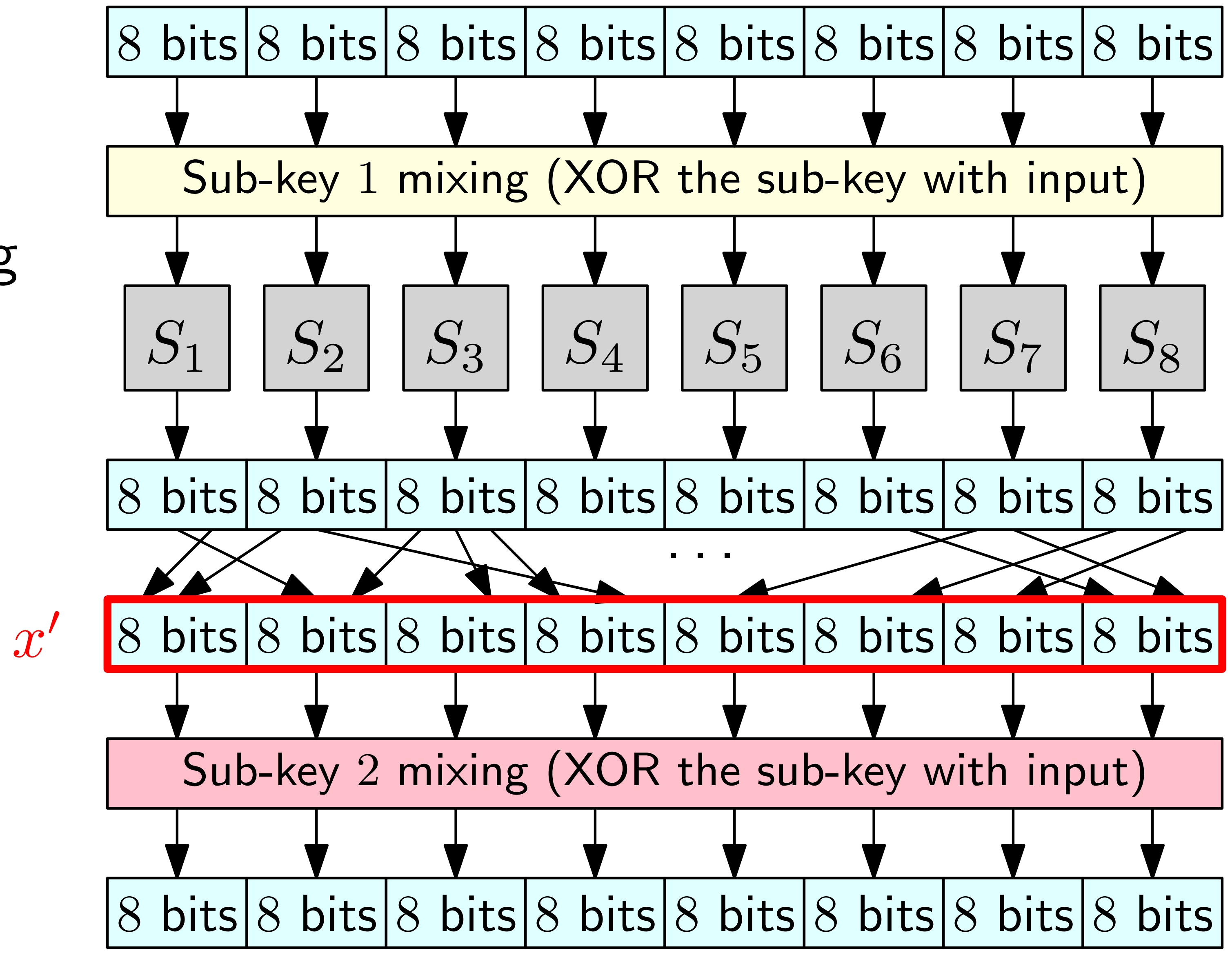

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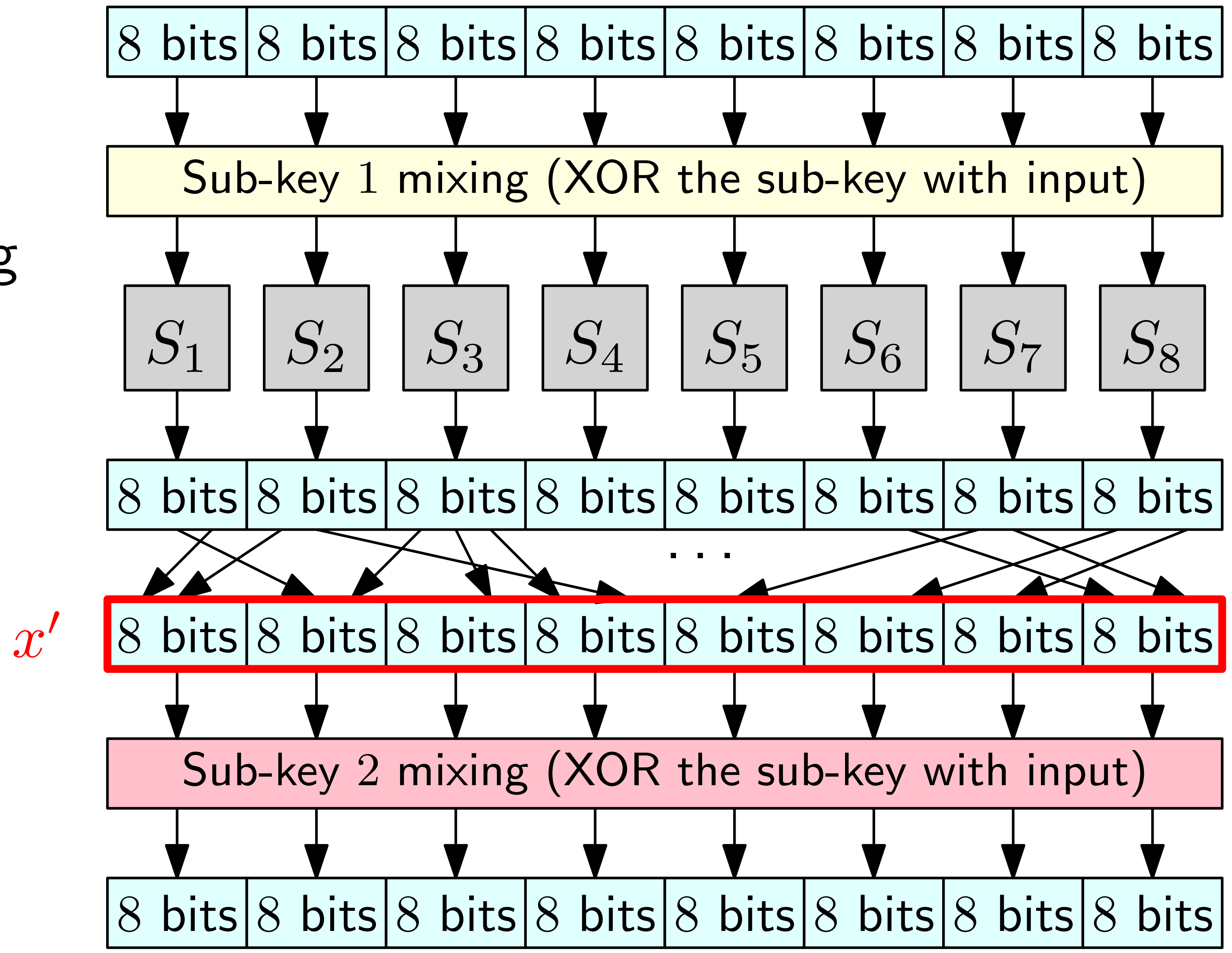

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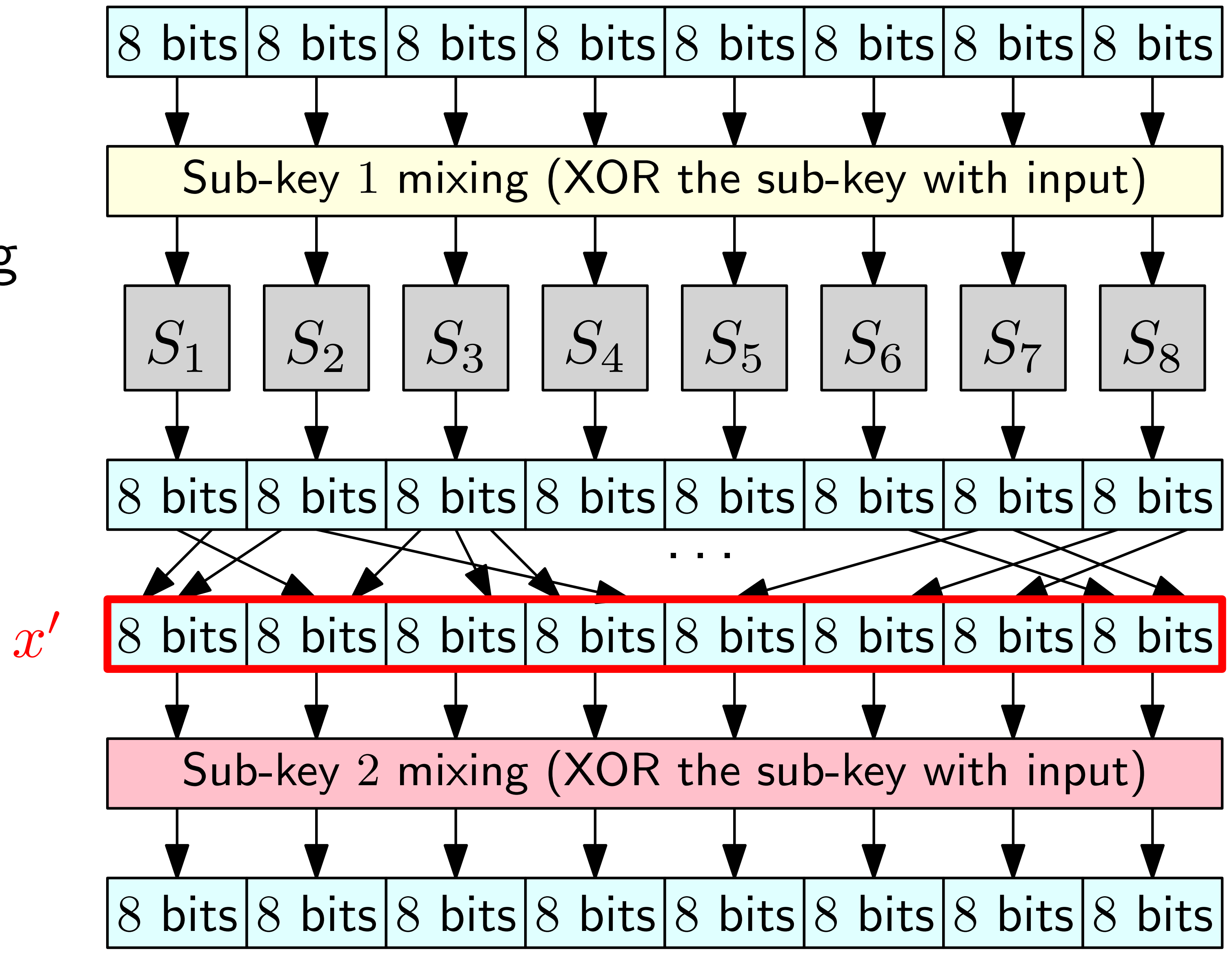

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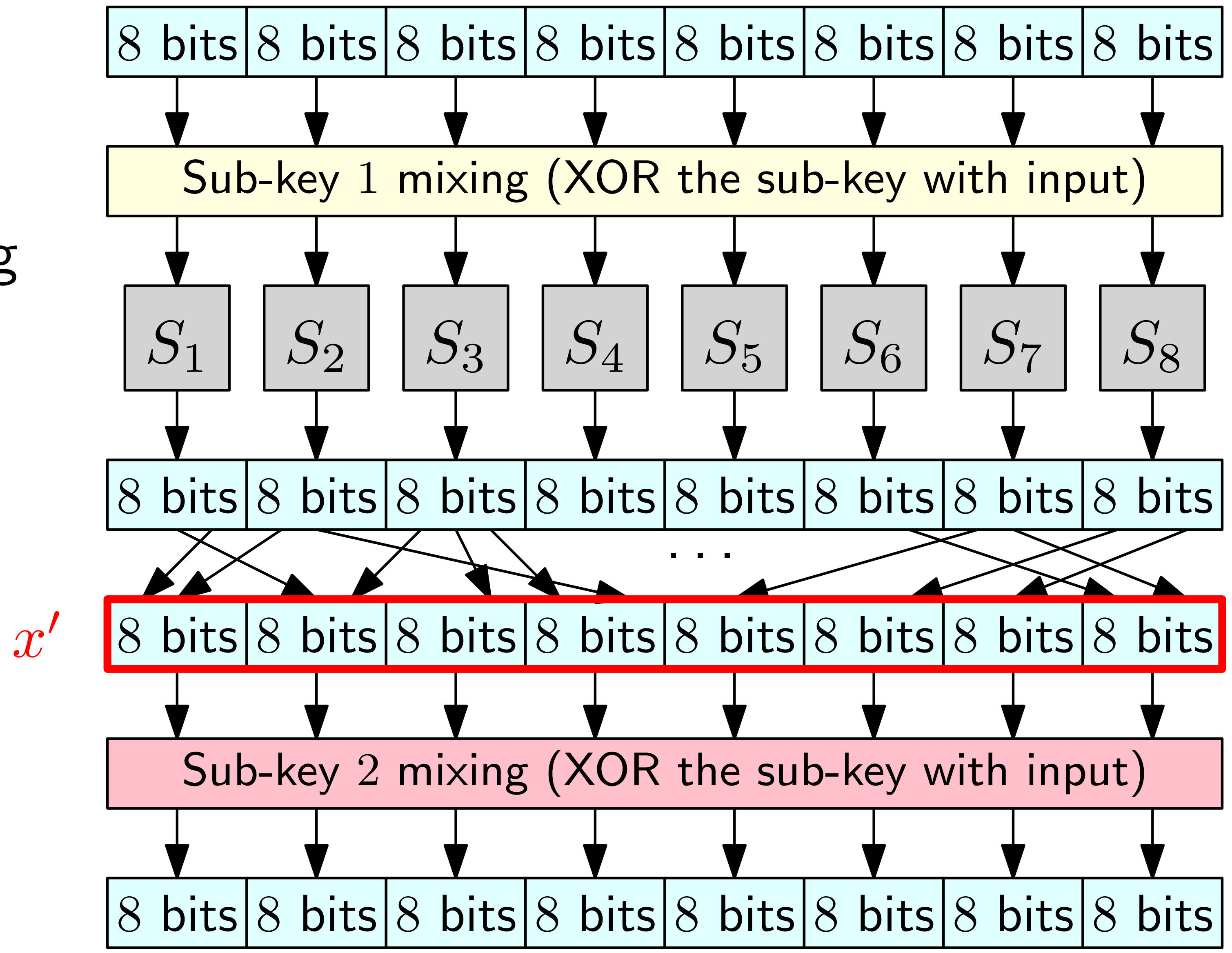

Indeed... we can design a better attack!

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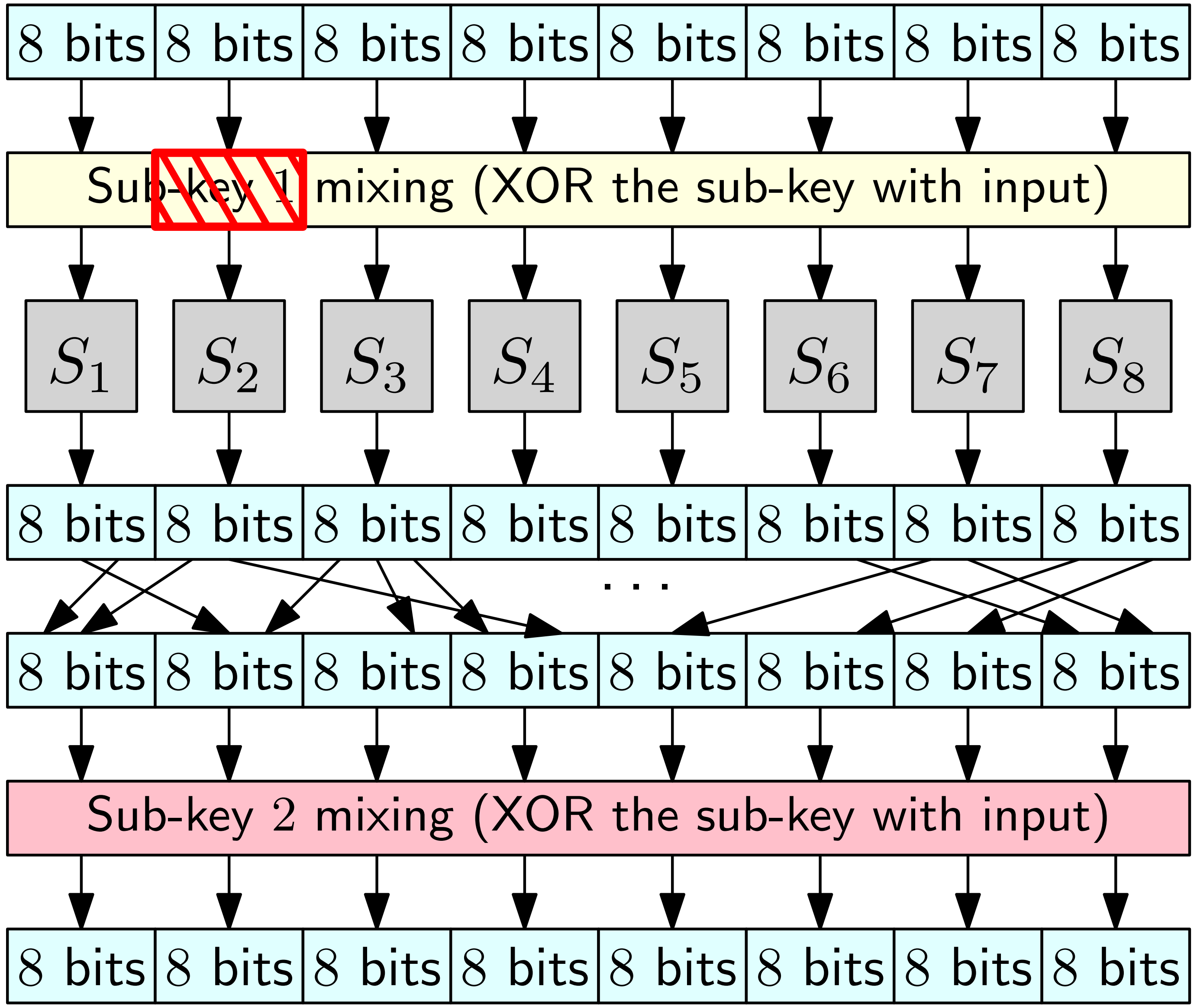
- Guess only the part of 1st mixing sub-key that contributes to the input of some S-box



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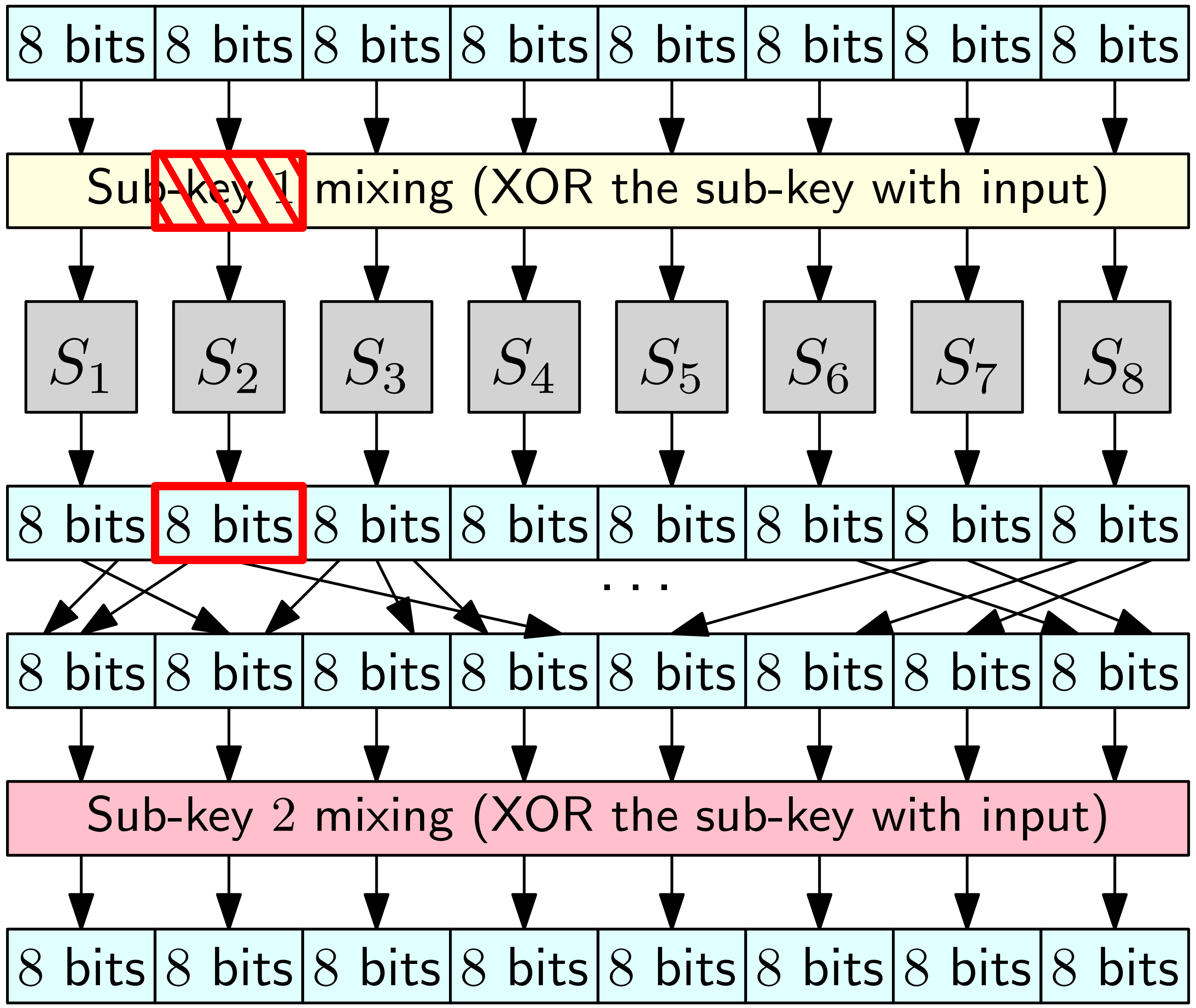

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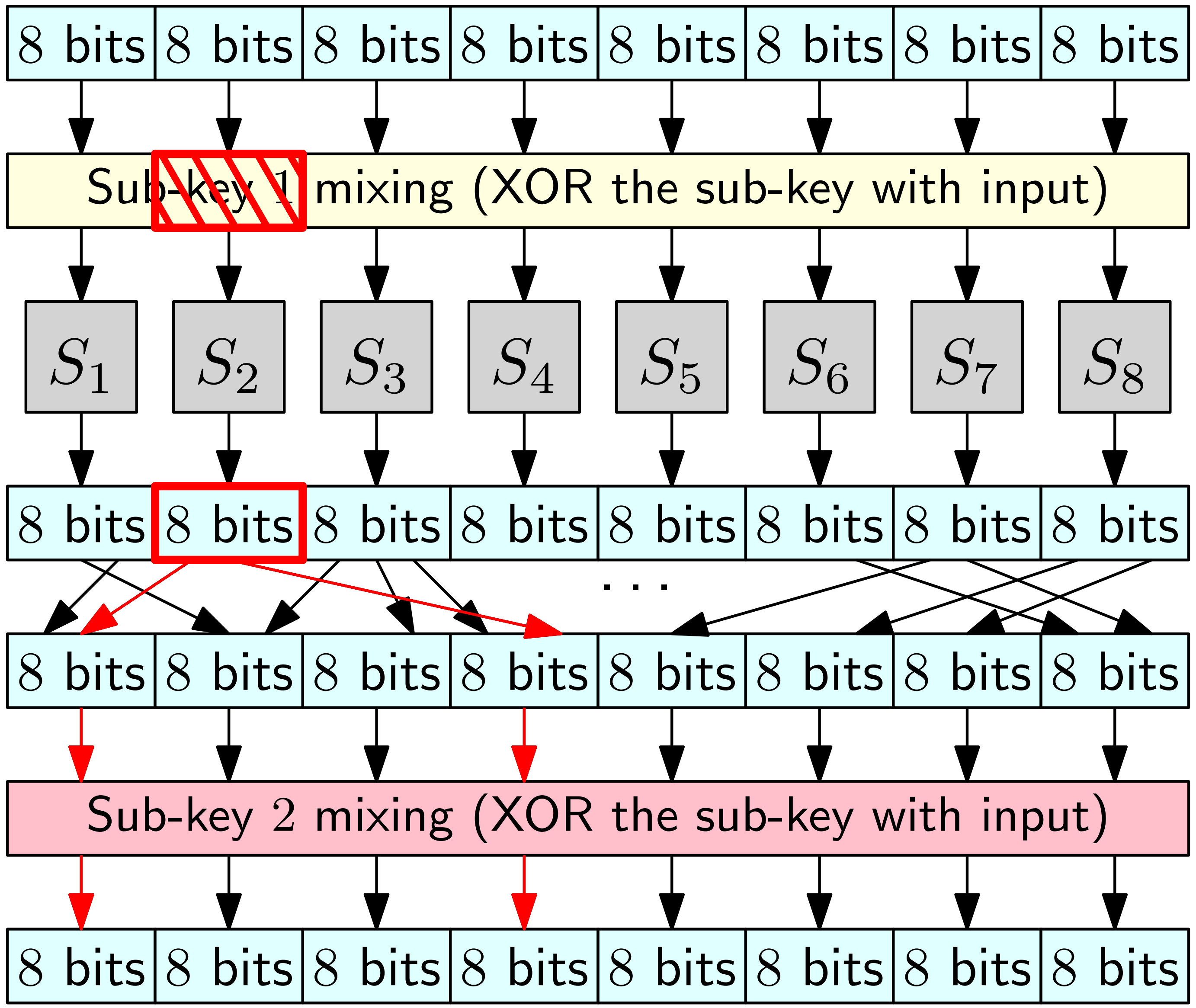

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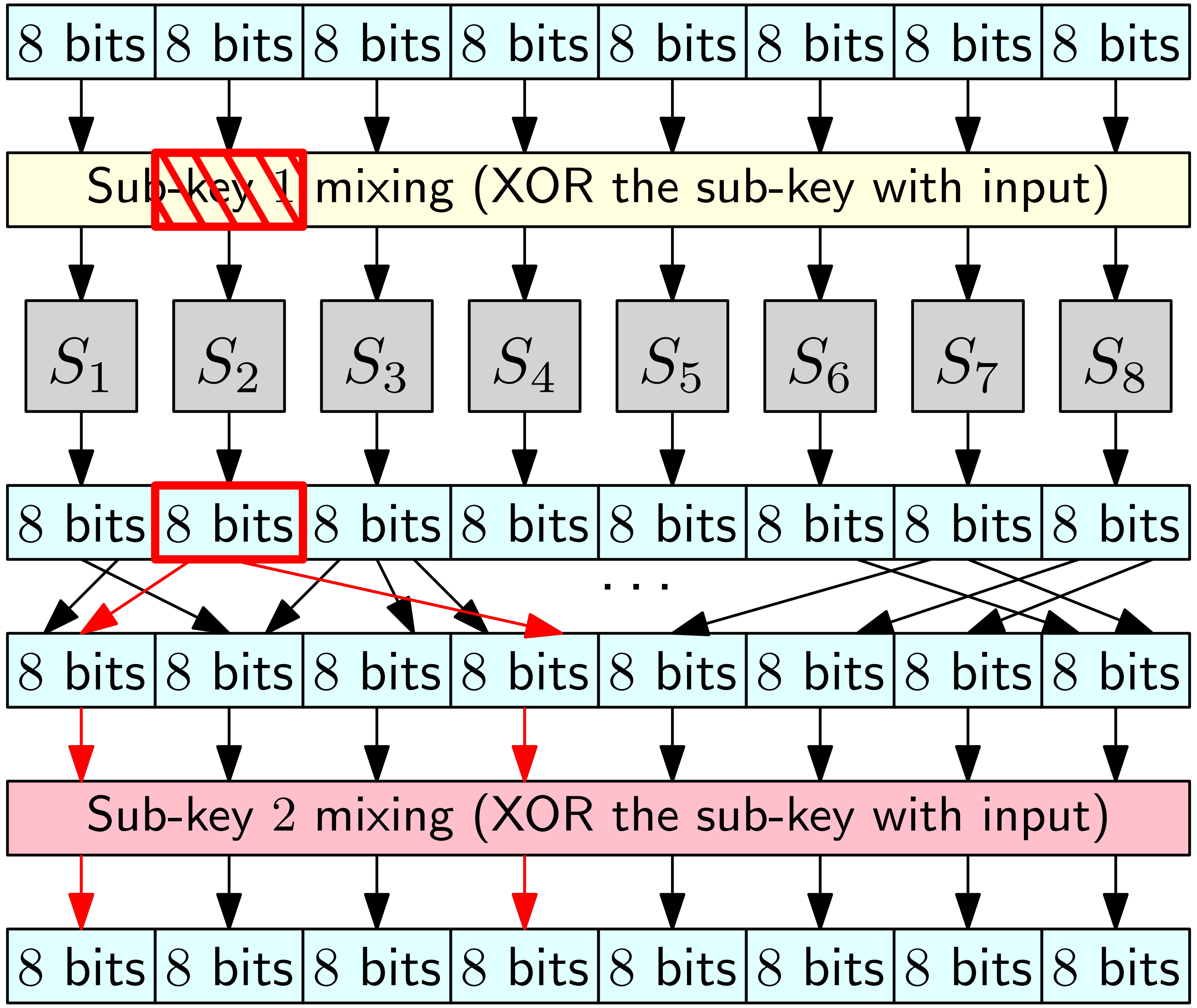

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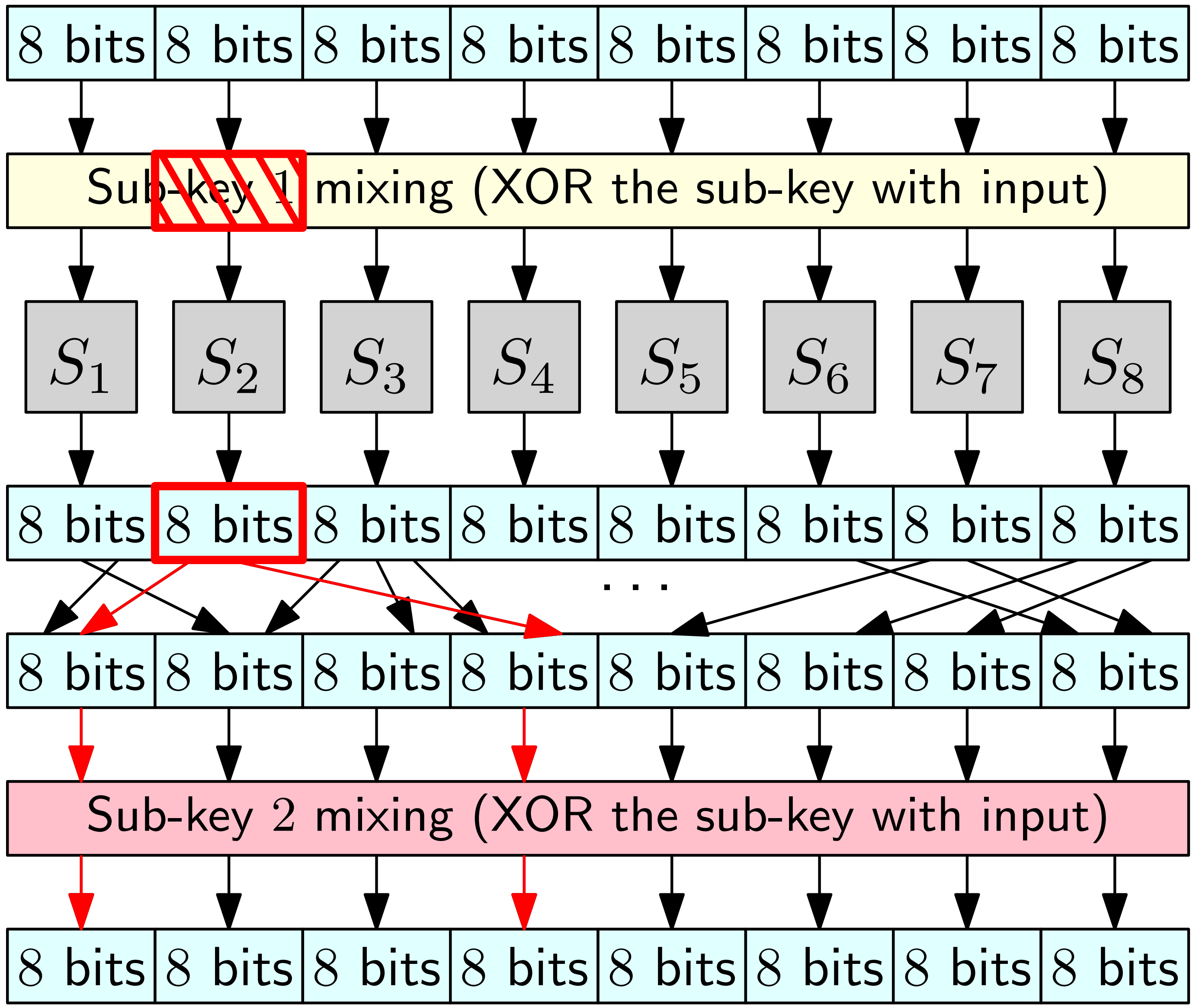

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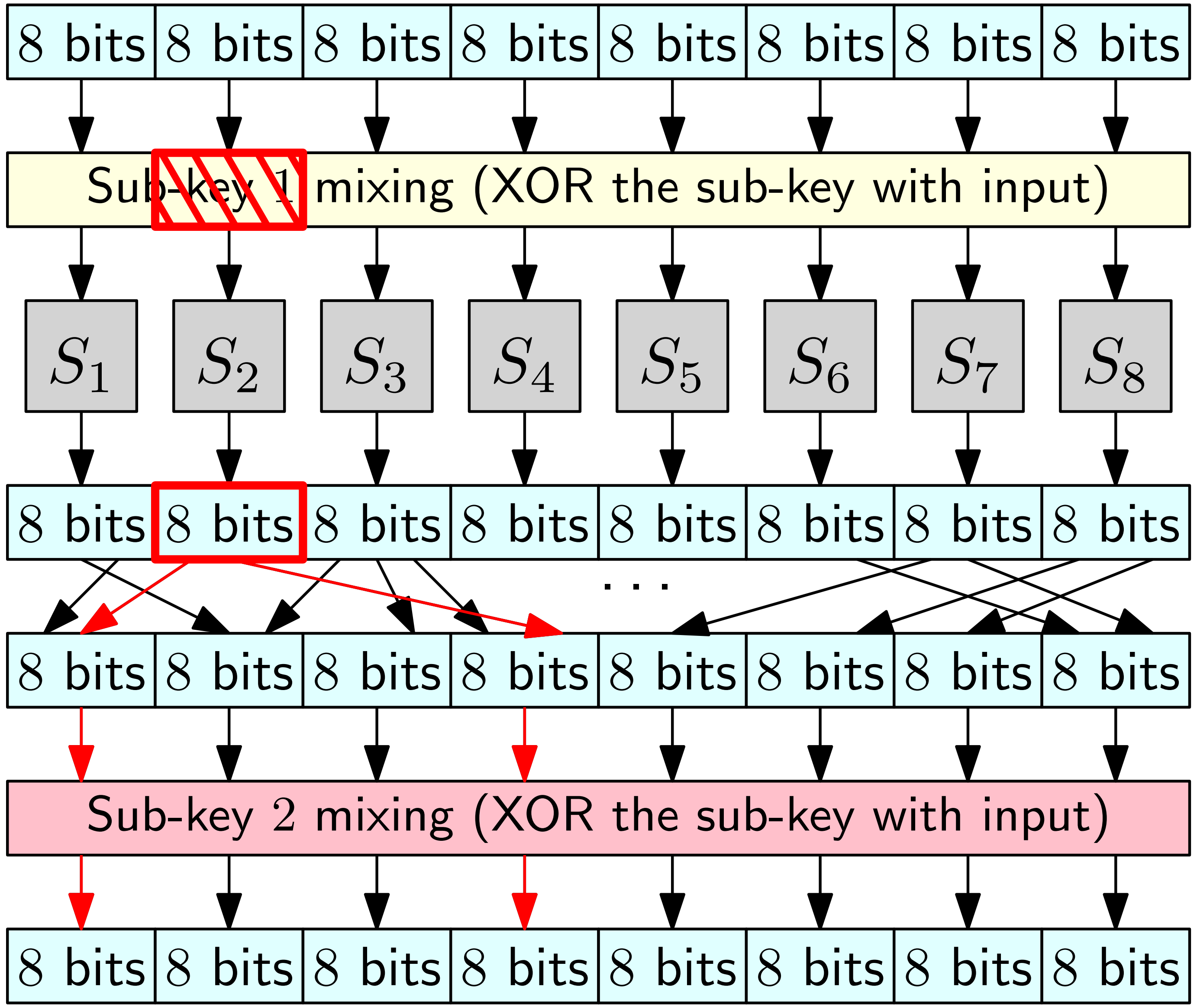

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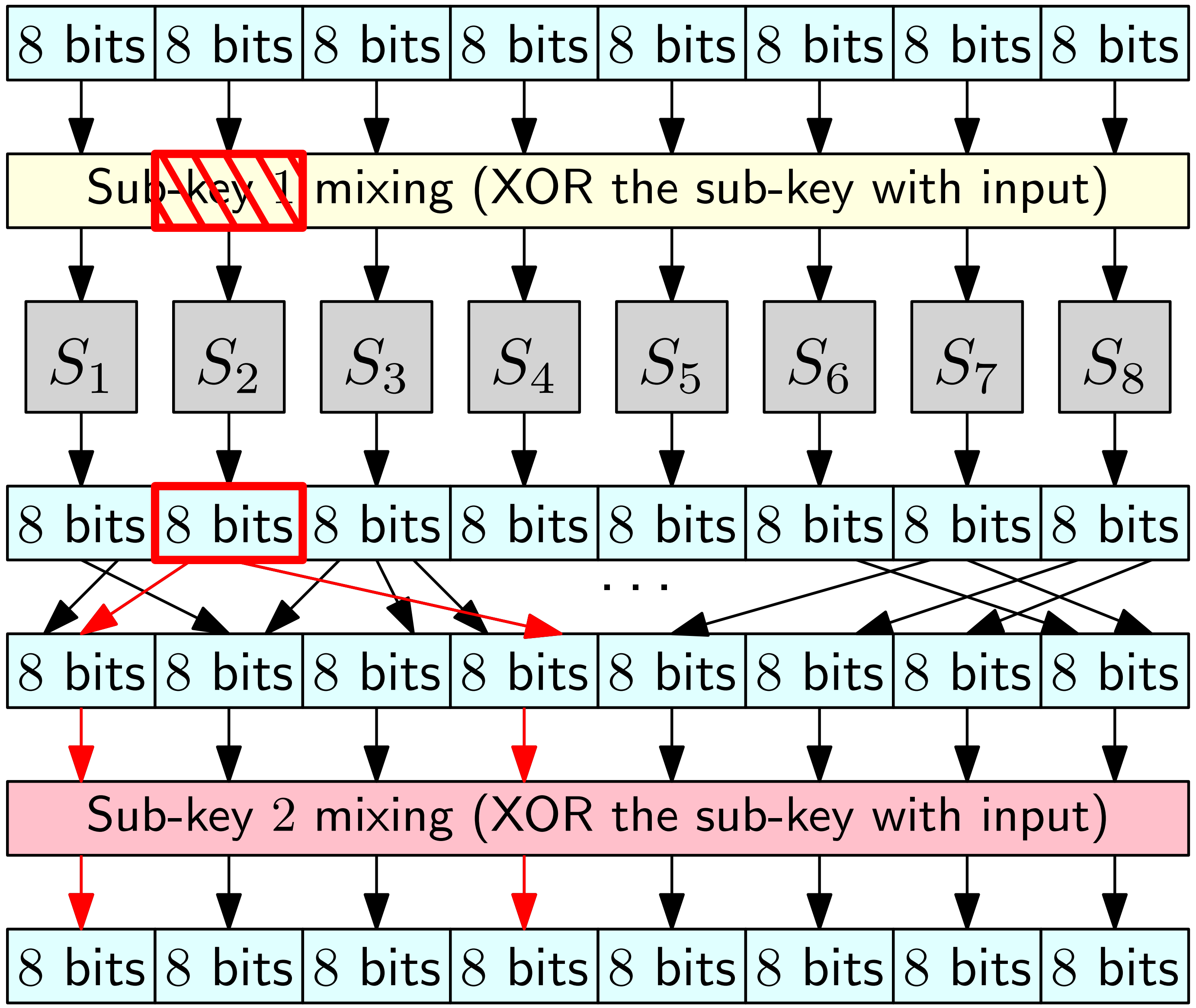

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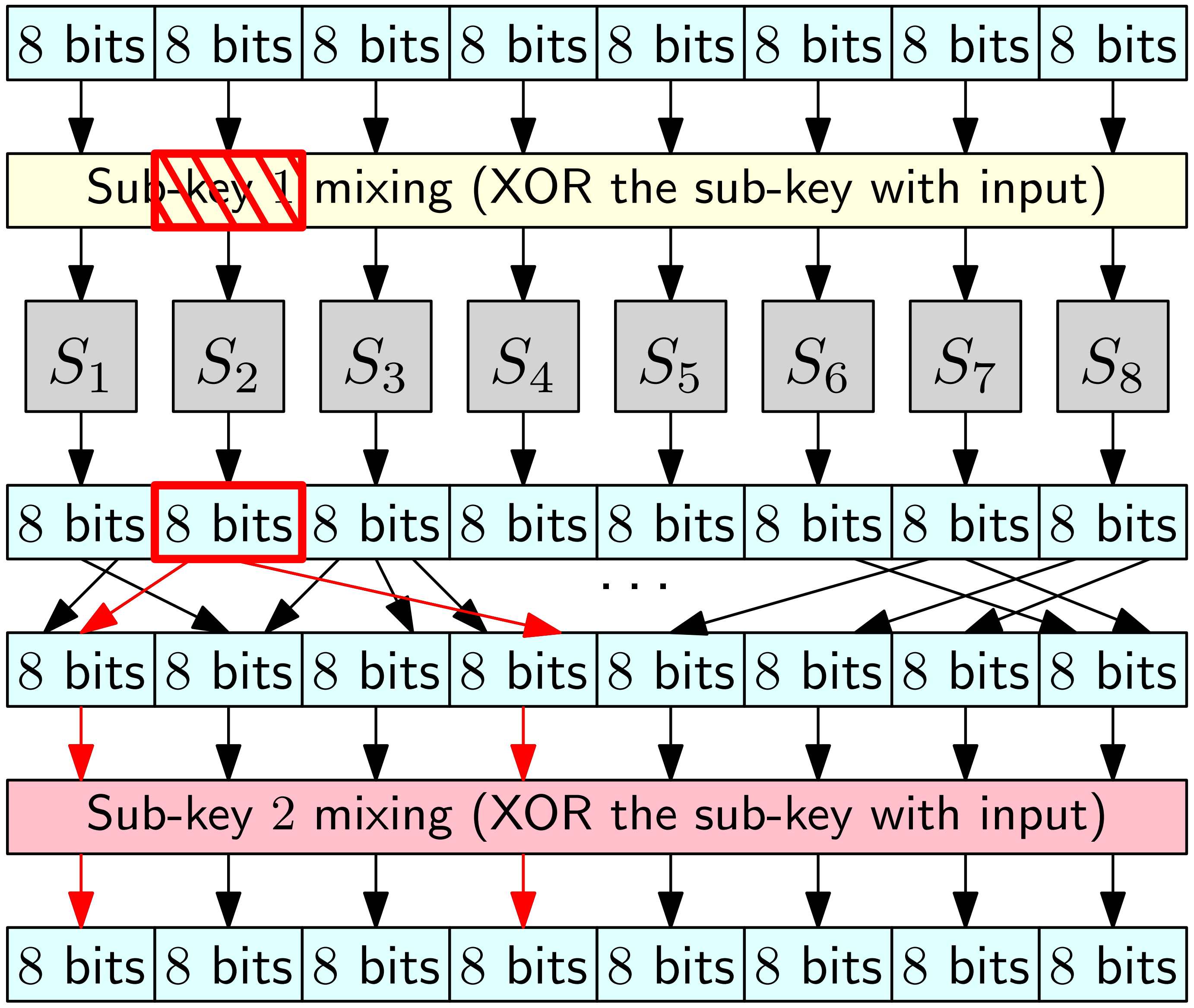

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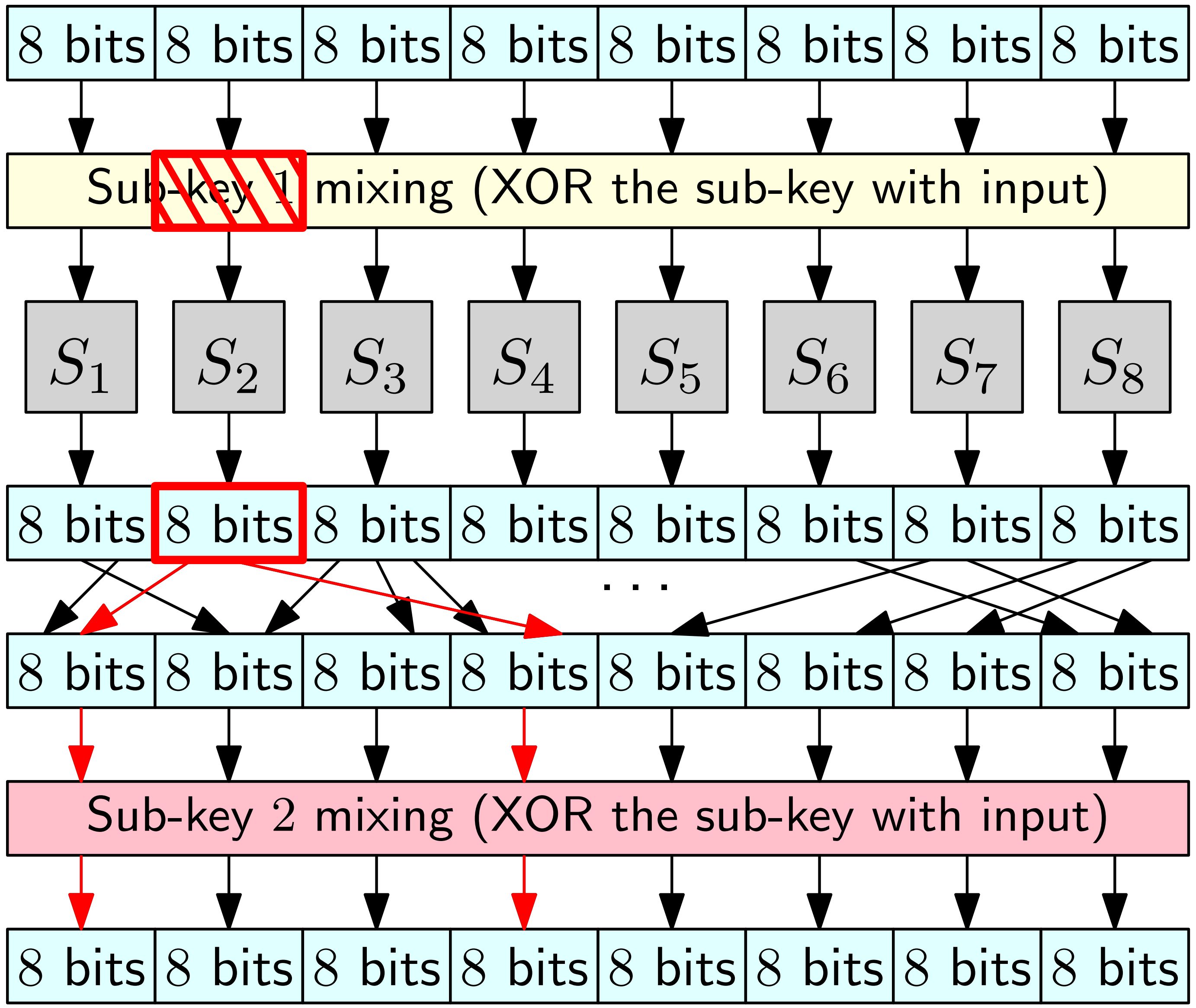

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In the example: $\approx 8 \cdot 2^8 = 2^{11}$

(intead of 2^{64} of the previous attack or 2^{128} of a naive bruteforce)

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It's common to see results of the form:

"A reduced version of [block cipher] using X instead of Y rounds has been broken"

Designing Block Ciphers

- To design a block cipher, we want the computed function to be "indistinguishable" from a uniform permutation over $\{0,1\}^\ell$

- If x and x' differ, even just by one bit, the outputs of $F_k(x)$ and $F_k(x')$ should look unrelated (except for $F_k(x) \neq F_k(x)$

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- To keep notation simple, define $f_i : \{0,1\}^{\ell/2} \to \{0,1\}^{\ell/2}$ as $f_i(x) = \widehat{f}(k_i, x)$, where k_i is the i-th sub-key

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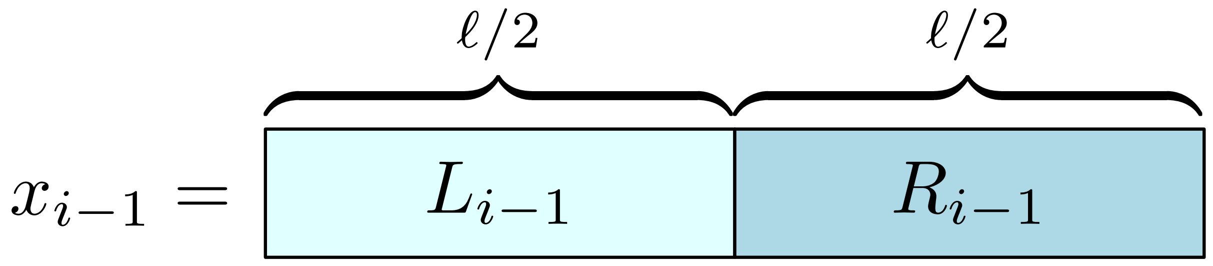

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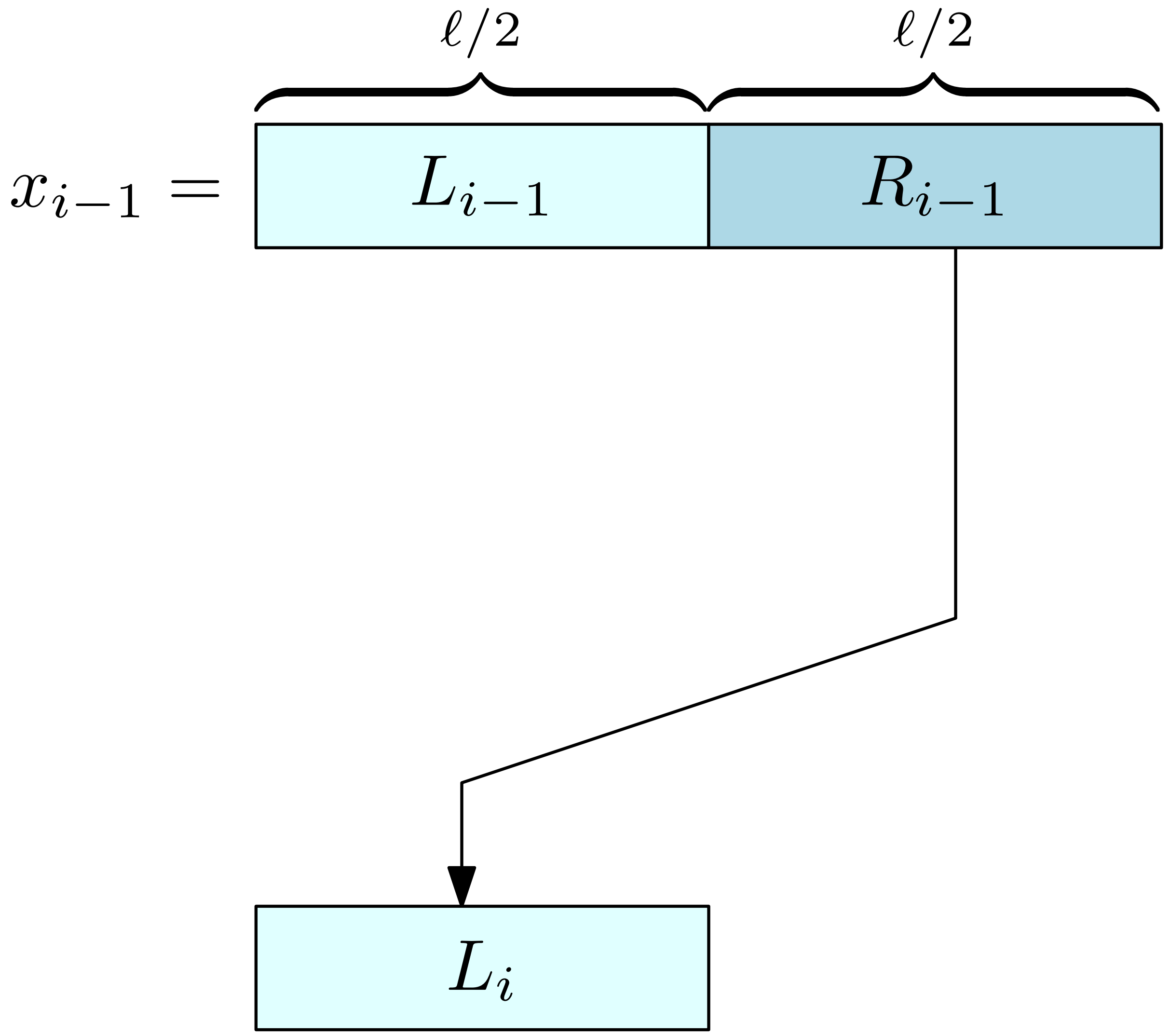

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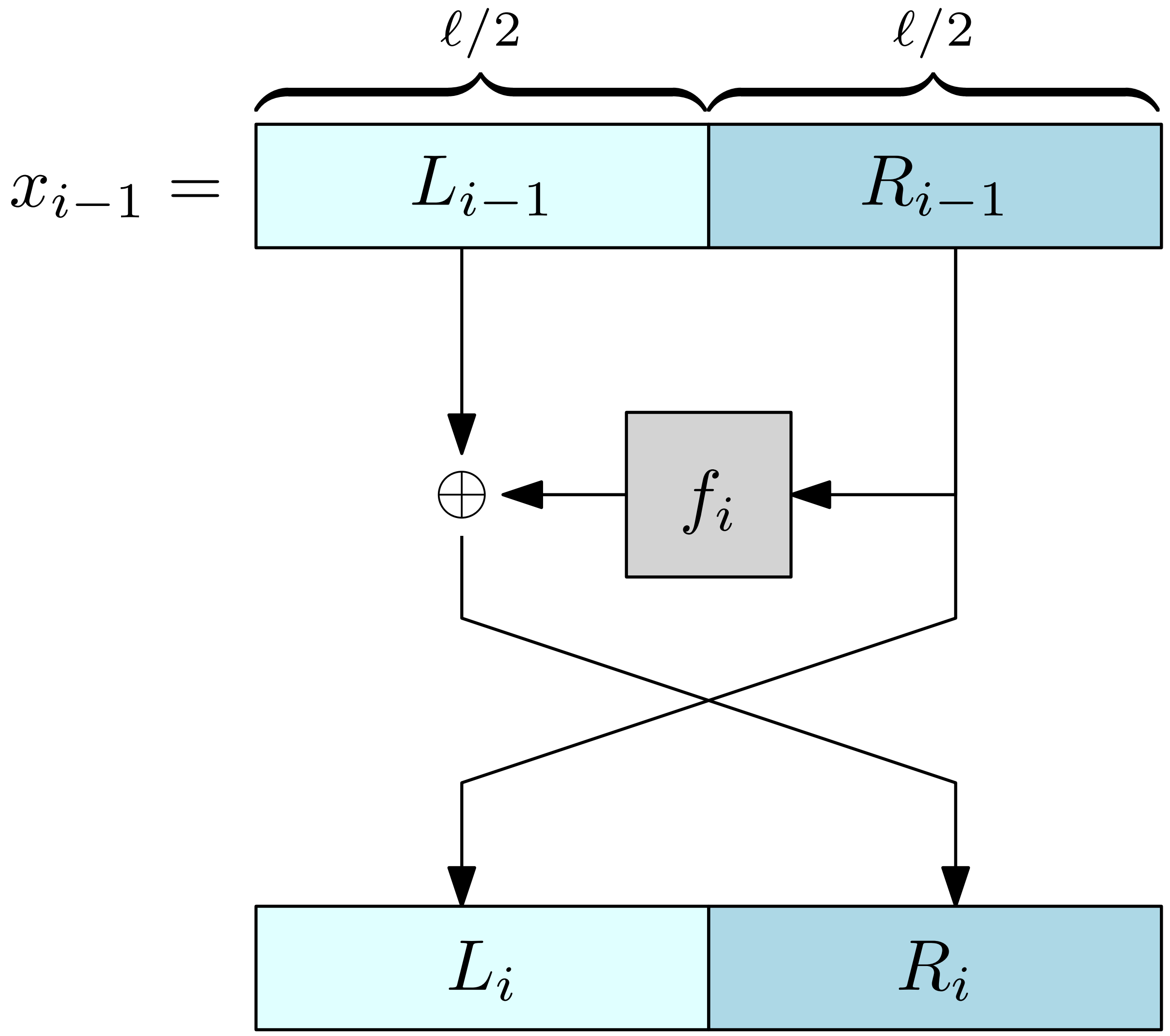

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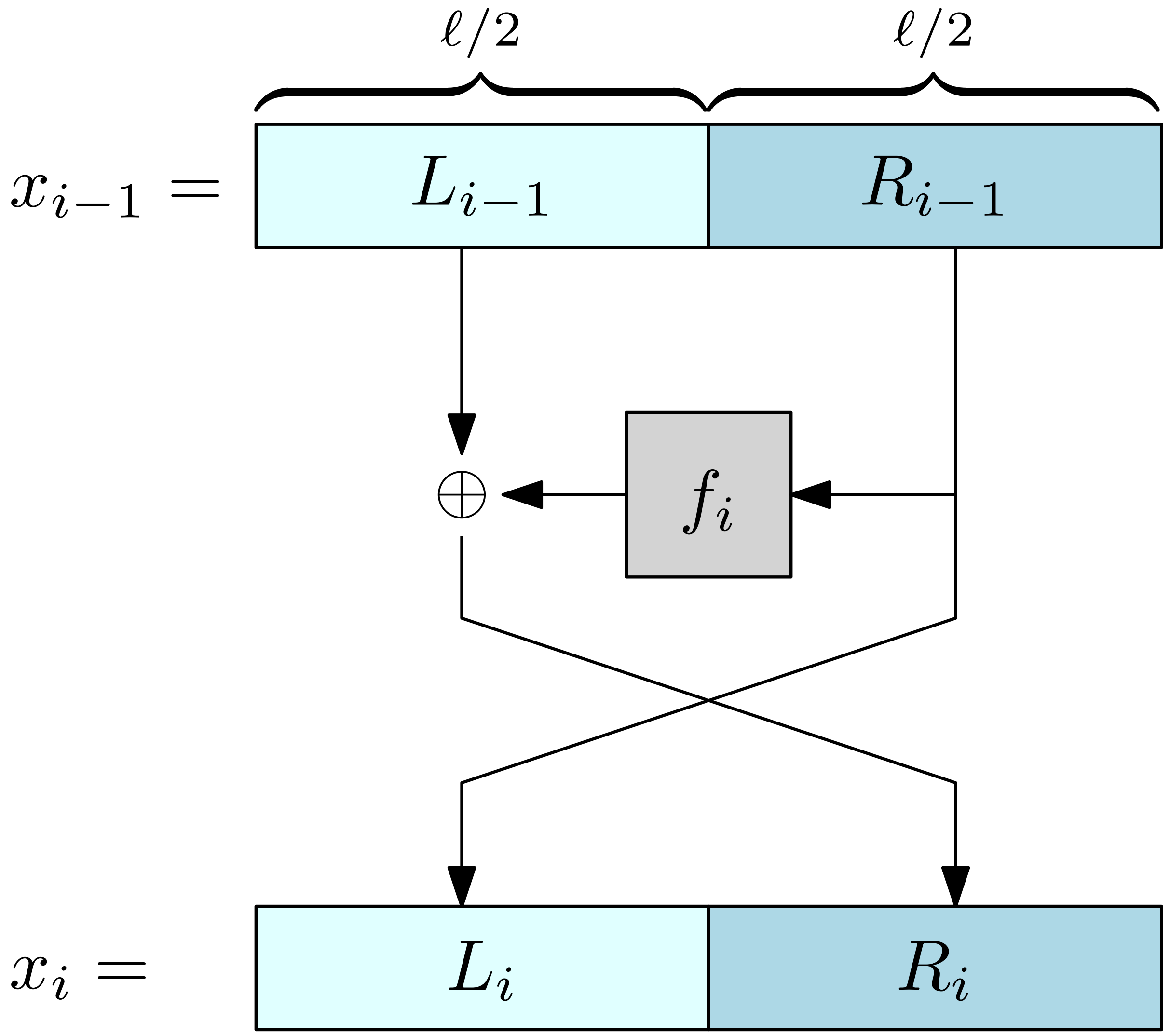
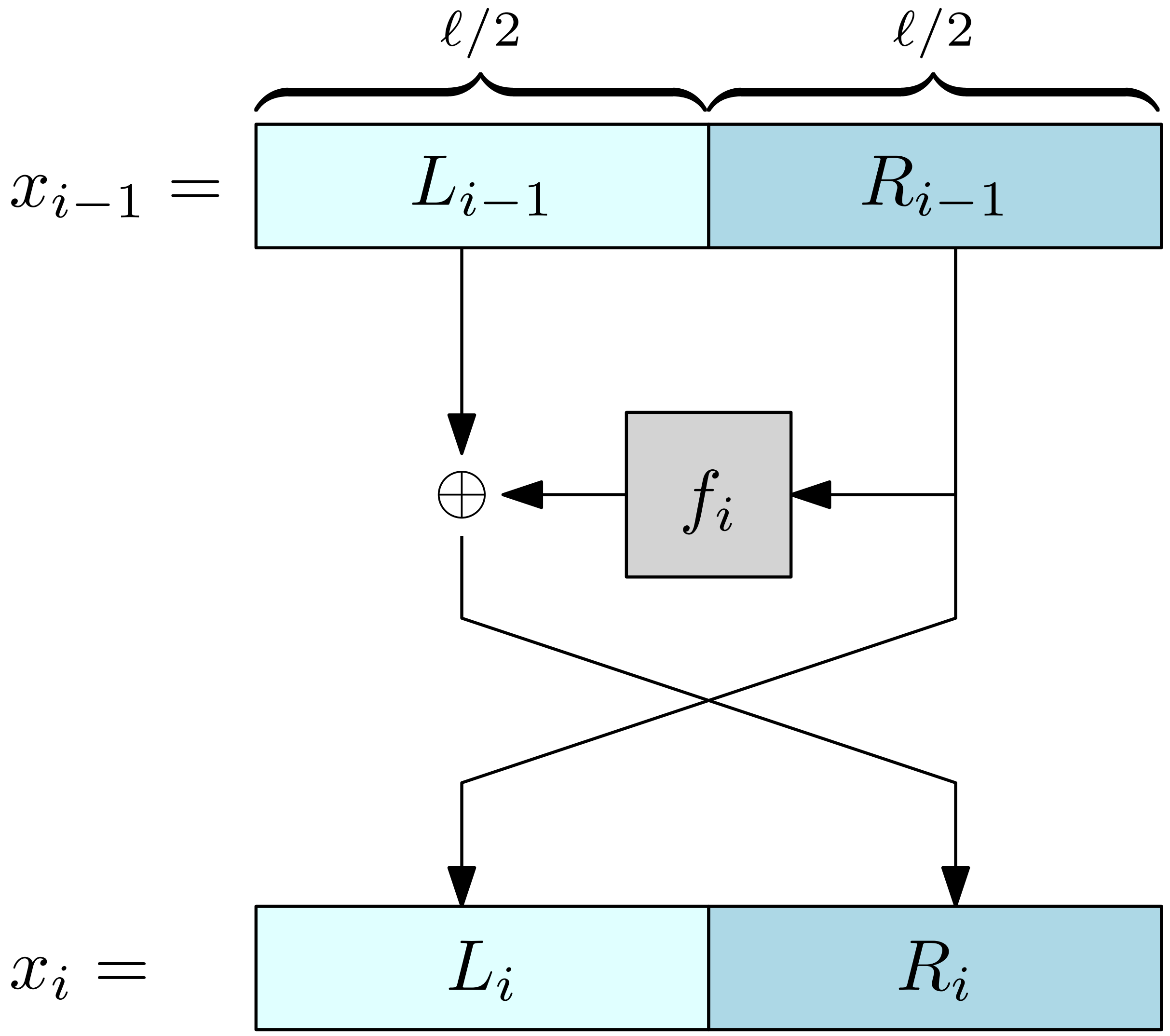


Feistel Network Rounds

- Let x_{i-1} and x_i be the input and the output of the i-th round of the Feistel Network, respectively

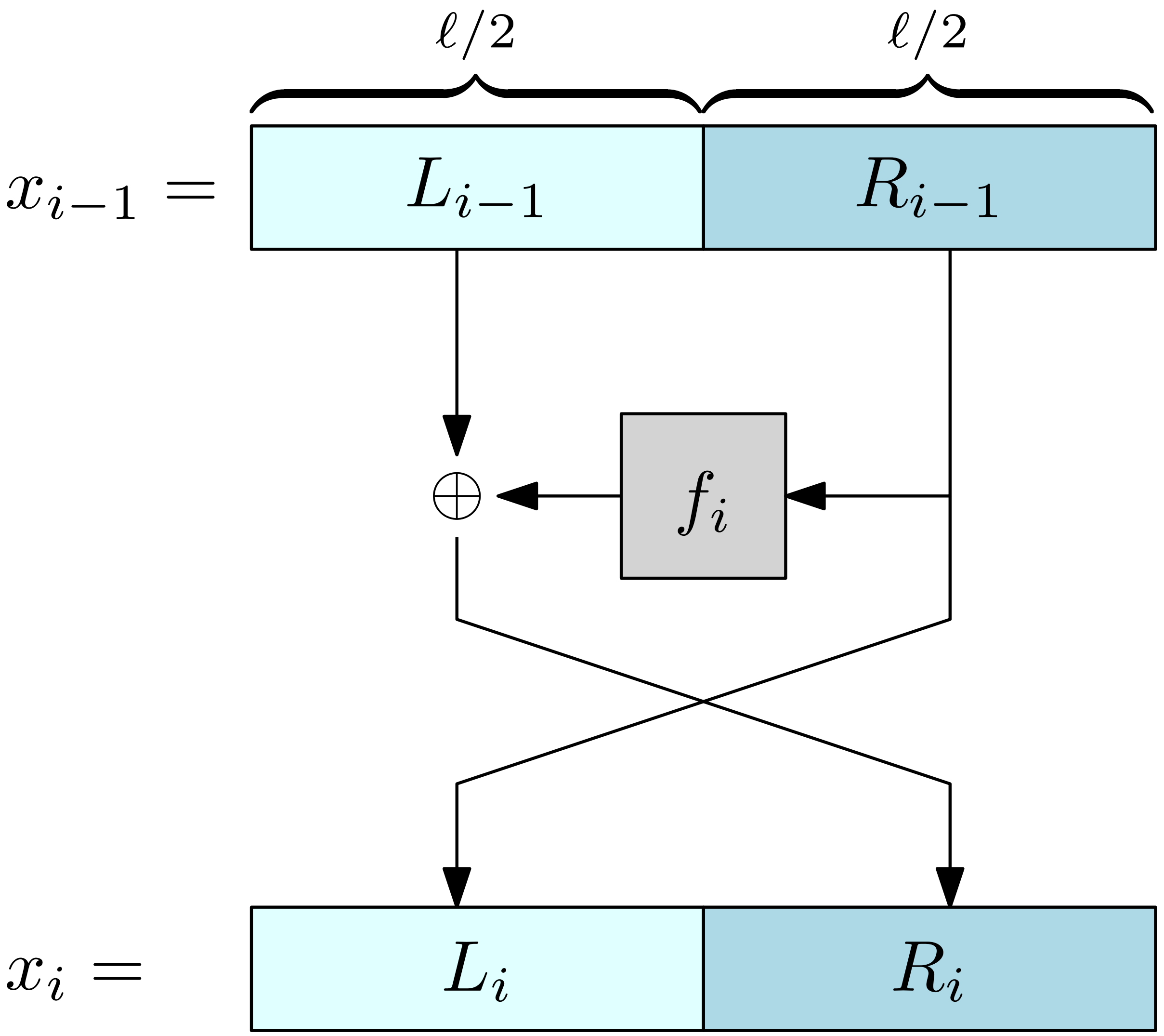
- Split each x_i into a "left side" L_i and a right side R_i, each of length $\ell/2$

- $x_{i-1} = L_{i-1} \| R_{i-1}$
- $L_i = R_{i-1}$
- $R_i = L_{i-1} \oplus f_i(R_{i-1})$
- $x_i = L_i \| R_i$



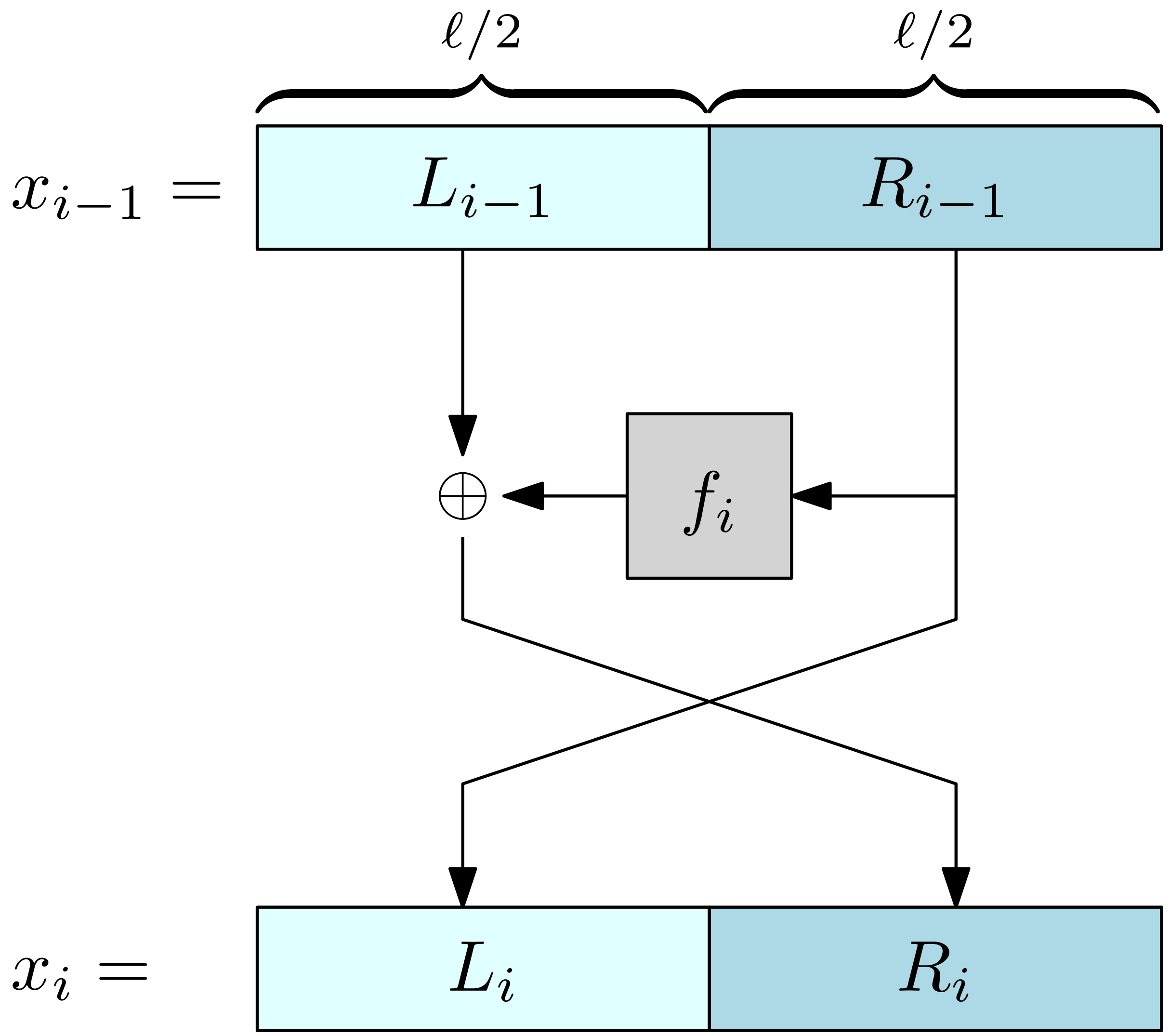
Is a Feistel Network round invertible? (How?)

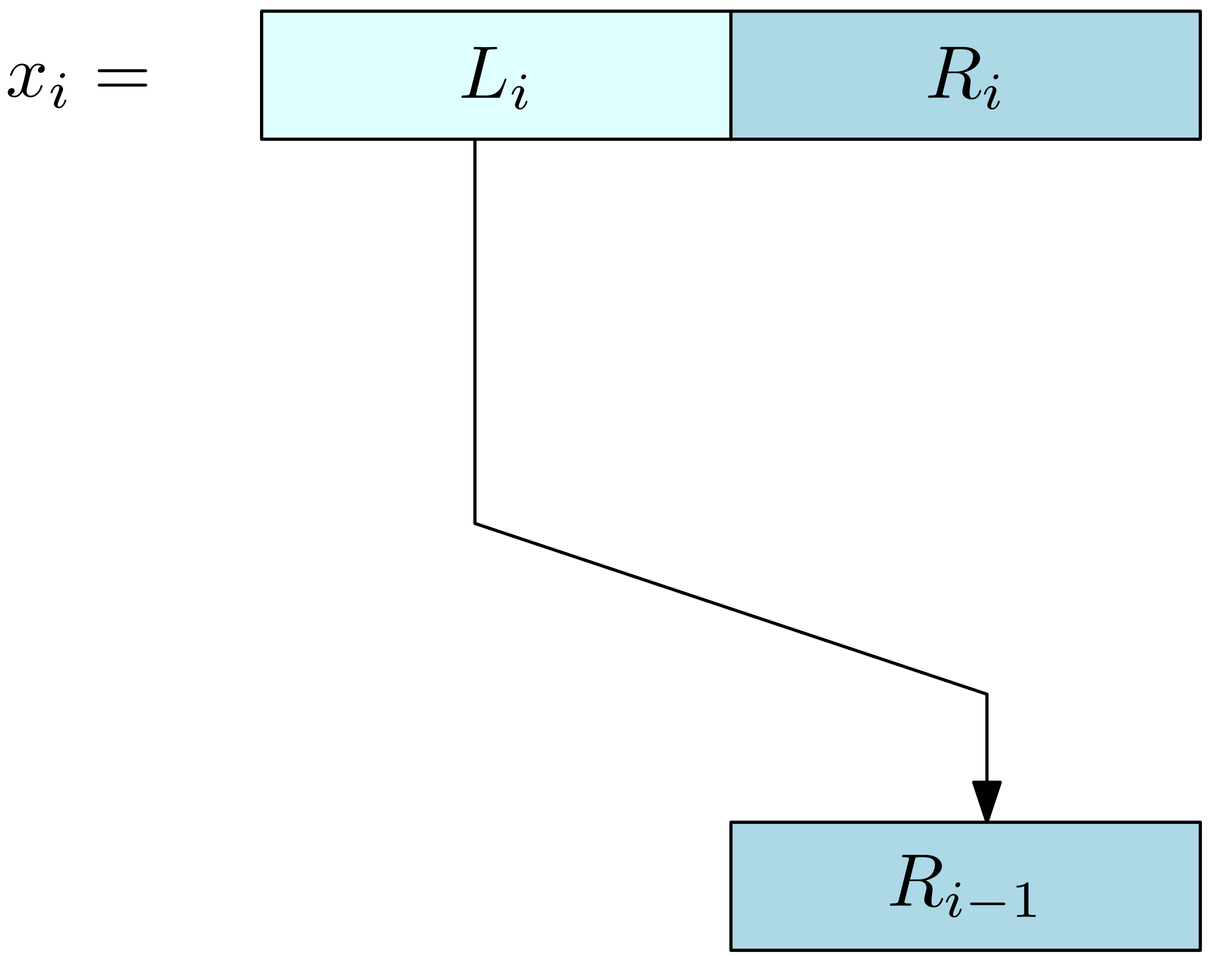
Inverting a Round of Feistel Network



- $L_i = R_{i-1}$
- $R_i = L_{i-1} \oplus f_i(R_{i-1})$

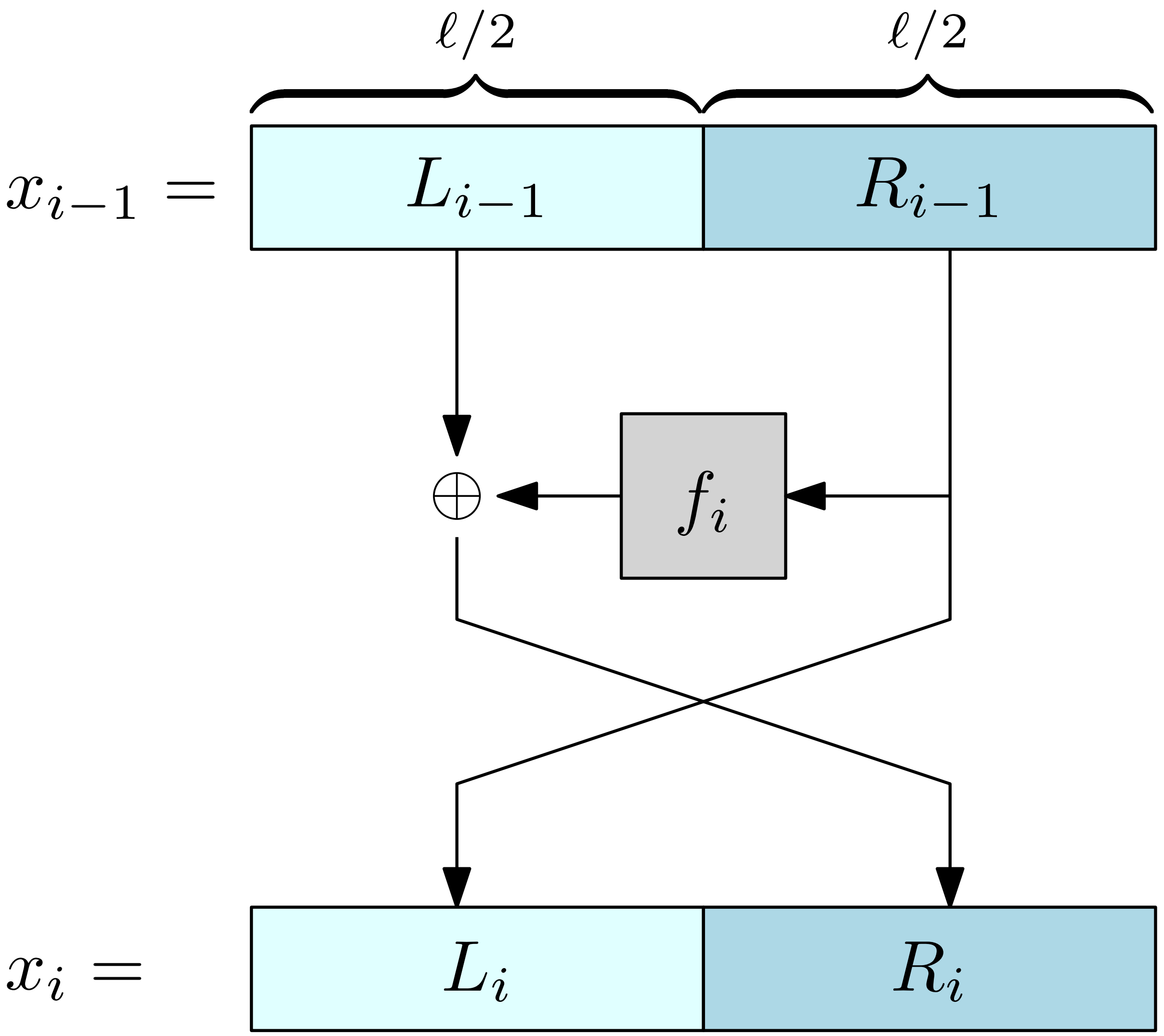
Inverting a Round of Feistel Network

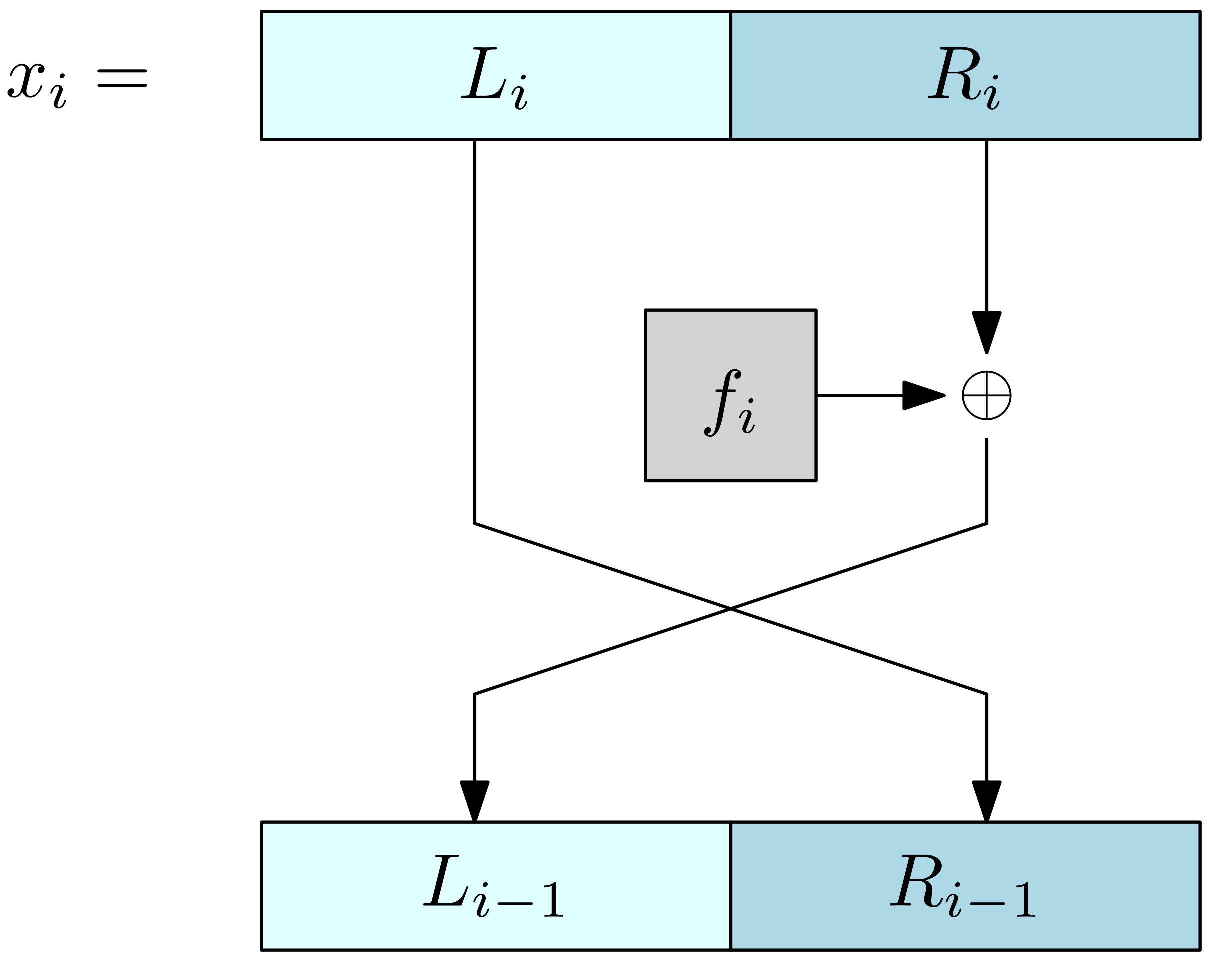




- $L_i = R_{i-1}$
- $R_i = L_{i-1} \oplus f_i(R_{i-1})$
- $R_{i-1} = L_i$

Inverting a Round of Feistel Network

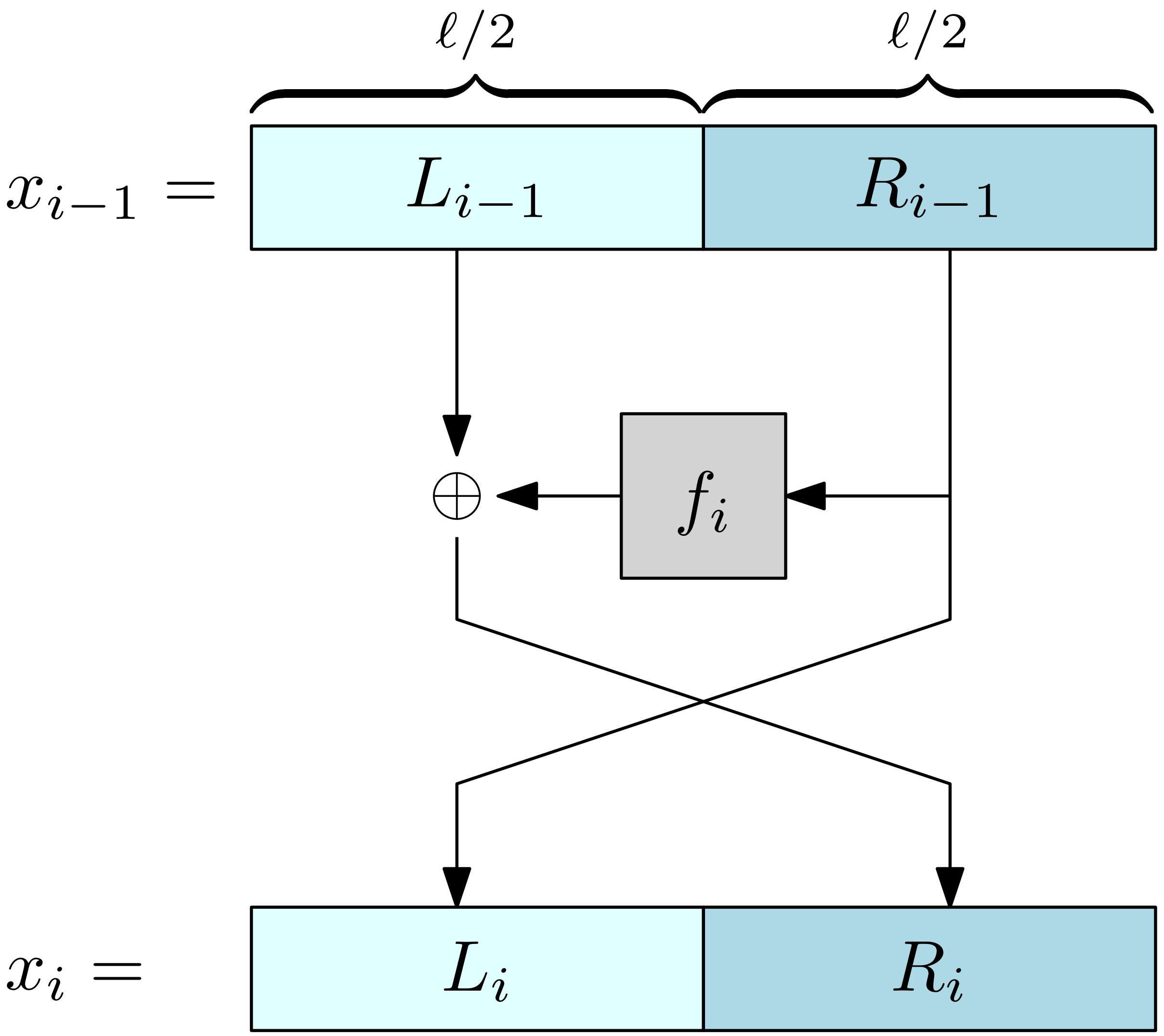


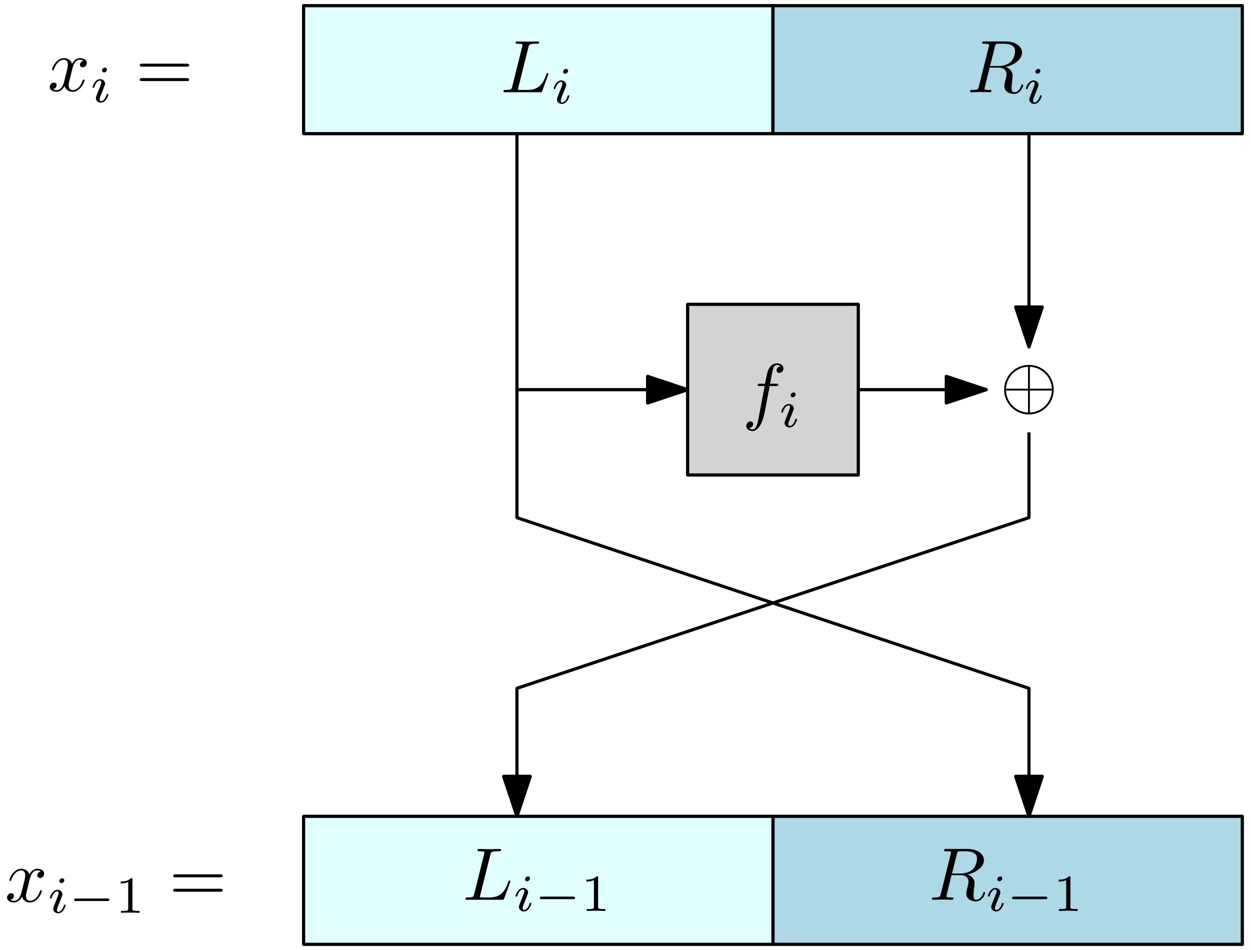


- $L_i = R_{i-1}$
- $R_i = L_{i-1} \oplus f_i(R_{i-1})$

- $R_{i-1} = L_i$
- $L_{i-1} = R_i \oplus f_i(R_{i-1})$

Inverting a Round of Feistel Network

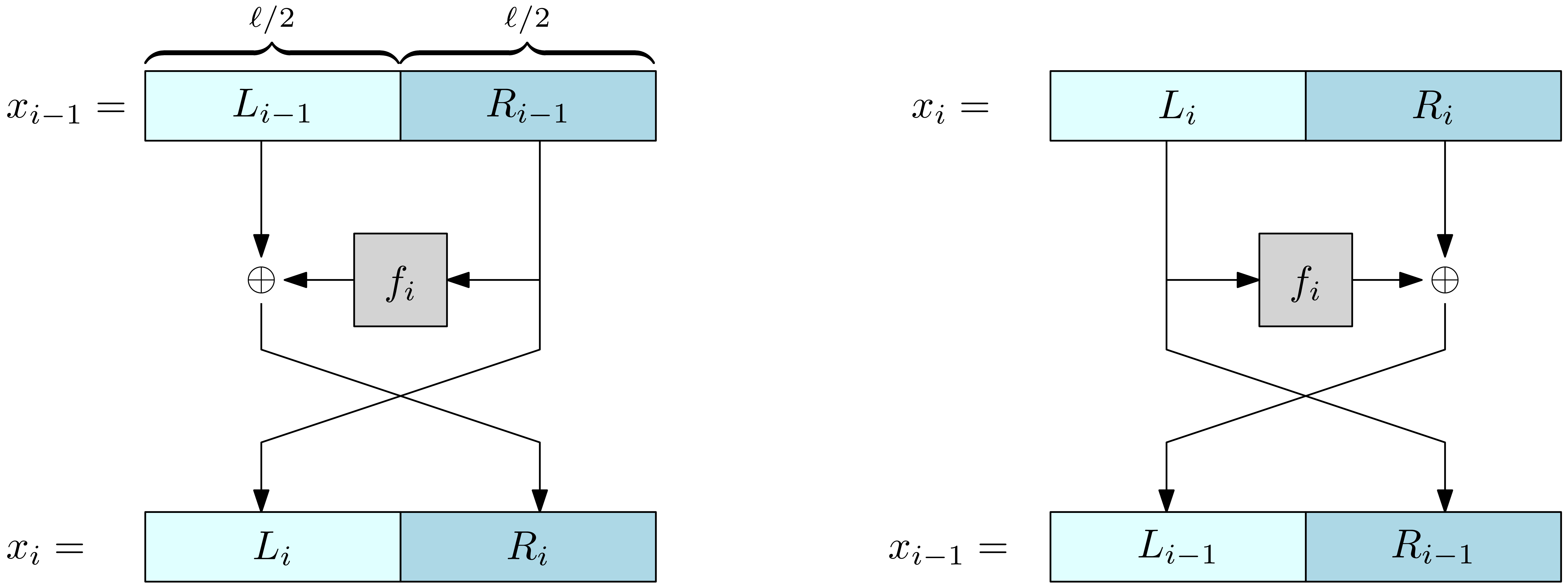




- $L_i = R_{i-1}$
- $R_i = L_{i-1} \oplus f_i(R_{i-1})$

- $R_{i-1} = L_i$
- $L_{i-1} = R_i \oplus f_i(R_{i-1}) \;= R_i \oplus f_i(L_i)$

Inverting a Round of Feistel Network

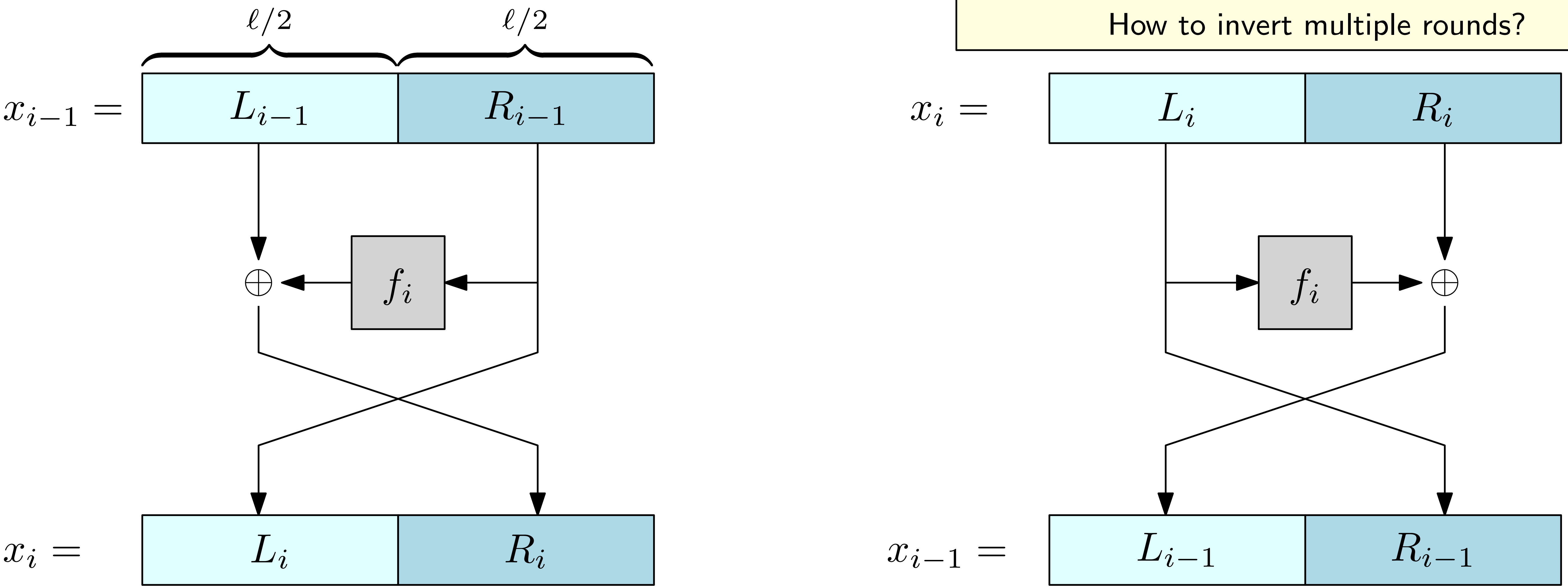


Let F be a keyed function defined by a Feistel network. Then regardless of the key schedule, the round functions $\widehat{f_i}$, and the number of rounds, F_k is a permutation for any k.

Inverting a Round of Feistel Network

F^{-1} is the same as F once the "left" and "right" sides are swapped!

How to invert multiple rounds?



Let F be a keyed function defined by a Feistel network. Then regardless of the key schedule, the round functions $\widehat{f_i}$, and the number of rounds, F_k is a permutation for any k.

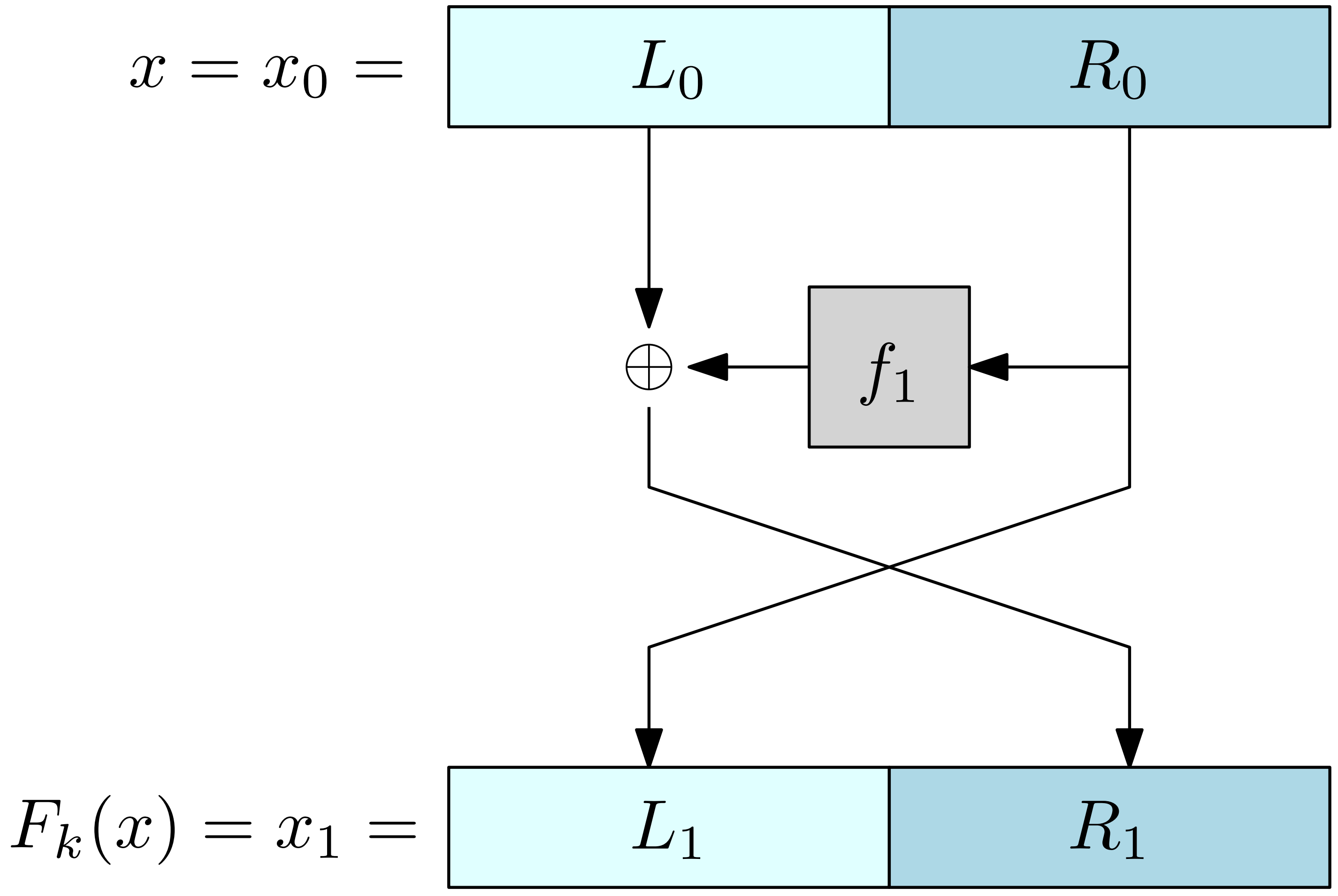
Security of 1-Round Feistel Networks

$F_k(L_0 \| R_0) = L_1 \| R_1$

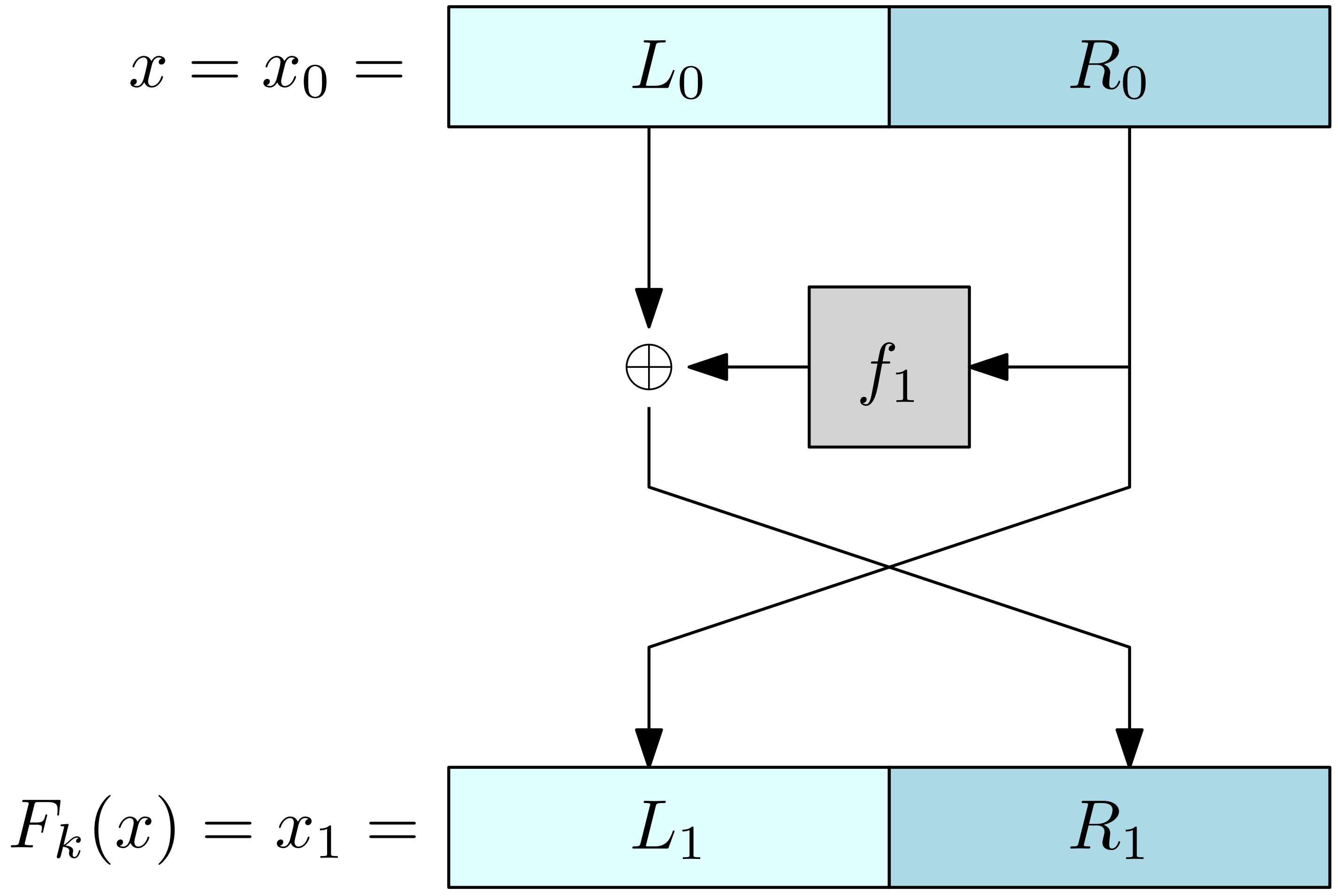
$L_1 = R_0$

$R_1 = L_0 \oplus f_1(R_0)$

Is this a Pseudorandom permutation?



Security of 1-Round Feistel Networks



$F_k(L_0 \,\|\, R_0) = L_1 \,\|\, R_1$

$L_1 = R_0$

$R_1 = L_0 \oplus f_1(R_0)$

Is this a Pseudorandom permutation?

- No! $F_k(x)$ can be easily distinguished from a random permutation

How?

Security of 1-Round Feistel Networks

$F_k(L_0 \| R_0) = L_1 \| R_1$

$L_1 = R_0$

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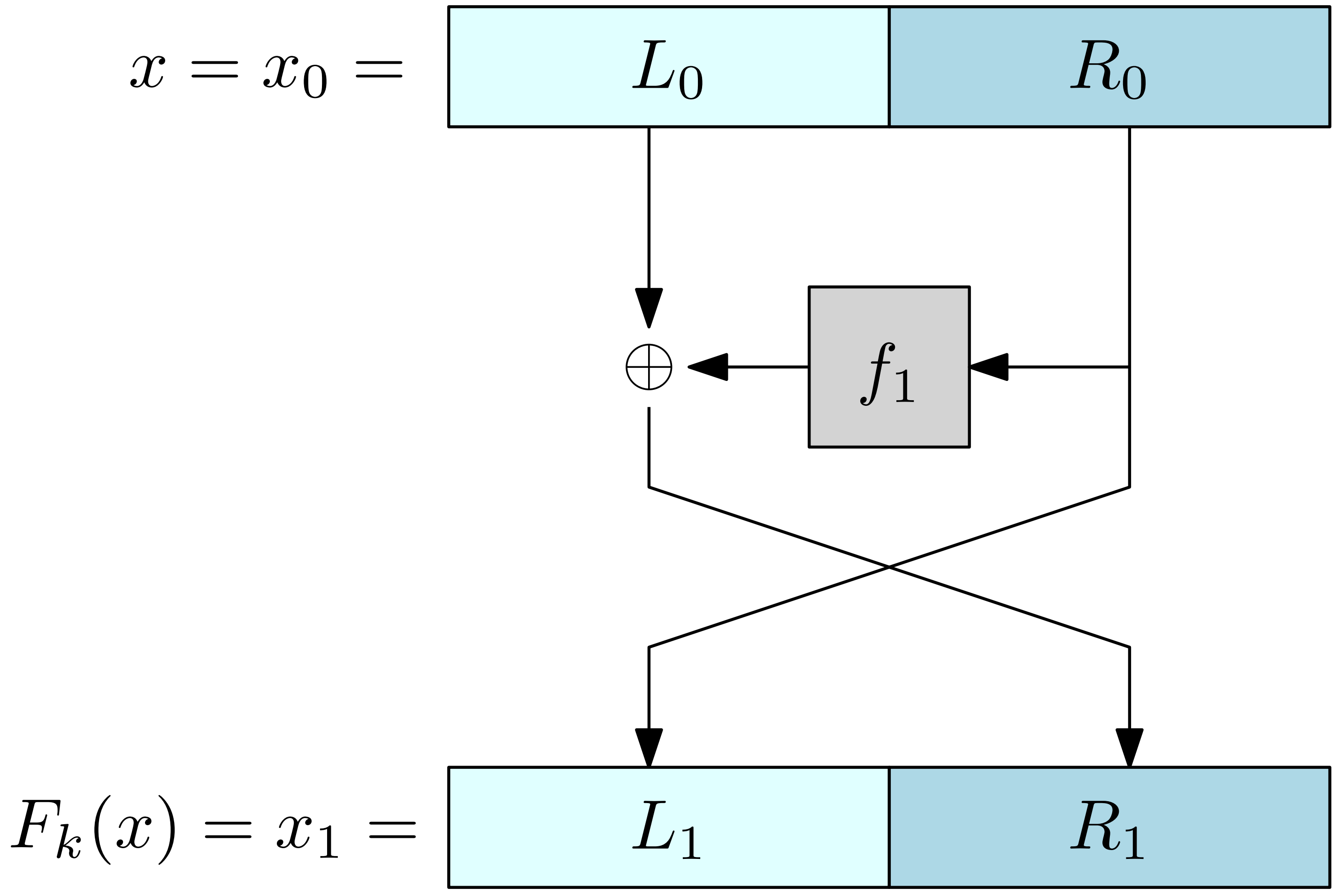


Is this a Pseudorandom permutation?

- No! $F_k(x)$ can be easily distinguished from a random permutation

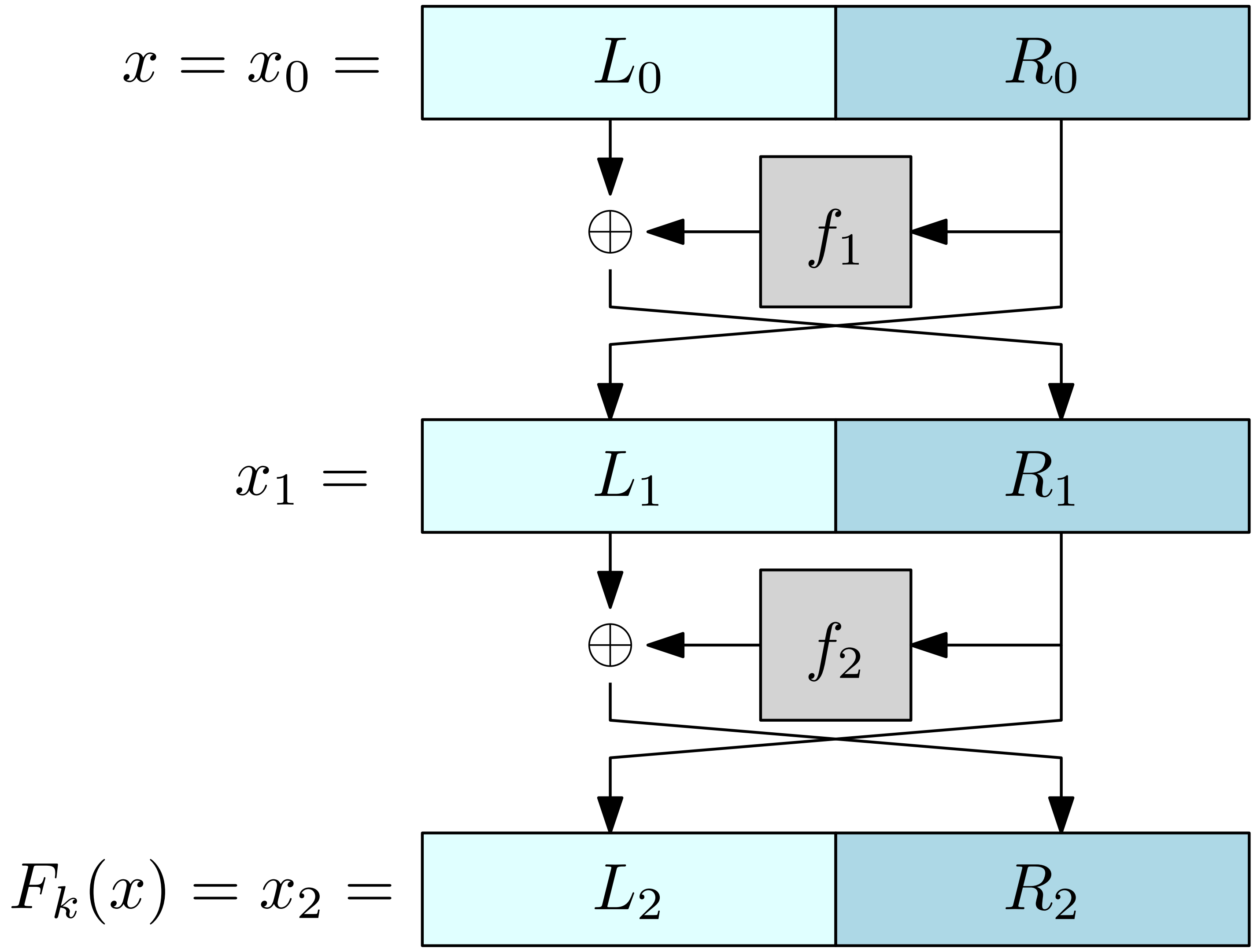
How?

- The adversary can simply query $x = 0^\ell$ and check whether the left $\ell/2$ bits of $F_k(x)$ are all 0

(or use any other string x and check whether the left half of $F_k(x)$ is equal to the right half of x)

Security of 2-Round Feistel Networks

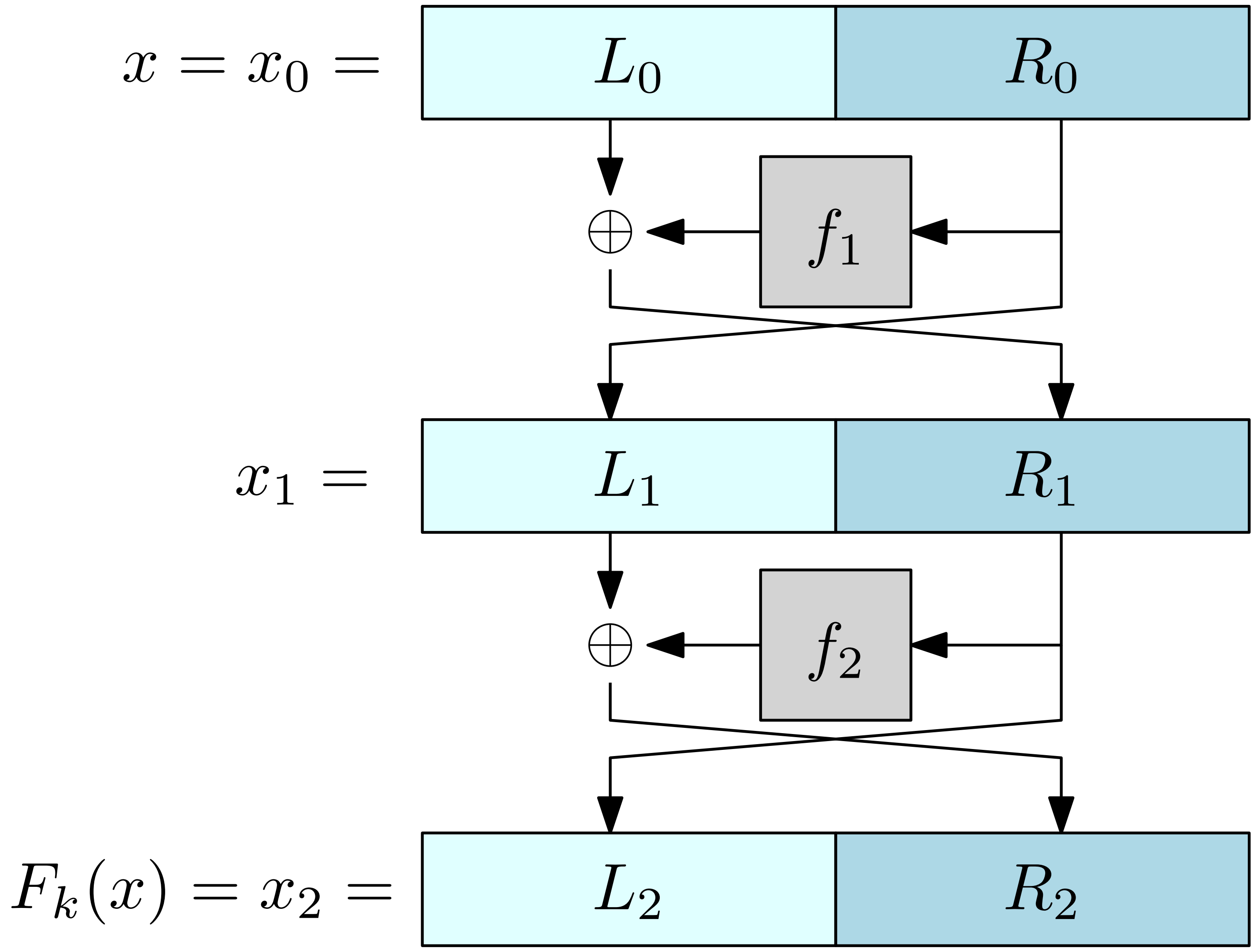
$F_k(L_0 \,\|\, R_0) = L_2 \,\|\, R_2$



Security of 2-Round Feistel Networks

$F_k(L_0 \,\|\, R_0) = L_2 \,\|\, R_2$

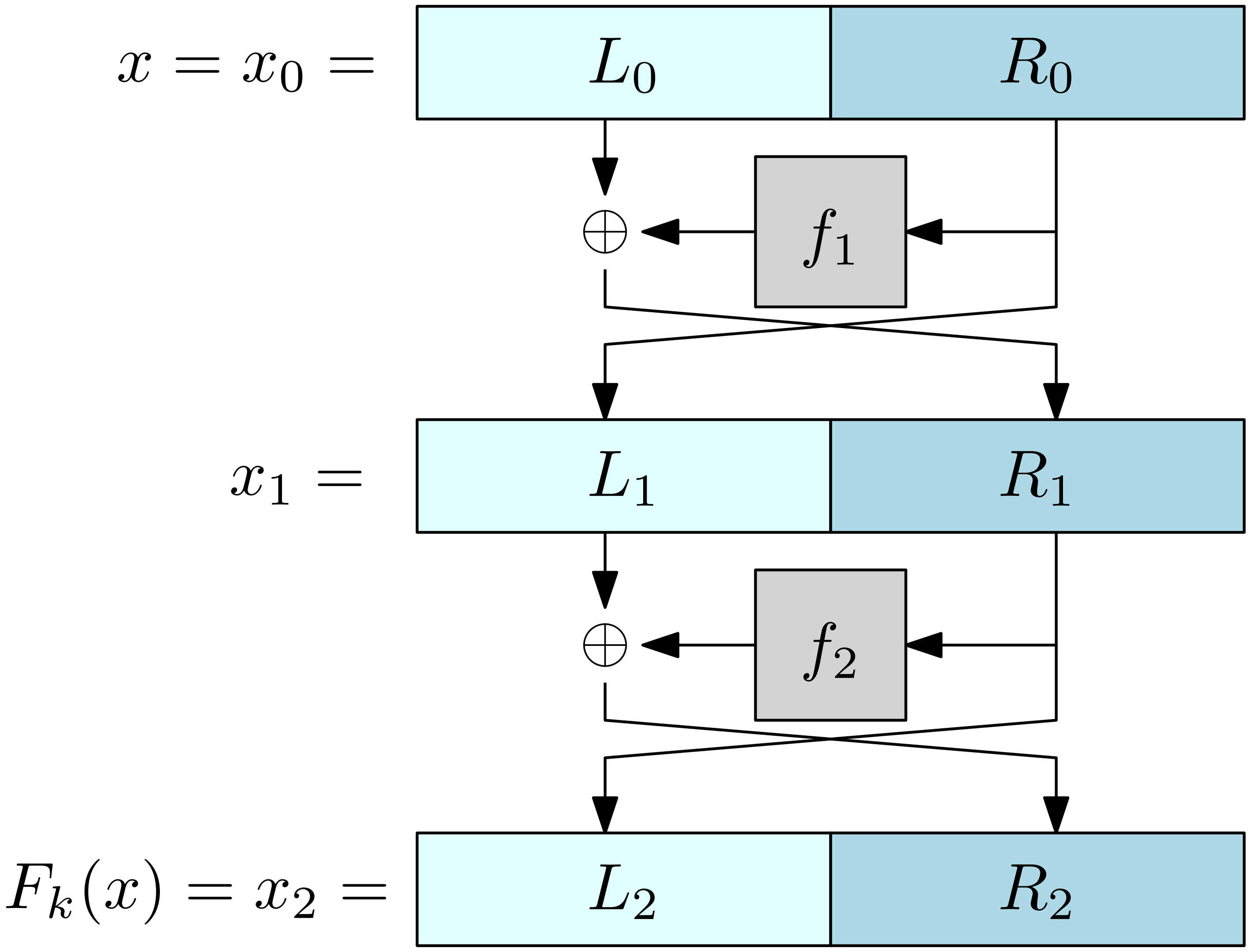
$L_2 = R_1$



Security of 2-Round Feistel Networks

$$F_k(L_0 \,\|\, R_0) = L_2 \,\|\, R_2$$

$$L_2 = R_1 = L_0 \oplus f_1(R_0)$$

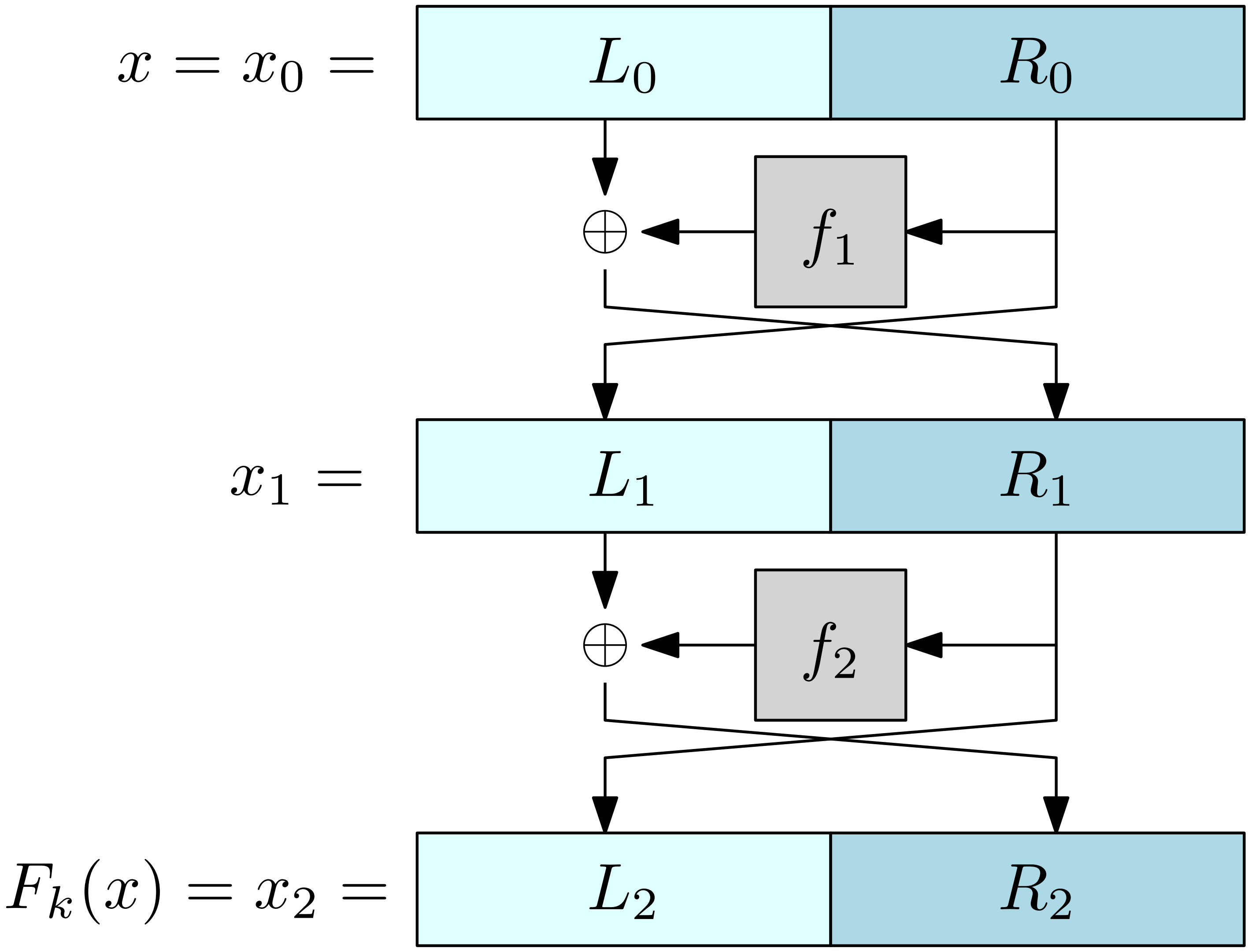


Security of 2-Round Feistel Networks

$F_k(L_0 \,\|\, R_0) = L_2 \,\|\, R_2$

$L_2 = R_1 = L_0 \oplus f_1(R_0)$

$R_2 = L_1 \oplus f_2(R_1)$



Security of 2-Round Feistel Networks

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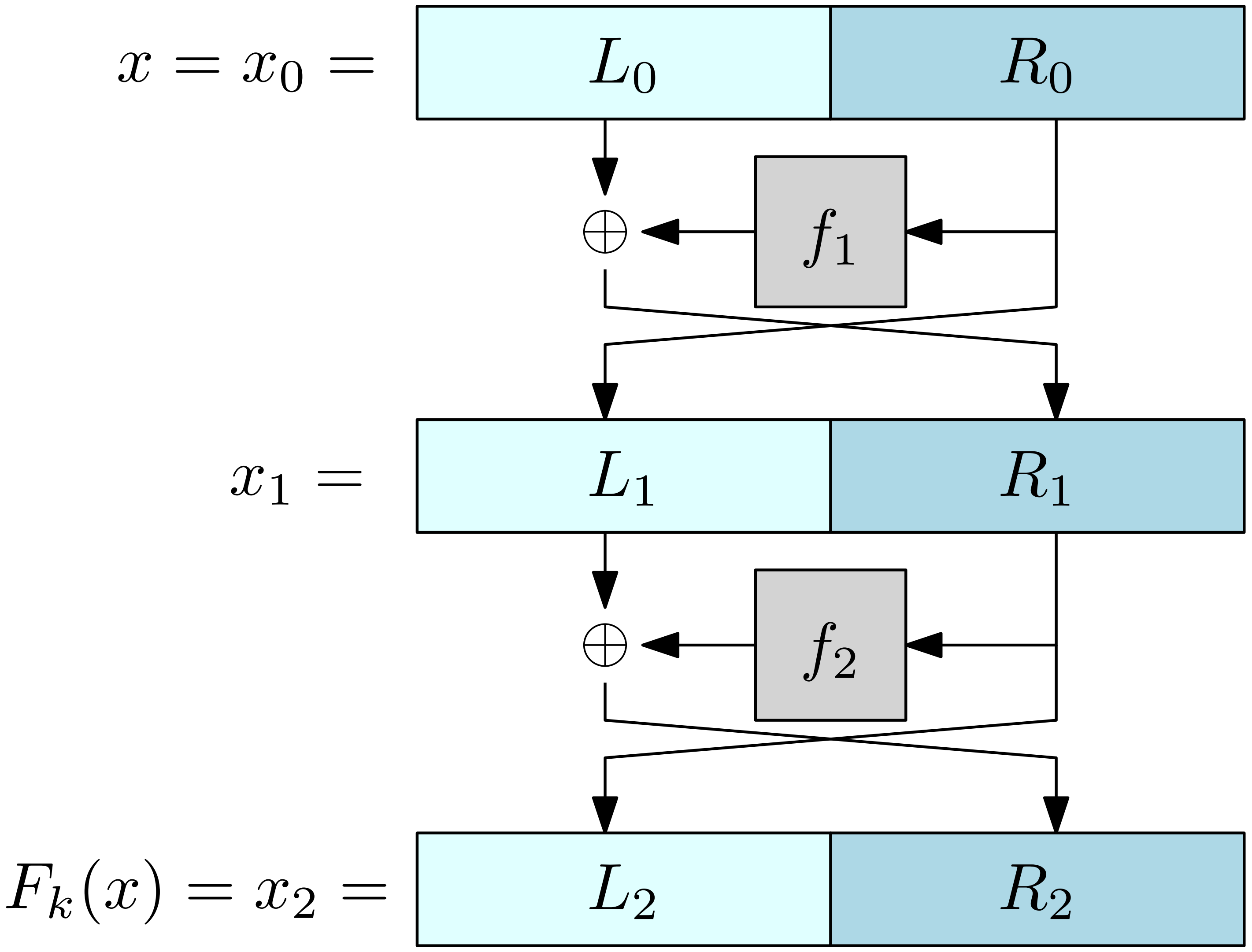
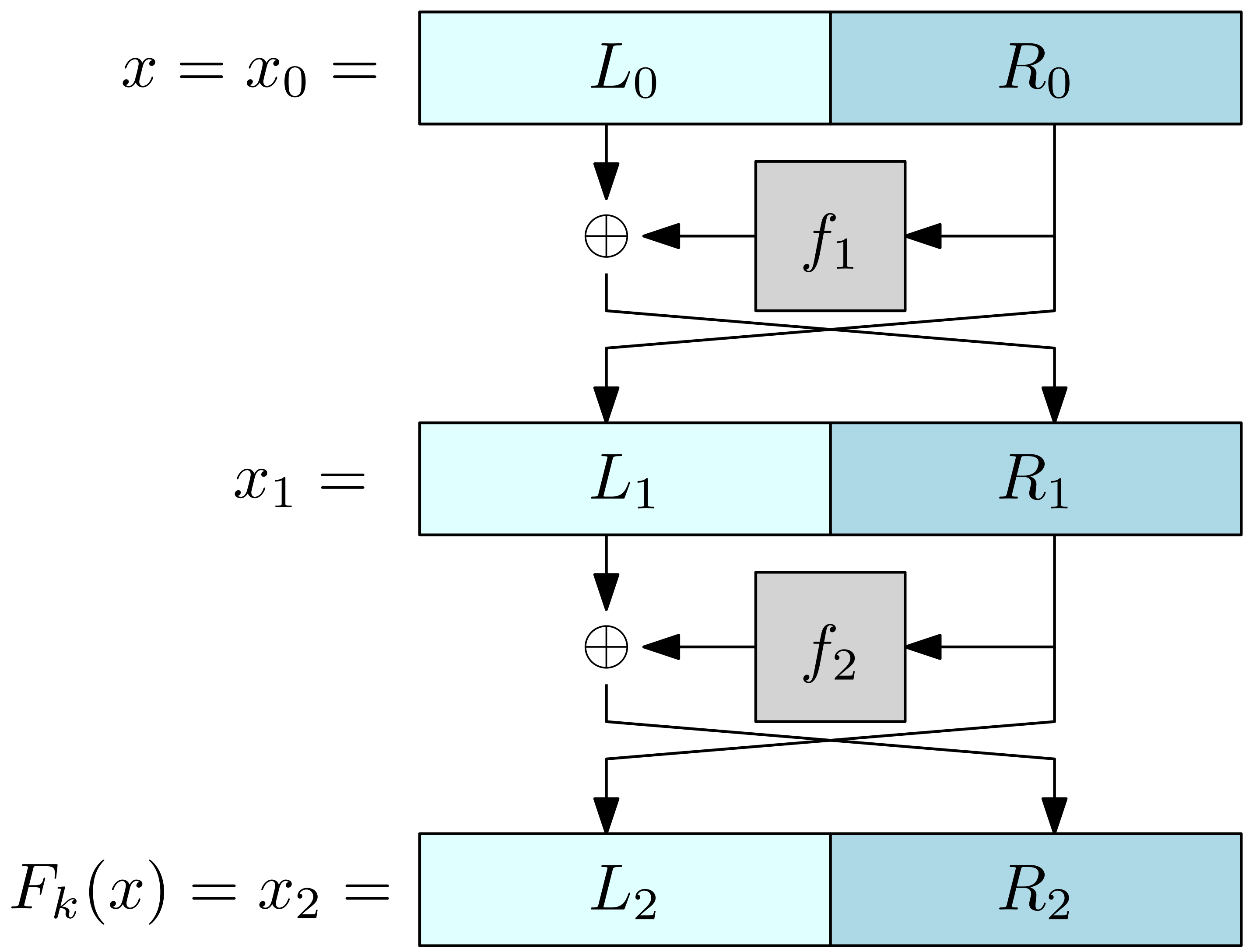


Security of 2-Round Feistel Networks

$F_k(L_0 \,\|\, R_0) = L_2 \,\|\, R_2$

$L_2 = R_1 = L_0 \oplus f_1(R_0)$

$$\begin{aligned} R_2 &= L_1 \oplus f_2(R_1) \\ &= R_0 \oplus f_2(R_1) \\ &= R_0 \oplus f_2(\, L_0 \oplus f_1(R_0)\,) \end{aligned}$$



Security of 2-Round Feistel Networks

$F_k(L_0 \| R_0) = L_2 \| R_2$

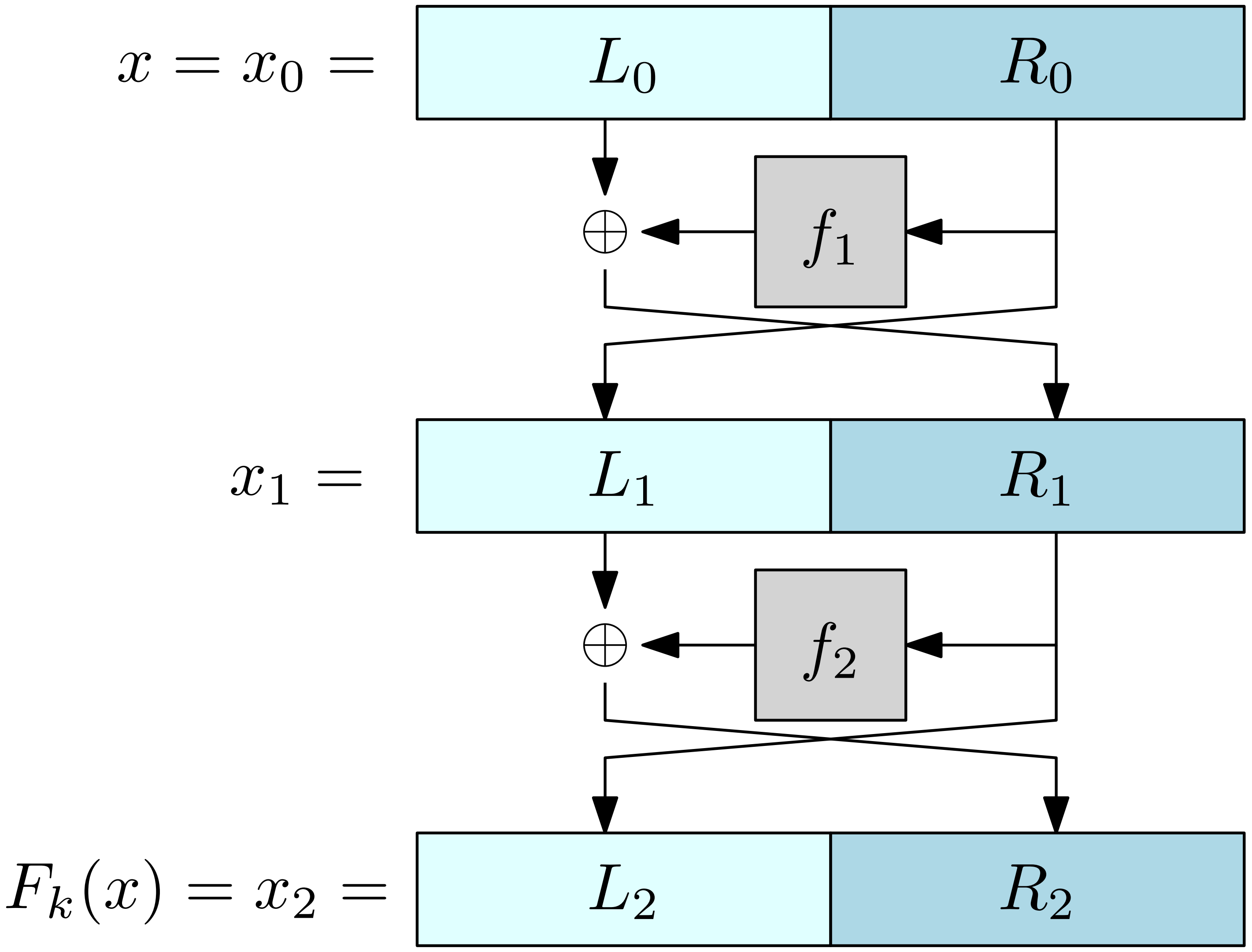
$L_2 = R_1 = L_0 \oplus f_1(R_0)$

$R_2 = L_1 \oplus f_2(R_1)$

$= R_0 \oplus f_2(R_1)$

$= R_0 \oplus f_2(\, L_0 \oplus f_1(R_0)\,)$

Is this a Pseudorandom permutation?



Security of 2-Round Feistel Networks

$F_k(L_0 \,\|\, R_0) = L_2 \,\|\, R_2$

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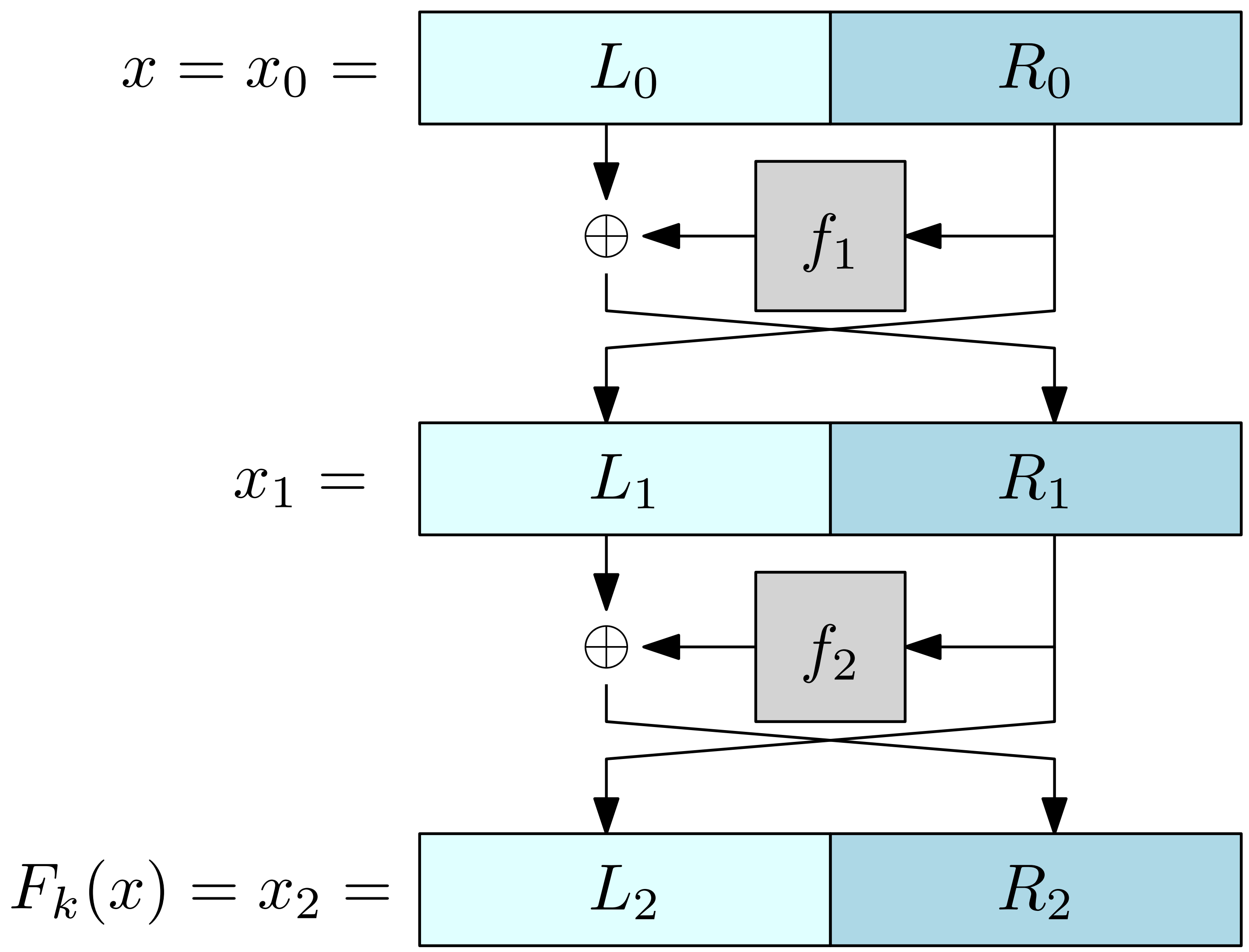
$= R_0 \oplus f_2(R_1)$

$= R_0 \oplus f_2(\, L_0 \oplus f_1(R_0)\,)$

Is this a Pseudorandom permutation?

No! Consider two different inputs $L_0 \| R_0$ and $L_0' \| R_0'$



Security of 2-Round Feistel Networks

$F_k(L_0 \,\|\, R_0) = L_2 \,\|\, R_2$

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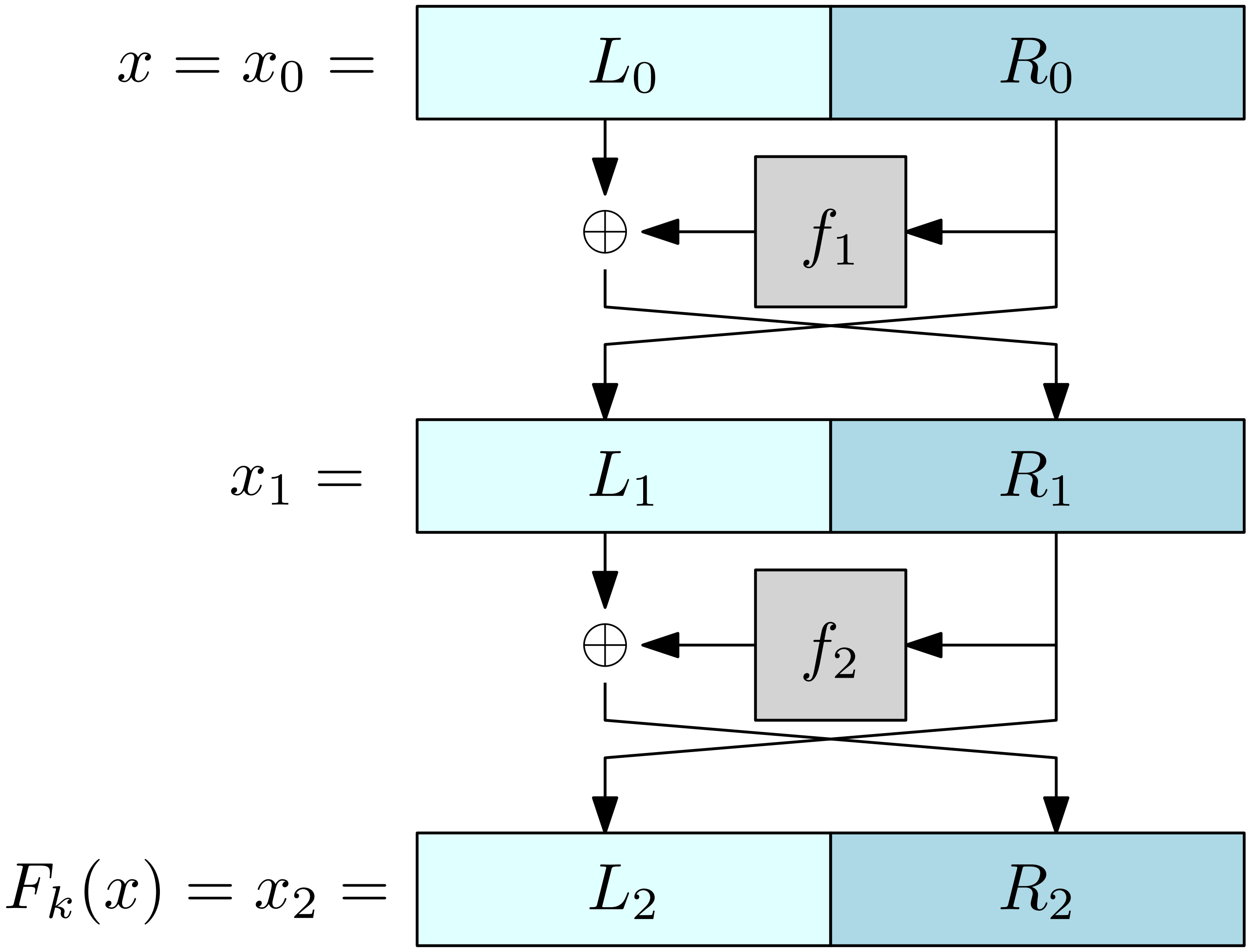

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No! Consider two different inputs $L_0 \| R_0$ and $L_0' \| R_0'$

$L_2 \oplus L_2'$

Security of 2-Round Feistel Networks

$F_k(L_0 \,\|\, R_0) = L_2 \,\|\, R_2$

$L_2 = R_1 = L_0 \oplus f_1(R_0)$

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$\quad = R_0 \oplus f_2(R_1)$

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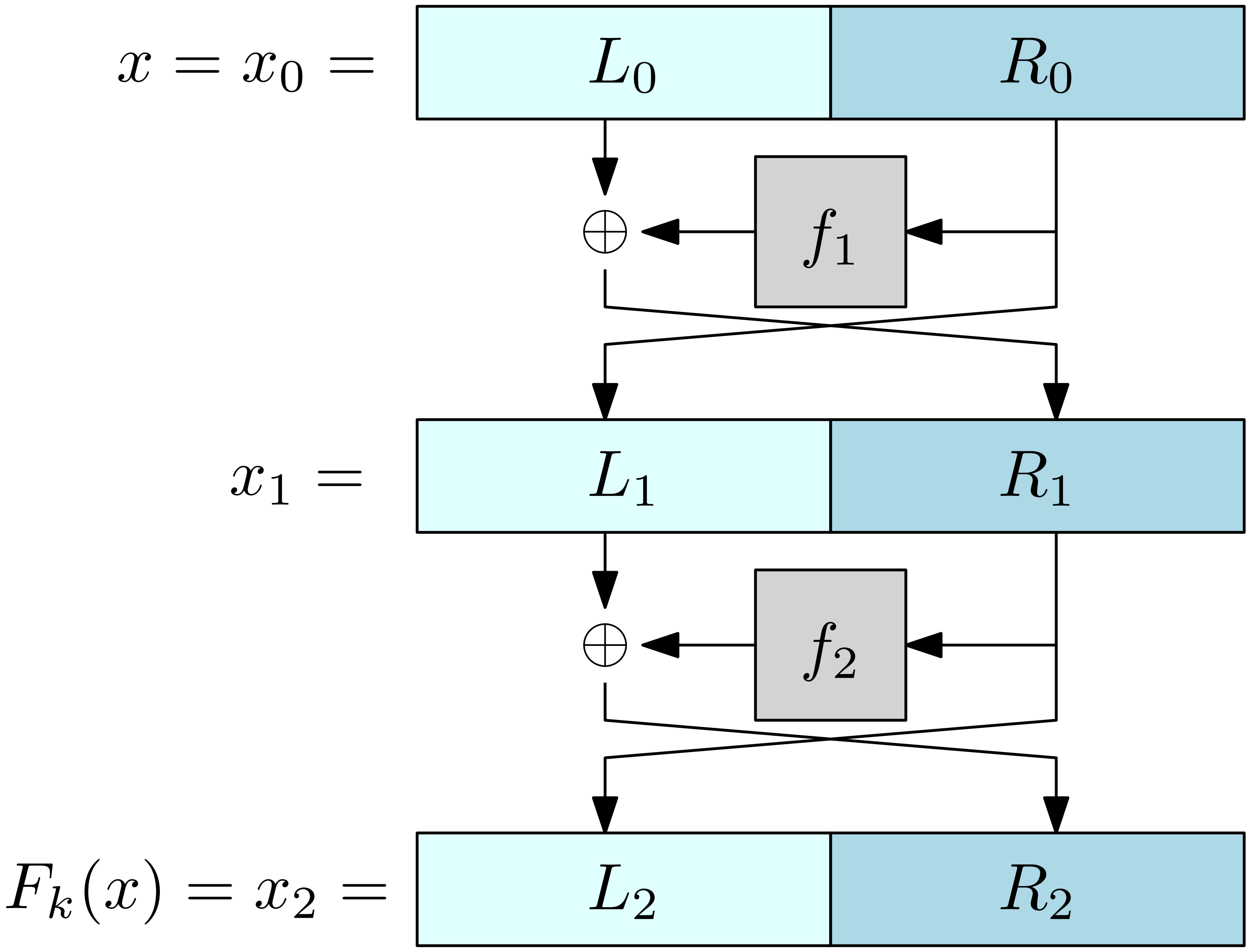


Is this a Pseudorandom permutation?

No! Consider two different inputs
$L_0 \| R_0$ and $L'_0 \| R'_0$

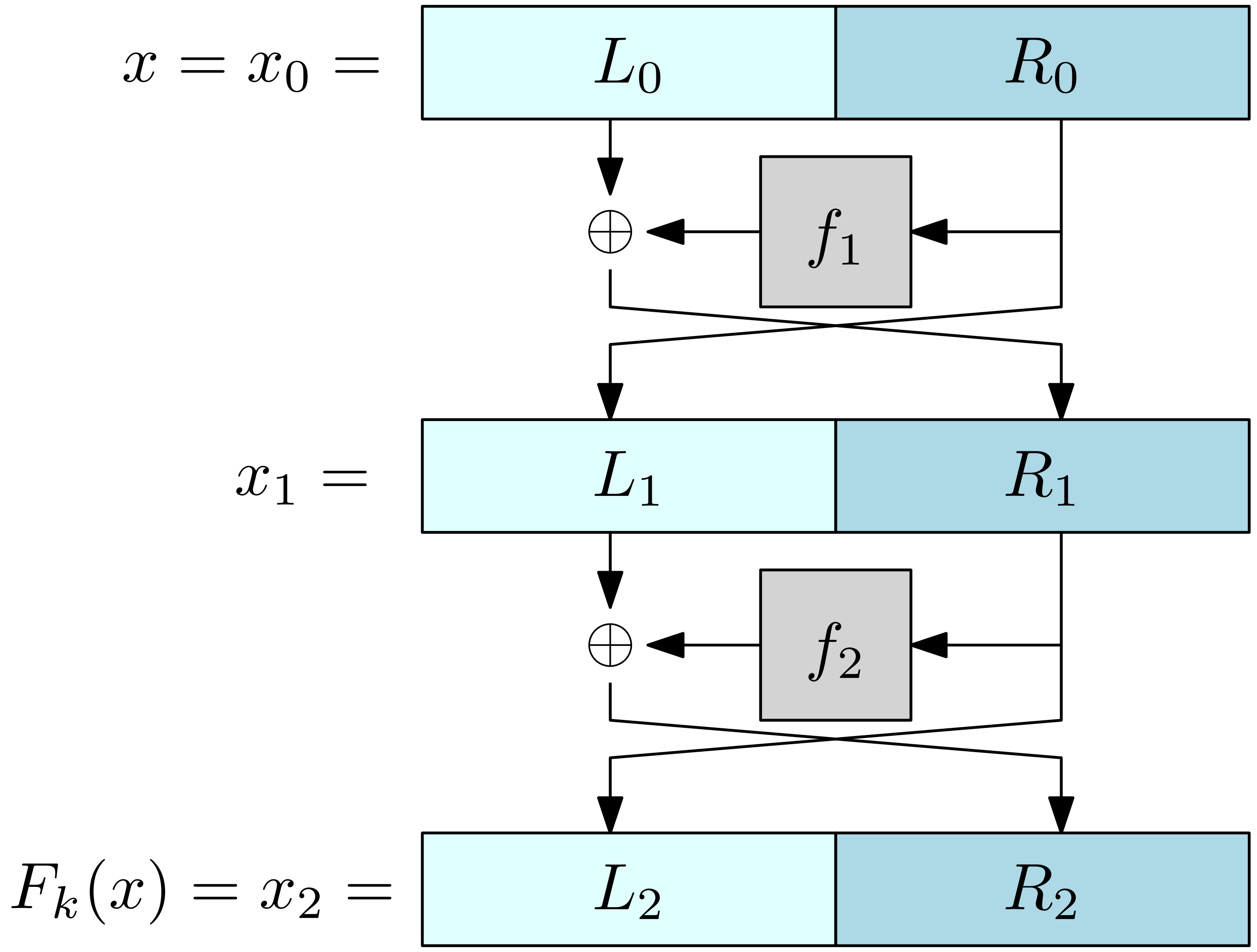
$L_2 \oplus L'_2 \; = L_0 \oplus f_1(R_0) \oplus L'_0 \oplus f_1(R'_0)$

Security of 2-Round Feistel Networks

$F_k(L_0 \,\|\, R_0) = L_2 \,\|\, R_2$

$L_2 = R_1 = L_0 \oplus f_1(R_0)$

$$
\begin{aligned}
R_2 &= L_1 \oplus f_2(R_1) \\
&= R_0 \oplus f_2(R_1) \\
&= R_0 \oplus f_2(\, L_0 \oplus f_1(R_0)\,)
\end{aligned}
$$



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$L_2 \oplus L'_2 \; = L_0 \oplus f_1(R_0) \oplus L'_0 \oplus f_1(R'_0)$

How can we exploit this?

Security of 2-Round Feistel Networks

$F_k(L_0 \,\|\, R_0) = L_2 \,\|\, R_2$

$L_2 = R_1 = L_0 \oplus f_1(R_0)$

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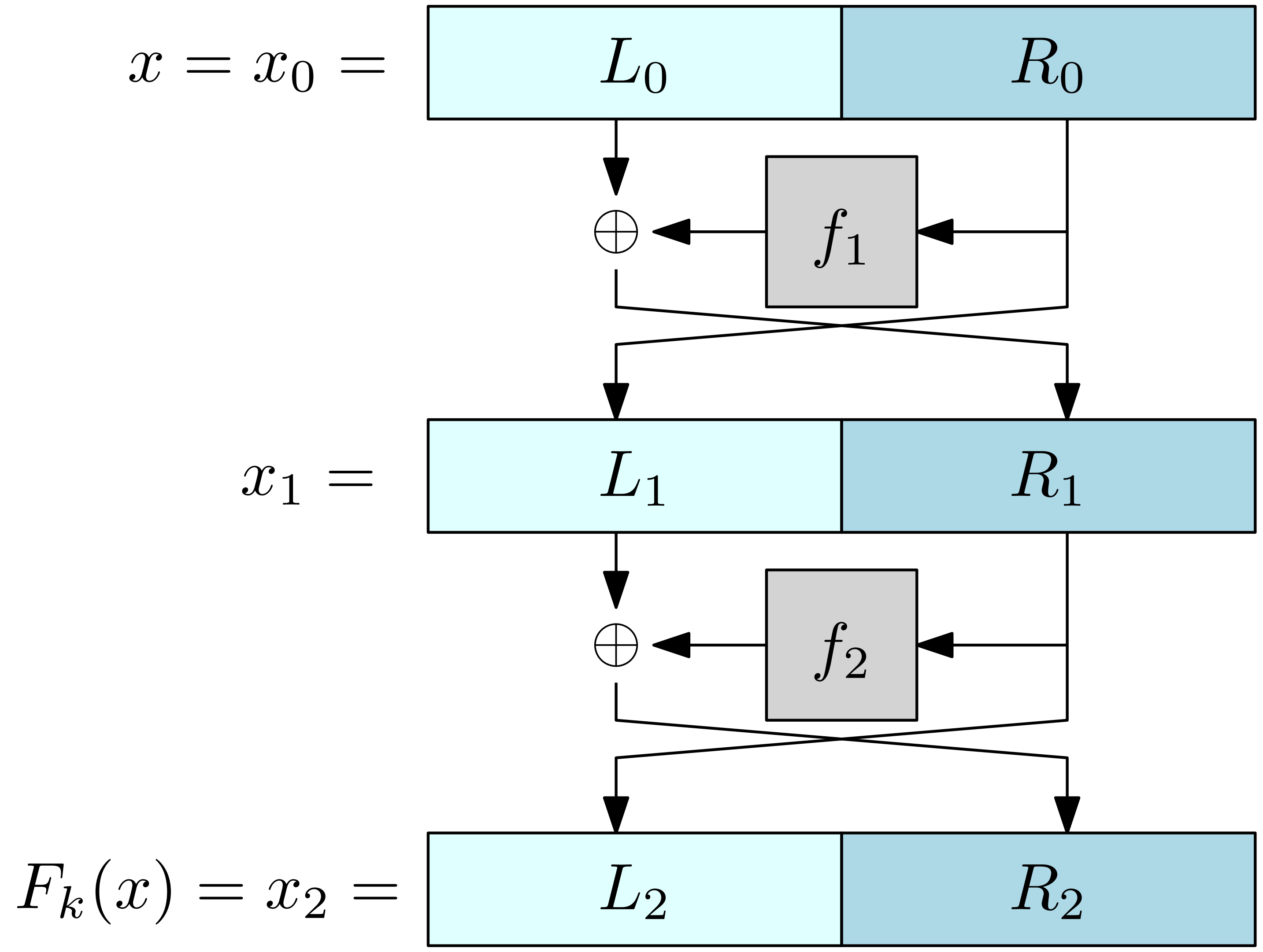

Is this a Pseudorandom permutation?

No! Consider two different inputs $L_0 \| R_0$ and $L_0' \| R_0'$

$L_2 \oplus L_2' = L_0 \oplus f_1(R_0) \oplus L_0' \oplus f_1(R_0')$

How can we exploit this? Pick $R_0 = R_0'$

Security of 2-Round Feistel Networks

$F_k(L_0 \,\|\, R_0) = L_2 \,\|\, R_2$

$L_2 = R_1 = L_0 \oplus f_1(R_0)$

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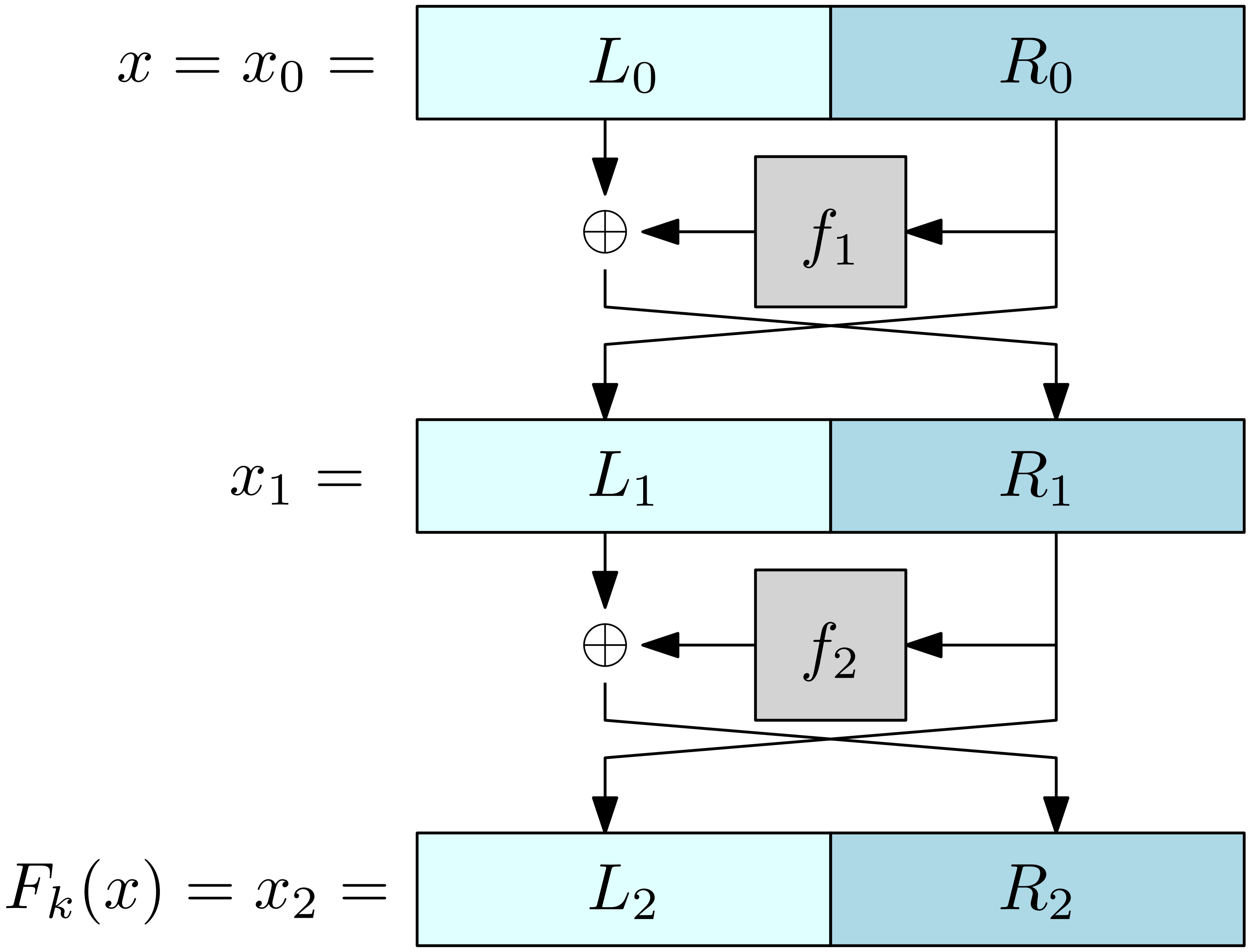

Is this a Pseudorandom permutation?

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$$\begin{aligned} L_2 \oplus L_2' &= L_0 \oplus f_1(R_0) \oplus L_0' \oplus f_1(R_0') \\ &= L_0 \oplus L_0' \end{aligned}$$

How can we exploit this? Pick $R_0 = R_0'$

Security of 2-Round Feistel Networks

$F_k(L_0 \| R_0) = L_2 \| R_2$

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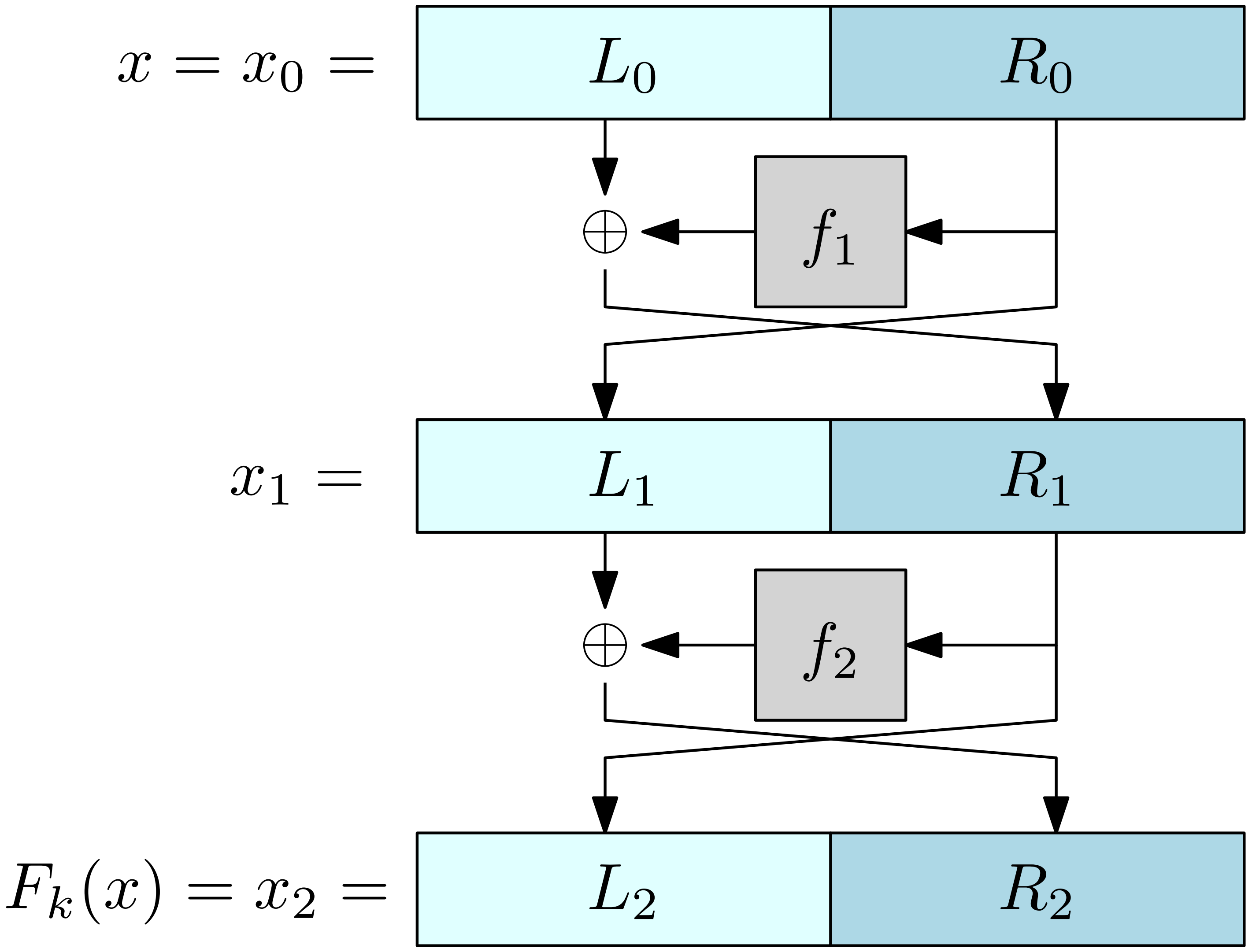

Is this a Pseudorandom permutation?

No! Consider two different inputs $L_0 \| R_0$ and $L'_0 \| R'_0$

$$L_2 \oplus L'_2 = L_0 \oplus f_1(R_0) \oplus L'_0 \oplus f_1(R'_0)$$
$$= L_0 \oplus L'_0$$

How can we exploit this? Pick $R_0 = R'_0$

This is easy to distinguish from a random function. E.g., pick $L_0 = 0^{\ell/2}$ and $L'_0 = 1^{\ell/2}$

Security of 2-Round Feistel Networks

$F_k(L_0 \,\|\, R_0) = L_2 \,\|\, R_2$

$L_2 = R_1 = L_0 \oplus f_1(R_0)$

$$R_2 = L_1 \oplus f_2(R_1)$$
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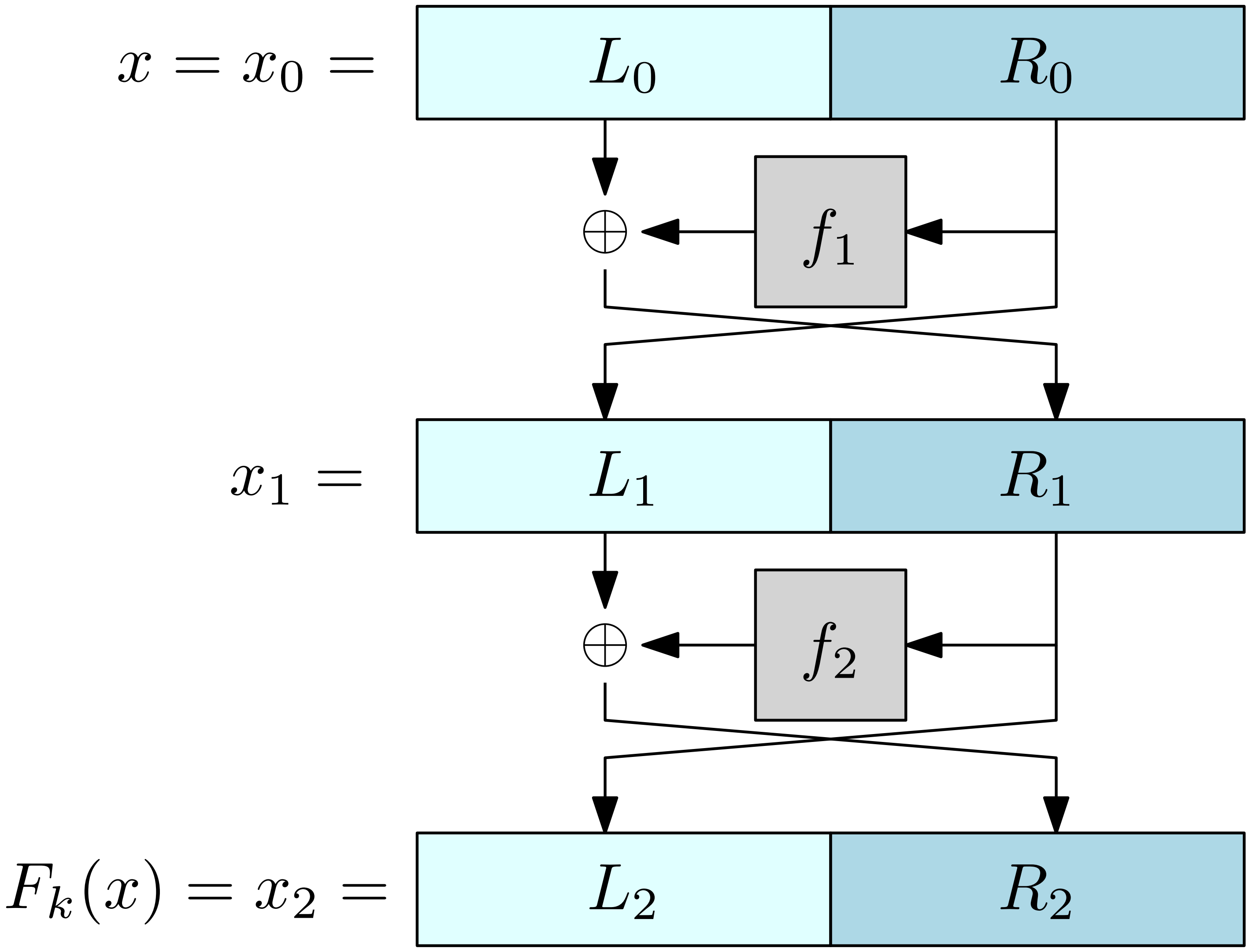

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$$= 0^{\ell/2} \oplus 1^{\ell/2}$$

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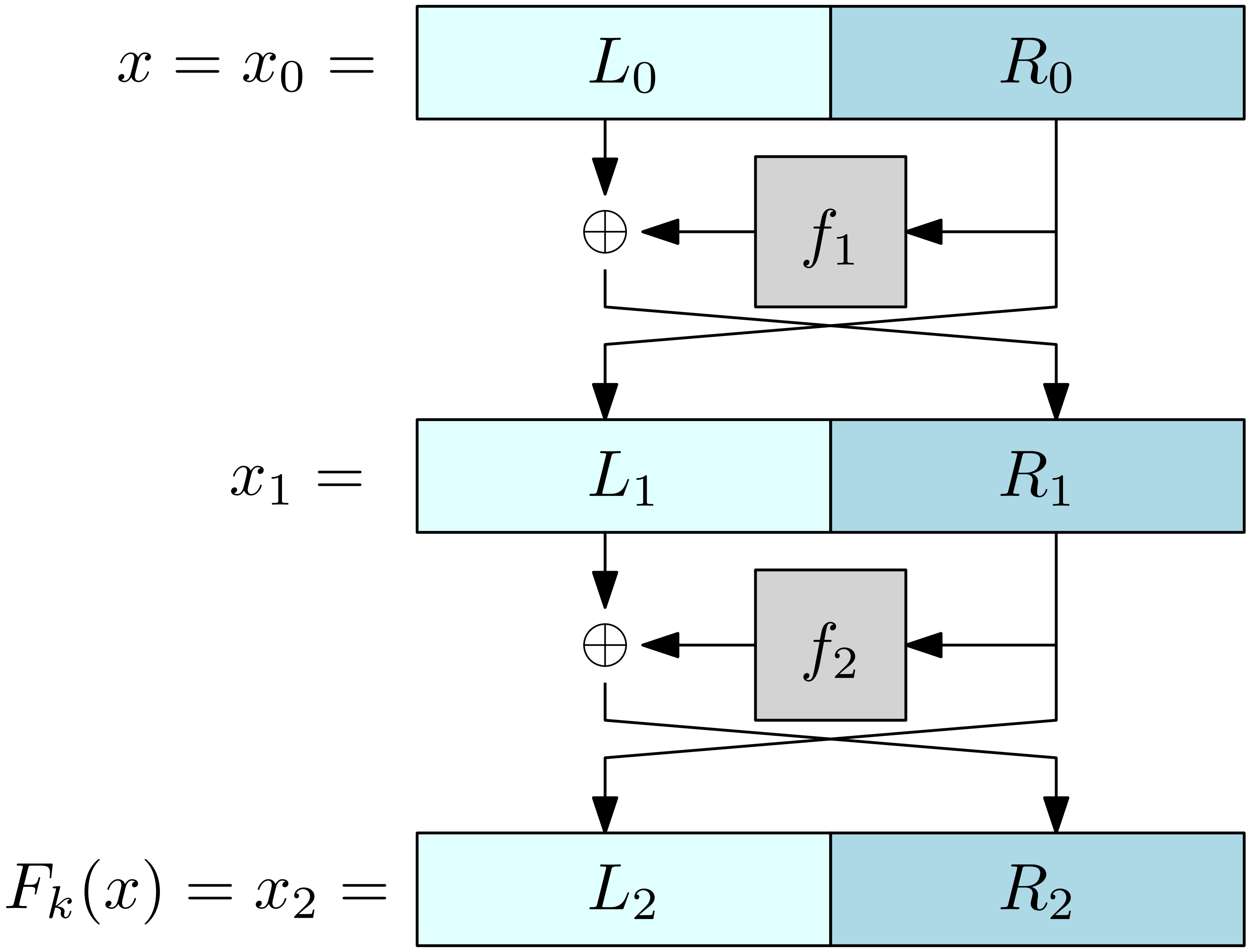
Security of 2-Round Feistel Networks

$F_k(L_0 \| R_0) = L_2 \| R_2$

$L_2 = R_1 = L_0 \oplus f_1(R_0)$

$$R_2 = L_1 \oplus f_2(R_1) = R_0 \oplus f_2(R_1) = R_0 \oplus f_2(L_0 \oplus f_1(R_0))$$



Is this a Pseudorandom permutation?

No! Consider two different inputs $L_0 \| R_0$ and $L'_0 \| R'_0$

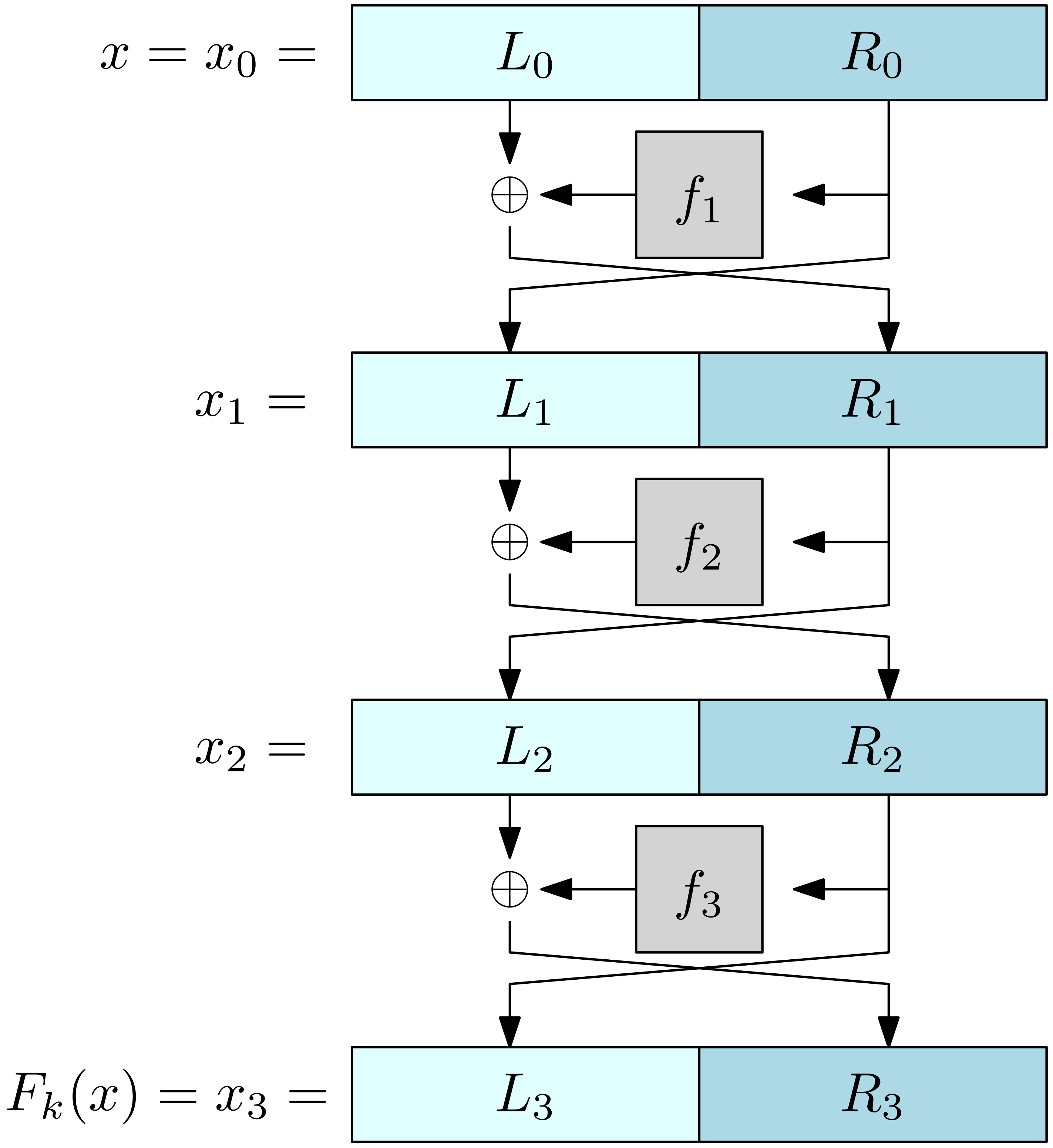
$$L_2 \oplus L'_2 = L_0 \oplus f_1(R_0) \oplus L'_0 \oplus f_1(R'_0) = L_0 \oplus L'_0 = 0^{\ell/2} \oplus 1^{\ell/2} = 1^{\ell/2}$$

How can we exploit this? Pick $R_0 = R'_0$

This is easy to distinguish from a random function. E.g., pick $L_0 = 0^{\ell/2}$ and $L'_0 = 1^{\ell/2}$

Security of 3-Round Feistel Networks

Is this a pseudorandom permutation?

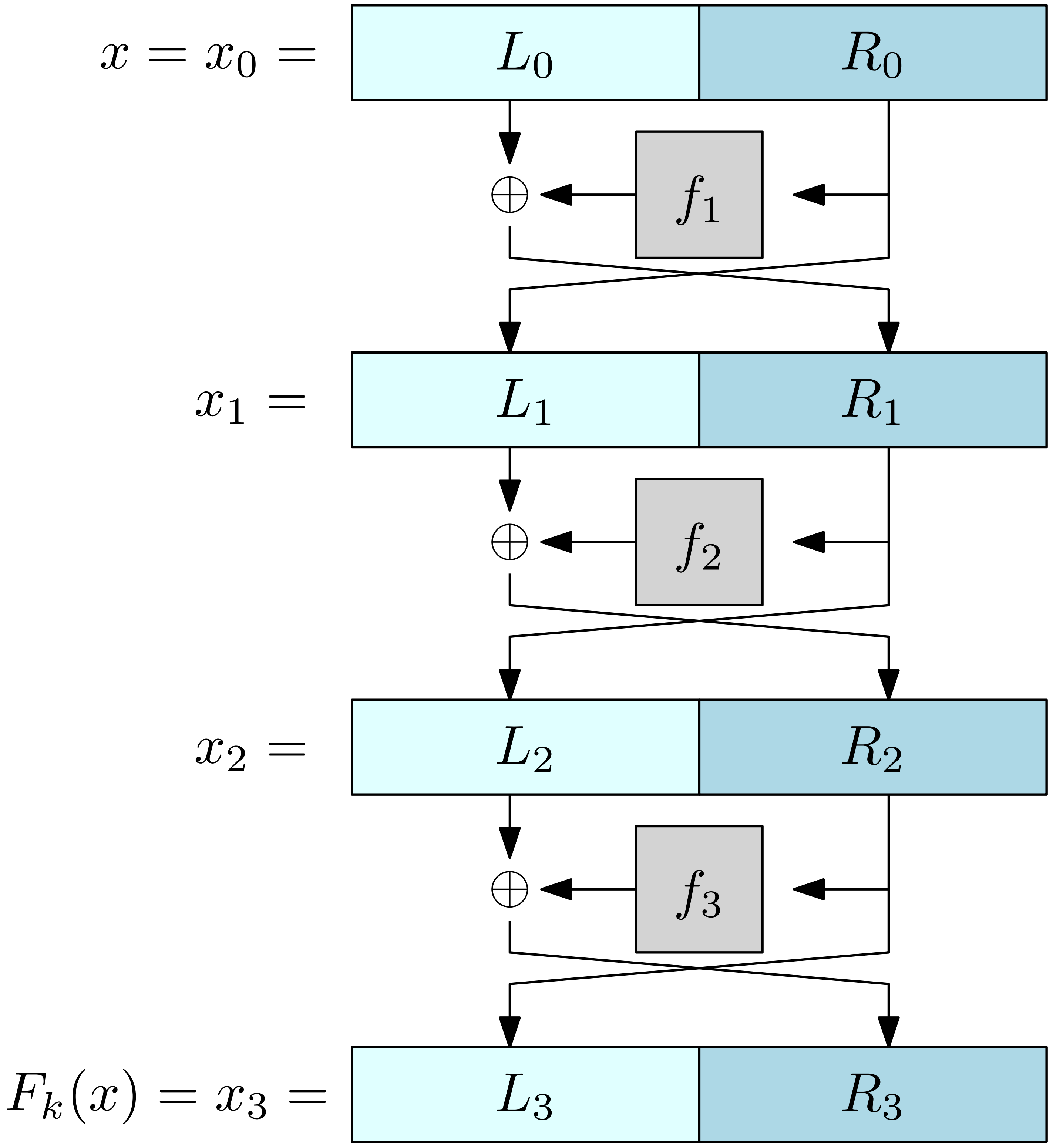


Security of 3-Round Feistel Networks

Is this a pseudorandom permutation?

- Yes!

  (If $f_i = F_{k_i}$ for some pseudorandom function F and the keys k_i are chosen independently and u.a.r.)



Security of 3-Round Feistel Networks

Is this a pseudorandom permutation?

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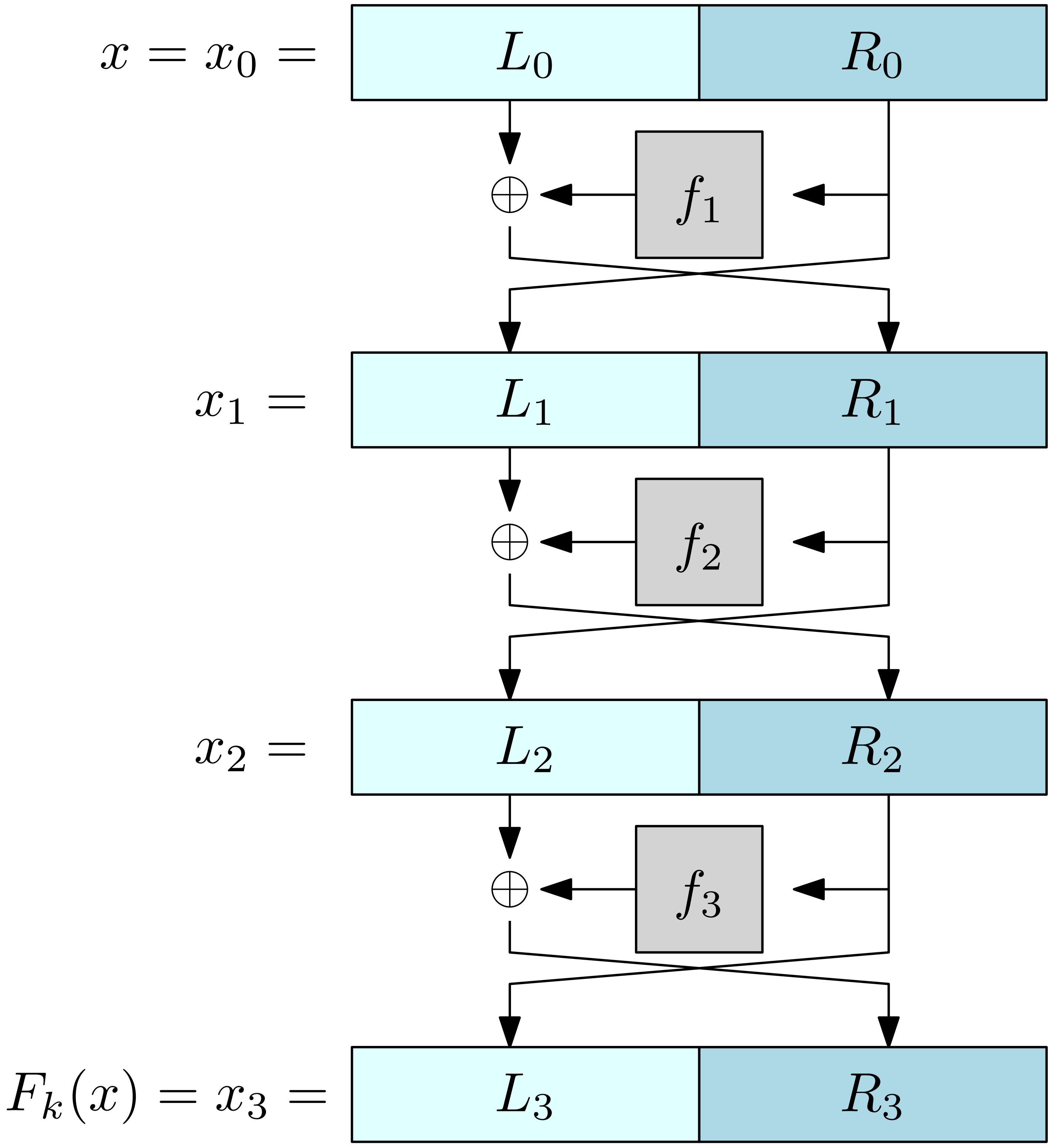
Is this a **strong** pseudorandom permutation?



Security of 3-Round Feistel Networks

Is this a pseudorandom permutation?

- Yes!

  (If $f_i = F_{k_i}$ for some pseudorandom function F and the keys k_i are chosen independently and u.a.r.)

Is this a **strong** pseudorandom permutation?

- No
- But 4-round Feistel networks are!