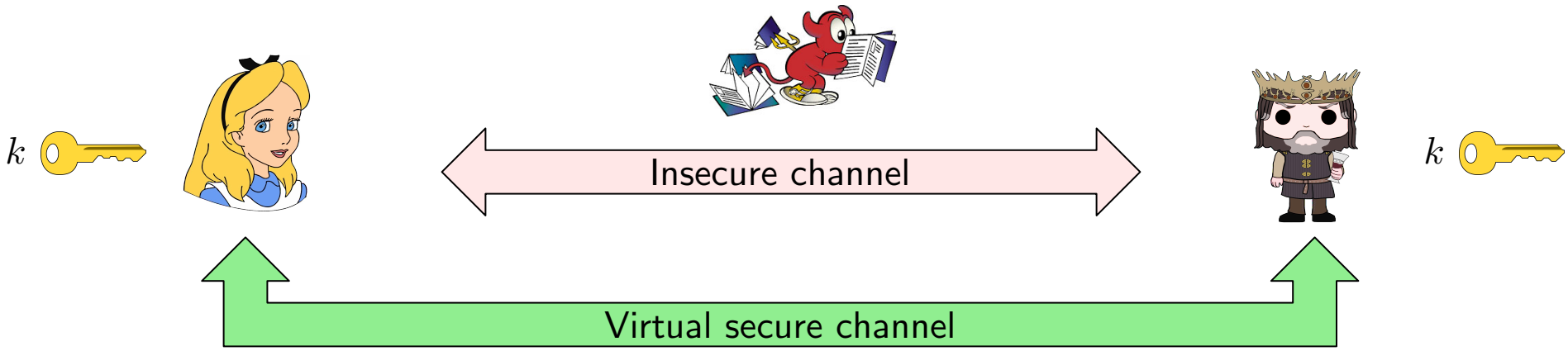
# Key Distribution




We have seen how Alice and Bob can use private-key criptography to establish a secure communication channel over an insecure one

To do this they need to share a secret key $k$

**How?**

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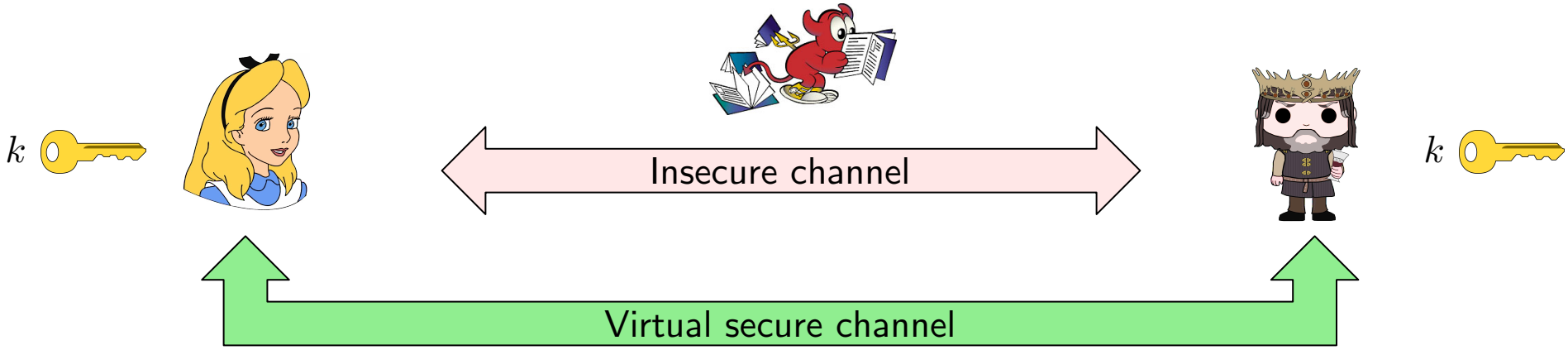


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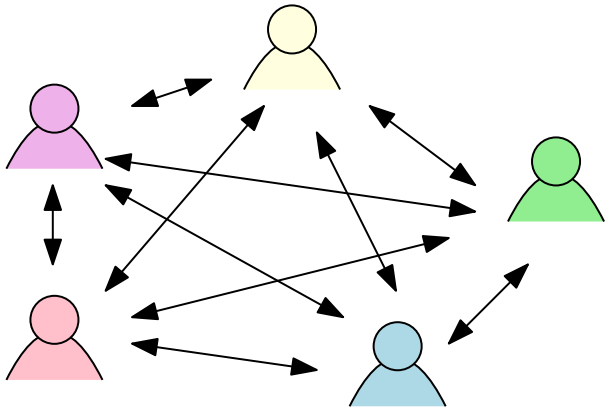
**How?**

- Alice and Bob can meet in person
- Alice and Bob can use a **trusted** courier

This can be difficult (or even impossible) to do, depending on the situation

# Key Management



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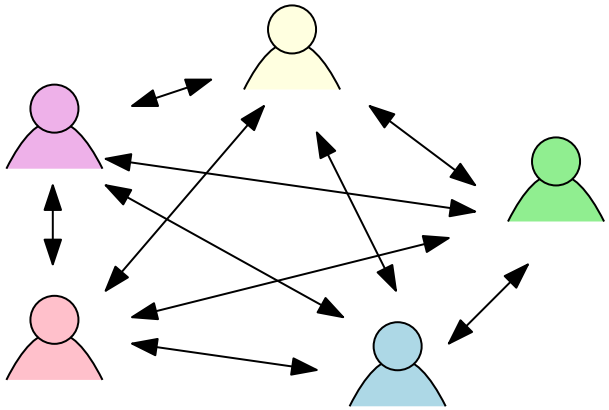

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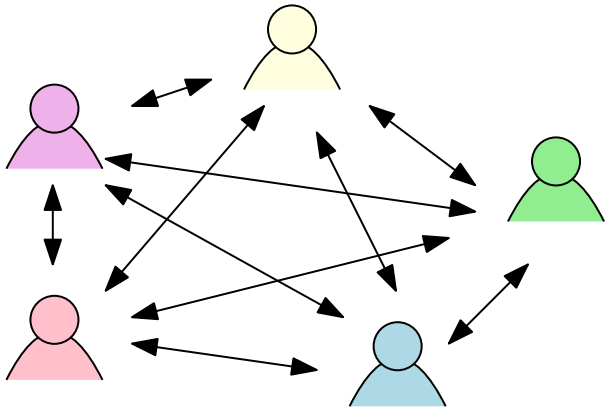

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A possible solution:

- Each pair of users $u, v$ generate their own secret key $k_{u,v}$
- There are $\Theta(N^2)$ keys in total
- If a new user joins the company, $\Theta(N)$ new keys are needed, and they need to be shared privately with each other user

**Not great...**

# A Partial Solution: Key-Distribution Centers ("offline solution")

Suppose that all the entities trust some central authority
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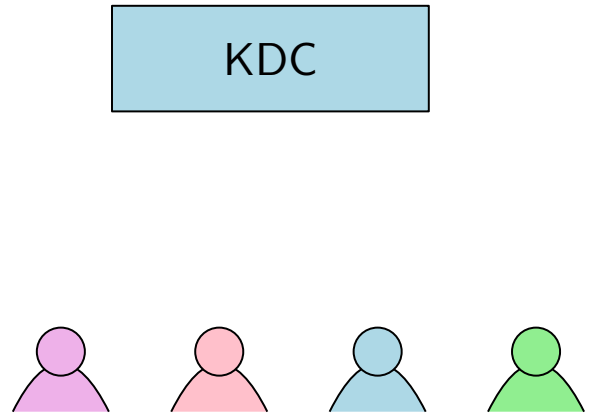

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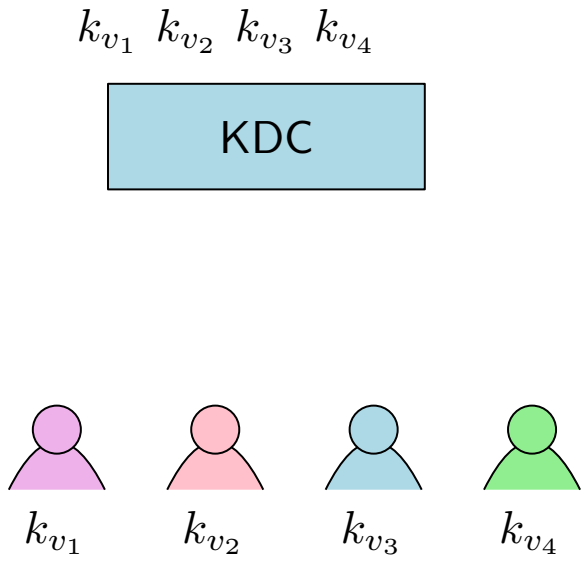

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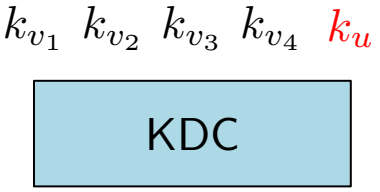


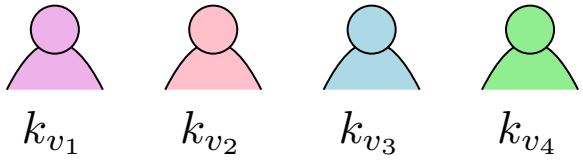

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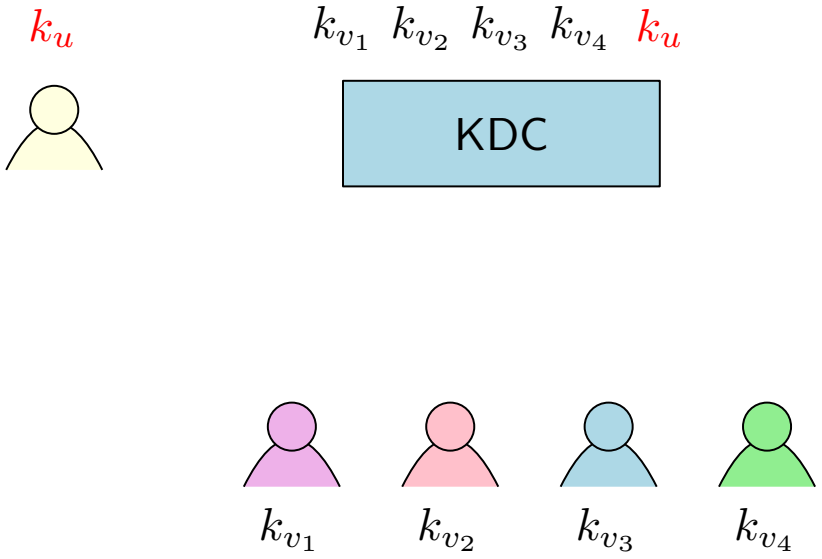

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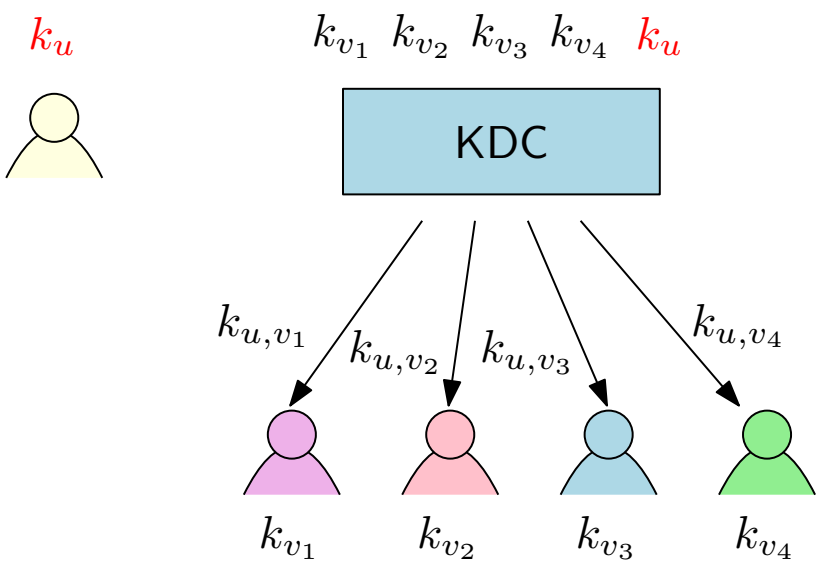

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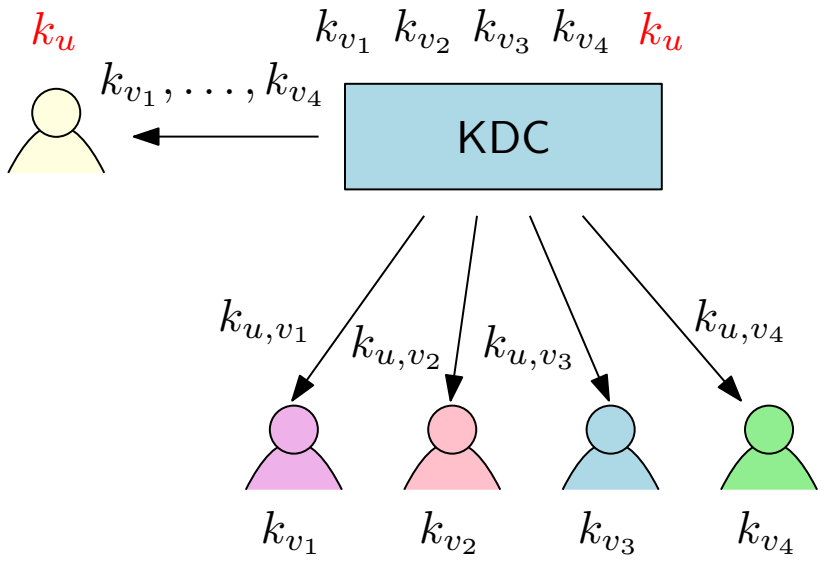

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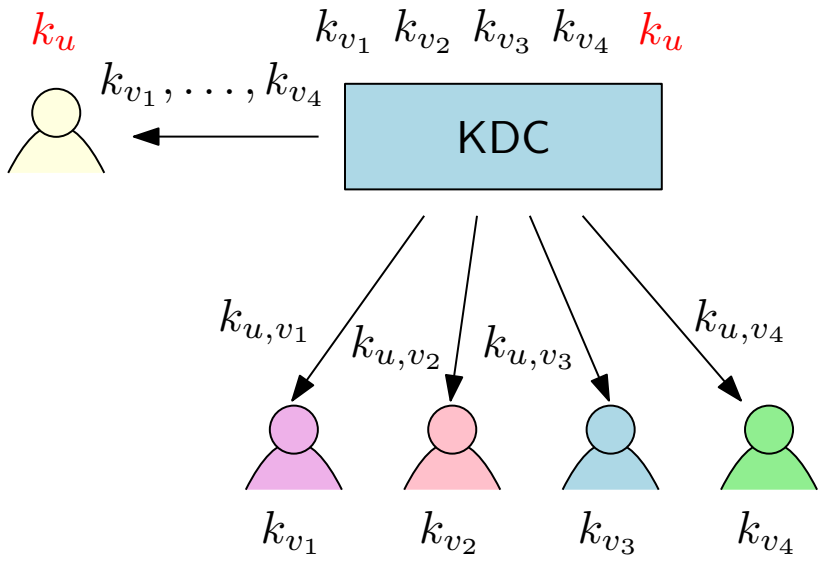


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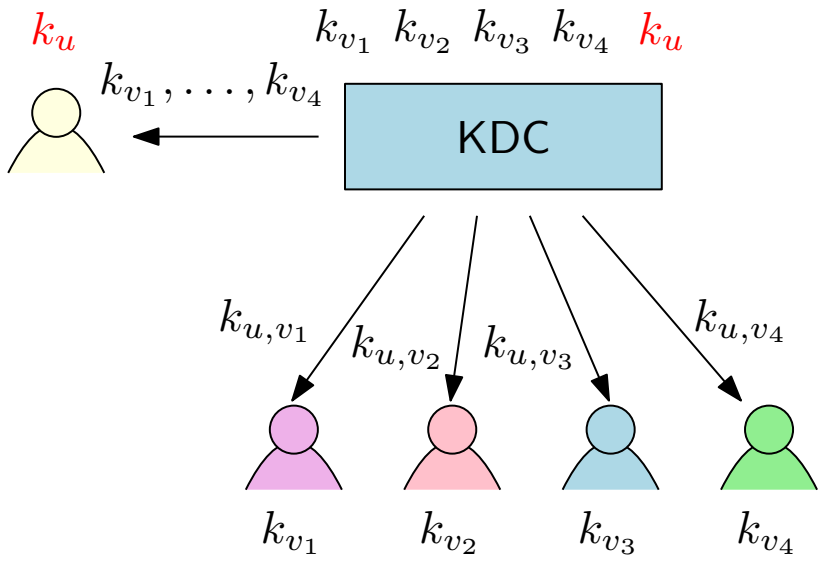


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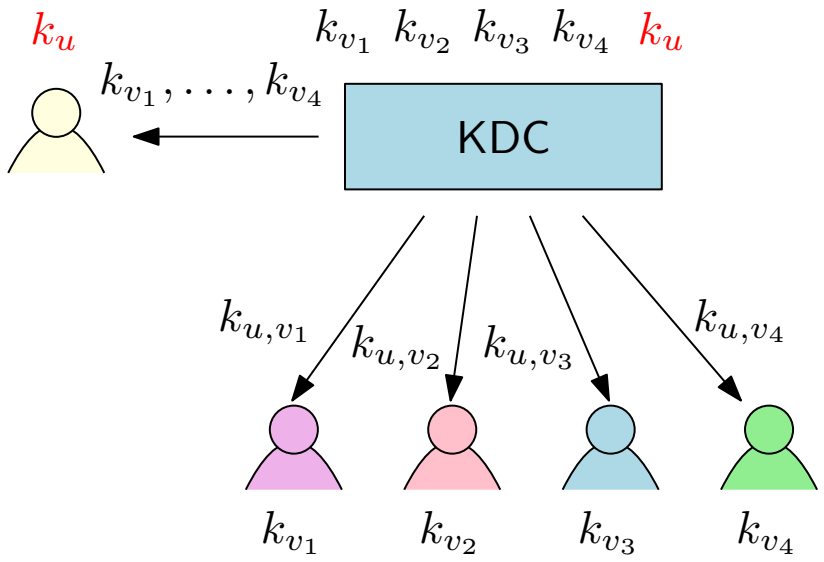


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**Con:** Still $\Theta(N^2)$ keys overall

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We can reduce the number of keys to $\Theta(N)$ using the KDC in a "online" fashion

- Each user $v$ shares a key $k_v$ which the KDC
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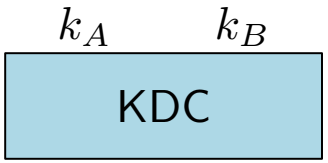


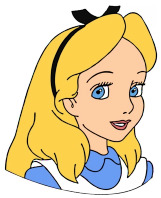

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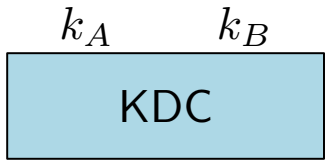


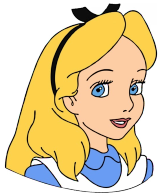

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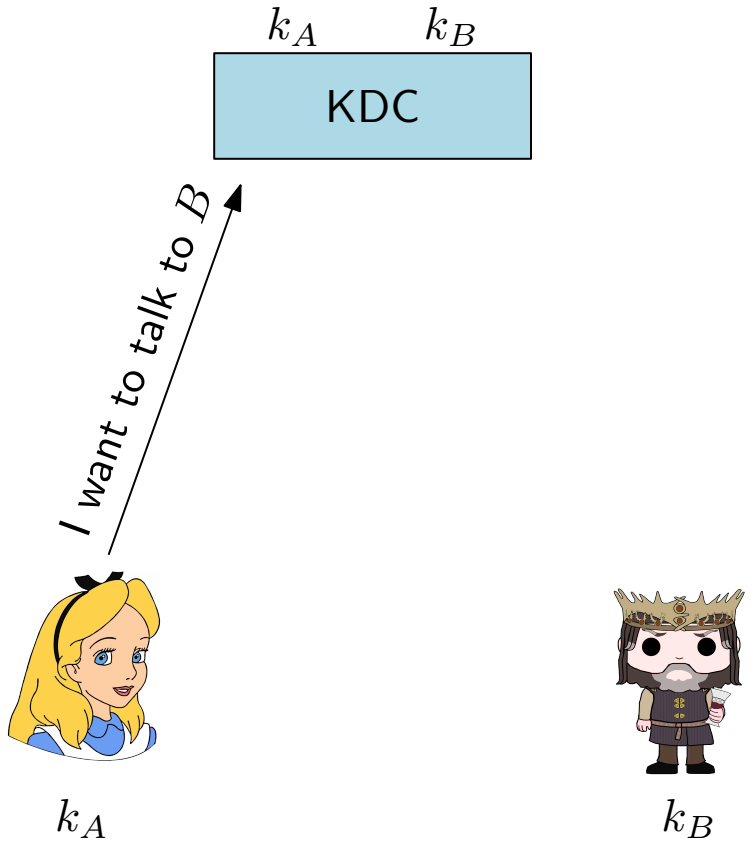

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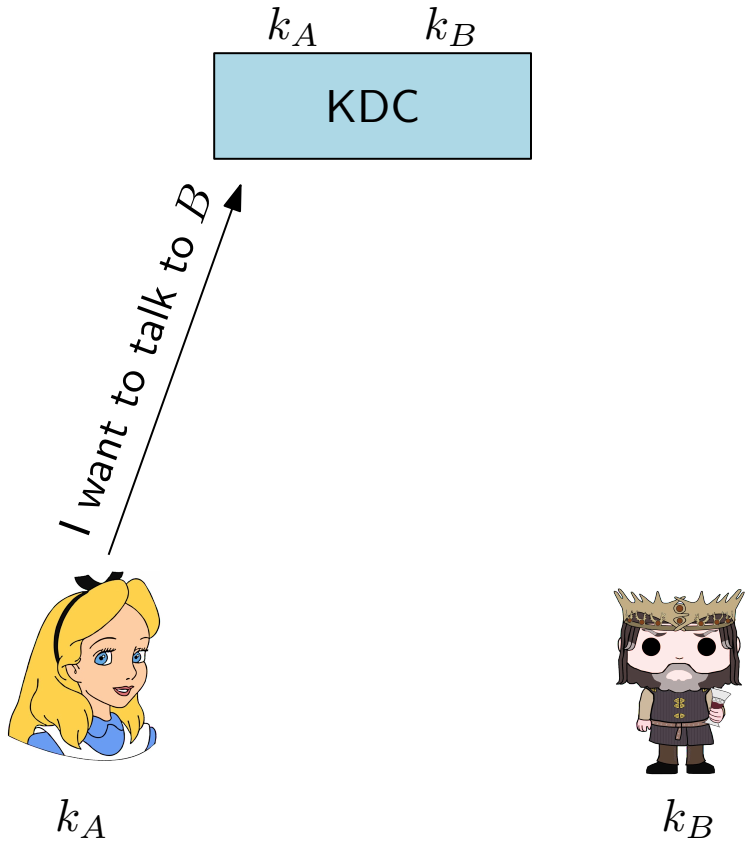

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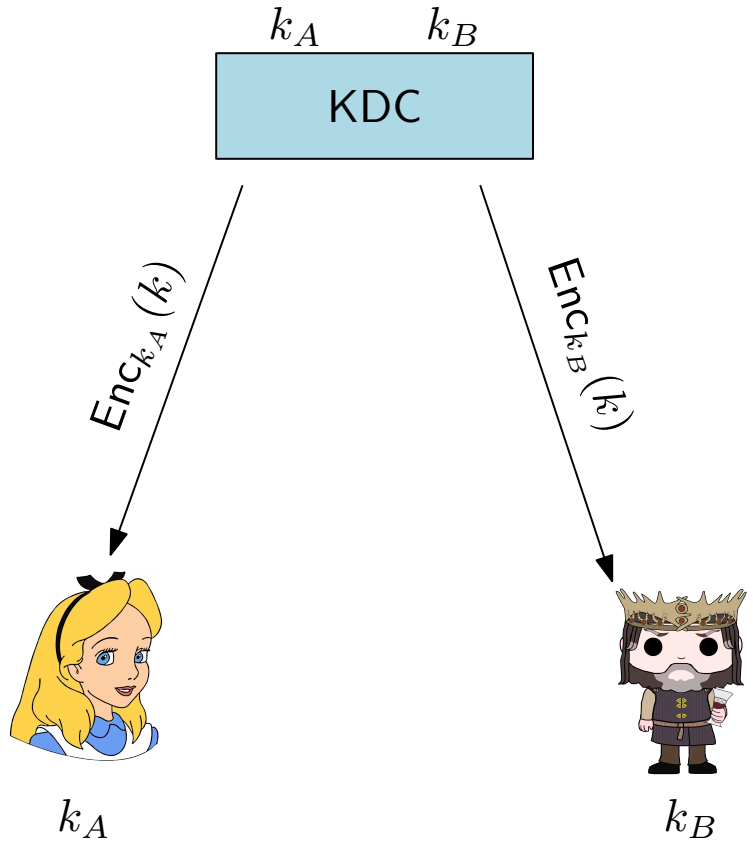

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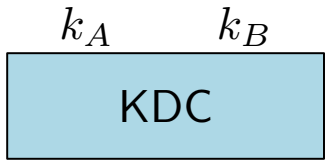


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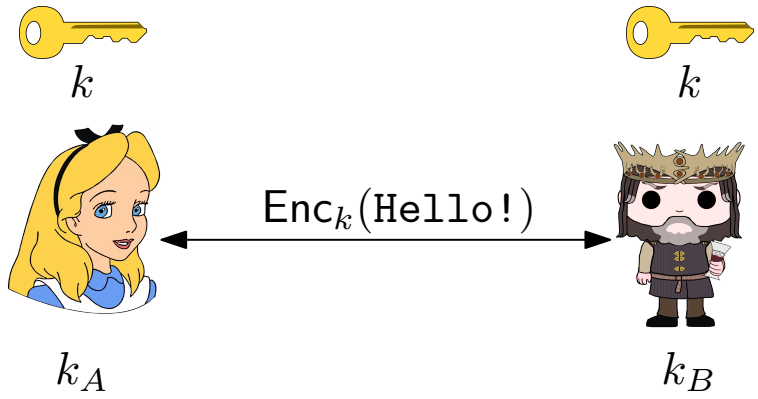

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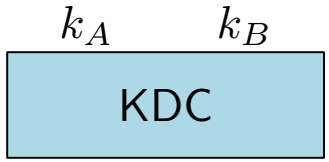


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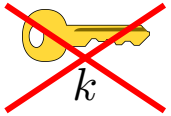


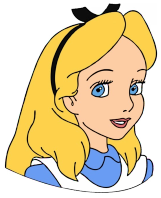

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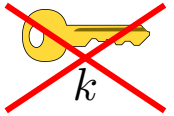


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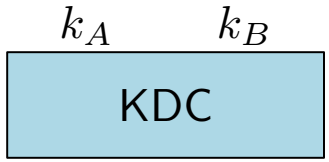


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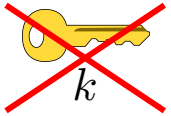


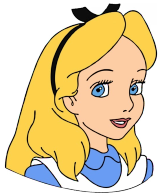

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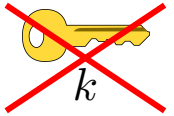


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**Con:** Relies on the KDC being online (KDC can be replicated at the cost of more complexity)

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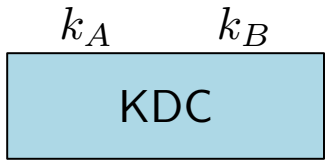
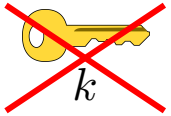
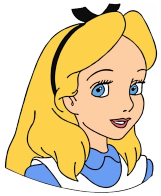



Suppose that a user $A$ (Alice) wants to talk to user $B$ (Bob)

- $A$ contacs the KDC and asks to talk to $B$
  (this message can be authenticated with a MAC, if desired)
- The KDC generates a temporary *session key* $k$
- The KDC sends $k$ to $A$, encrypting the message with $k_A$
- The KDC sends $k$ to $B$, encrypting the message with $k_B$
- $A$ and $B$ start communicating using the session key $k$
- At the end of the communication session, $A$ and $B$ destroy $k$



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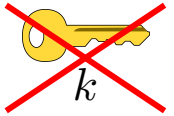


$k_B$

**Con:** Relies on the KDC being online (KDC can be replicated at the cost of more complexity)

**Con:** A data breach on the KDC compromises all communications

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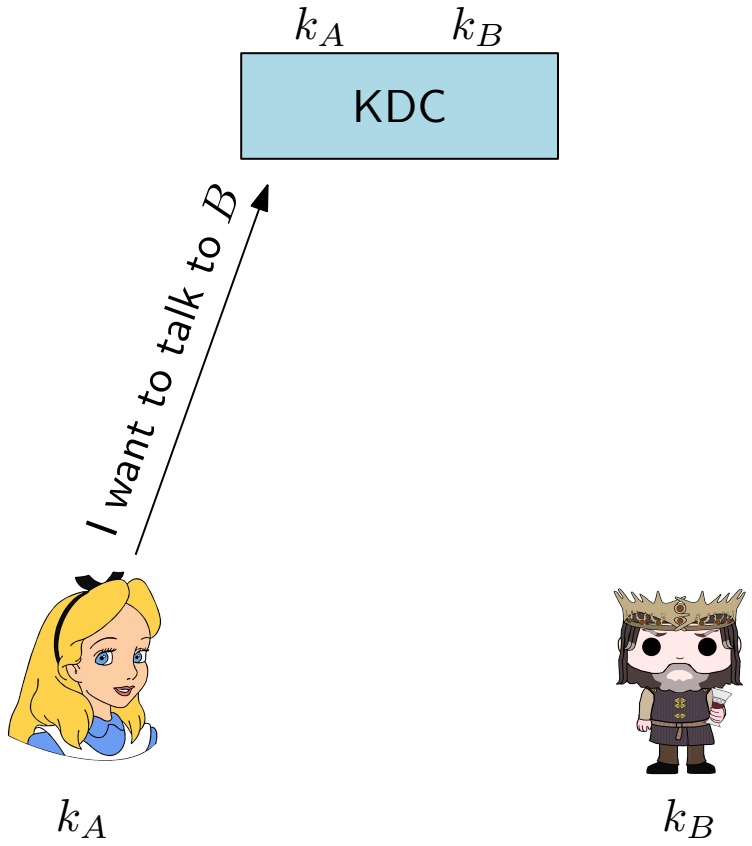

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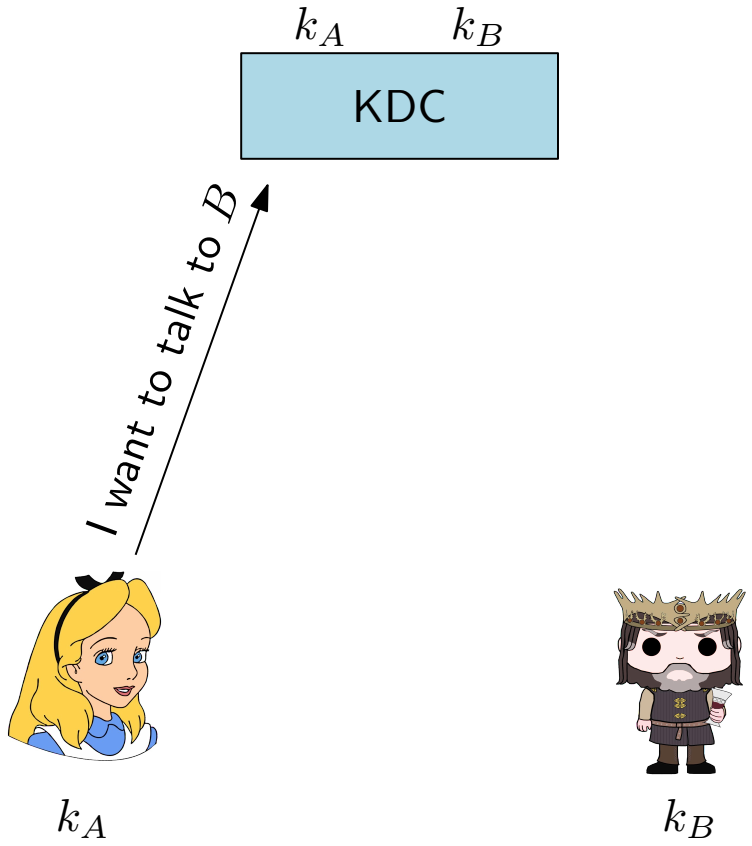

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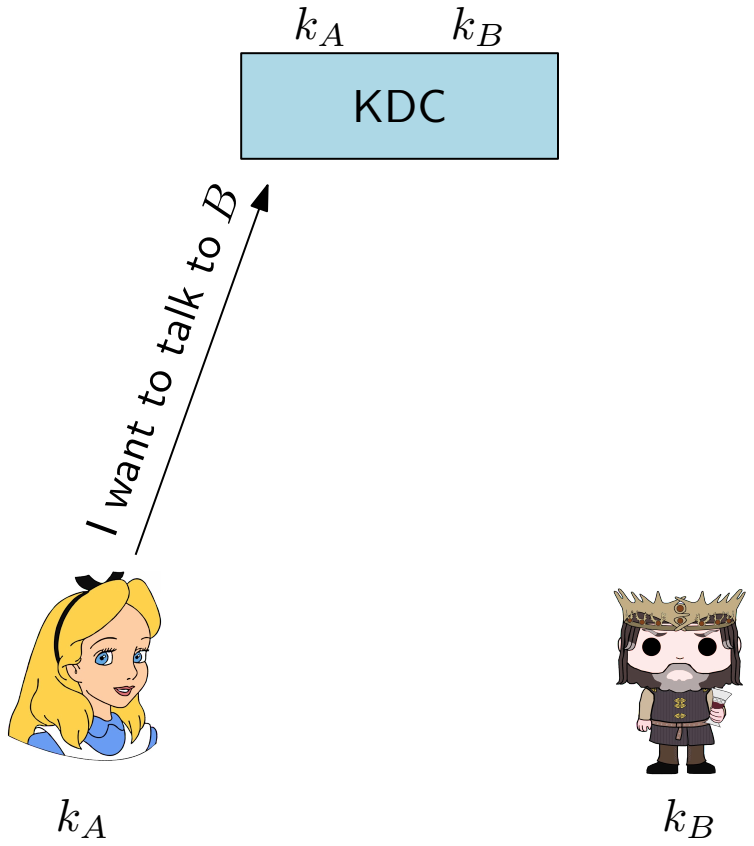

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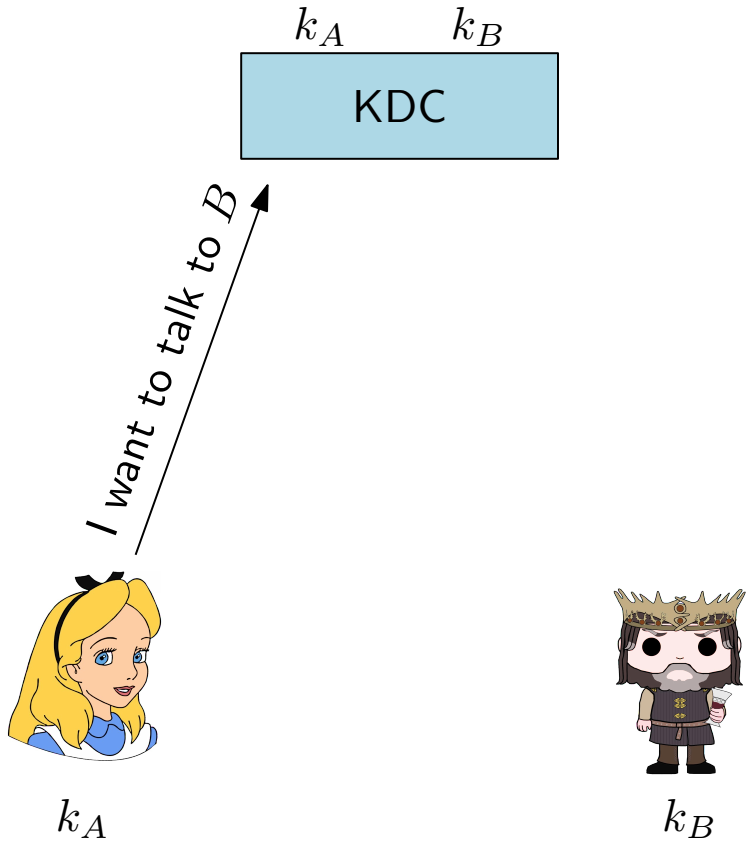

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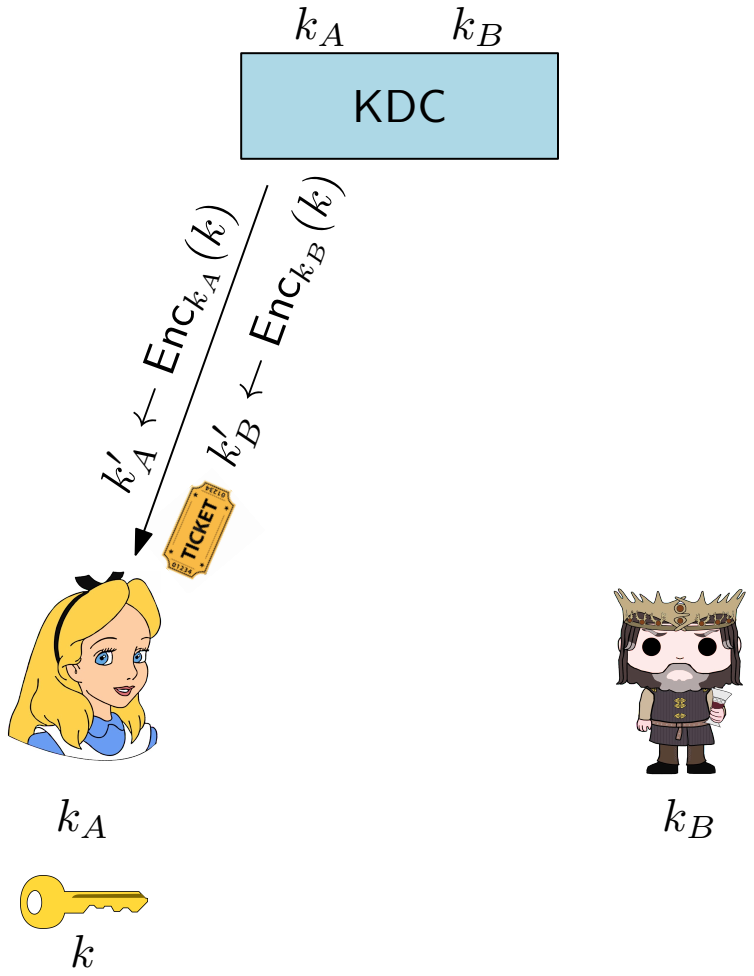

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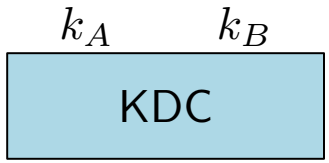


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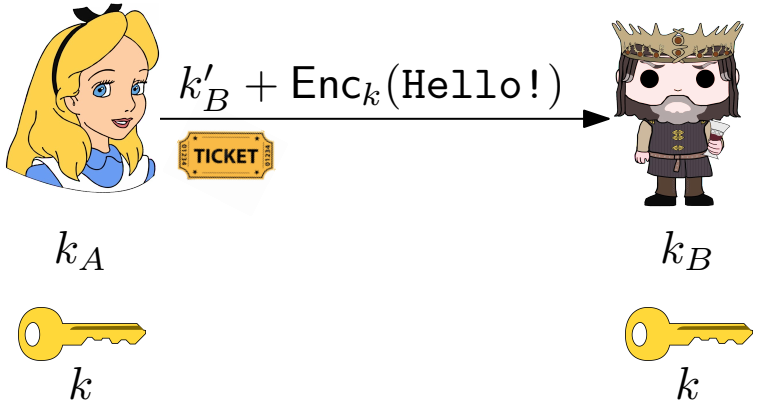

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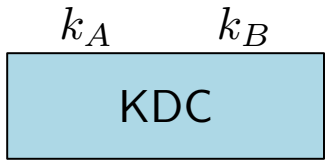


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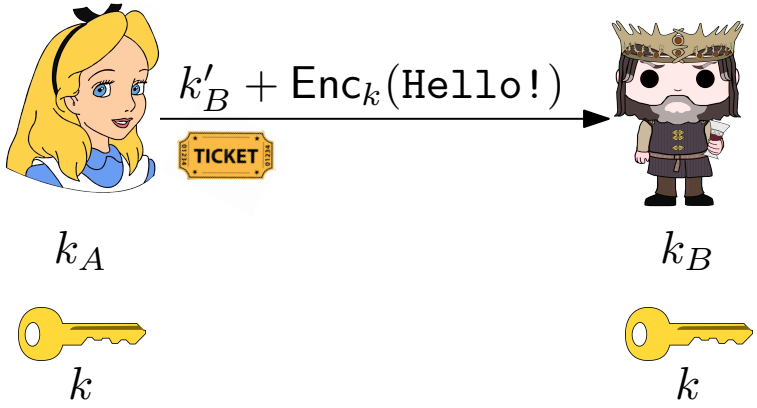

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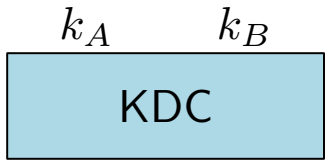


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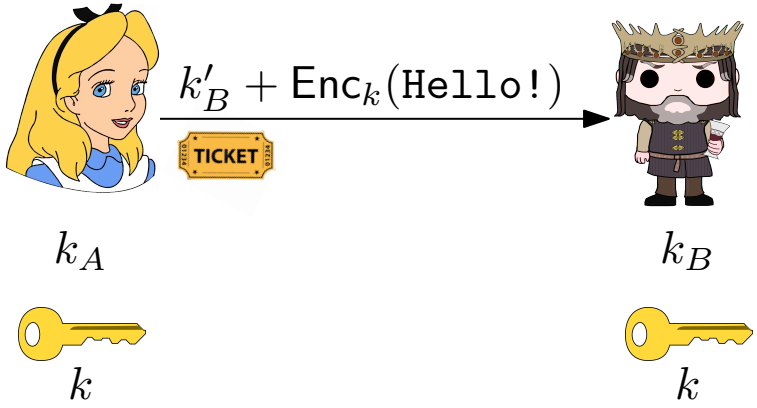

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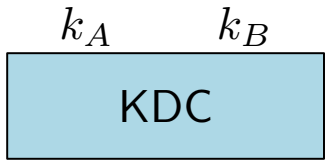


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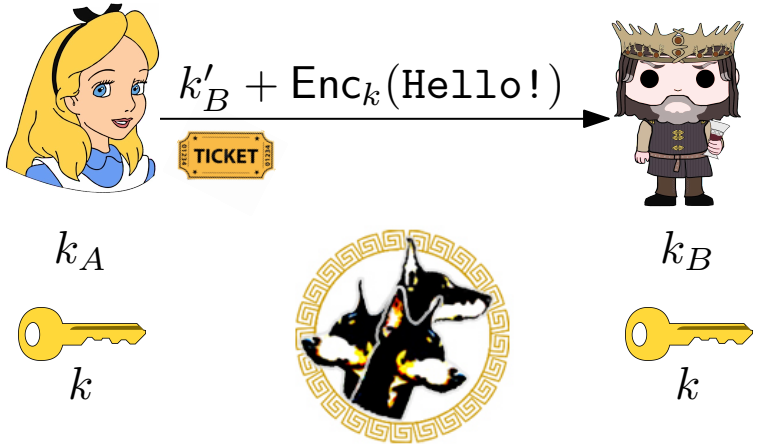
This approach is used, e.g., in the Kerberos protocol.



# Open systems

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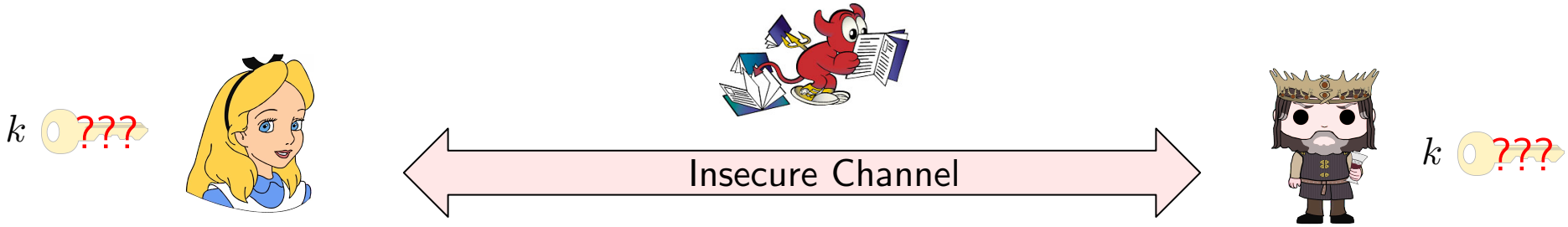


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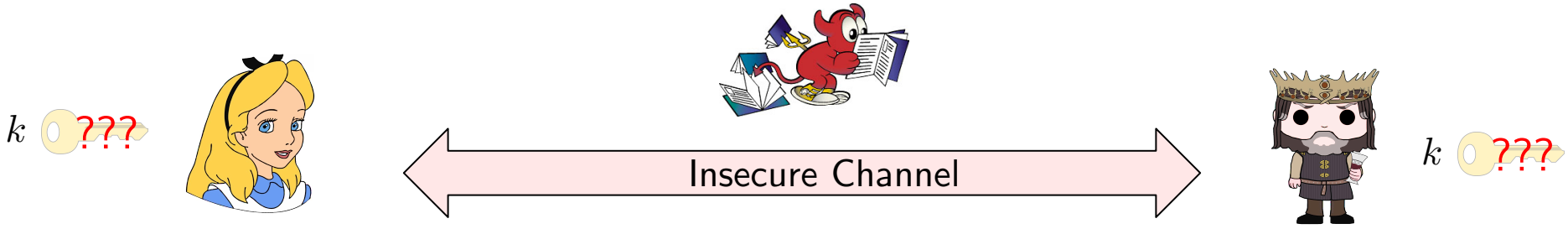


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This happens all the time

- Sending an instant-message to someone for the first time
- Visiting a new website

**How do we handle this case?**

# Key Exchange

Classical private-key cryptography does not offer a satisfactory solution to these problems

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- Easy to compute, hard to invert (Candidate examples?)
- Use asymmetry in a clever way, to enable two parties to agree on a secret key over an insecure channel

# Key Exchange: Informal Security Definition

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**Idea:**

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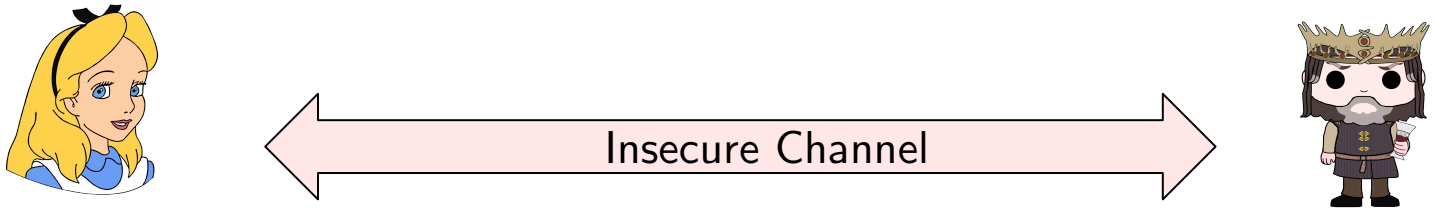

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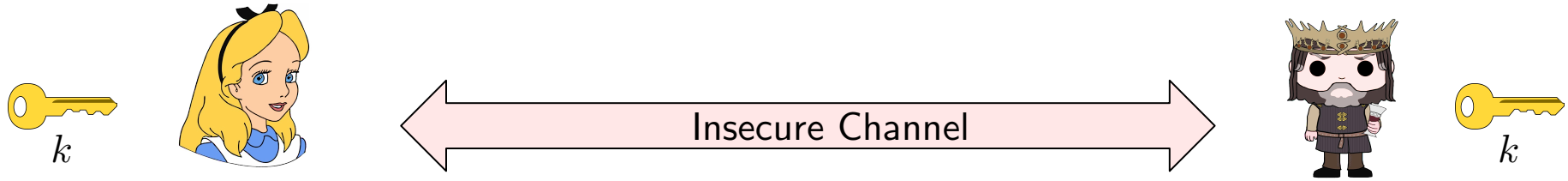


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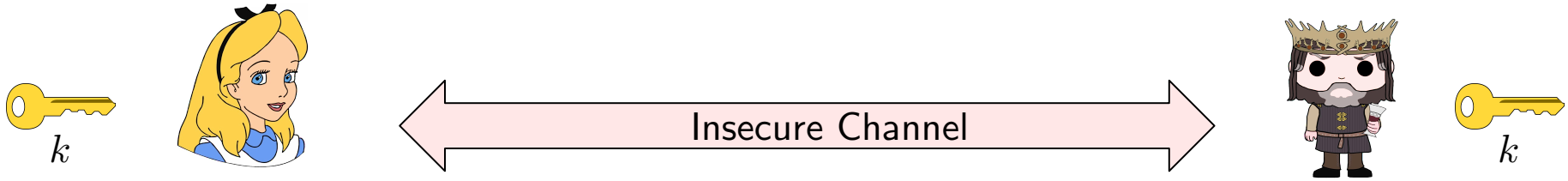


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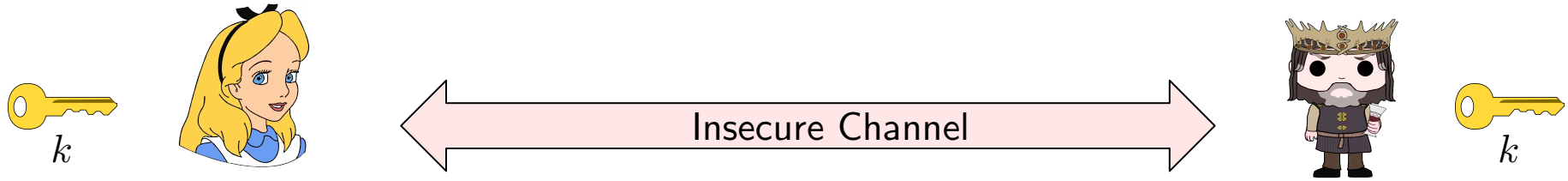


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**Security goal (informal):** even after observing the transcript, the shared key $k$ should be indistinguishable from a uniform key (to any polynomial-time adversary)

# The Key Exchange Experiment

Fix a key exchange protocol $\Pi$ and an attacker $\mathcal{A}$

We define the **key-exchange experiment** $\mathsf{KE}^{\mathsf{eav}}_{\mathcal{A},\Pi}(n)$ as follows:

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The outcome of the experiment is defined to be $1$ if $b' = b$ and $0$ otherwise

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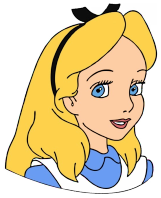

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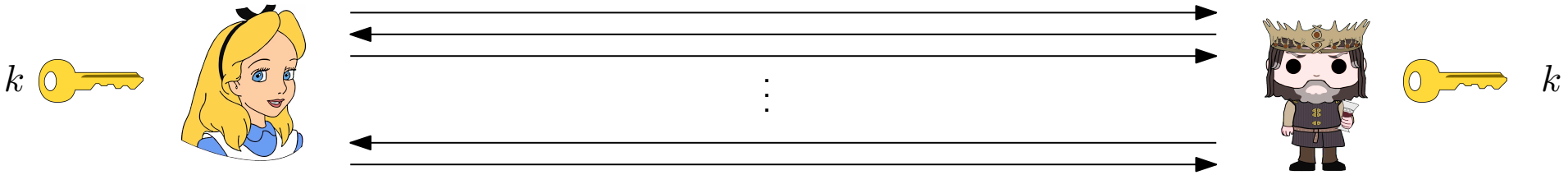


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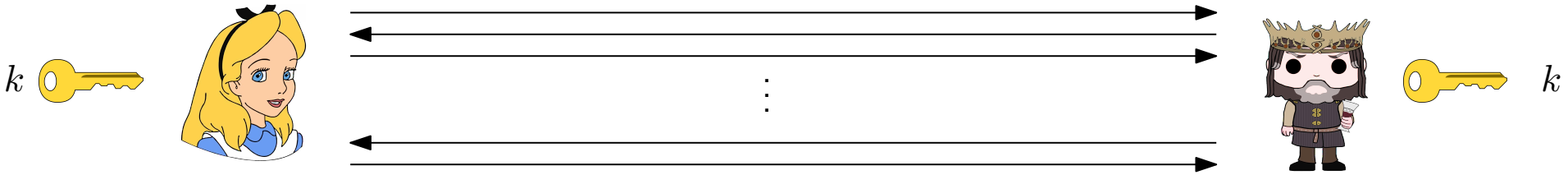


$b \leftarrow \{0,1\}$

$$k' = \begin{cases} k & \text{if } b = 0 \\ \text{random } n\text{-bit string} & \text{if } b = 1 \end{cases}$$

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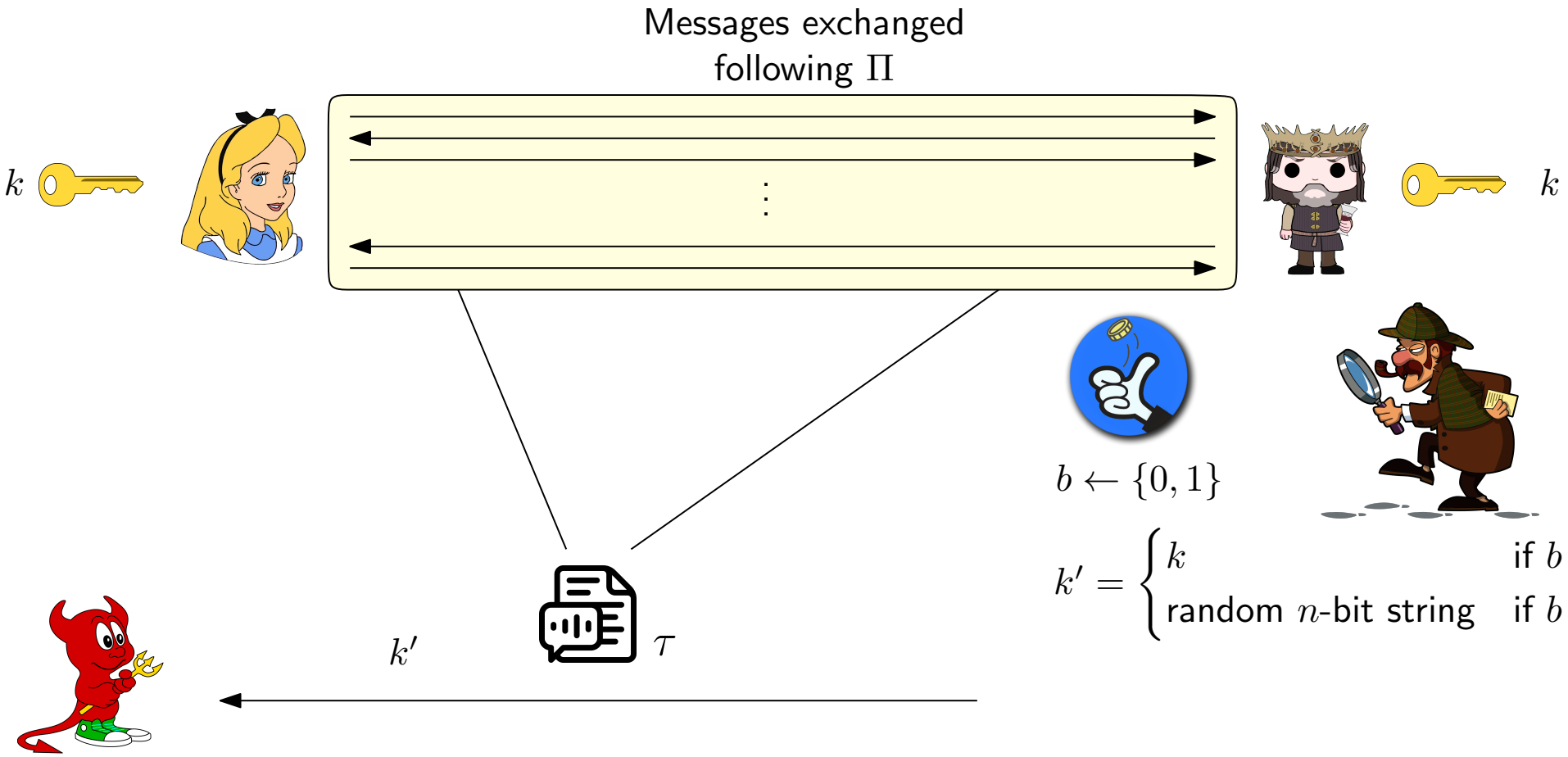

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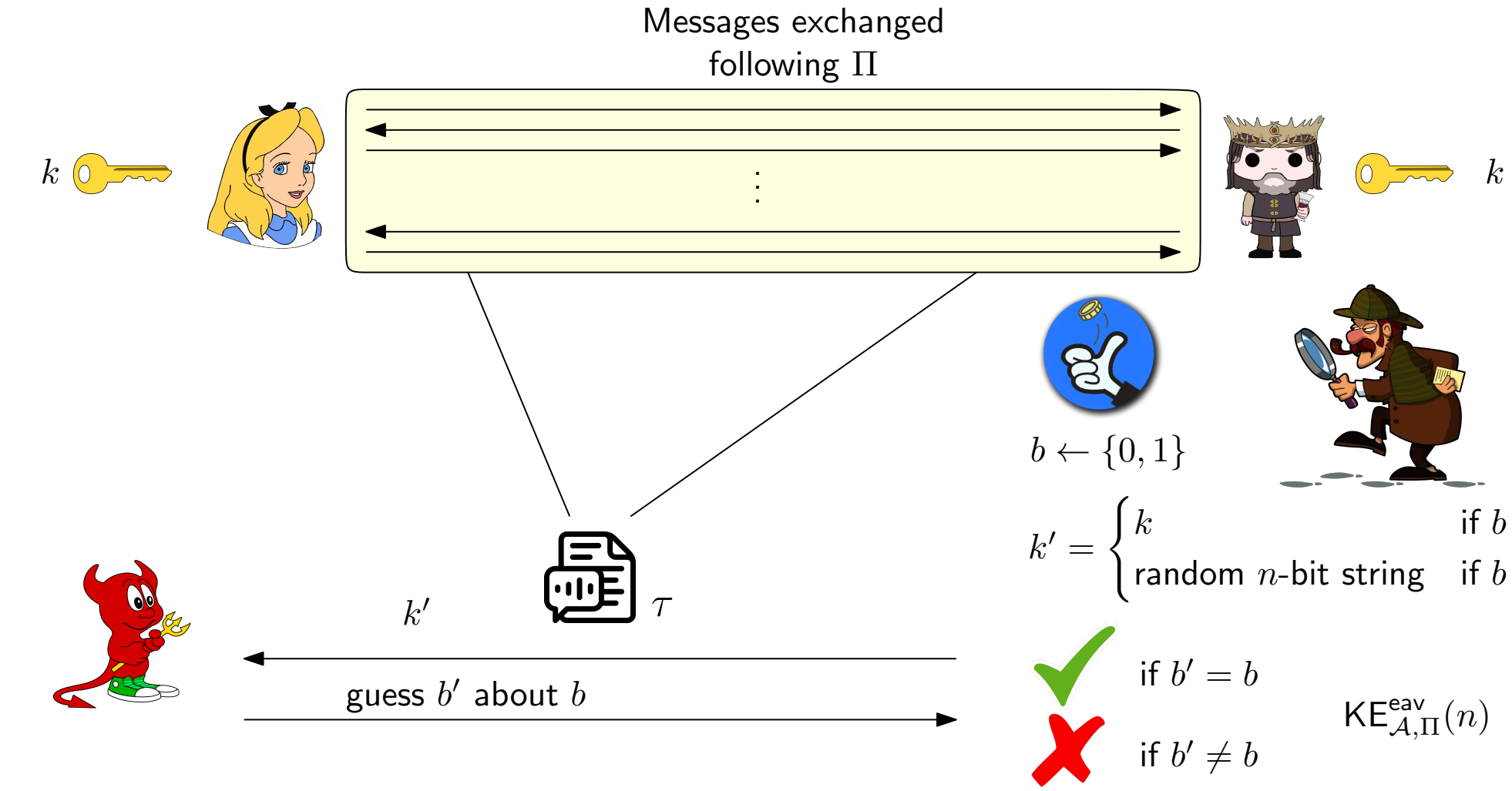

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Notice that being unable to compute $k$ from the transcript $\tau$ is not a strong enough security guarantee

- The requirement we impose is **stronger**. Namely $k$ must look just like a random string.
- This is necessary since we are going to use $k$ for private-key cryptography.