Reminder: Security of Plain RSA

Is plain RSA secure (assuming the RSA assumption)?

Observation 1:

- The RSA assumption is defined with respect to the RSA-ind experiment
- In the RSA-ind experiment the adversary wins by computing the e-th root of a **random group element** $x \in Z_N^*$
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- Receive the public key $pk = \langle N, e \rangle$
- Output two messages m_0, m_1 and receive the challenge ciphertext c
- Query the decryption oracle with $c' = 2^e \cdot c \pmod{N}$ to obtain $m' = 2 \cdot m_b$ of
- If $m' = 2 \cdot m_0 \pmod{N}$ return $b' = 0$, otherwise return $b' = 1$

“Fixing” RSA

How do we make RSA secure?

Option 1: Use randomized encoding

- Make sure that the message is “random enough”
- Choose some **randomized encoding** between messages and group elements
- To encrypt: encode the message, then encrypt the group element
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To decrypt c:

Dec$_{sk}(c)$: (where $sk = \langle N, d \rangle$)

- Compute $\hat{m} = c^d \pmod N$
- Return the $\eta - \ell(n)$ least significant bits of $\hat{m}$

Security of Padded RSA

The security of padded RSA depends on the choice of $\ell(n)$

- An attacker can win the $\mathsf{PubK}^{\mathsf{cpa}}_{\mathcal{A},\Pi}(n)$ experiment by guessing $r \in \{0,1\}^{\ell(n)}$ using a brute-force attack

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In any case, Padded RSA is not CCA-secure!

PKCS #1 v1.5

Variant of Padded RSA standardized in 1993 (PKCS stands for Public-Key Cryptography Standard)

It uses paddings of specific lengths and formats

- For a public key $pk = \langle N, e \rangle$, let k be the length of N in bytes
- The message m is required to have an integral number D of bytes between 1 and $k - 11$
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- The choice of the padding ensures that $\hat{m} < N$ and that m can be unambiguously recovered from $\hat{m}$

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If $(2^{75})^e \leq N$ (for example when N has more than $75e$ bits) the “short message” attack applies.

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A word of caution:

- The recipient needs to check that the leading bytes of the decoded plaintext are `0x00 0x02`
- If this is not the case, the decryption returns an error
- If not done properly, this might provide access to a **padding oracle**!
- The whole message m can be recovered!

RSA-OAEP (Encryption)

Optimal asymmetric encryption padding is a (more complex) randomized encoding of messages that results in a **CCA-secure** version of RSA, called **RSA-OAEP**

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$$G : \{0,1\}^{k(n)} \to \{0,1\}^{\ell(n)+k(n)} \qquad \text{and} \qquad H : \{0,1\}^{\ell(n)+k(n)} \to \{0,1\}^{k(n)}$$

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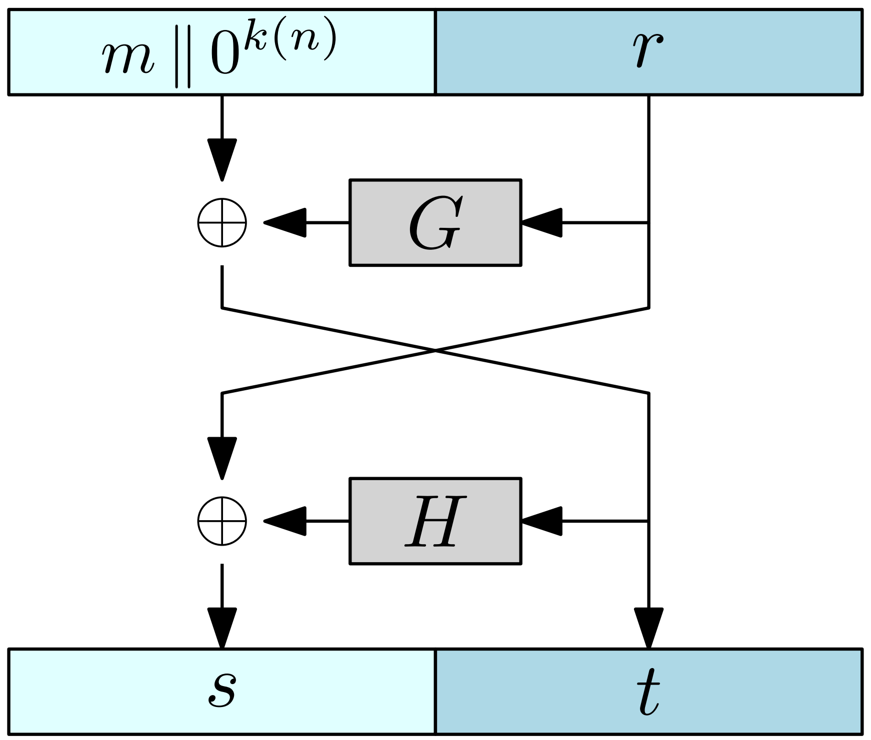

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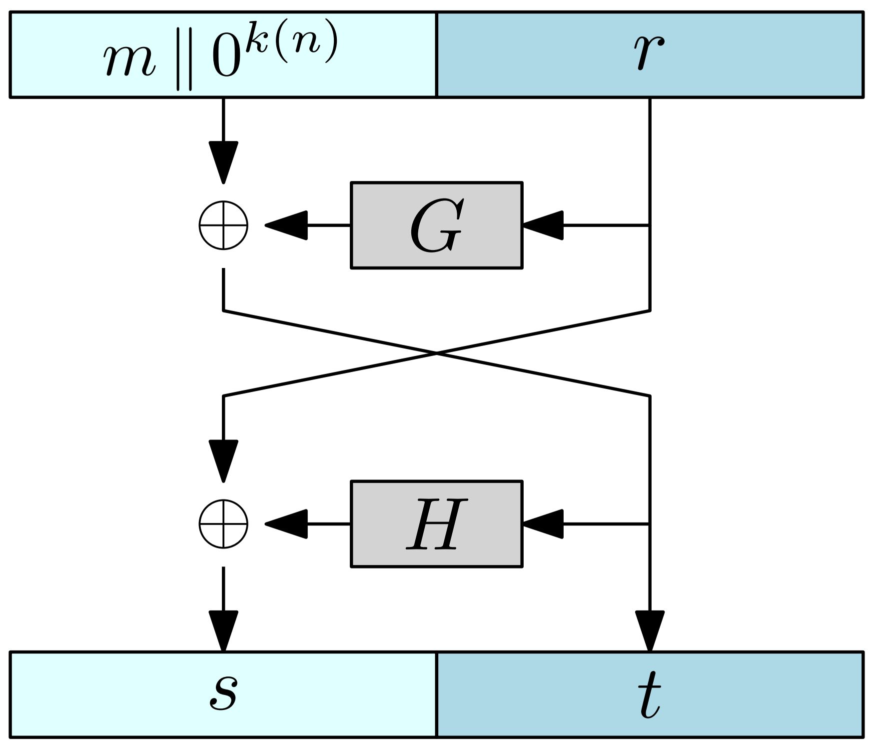
padding

actual encryption



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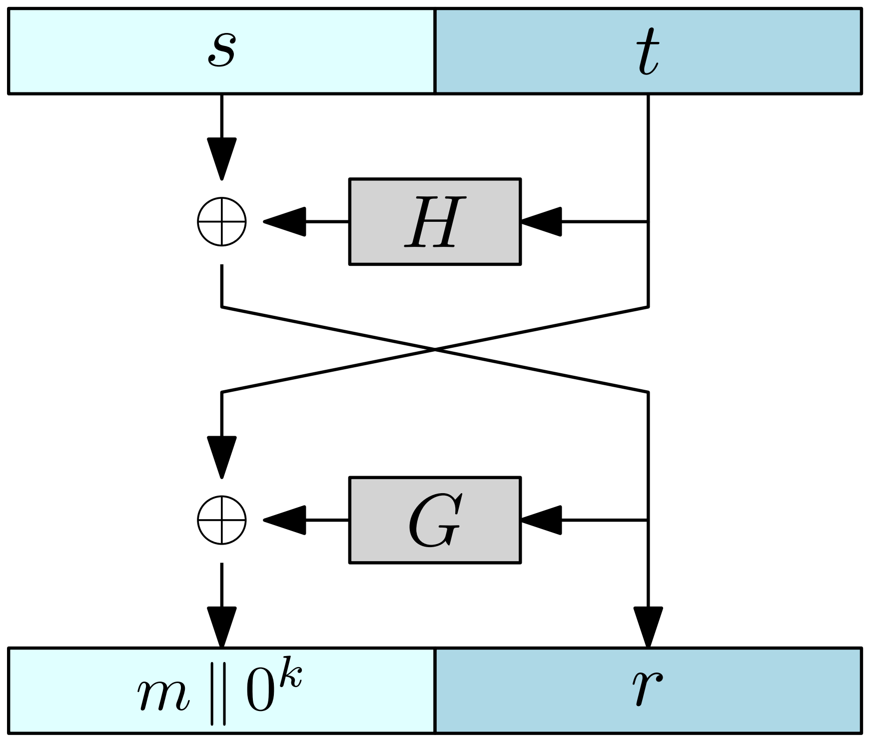

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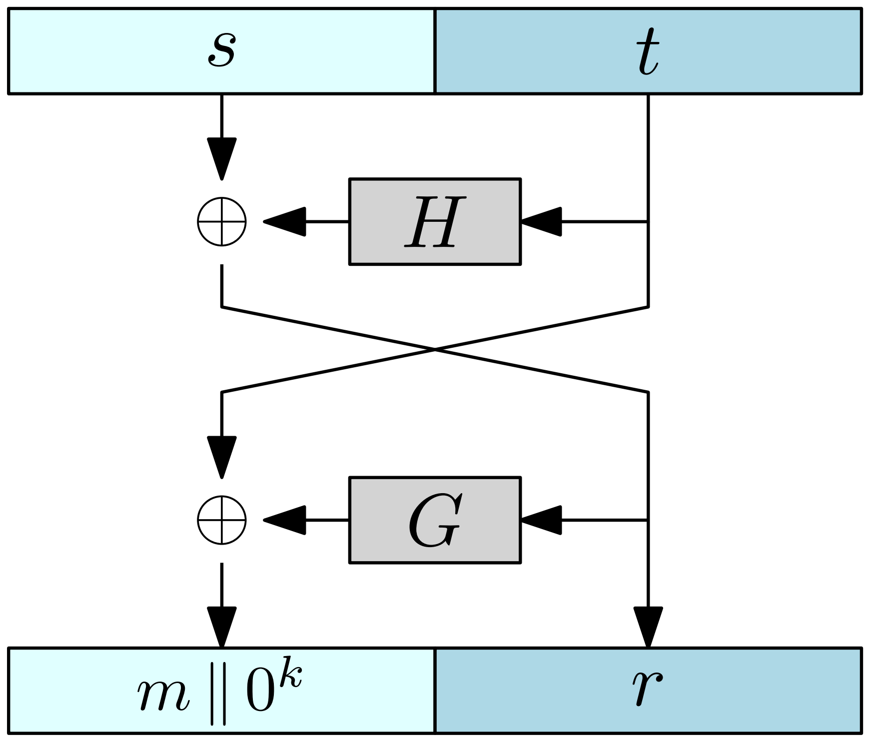

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RSA-OAEP (Decryption)

To decrypt c, the recipient can compute $\hat{m} = c^d \pmod N$ and then invert the Feistel network to recover $(m \,\|\, 0^k) \,\|\, r$

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- If $\hat{m}$ has more than $\ell(n) + 2k(n)$ bits: return $\perp$
- Interpret $\hat{m}$ as $s \,\|\, t$, where s has $k(n)$ bits and t has $\ell(n) + k(n)$ bits
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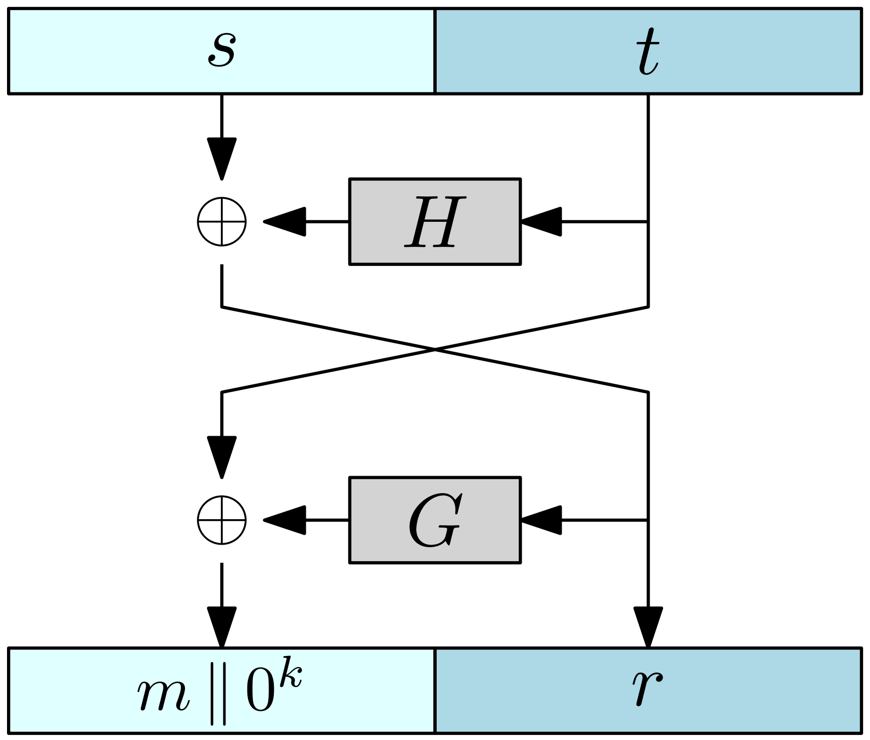

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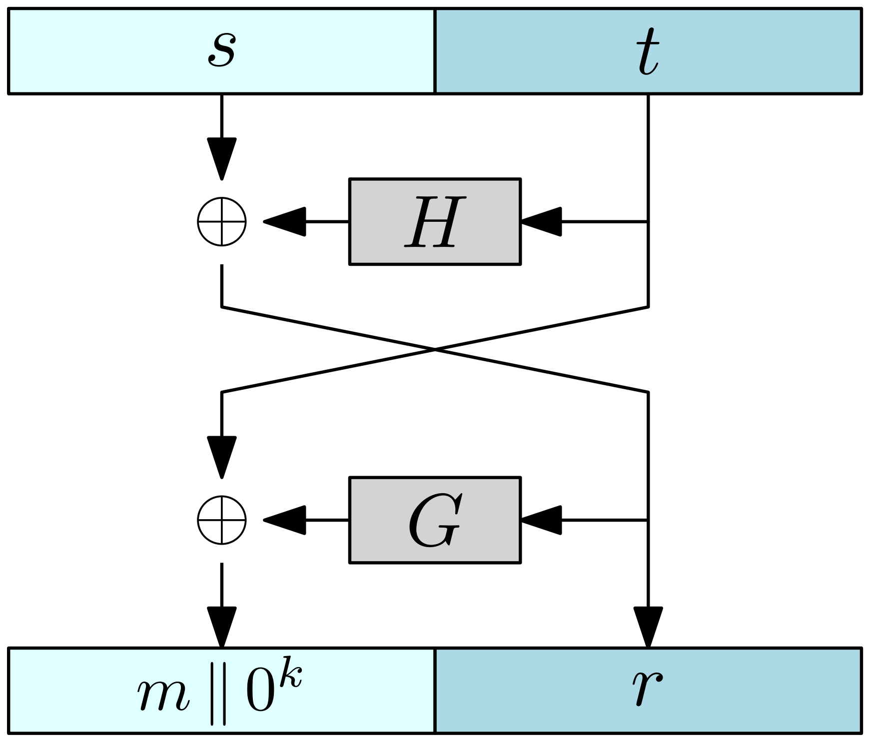

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Side channel attacks: Even if the error is the same, an attacker might be able to distinguish the two cases by observing the time elapsed before an error is returned.

“Fixing” RSA

How do we make RSA secure?

Option 1: Use randomized encoding

- Make sure that the message is “random enough”
- Choose some **randomized encoding** between messages and group elements
- To encrypt: encode the message, then encrypt the group element
- To decrypt: decrypt the ciphertext to recover the group element, then decode the group element

Option 2: Use RSA as a KEM **with a key derivation function**

RSA as a KEM

We can design a KEM based on RSA:

- We will encrypt **a random group element**
- We will use a key derivation function $H : \mathbb{Z}^* \to \{0,1\}^n$, modeled as a random oracle

Gen(1^n):

- $(N, e, d) \leftarrow \mathsf{GenRSA}(1^n)$
- Return (pk, sk) where $pk = \langle N, e\rangle$ and $sk = \langle N, d\rangle$

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Theorem: If the RSA problem is hard relative to GenRSA and H is modeled as a random oracle, then the RSA-based KEM described before is CCA-secure.

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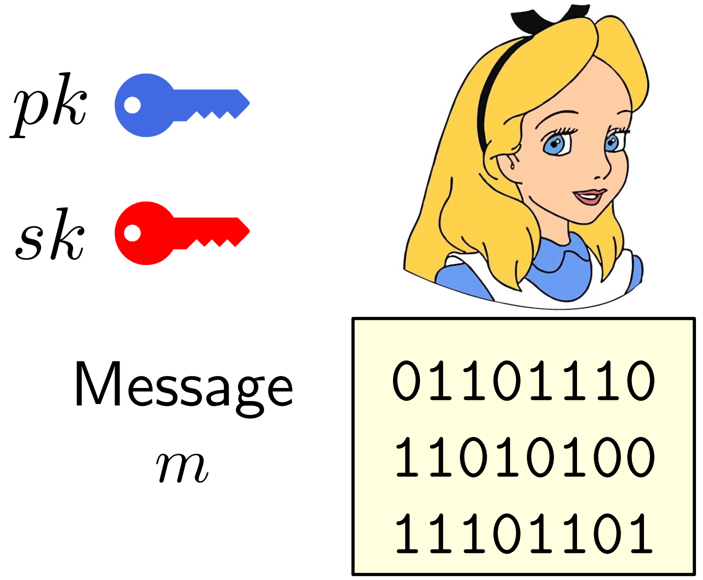
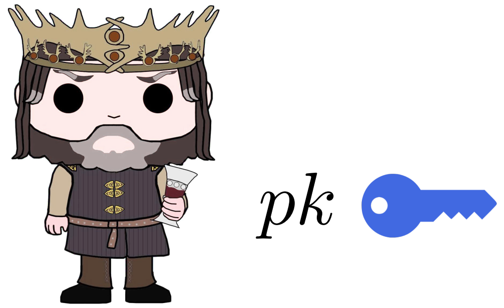
- Even if an attacker can compute a ciphertext c' related to c, the decapsulation oracle returns $H(r')$, where $r' = (c')^d \pmod N$
- Since $r' \neq r$ and H is modeled as a random oracle, $H(r')$ is a random binary string independent of $H(r)$

Digital Signatures

We can guarantee authentication and integrity in the public-key setting by using **digital signatures**





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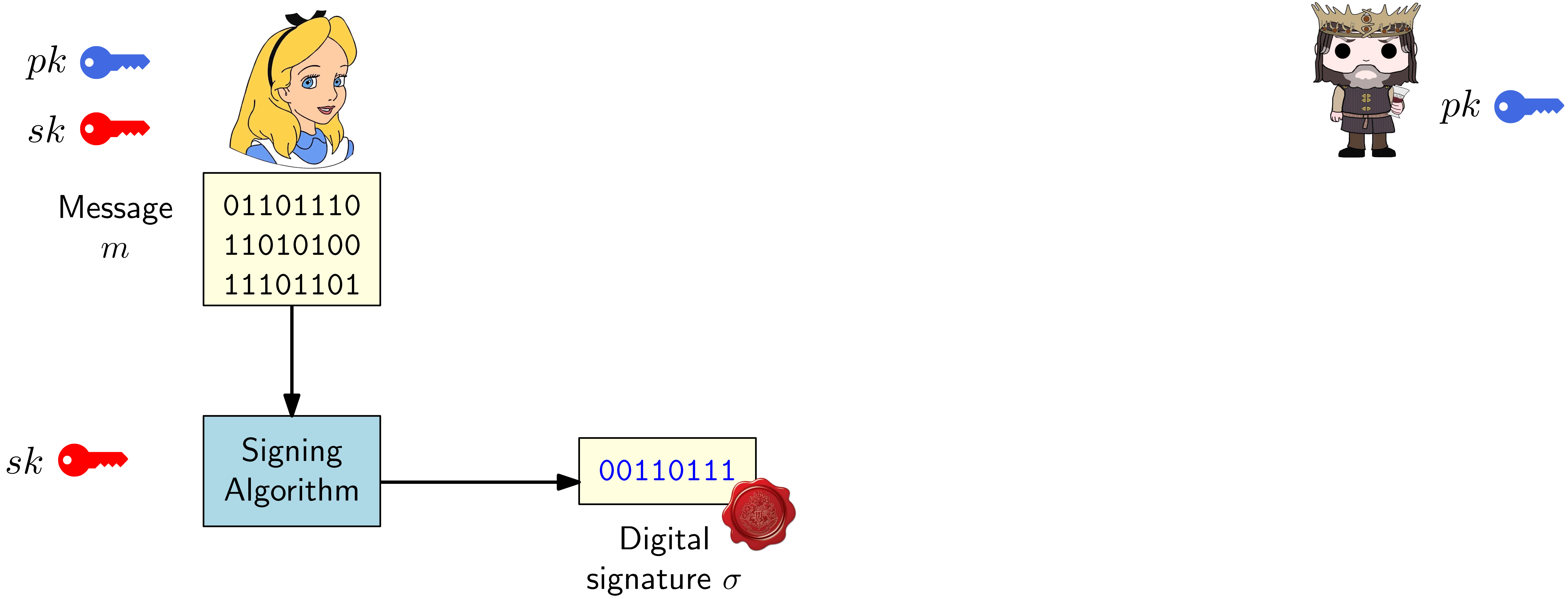

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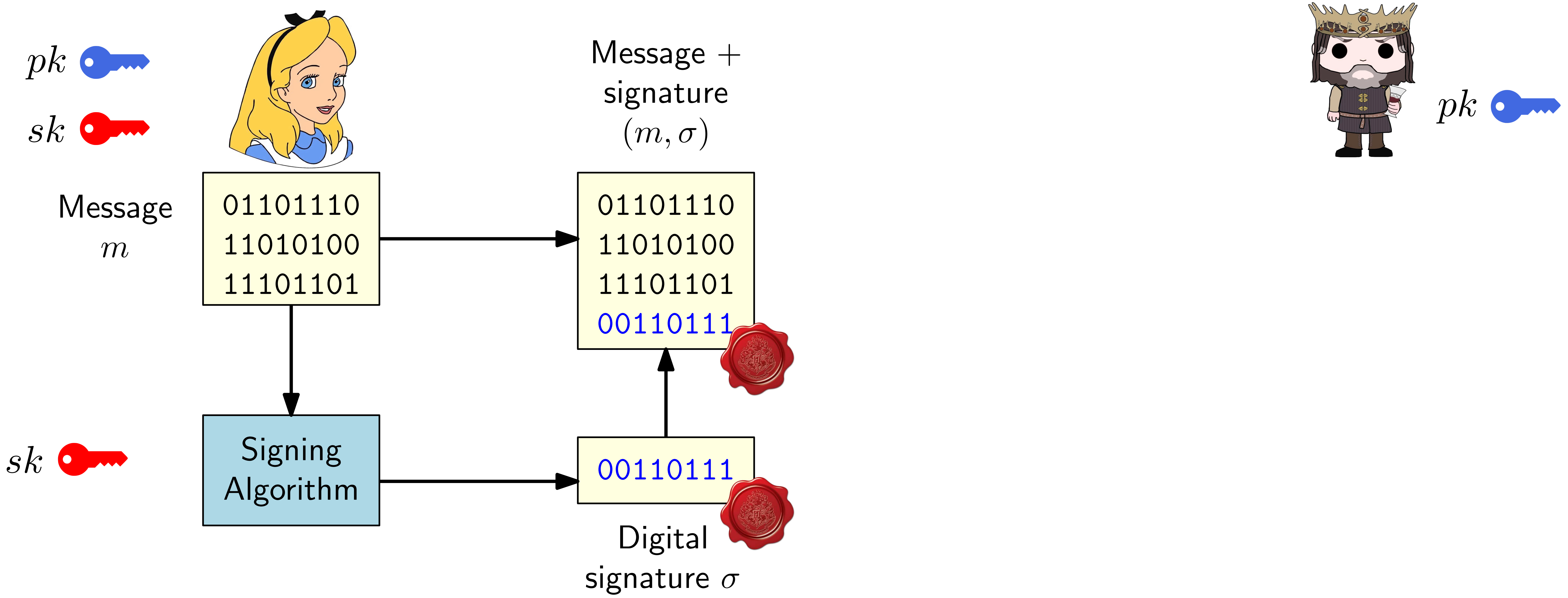

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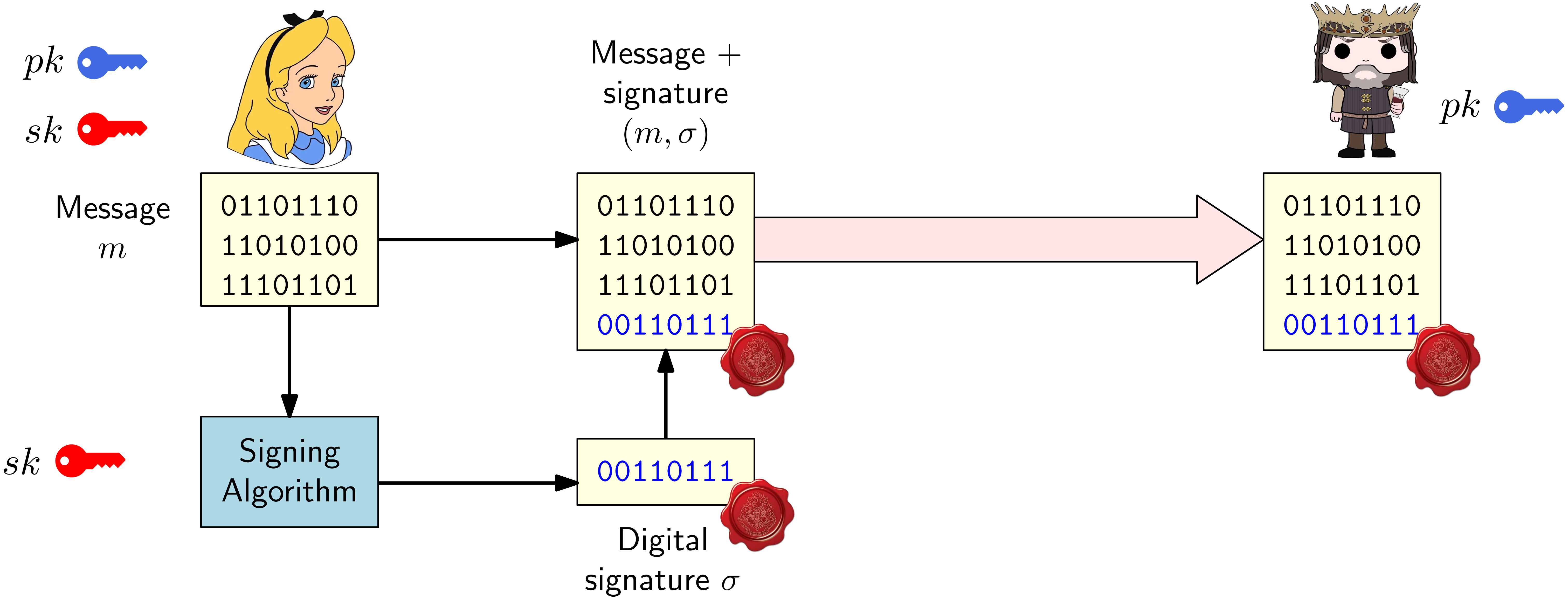

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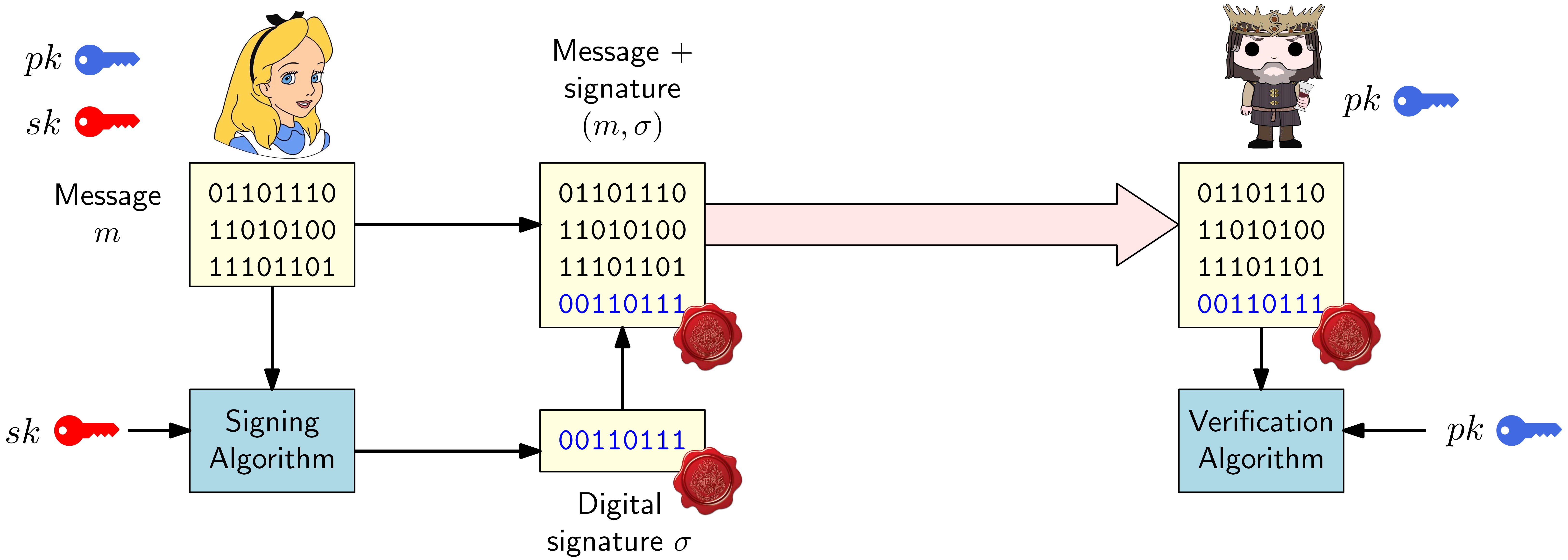

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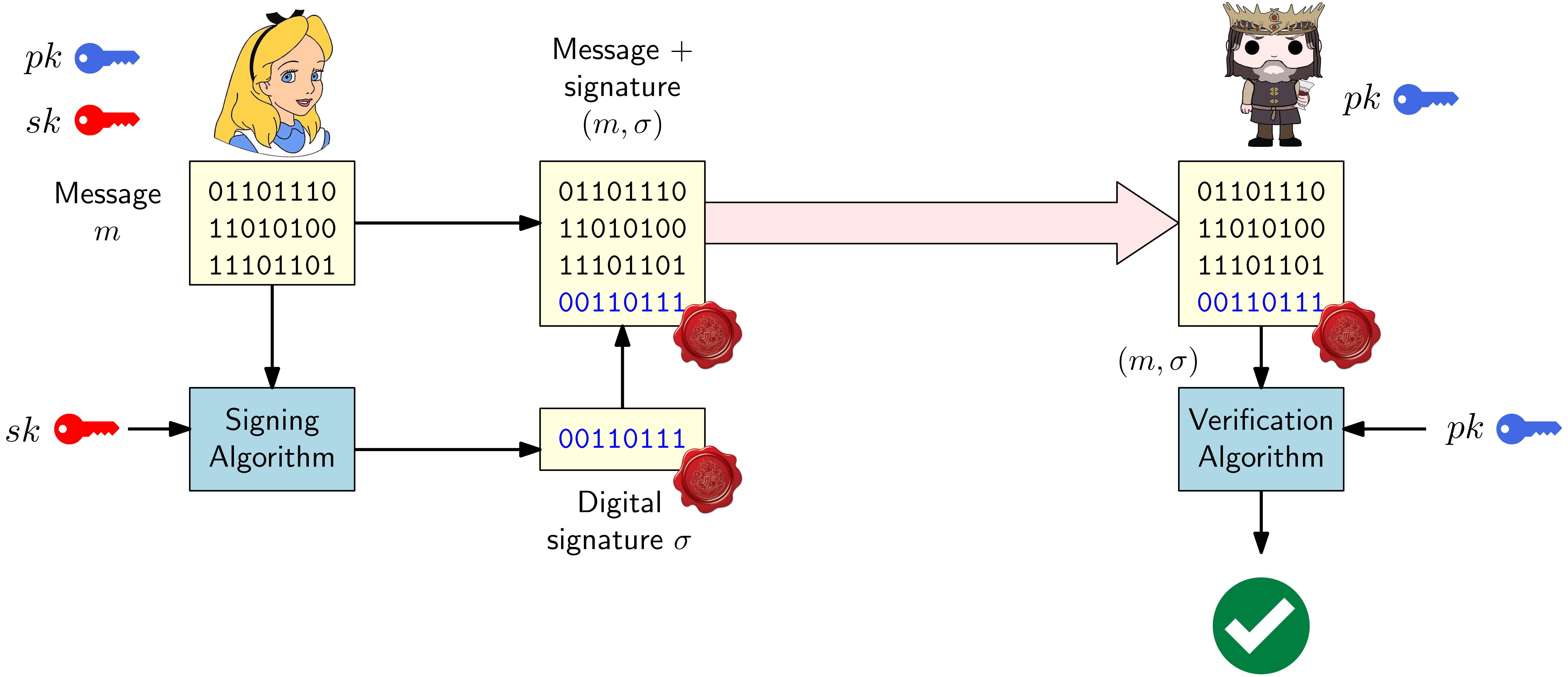

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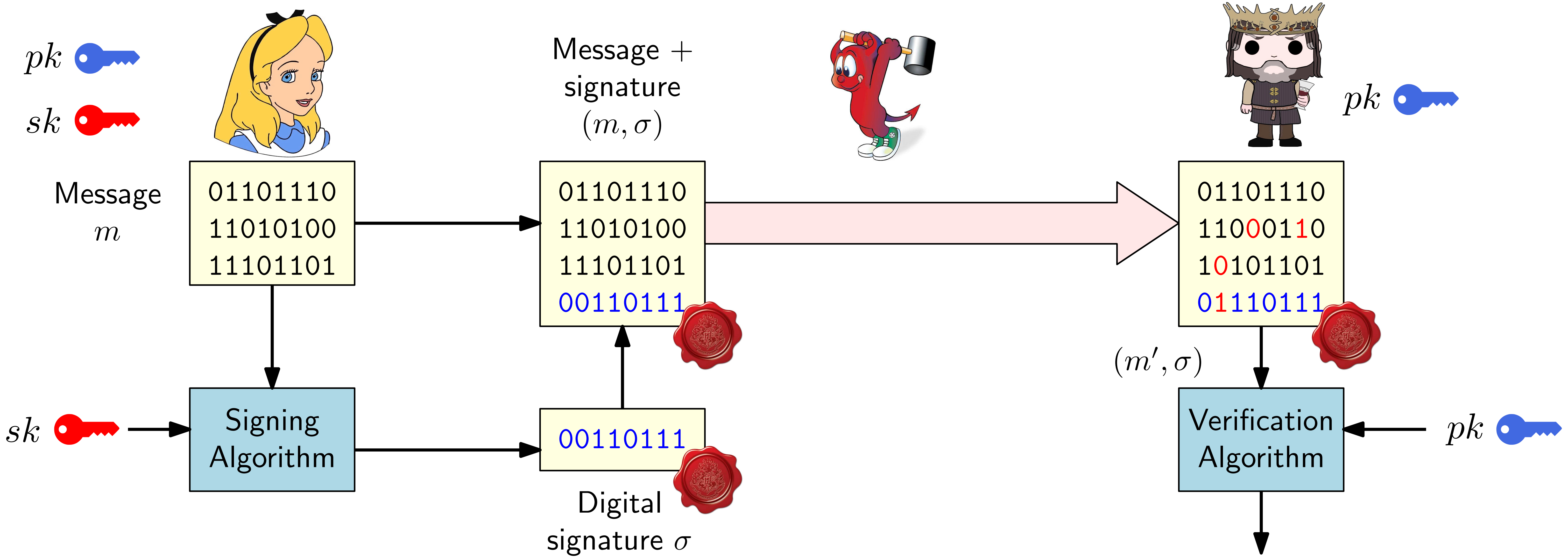

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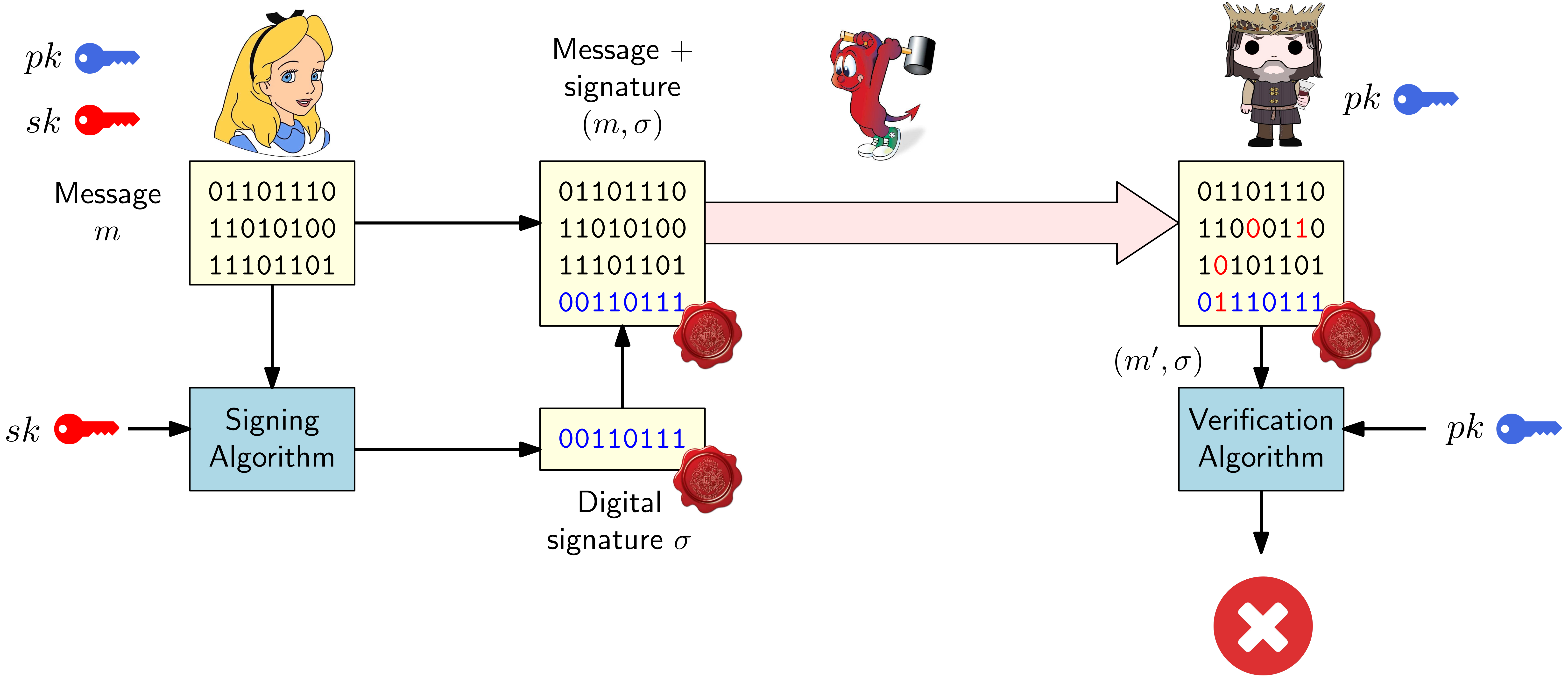

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Slower, Longer signatures

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MACs:

Faster (by 2,3 order of magnitude), shorter tags

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Digital Signature Schemes: Formal Definition

A **Digital Signature Scheme** is a triple of algorithms (Gen, Sign, Vrfy)

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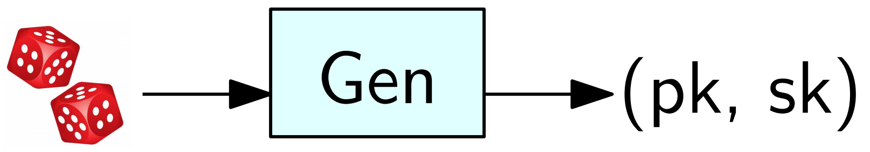

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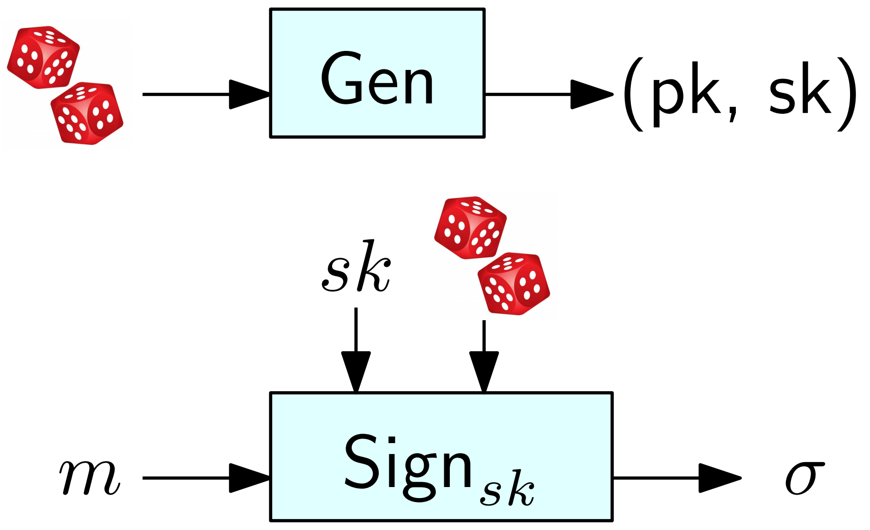

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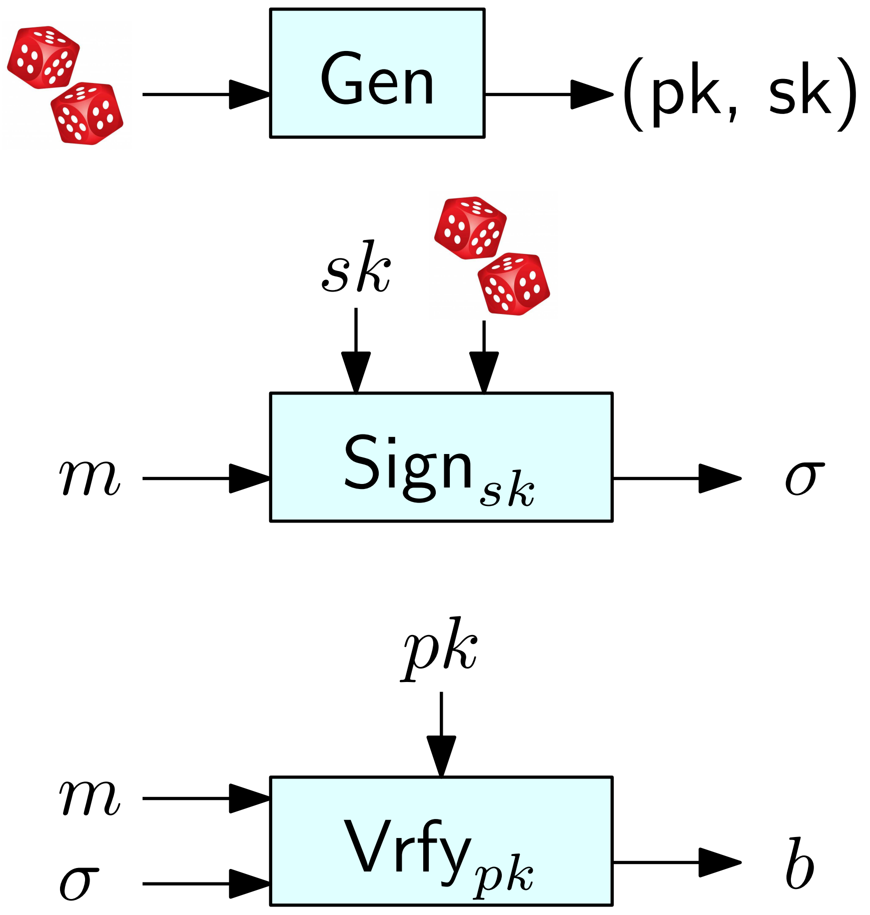

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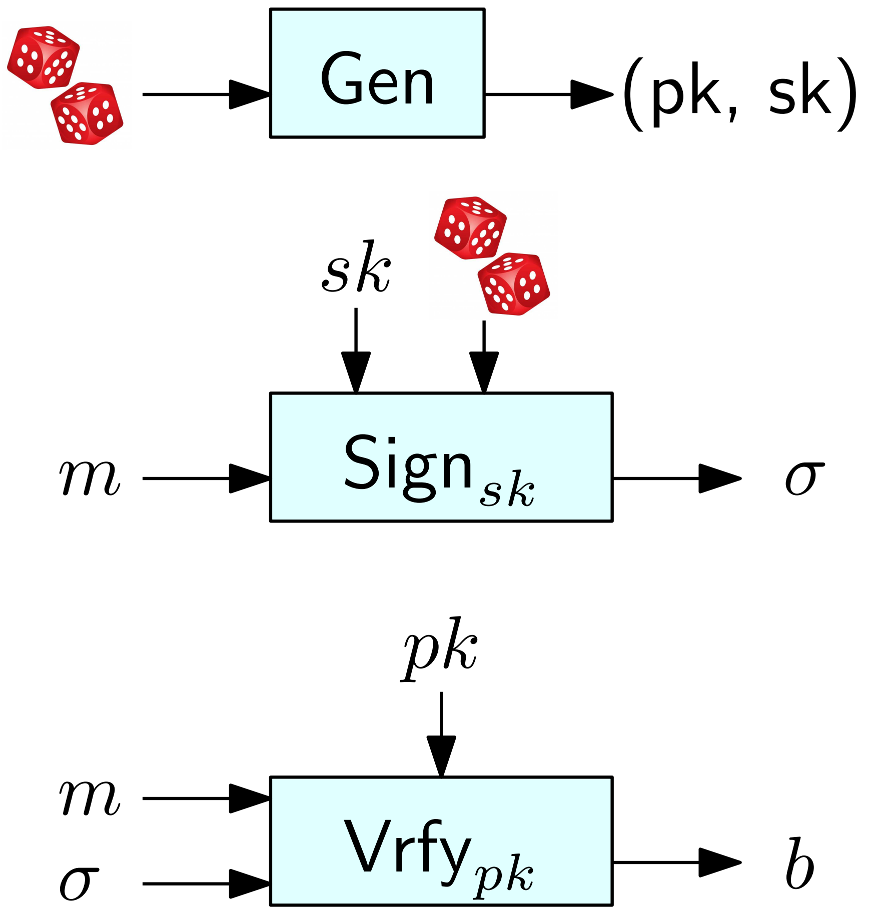

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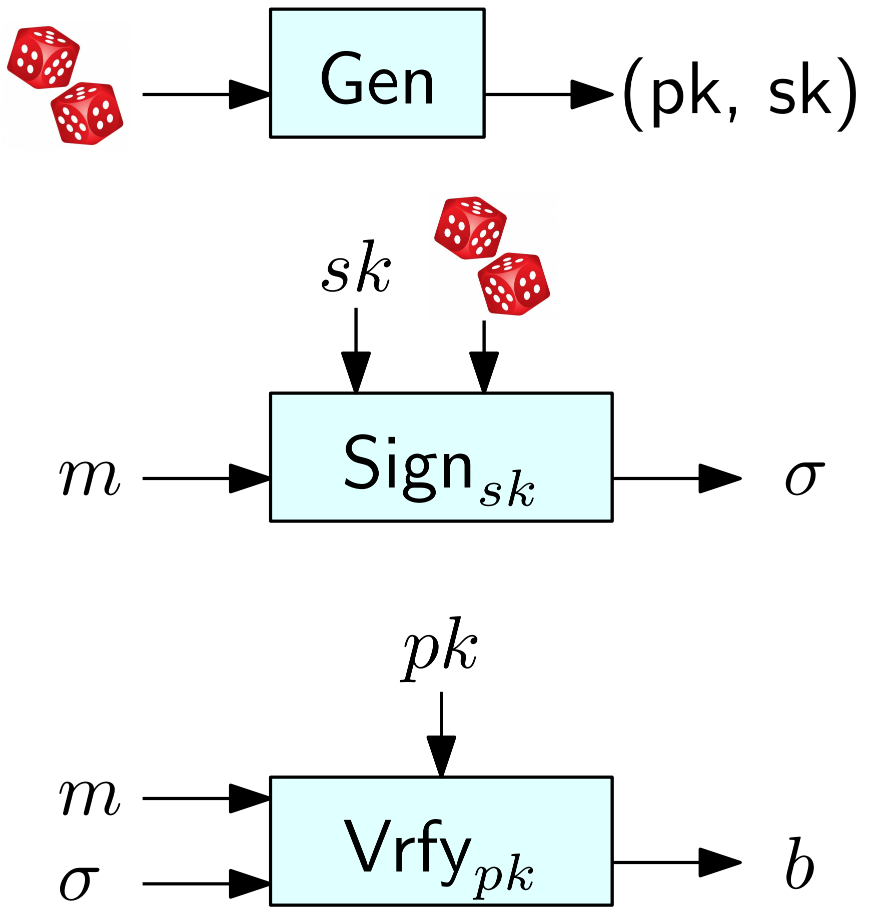

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If the message space is $\{0,1\}^{\ell(n)}$, we call $(\mathsf{Gen}, \mathsf{Sign}, \mathsf{Vrfy})$ a signature scheme for messages of length $\ell(n)$.

Digital Signature Schemes: Security Definition

Let $\Pi = (\mathsf{Gen}, \mathsf{Sign}, \mathsf{Vrfy})$ be a digital signature scheme. We name the following experiment $\mathsf{Sig\text{-}forge}_{\mathcal{A},\Pi}(n)$:

- A key-pair (pk, sk) is generated using $\mathsf{Gen}(1^n)$
- The public key pk is sent to the adversary
- The adversary can interact with an oracle that can be queried with a message m' and outputs a signature σ' obtained by running $\mathsf{Sign}_{sk}(m')$
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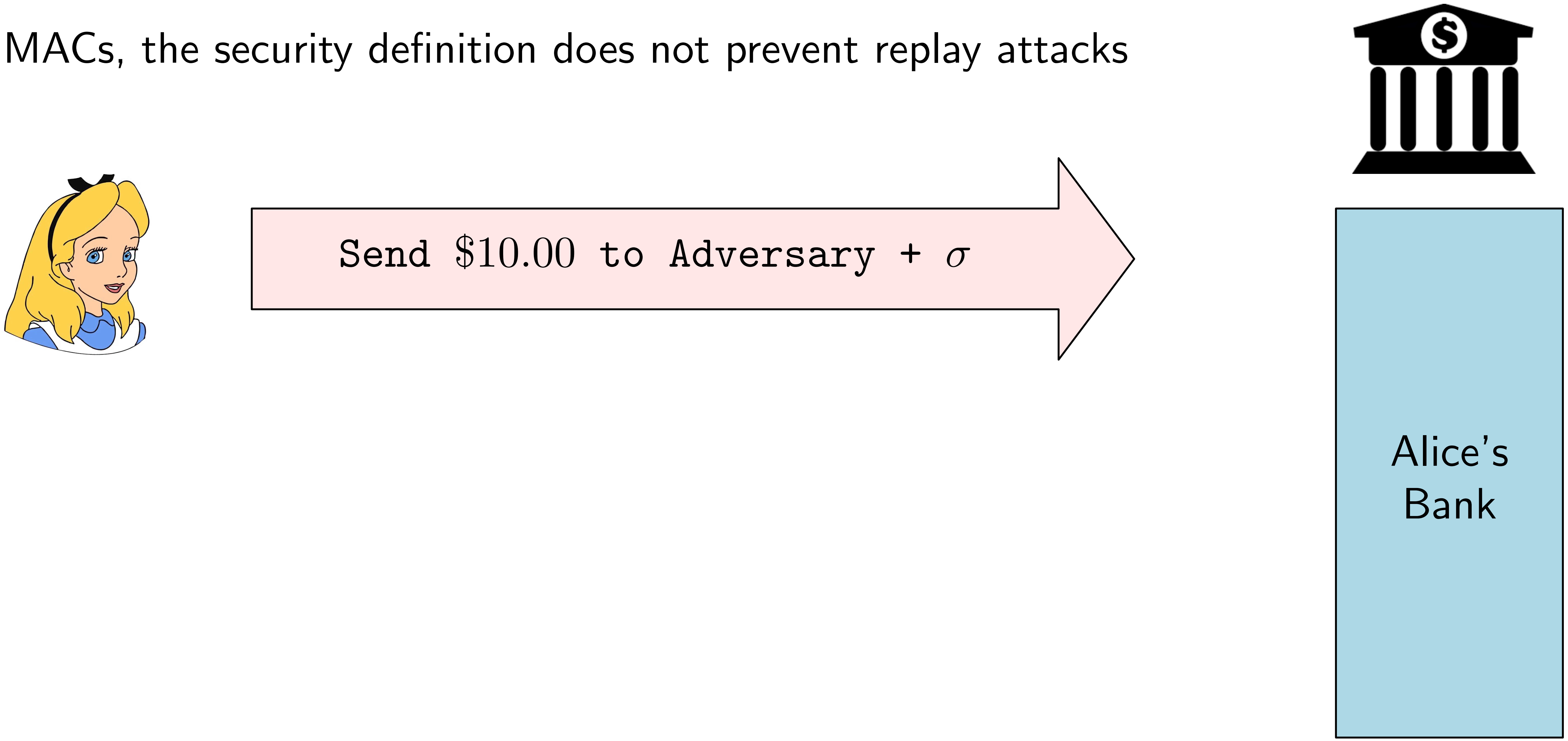
> **Definition**: *A digital signature scheme Π is existentially unforgeable under an adaptive chosen-message attack (is* **secure***) if, for every probabilistic polynomial-time adversary $\mathcal{A}$, there is a negligible function ε such that:*
>
> $$\Pr[\textit{Sig-forge}_{\mathcal{A},\Pi}(n) = 1] \leq \varepsilon(n)$$

Some Remarks on the Security Definition

Just like for MACs, the security definition does not prevent replay attacks



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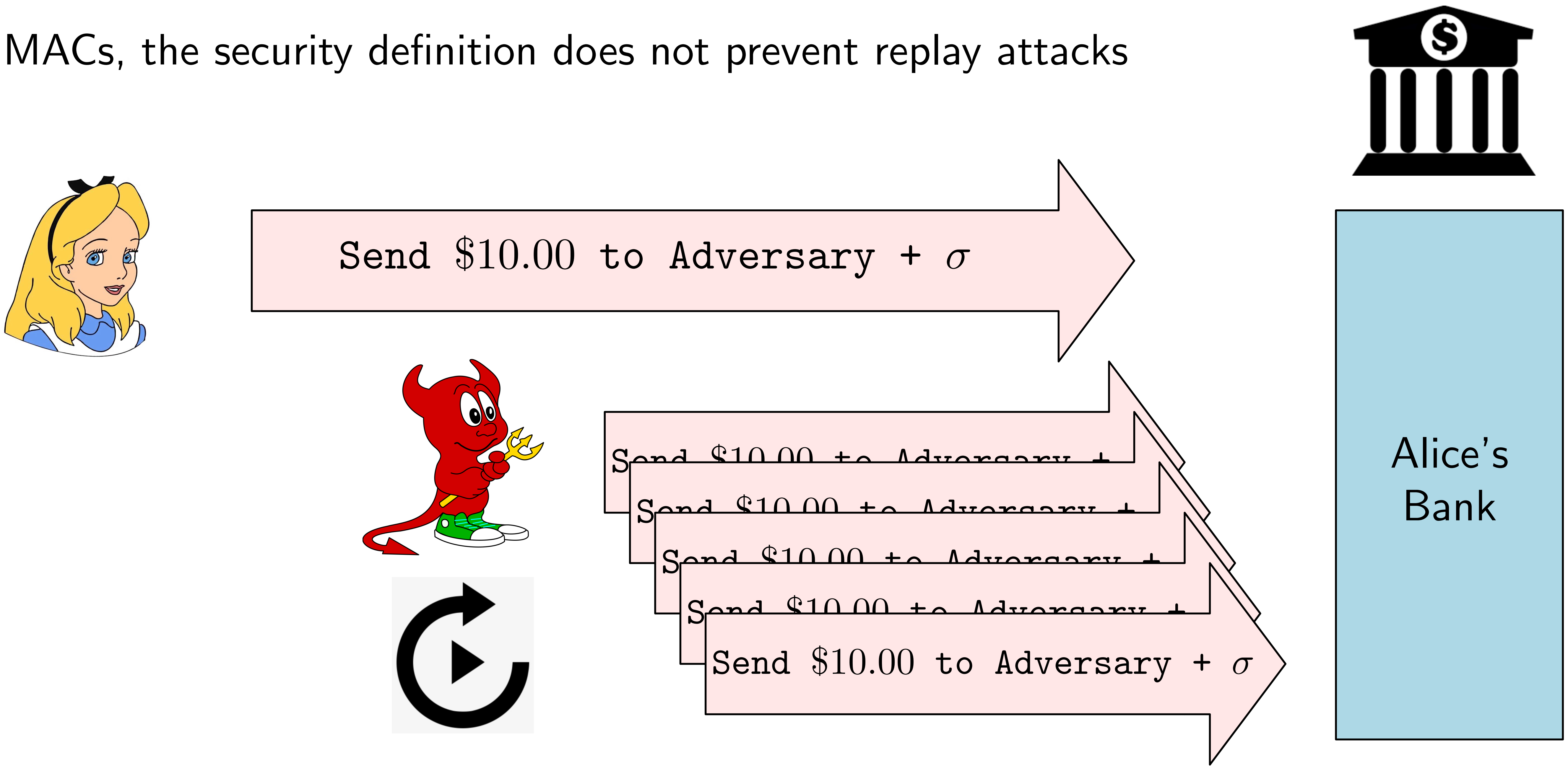

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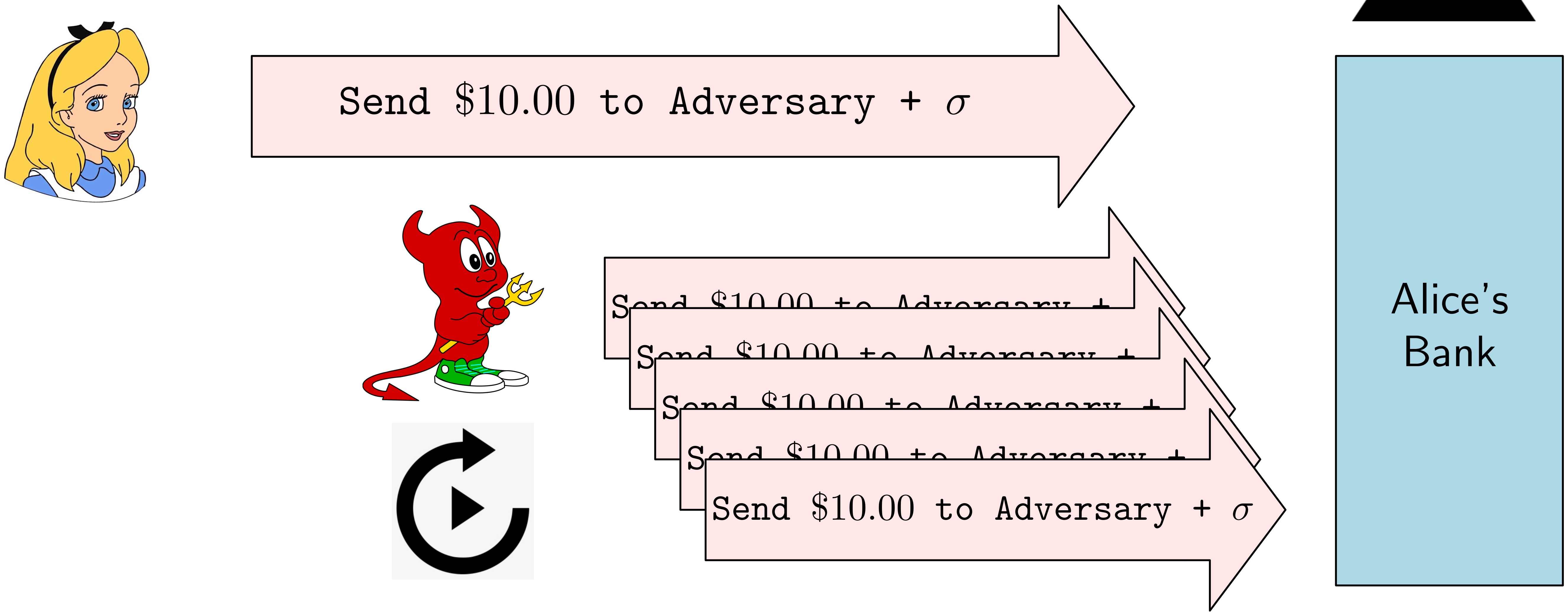

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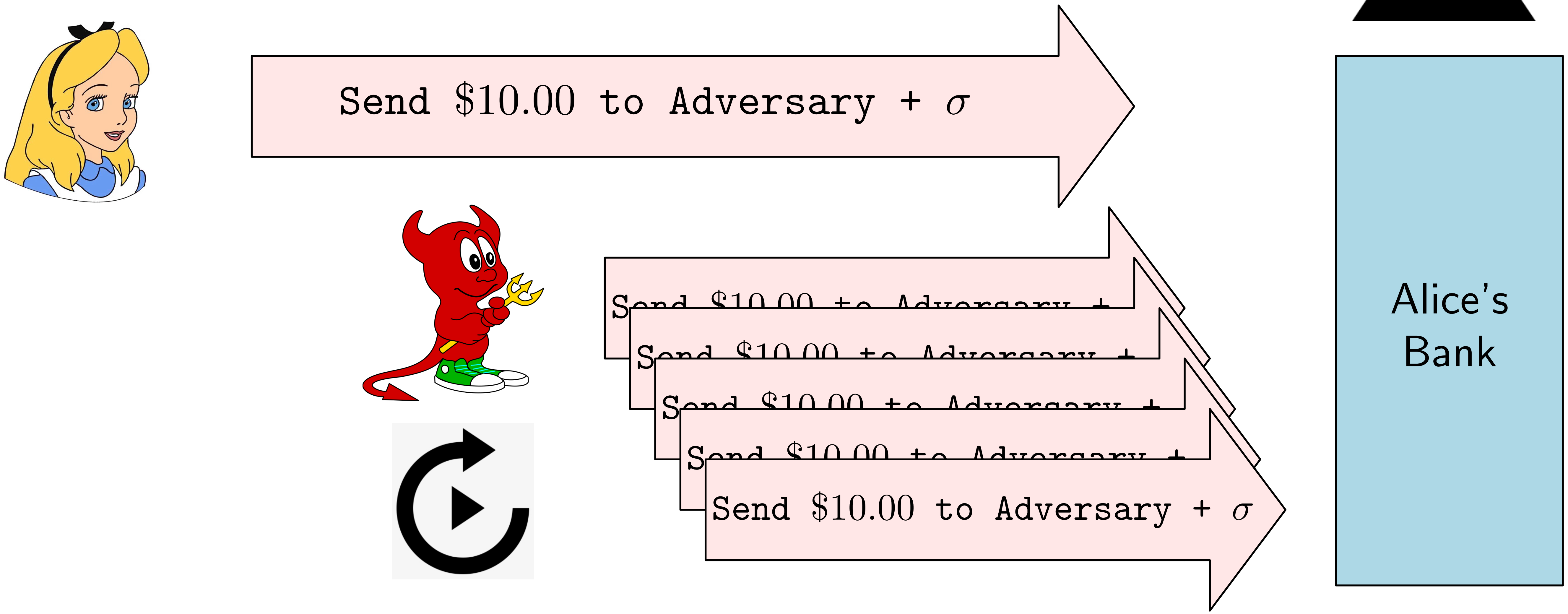

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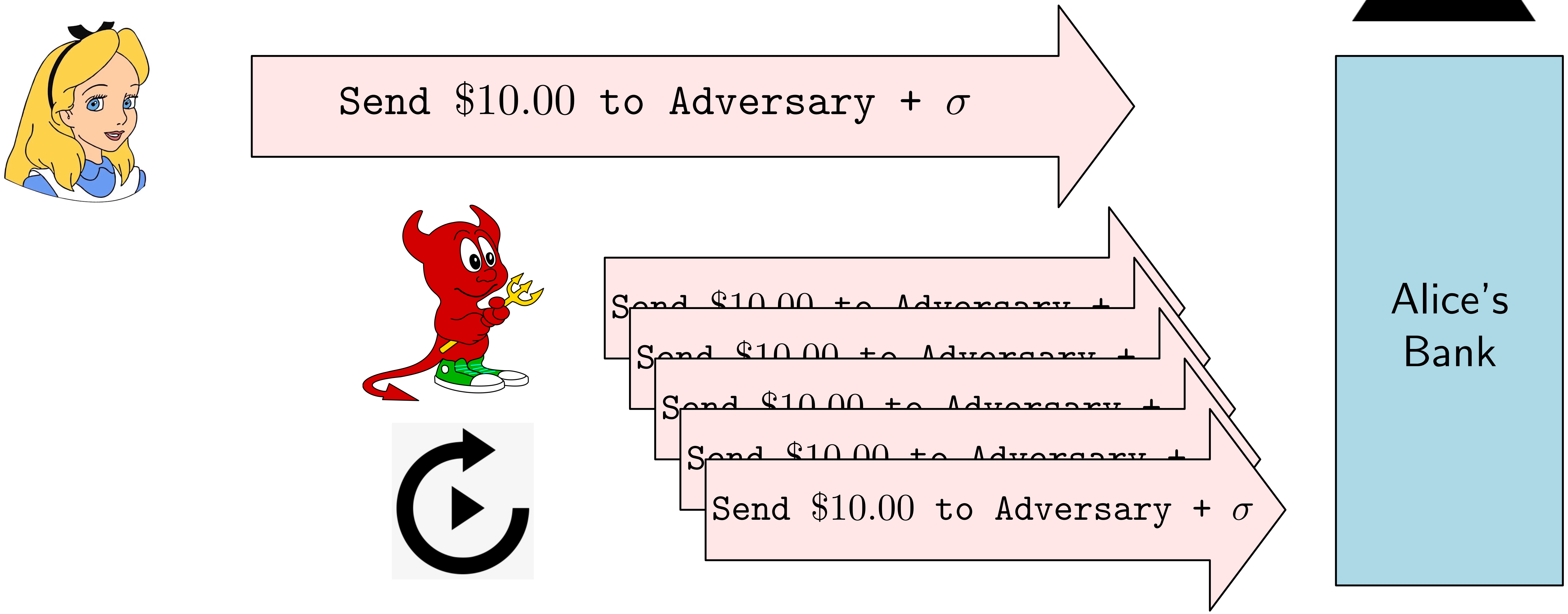

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Strongly Secure Digital Signature schemes can be defined to account for this (similarly to the modified experiment for MACs)

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We can focus on designing fixed-length digital signature schemes.

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Can be proven secure under the RSA assumption in the random oracle model

Longer signatures, slower

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Each of the above approaches suffices to win the Sig-forge$_{\mathcal{A},\Pi}(n)$ experiment!

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How can an attacker use this?

- Pick a malicious message m and factor it into $m_1 \cdot m_2$ (recall that factoring is easy on average)
- Convince an honest party to sign the two "innocuous"-looking messages m_1 and m_2 separately
- Forge a signature σ for m
- Use (m, σ) to convince a third party (e.g., a judge) that the honest party signed m (e.g., a contract)

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RSA-FDH natively handles long messages without using the Hash-and-Sign paradigm

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The RSA PKCS #1 v2.1 standard includes a randomized variant of RSA-FDH

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Both approaches have problems if used in this naive way

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Encrypt-then-authenticate:

- Alice computes $c \leftarrow \mathsf{Enc}_{ek}(\text{“Alice”} \,\|\, m)$
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Digital Certificates

Digital Certificates & Public-Key Infrastructure

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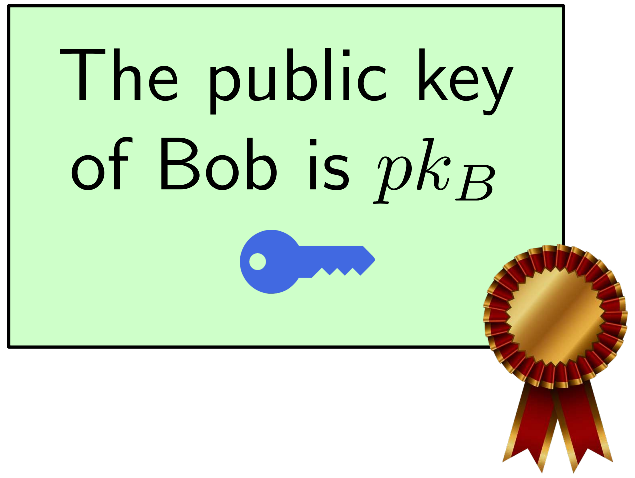
...and send the resulting signature, called a **digital certificate**, to Bob



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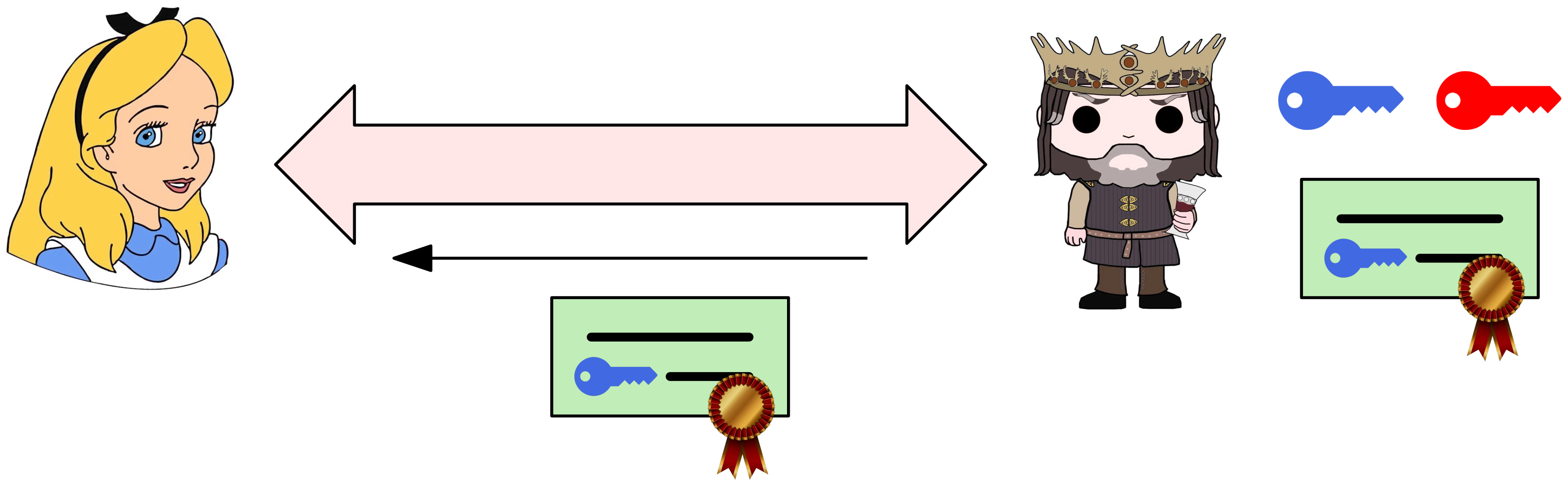

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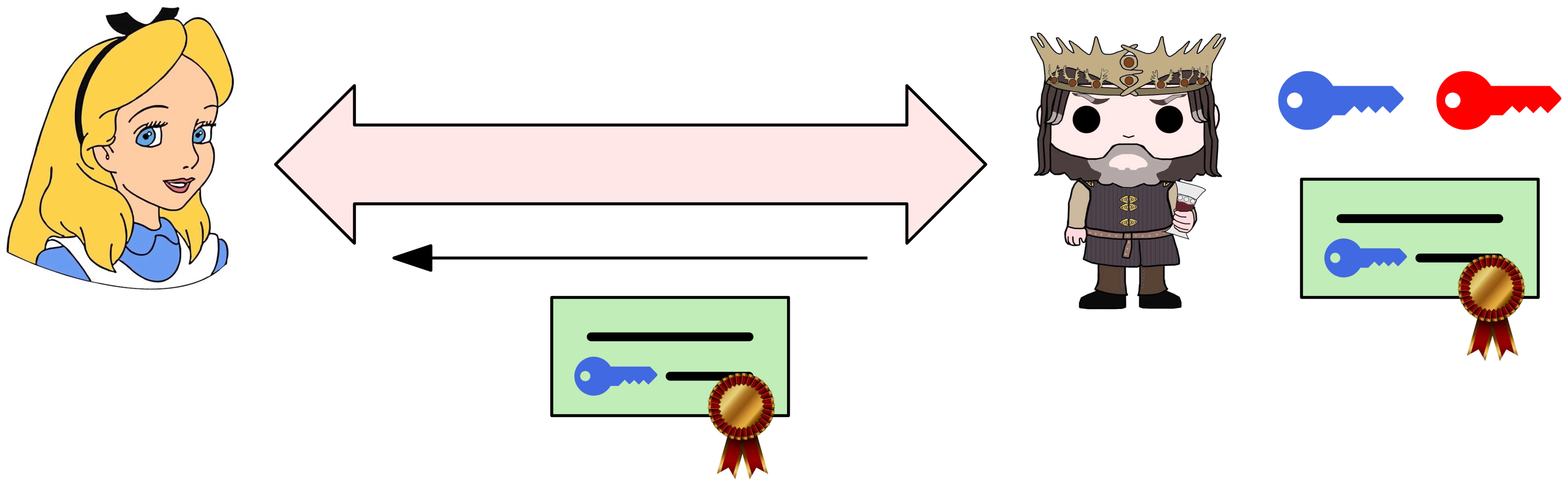

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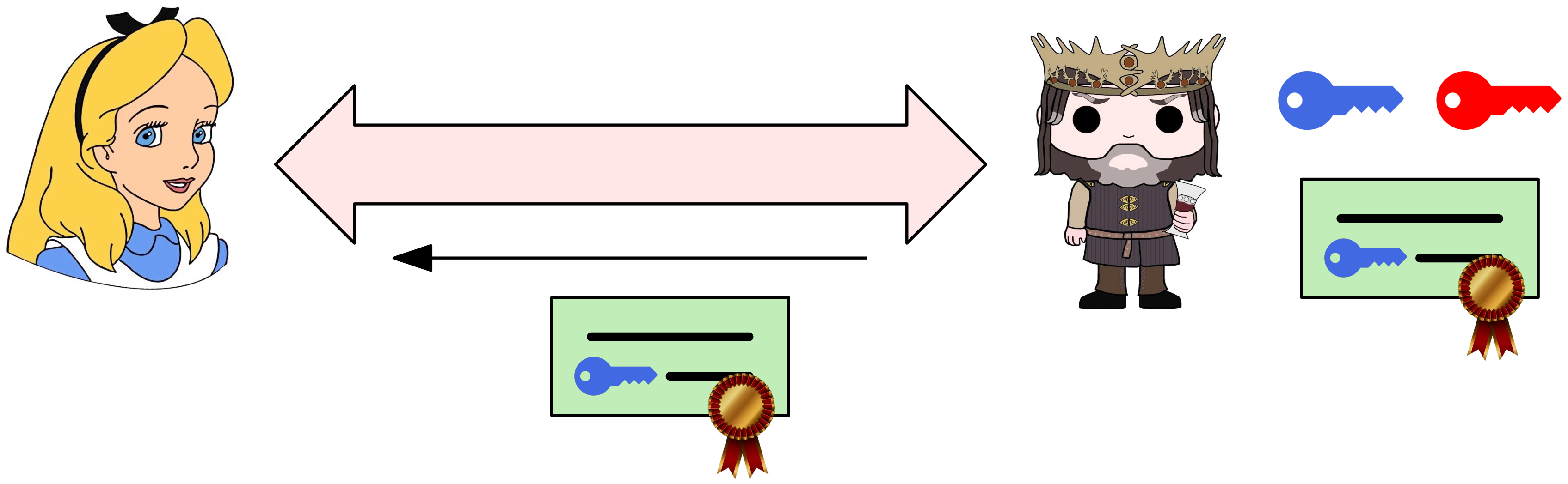

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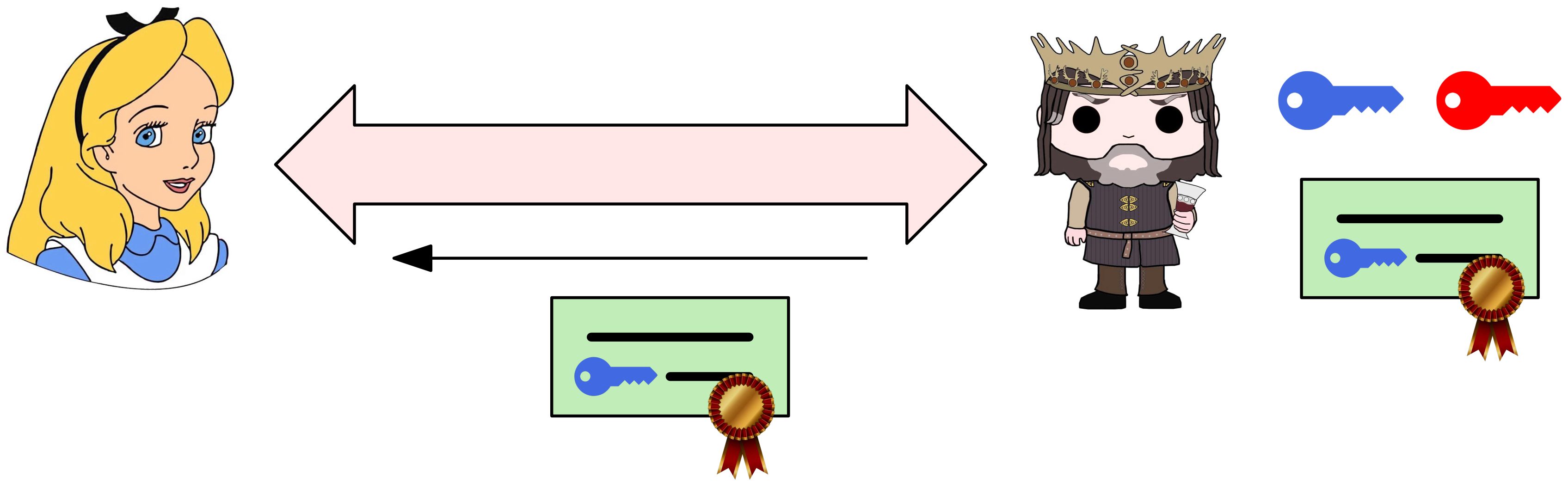

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- In practice pk_C is bundled together with some piece of software

  E.g., Web browsers have a default trusted list of CAs and are shipped with their public keys

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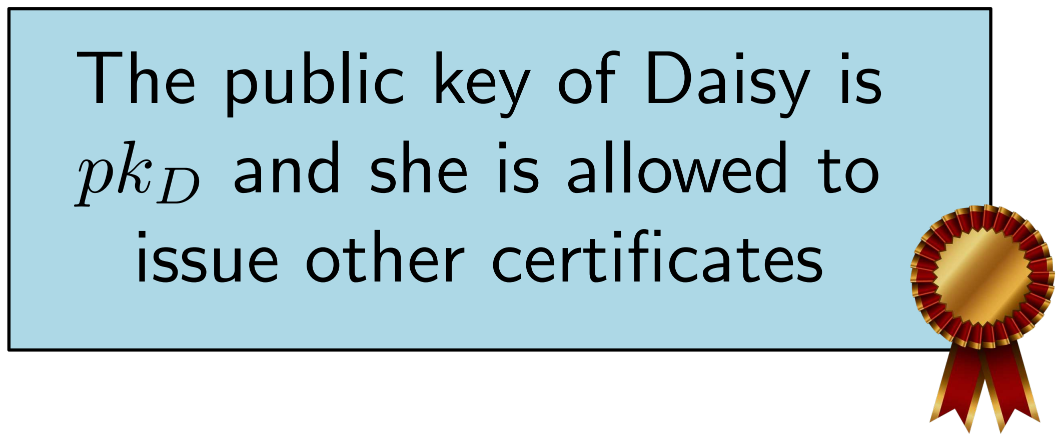

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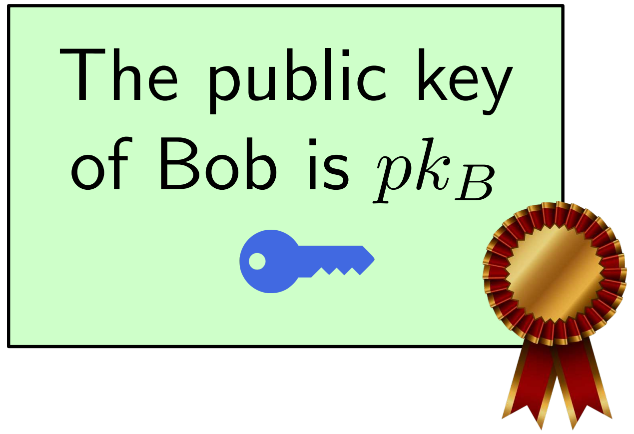

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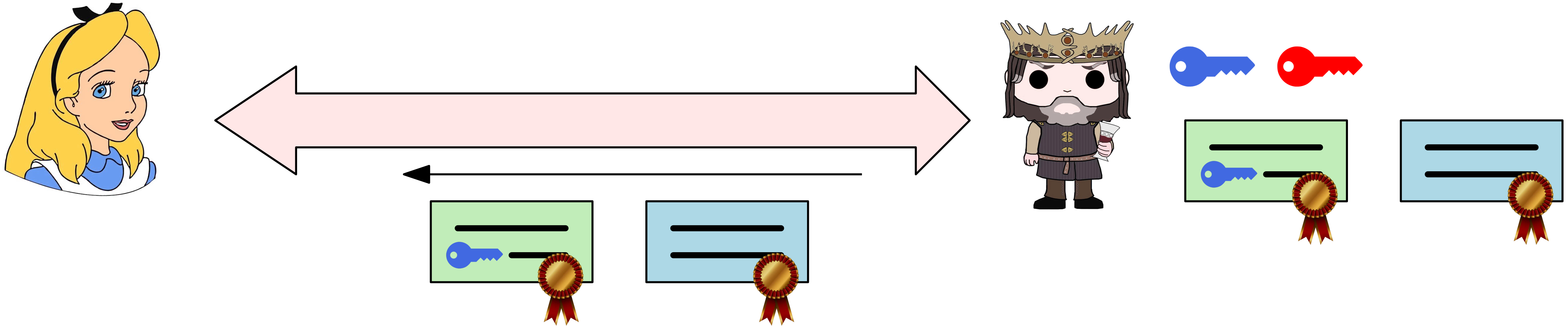

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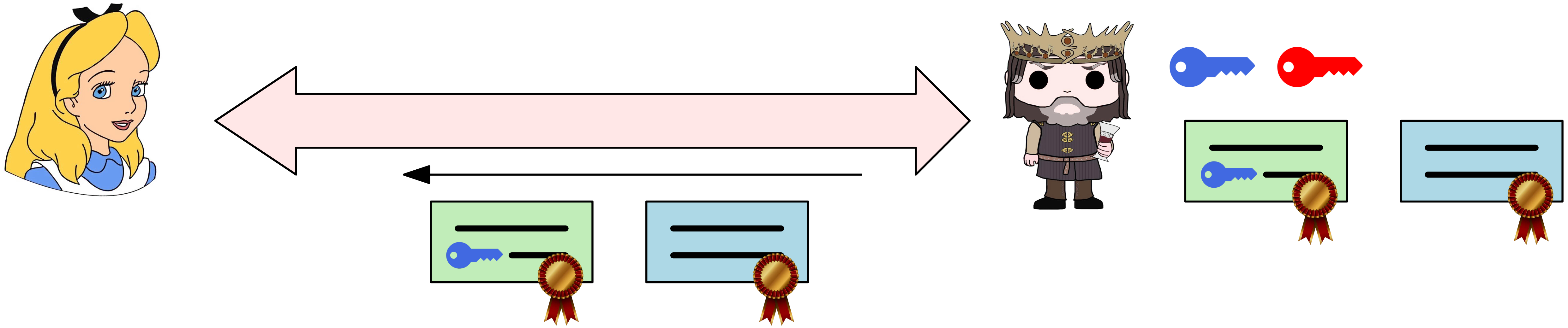

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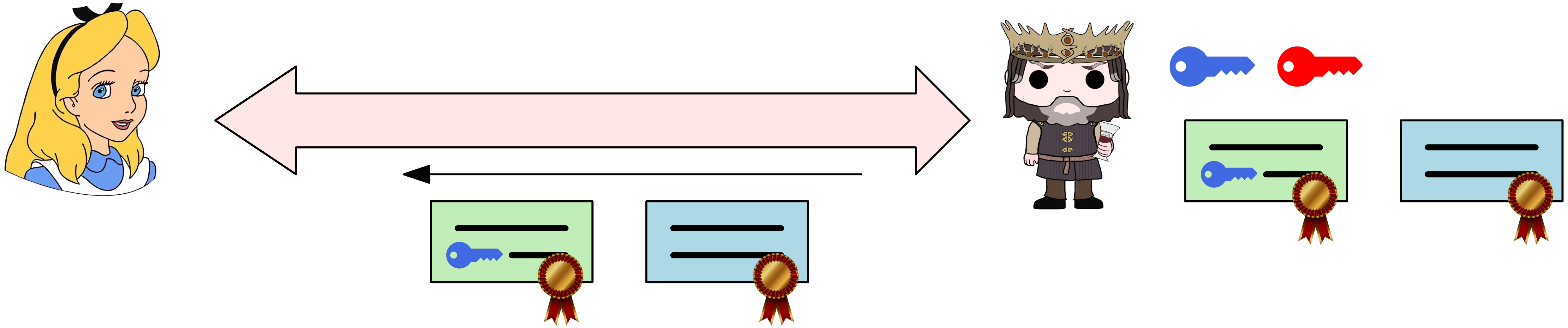

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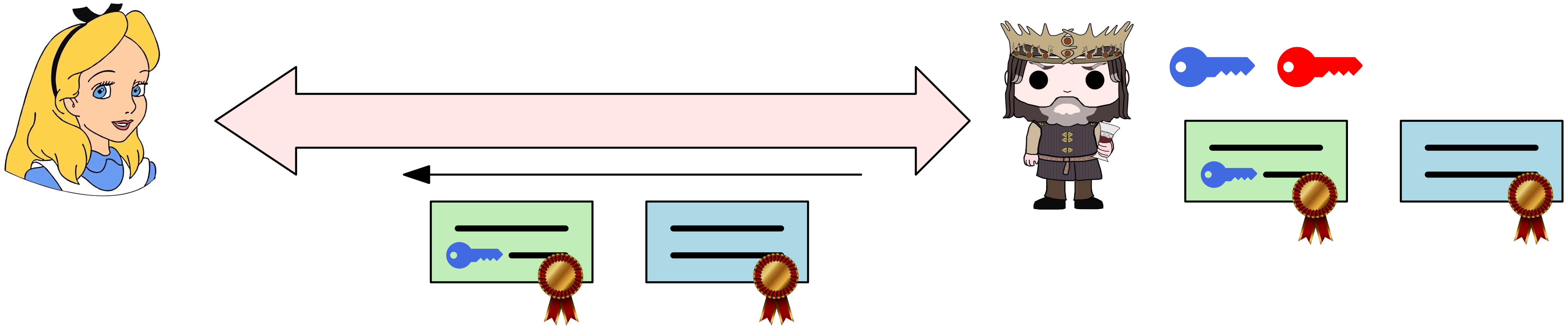

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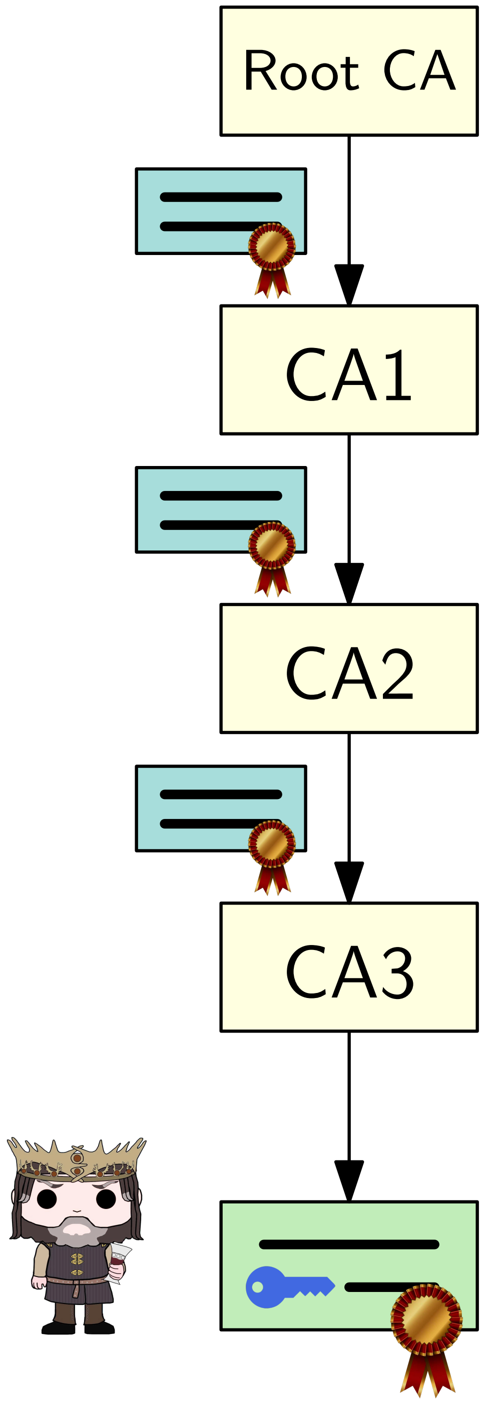

The same idea generalizes to any number of intermediate CAs

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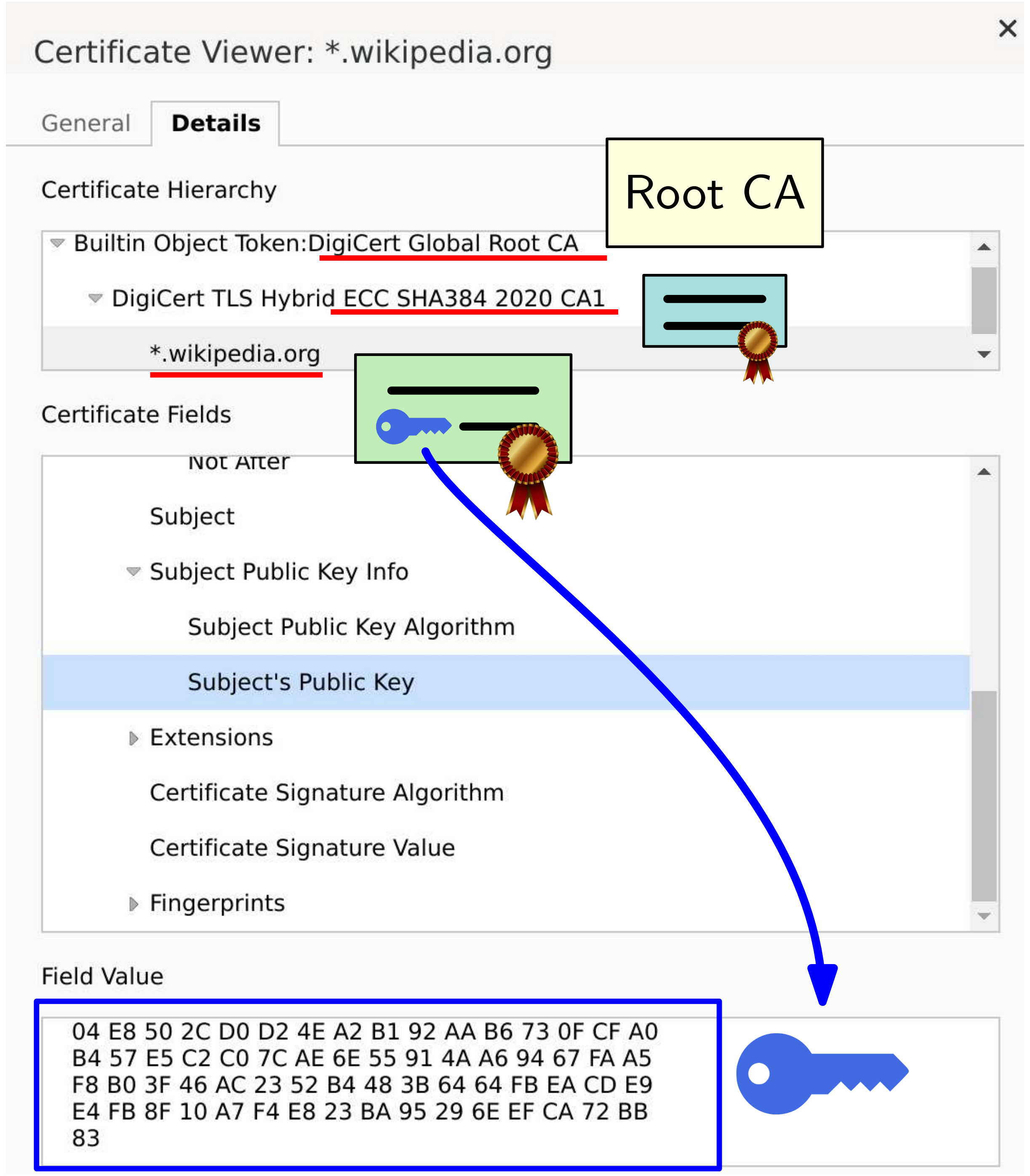

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- **Revocation**

  — The CA signs the message ("The public key of Bob is pk_b, serial_no), where serial_no is a unique serial number

  — The CA periodically (e.g., daily) creates a **revocation list** of all the serial numbers of the certificates it issued but need to be invalided, signs it, and publishes the signed list

  — When Alice checks Bob's certificate, she also checks that the serial number is not in the revocation list

  — Disadvantage: Alice needs to keep an updated copy of the revocation list

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- **Handshake protocol:** performs authenticated key exchange to establish two shared *symmetric* private keys k_S and k_C
- **Record-layer protocol:** uses the shared keys to encrypt and authenticate the communication

— Messages from client to server and vice-versa are encrypted using an **authenticated encryption scheme**

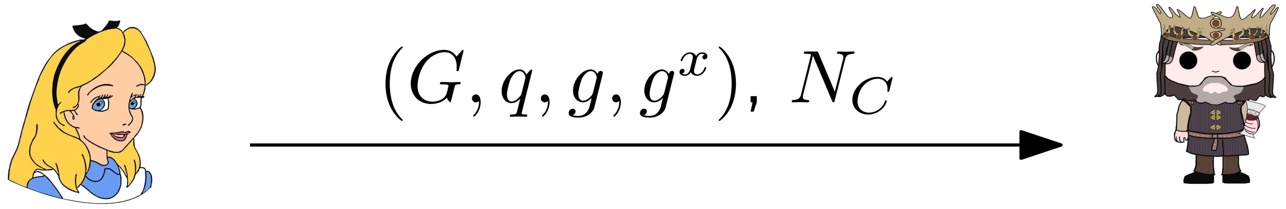
— The client encrypts with key k_C (and decrypts with key k_S)

— The server encrypts with key k_S (and decrypts with key k_C)

— Sequence numbers are used to prevent replay attacks

TLS: Handshake protocol

- The client initiates the handshake by sending the initial message (G, q, g, g^x) of the Diffie-Hellman key-exchange protocol and a nonce N_C chosen u.a.r. from $\{0,1\}^n$



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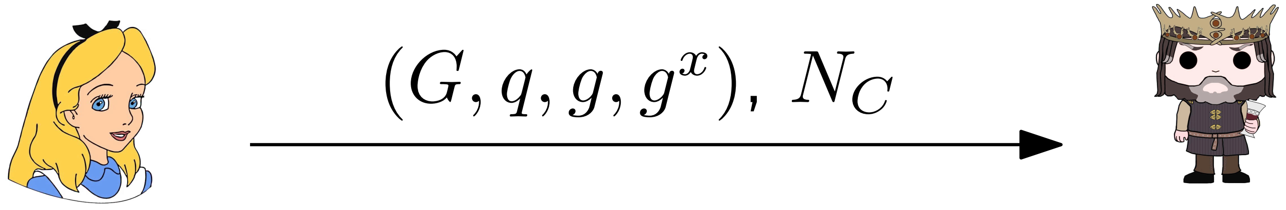

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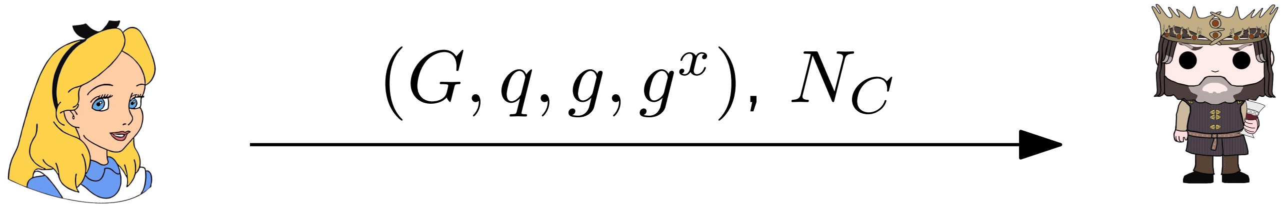

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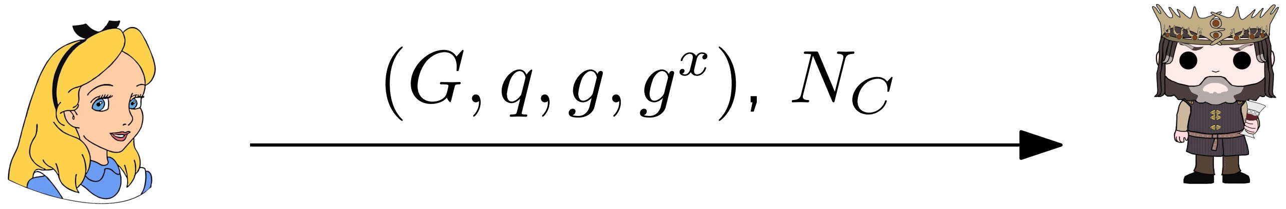

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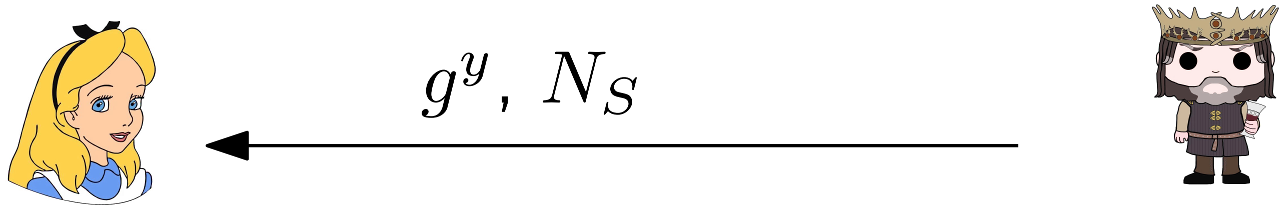

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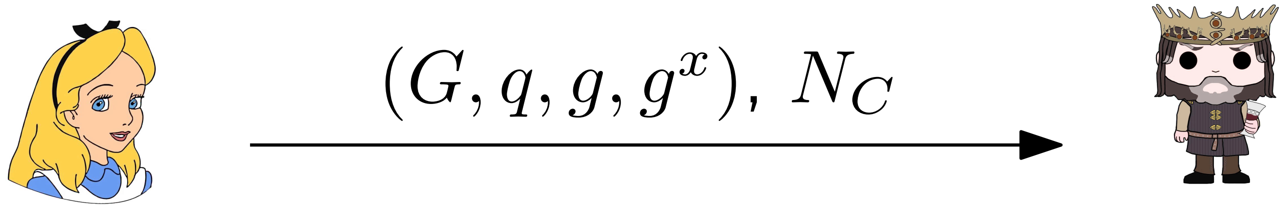

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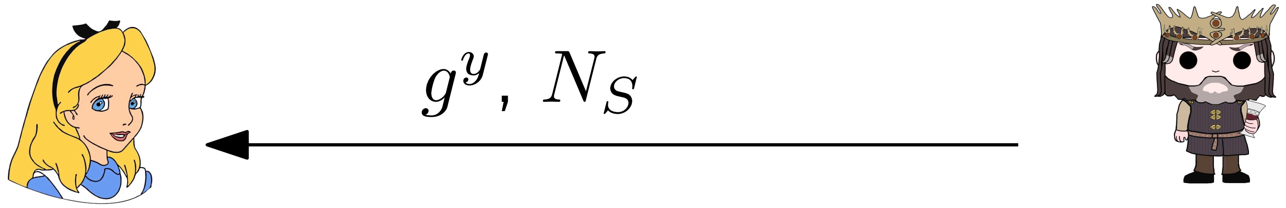

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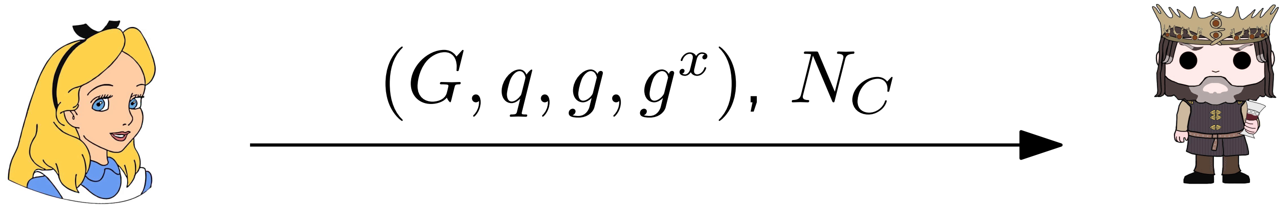

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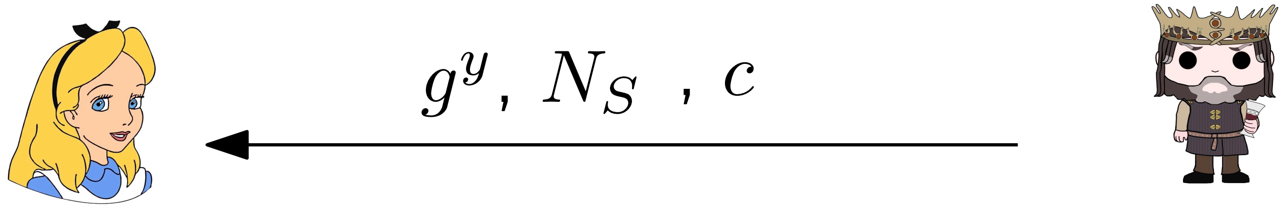
  The server uses pk_S to compute a digital signature σ of all the handshake messages exchanged so far. The server encrypts pk_S, cert, and σ with the symmetric key k'_S and sends the resulting ciphertext c to the client.



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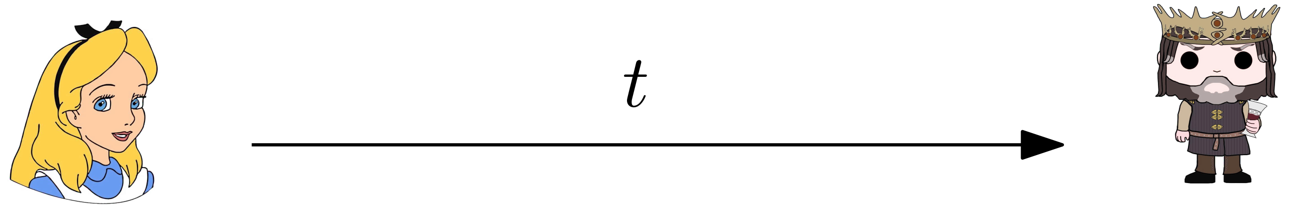

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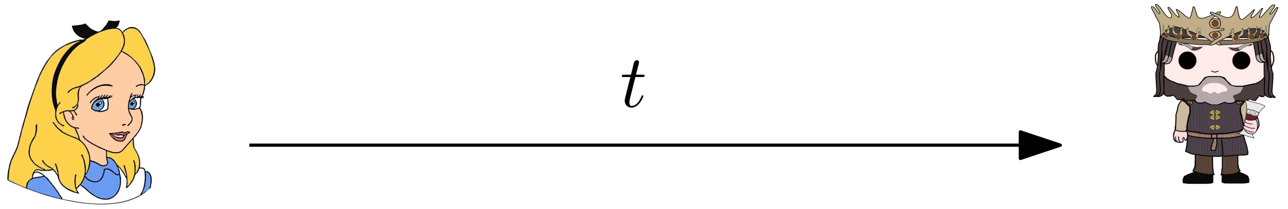
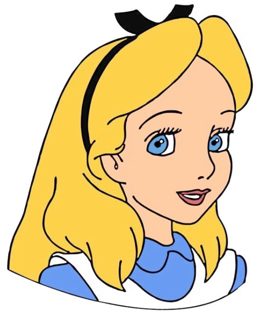
  The server checks t. If verification fails, the server aborts the protocol. Otherwise the actual communication starts using the record-layer protocol with keys k_S and k_C
  (the keys k'_S and k'_C are destroyed: they are needed during the handshake phase)

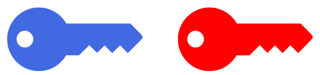


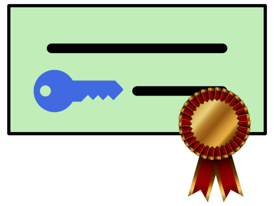
TLS: Handshake protocol: Security (informal)



$(G, q, g, g^x),\ N_C$ →

pk_S sk_S





cert

Derive k'_S, k'_C, k_S, k_C from $K = g^{xy}$

$\sigma \leftarrow \mathsf{Sign}_{sk_S}((G, q, g, g^x, N_C, g^y, N_S))$

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← $g^y,\ N_S,\ c$

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Validate cert

$\mathsf{Vrfy}_{pk_S}((G, q, g, g^x, N_C, g^y, N_S), \sigma)$

$t \leftarrow \mathsf{Mac}_{k'_C}(G, q, g, g^x, N_C, g^y, N_S, c)$

t →

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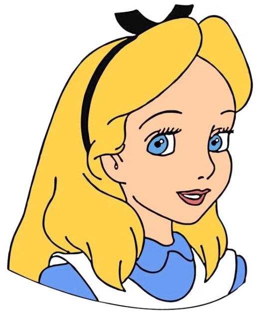
TLS: Handshake protocol: Security (informal)



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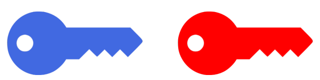

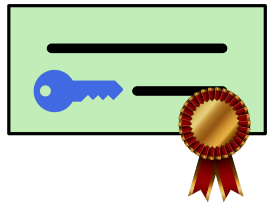

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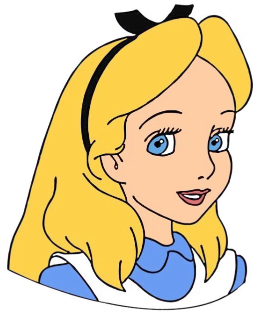

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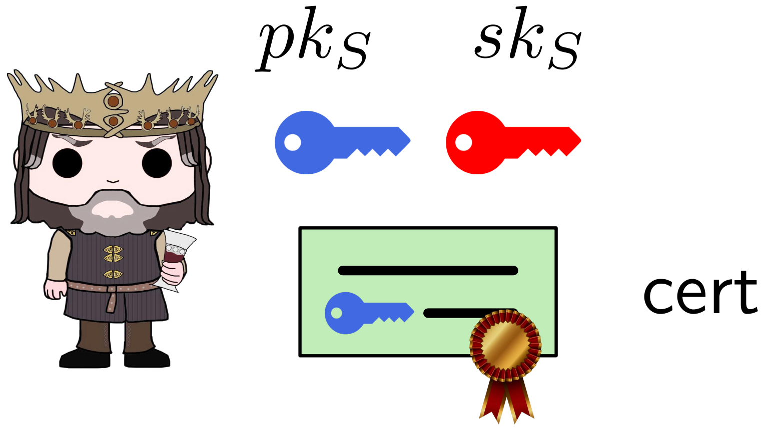

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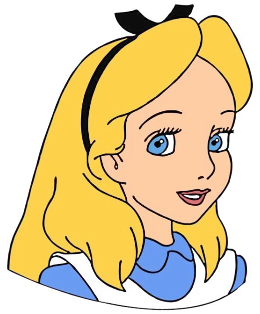

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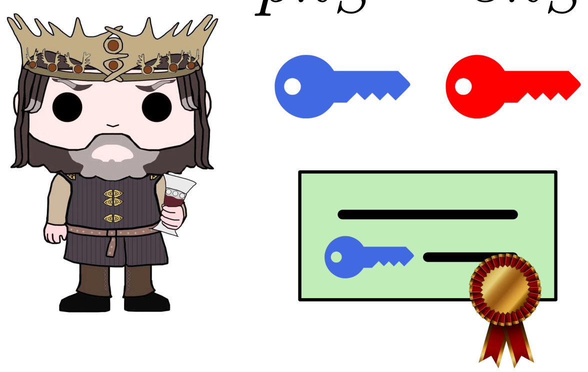

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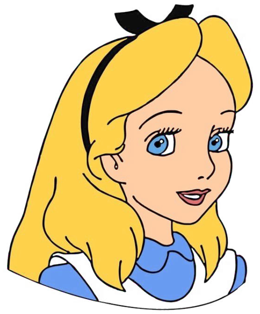

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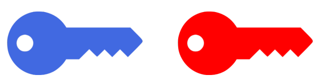

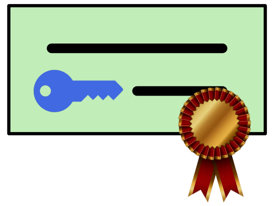

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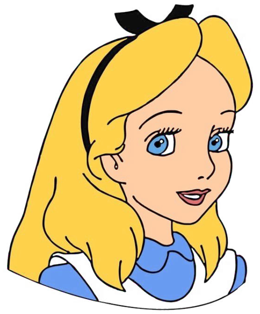

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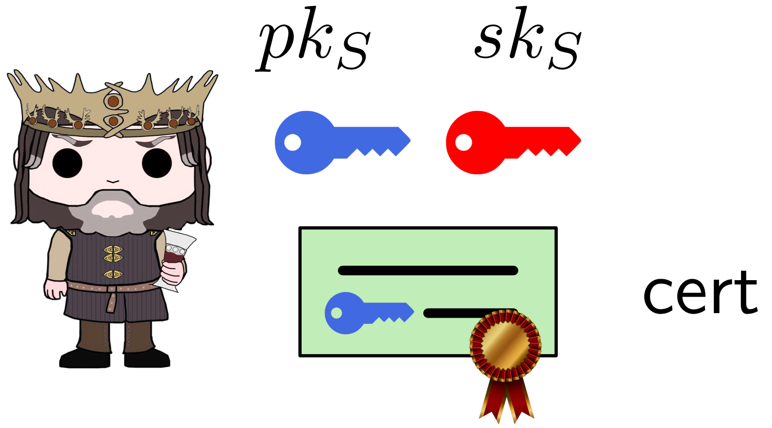

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Diffie-Hellman protocol: the adversary learns nothing about K (and k'_S, k'_C, k_S, k_C)

TLS: Handshake protocol: Security (informal)

Why do we use Diffie-Hellman?

Consider the following alternative protocol:

- The client verifies the server certificate
- The client picks a random secret key K
- The client encrypts K with pk_S ands sends the ciphertext c to the server
- The server decrypts c with sk_S, and replies using an authenticated encryption scheme Π with key K
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- Using the Diffie-Hellman key-exchange, the symmetric keys k'_S, k'_C, k_S, k_C are **ephemeral** and can be erased at the end of the handshake/session.