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Is there a secure private-key encryption scheme (with short keys) according to this new definition?

Recap: Pseudorandom Generators (formal)

Let G be a deterministic polynomial-time algorithm such that for any n and any input $s \in \{0,1\}^n$, the output $G(s)$ is a string of length $\ell(n)$

Expansion factor of G

G is a **pseudorandom generator (PRG)** if the following conditions hold:

- **Expansion:** For every $n \geq 1$, $\ell(n) > n$
- **Pseudorandomness:** For any probabilistic polynomial-time algorithm D, there is a negligible function ε such that

$$\left| \Pr[D(\, G(s)\,) = 1] - \Pr[D(r) = 1] \right| \leq \varepsilon(n)$$

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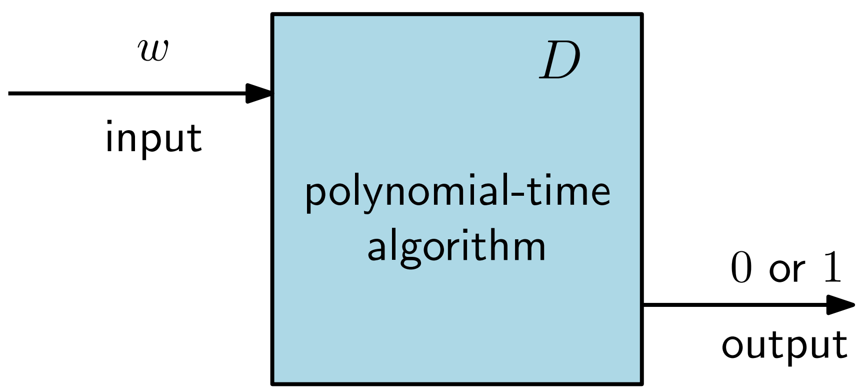

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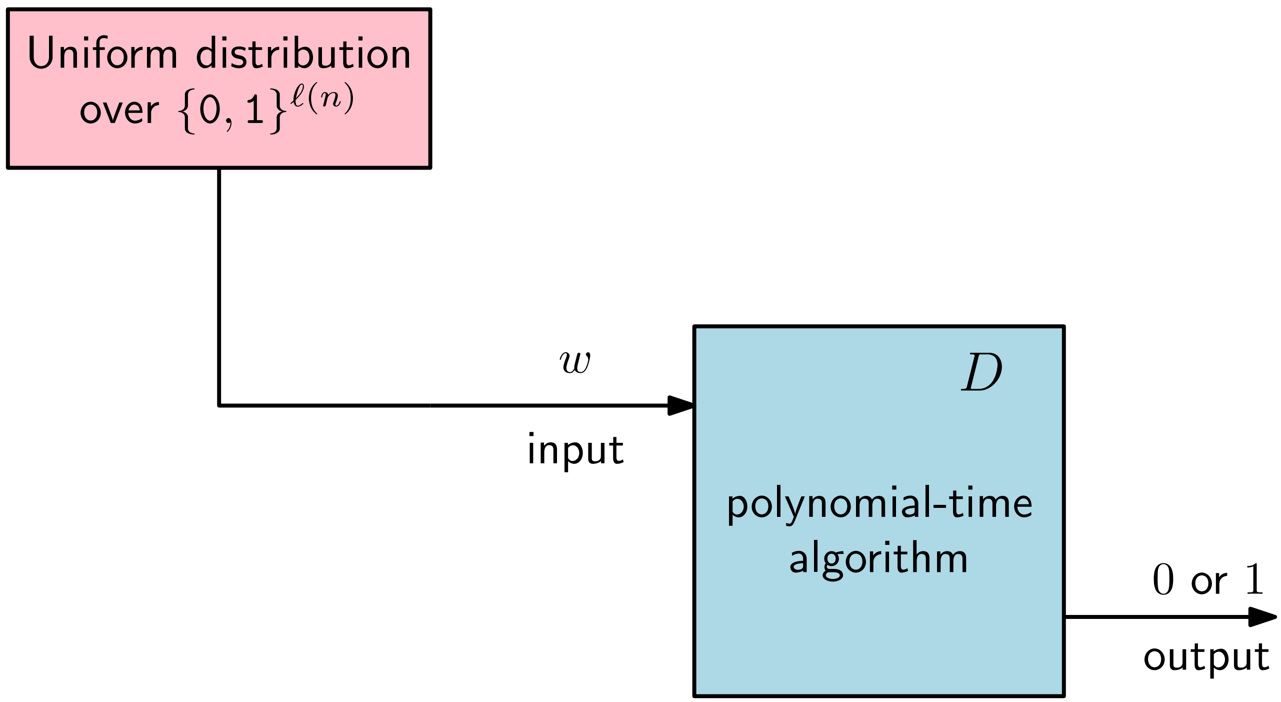

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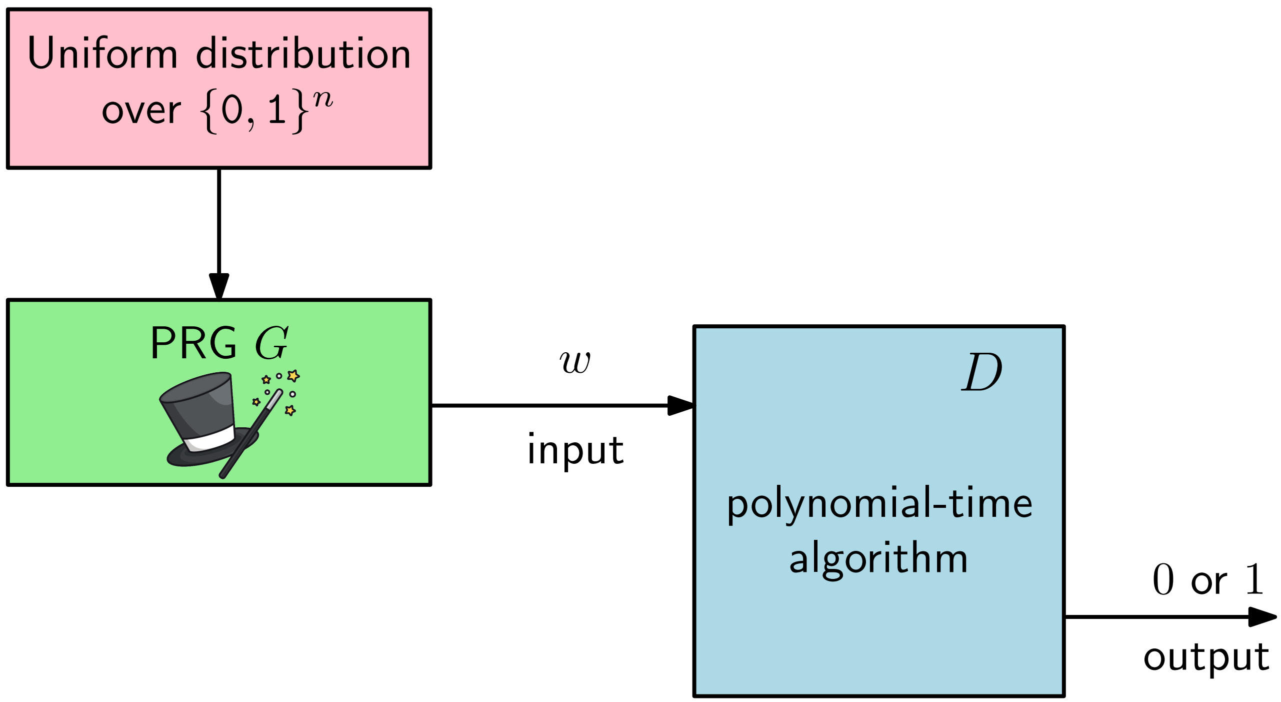

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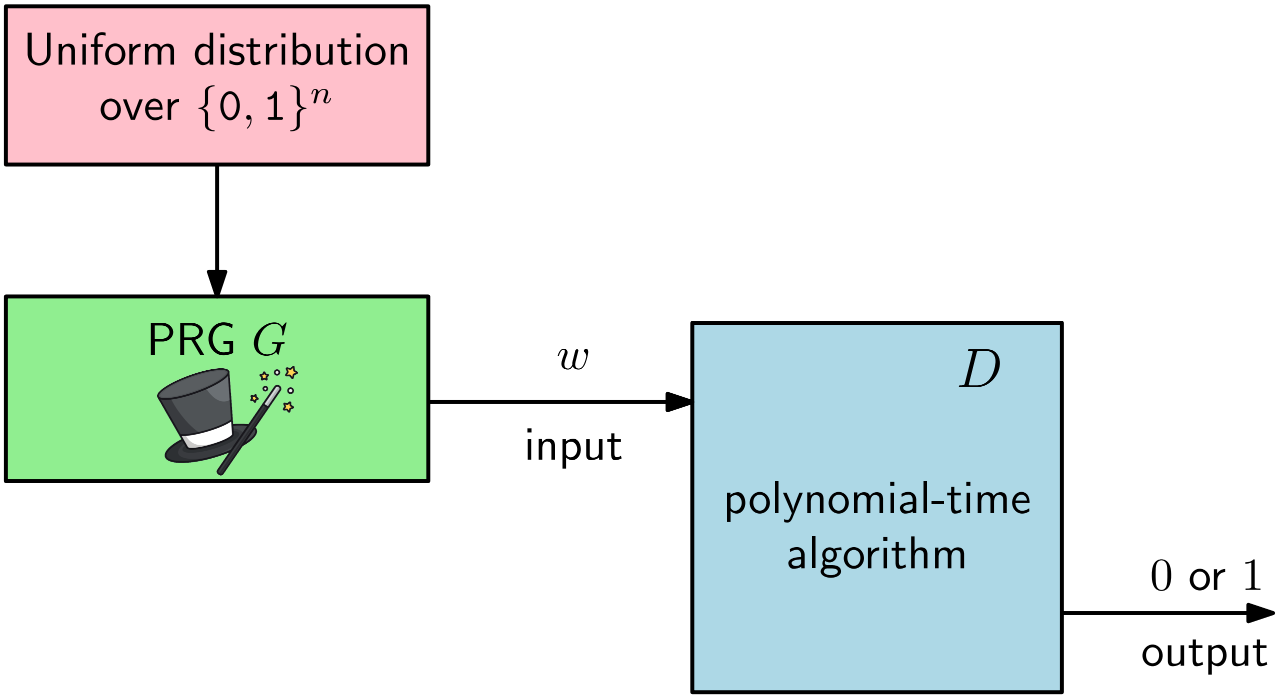

Regardless of how the input x is generated, the probability that D outputs 1 should be almost the same (the two probabilities differ by at most a negligible function)

Examples

Consider a polynomial-time algorithm G that, with input $s = s_1 s_2 \ldots s_n$ outputs $G(s) = s \,\|\, \bigvee_{i=1}^{n} s_i$

$$s = 000000 \longrightarrow G(s) = 0000000$$

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$$\left|1 - \frac{1}{2^n} - \frac{1}{2}\right| = \frac{1}{2} - \frac{1}{2^n} \text{ is not negligible}$$

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Consider a (polynomial-time) algorithm G that takes a binary string $s = s_1 \ldots s_n \in \{\texttt{0},\ \texttt{1}\}^n$ and outputs a string in $f(s) \in \{\texttt{0},\ \texttt{1}\}^{n+1}$ such that:

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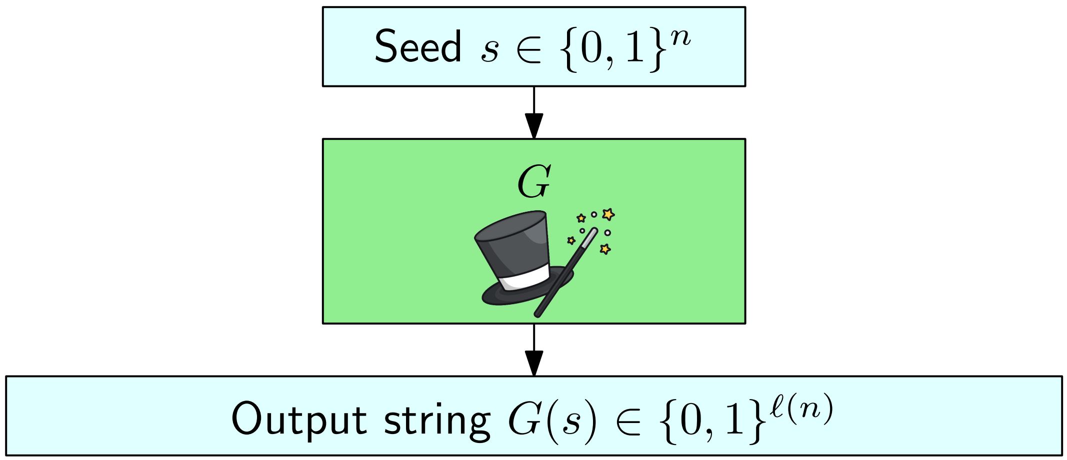
Not negligible!

Why are PRGs useful?

As far as polynomial-time algorithms are concerned, the output of $G(s)$ with a random seed s is indistinguishable (up to some negligible probability) from a random string r

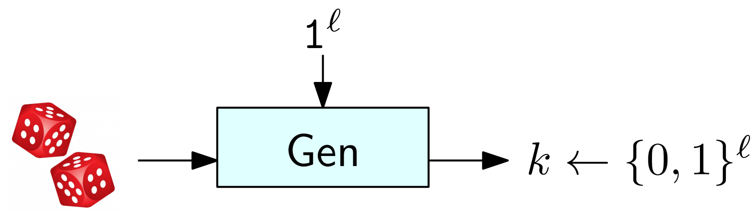


If we have a randomized polynomial-time algorithm that uses r random bits, and we replace those random bits with the output of $G(s)$, the resulting (randomized) algorithm "behaves the same" except for a negligible probability

One-time pad (redefined with security parameter)

security parameter ℓ = length of the message (for convenience we name the security parameter ℓ instead of n)

- Gen(1^ℓ): return a key k chosen u.a.r. from $\{0,1\}^\ell$



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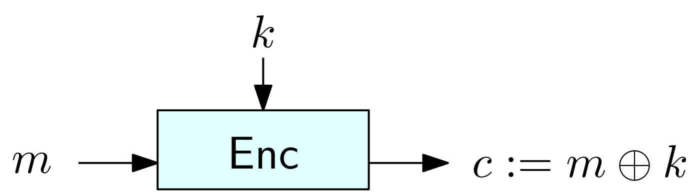

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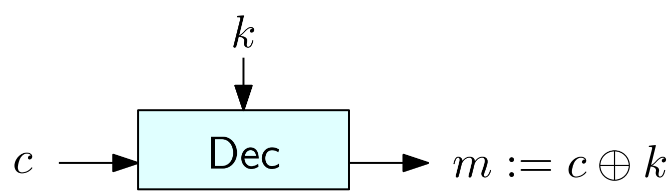

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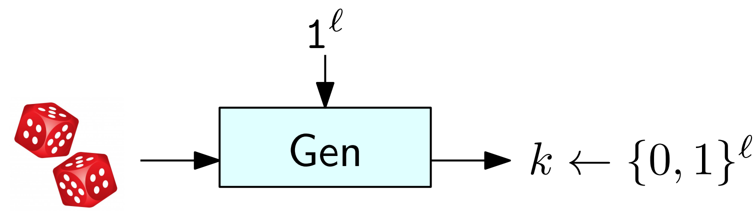

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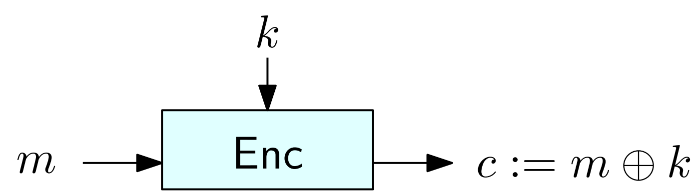

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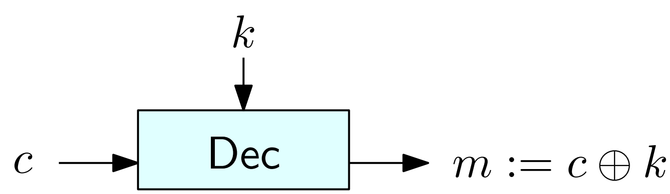

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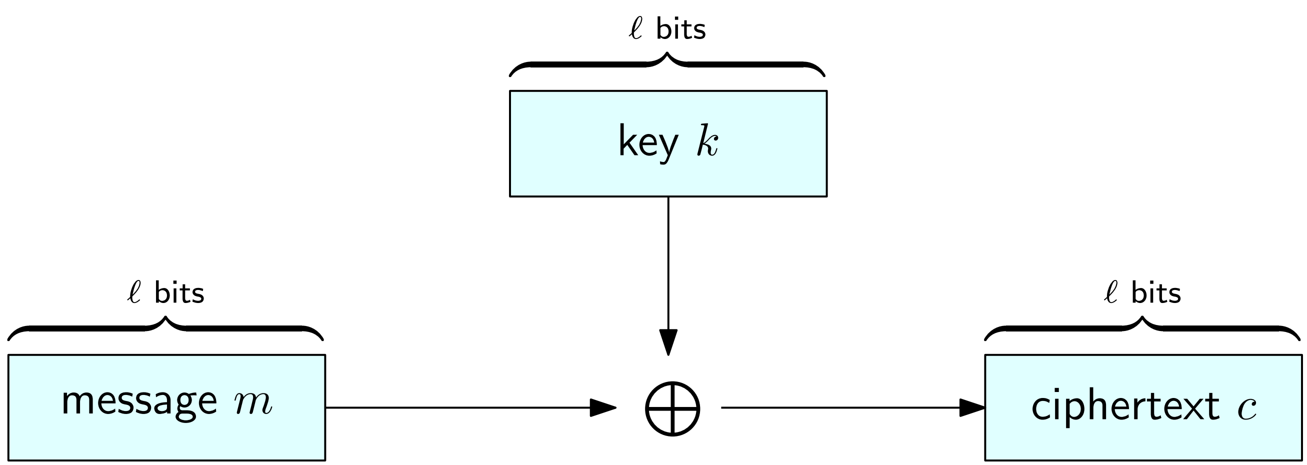

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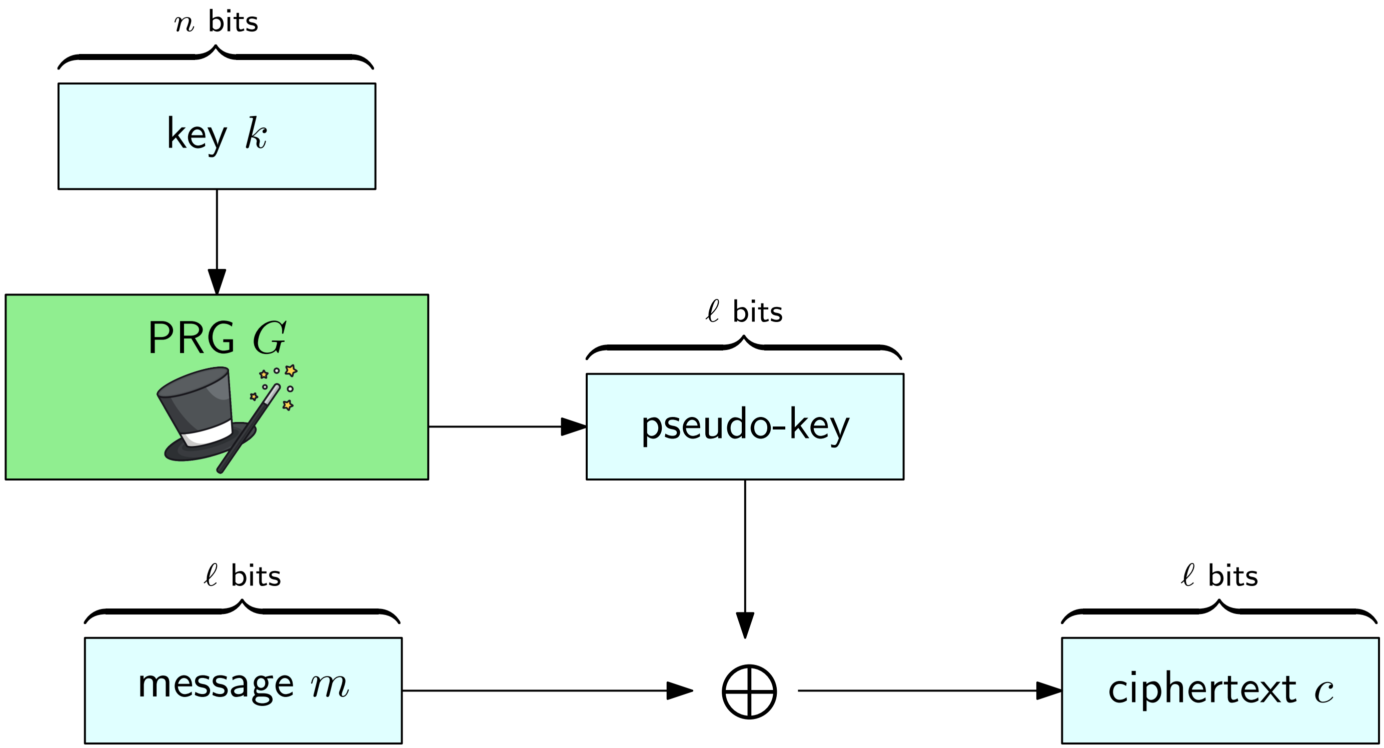

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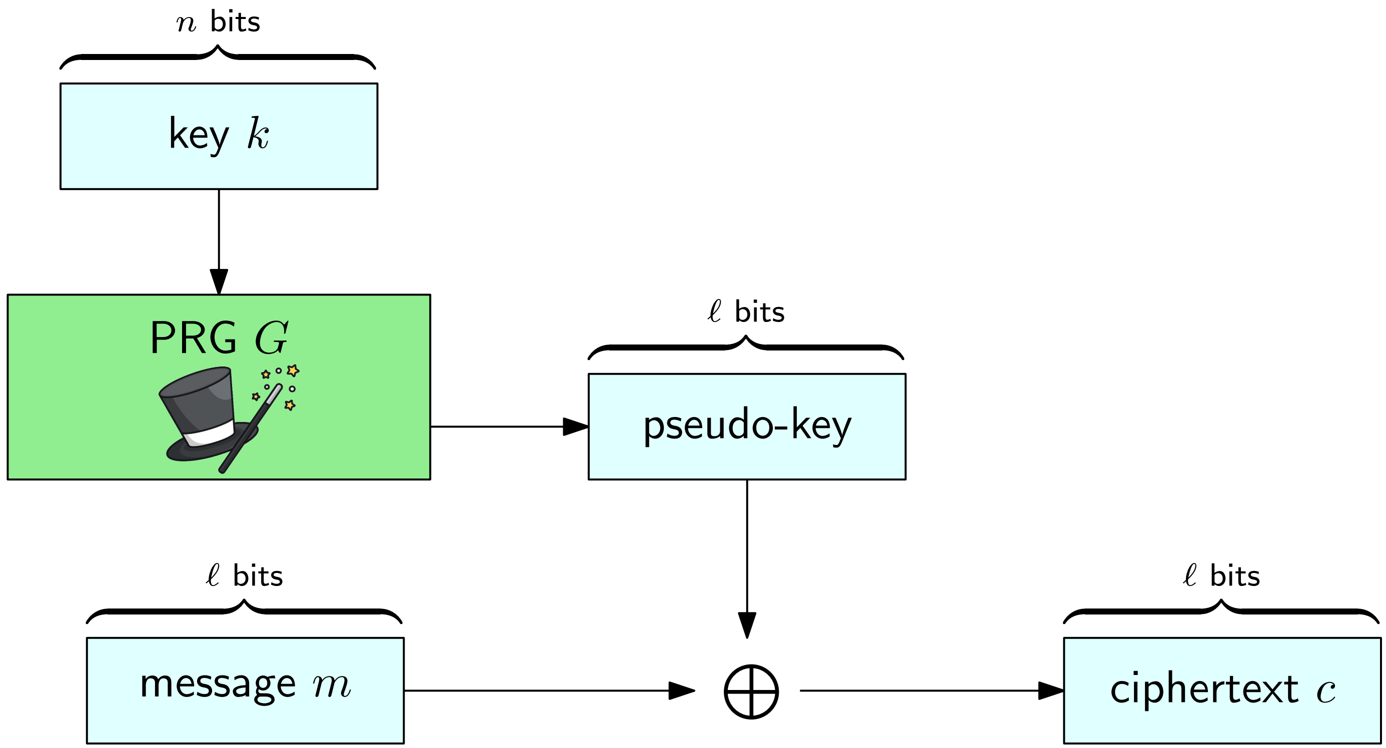

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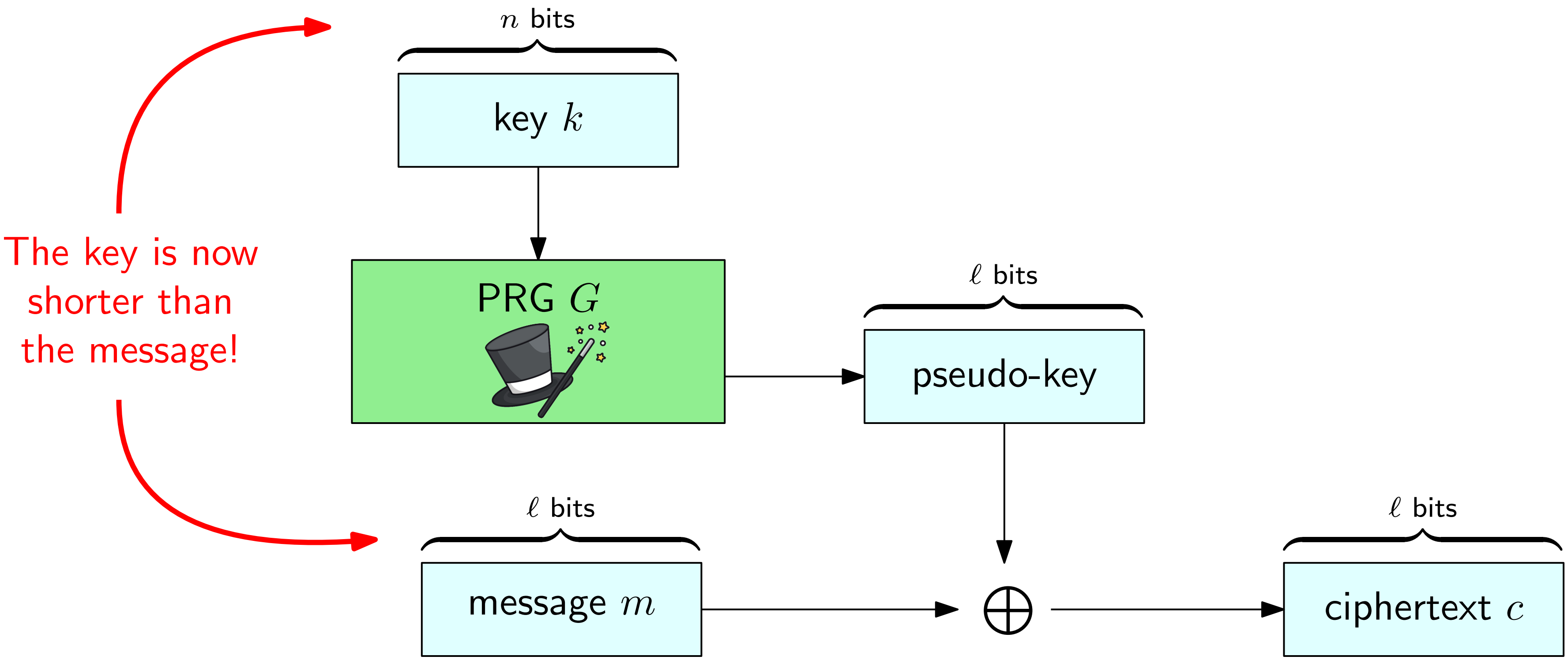

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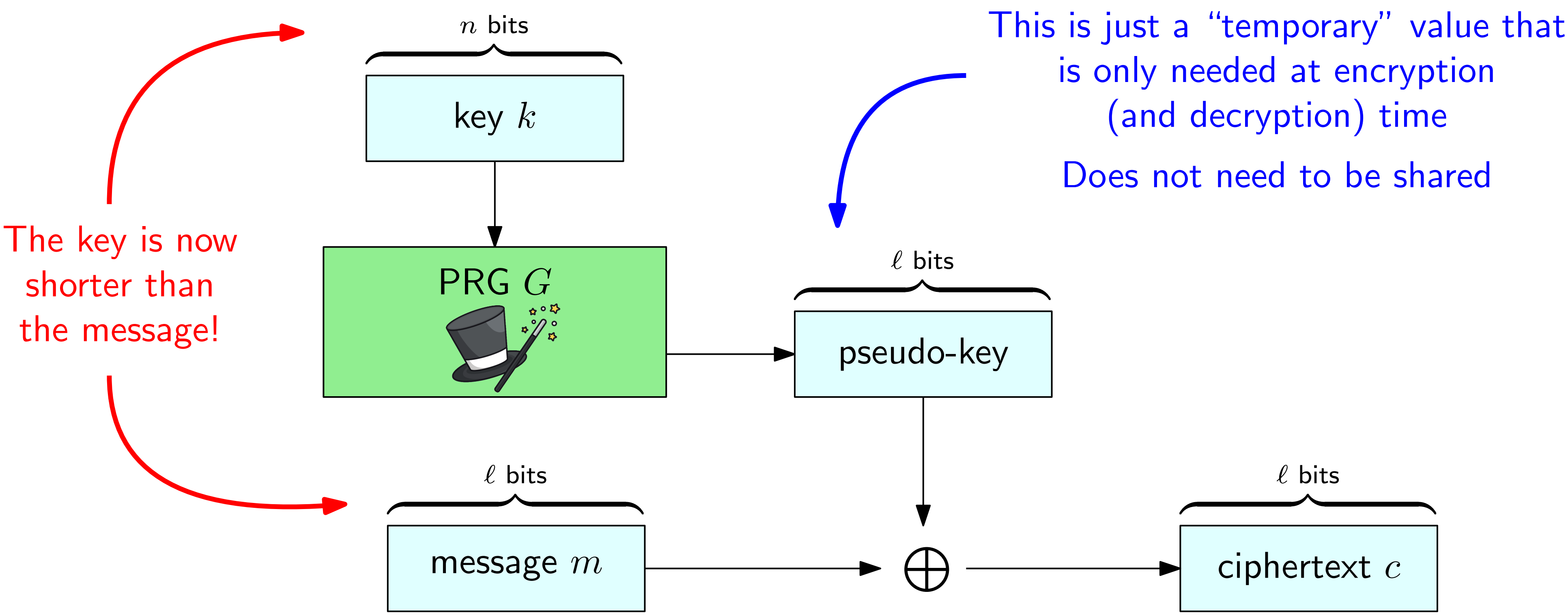

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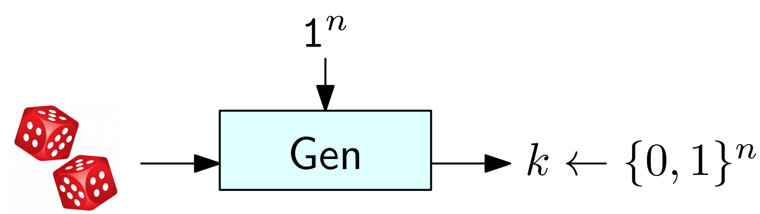

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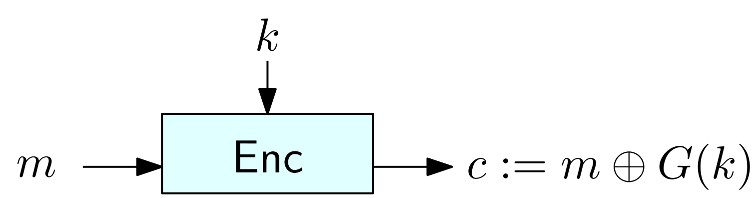

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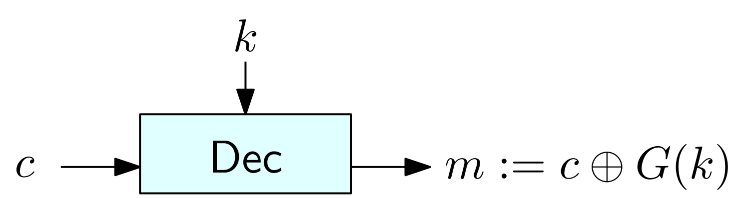

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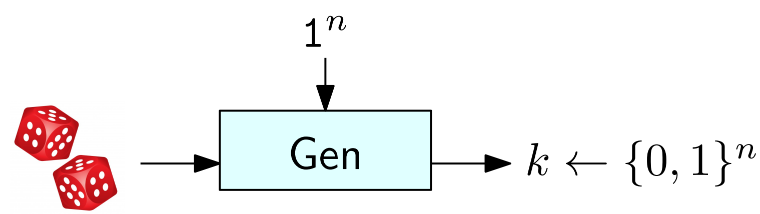

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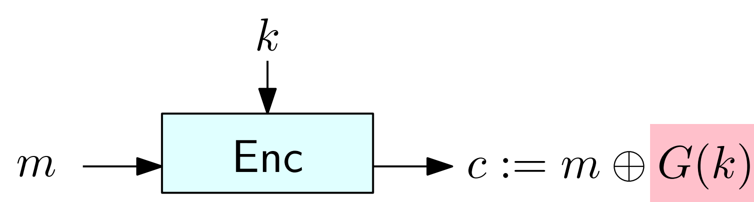

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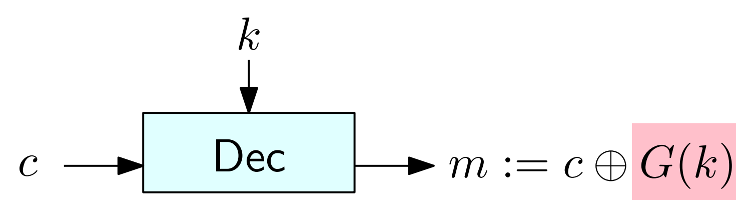

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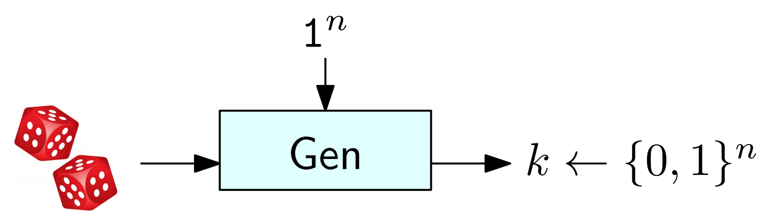

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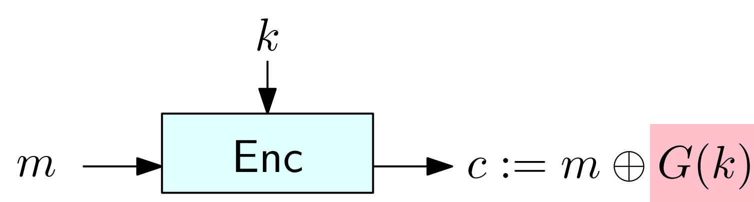

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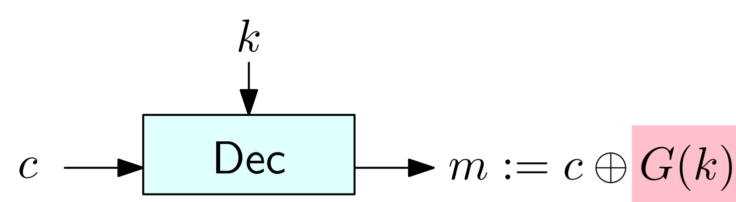

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In general, even stronger cryptographic assumptions might be needed to prove that a scheme is secure

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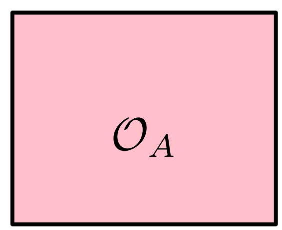

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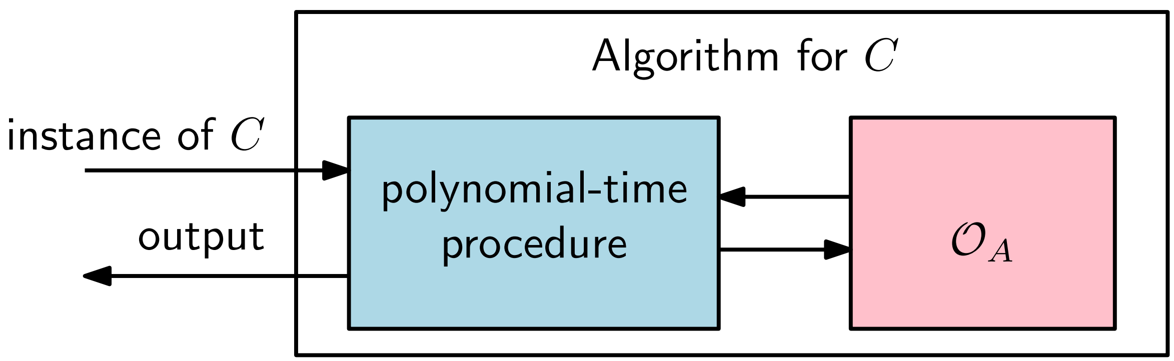

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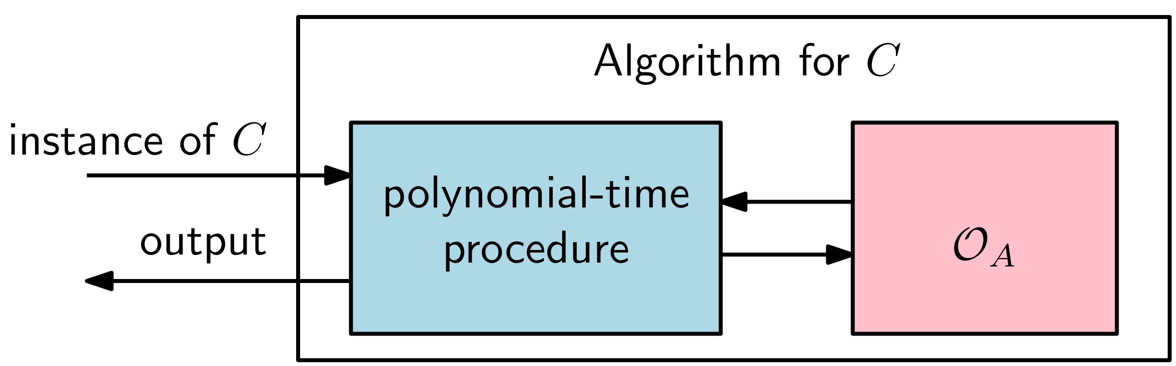

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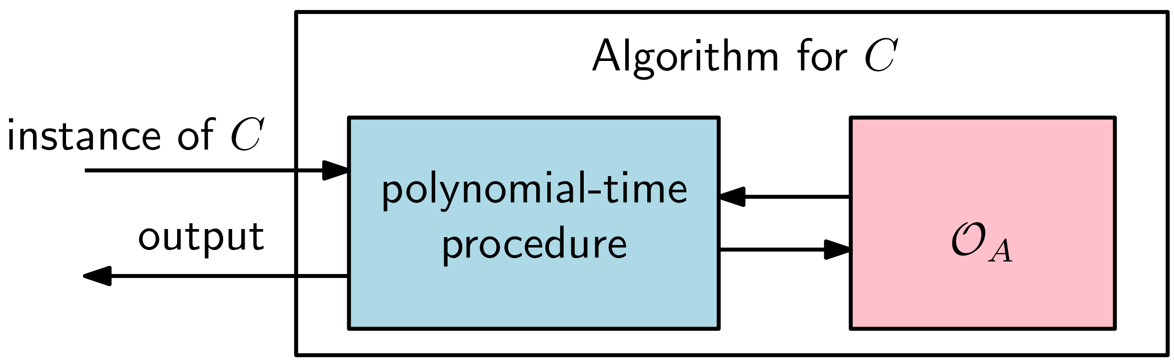

Security reductions

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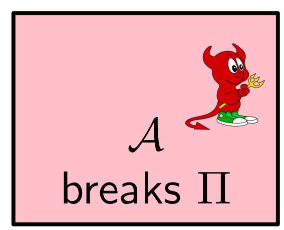

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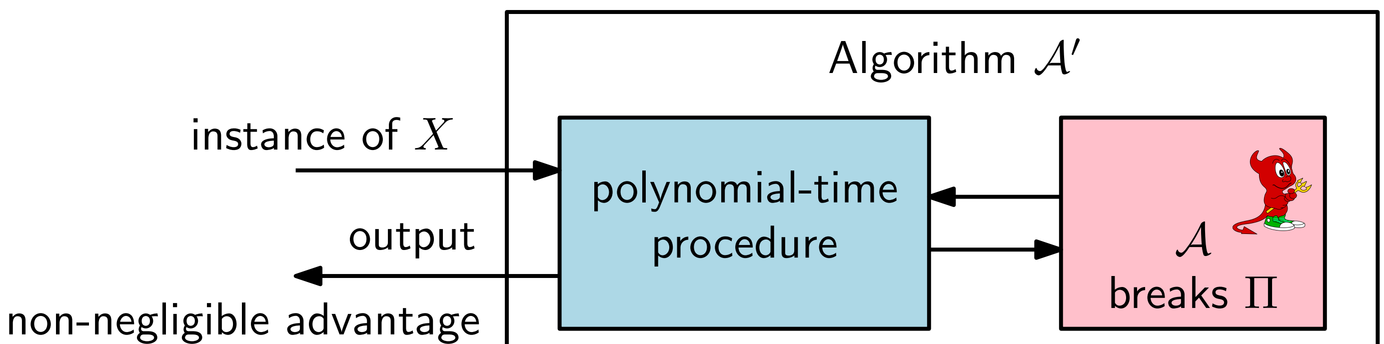

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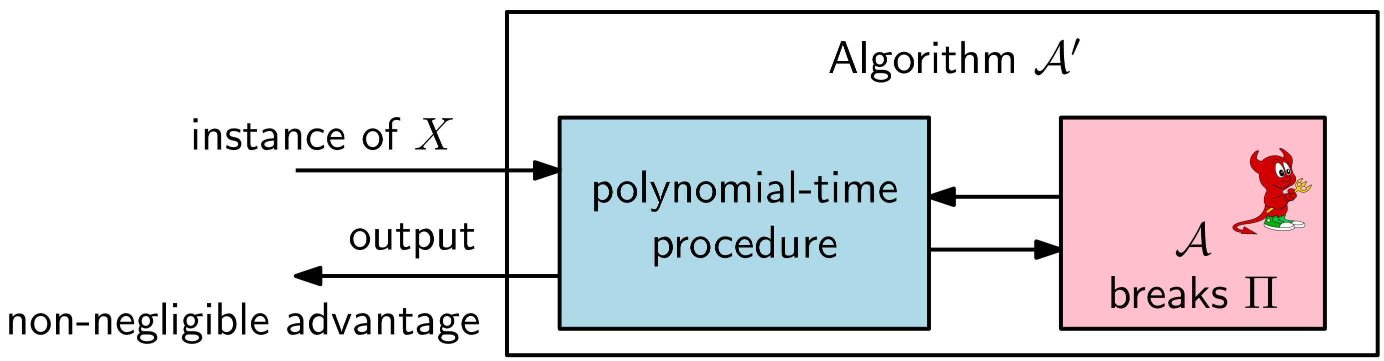

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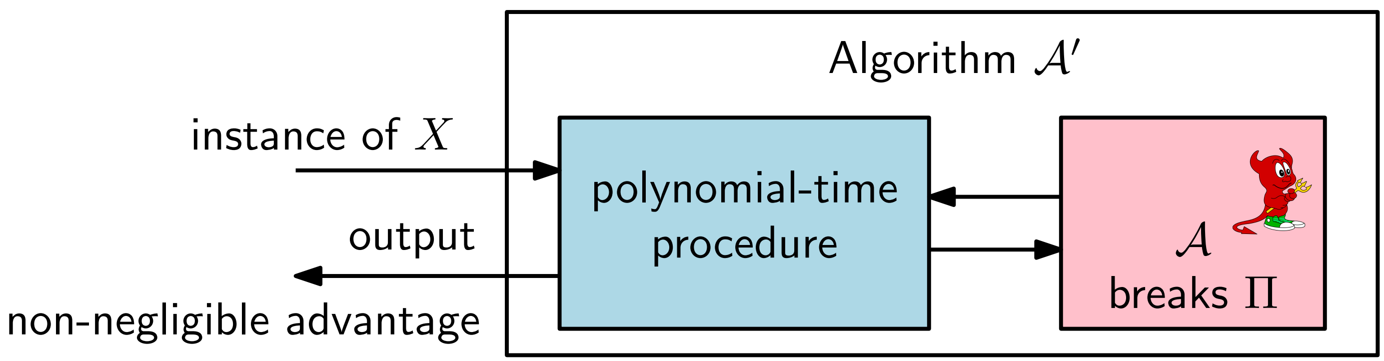

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Theorem: If G is a pseudorandom generator with expansion factor $\ell(n)$, then pseudo OTP is an EAV-secure, fixed-length private-key encryption scheme for messages of length $\ell(n)$.

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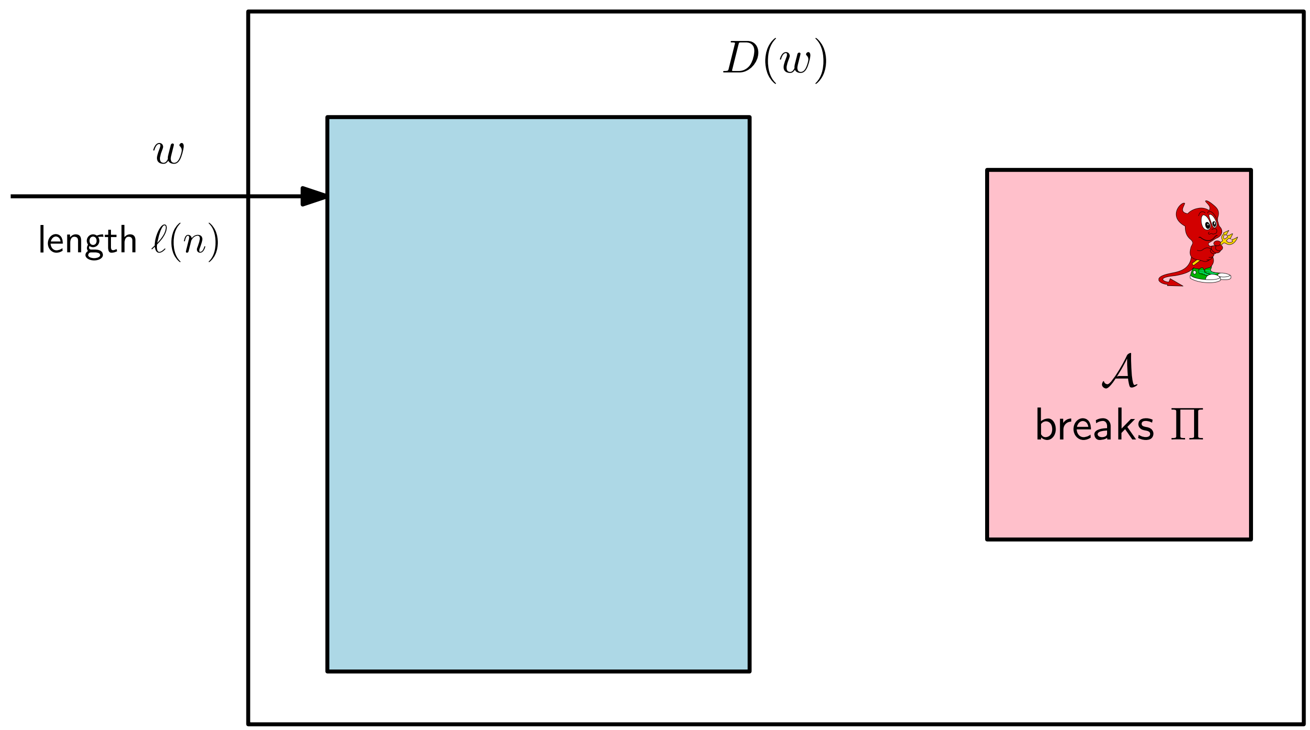

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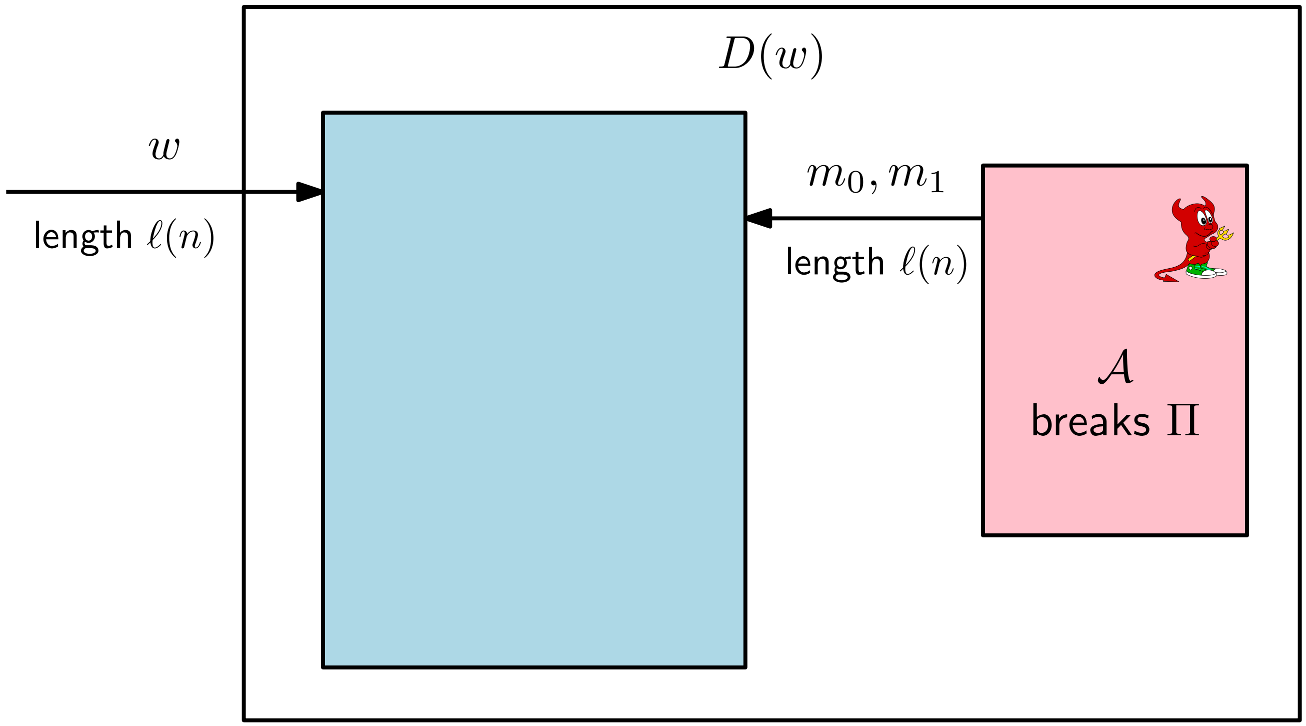

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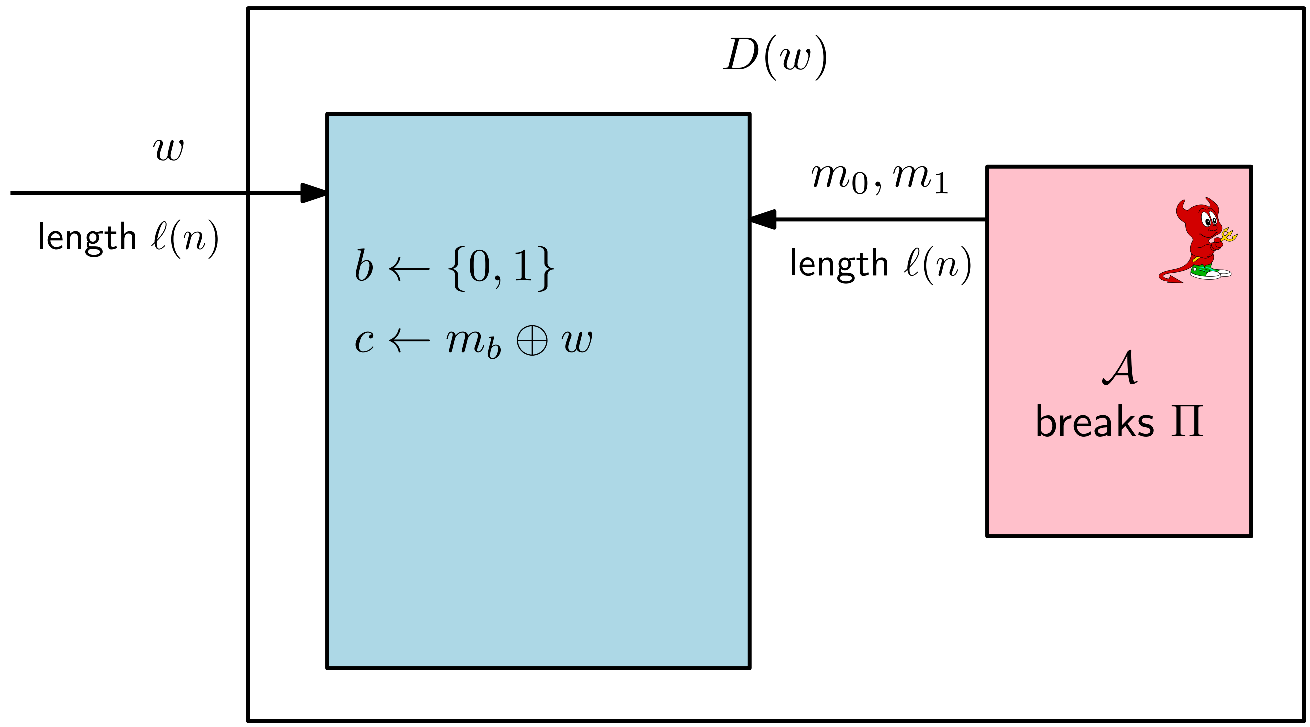

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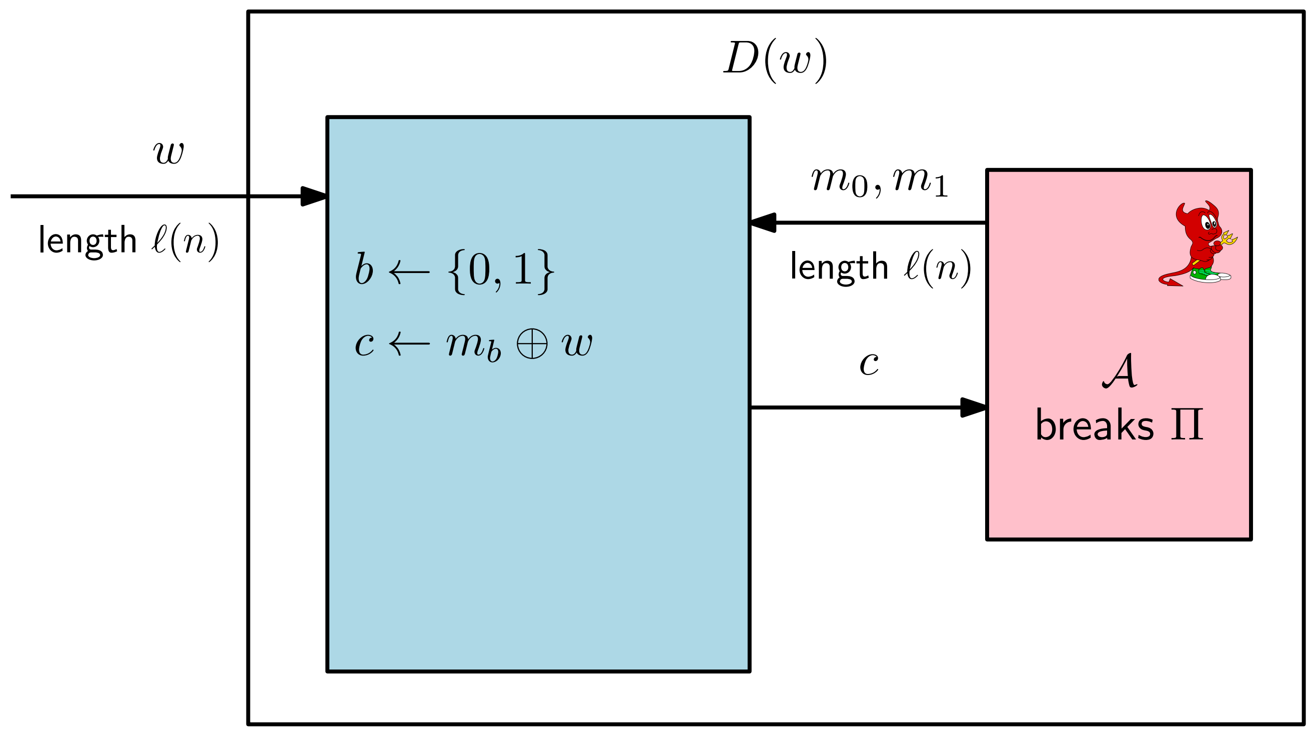

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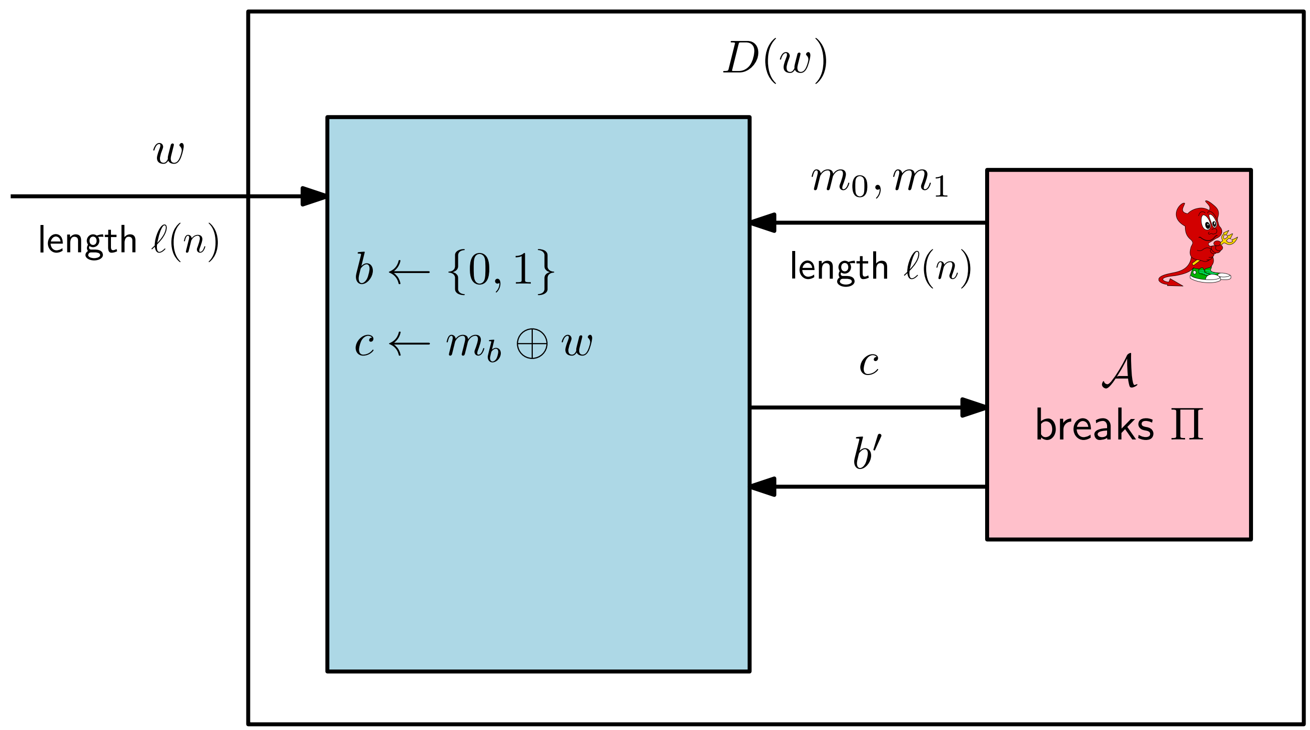

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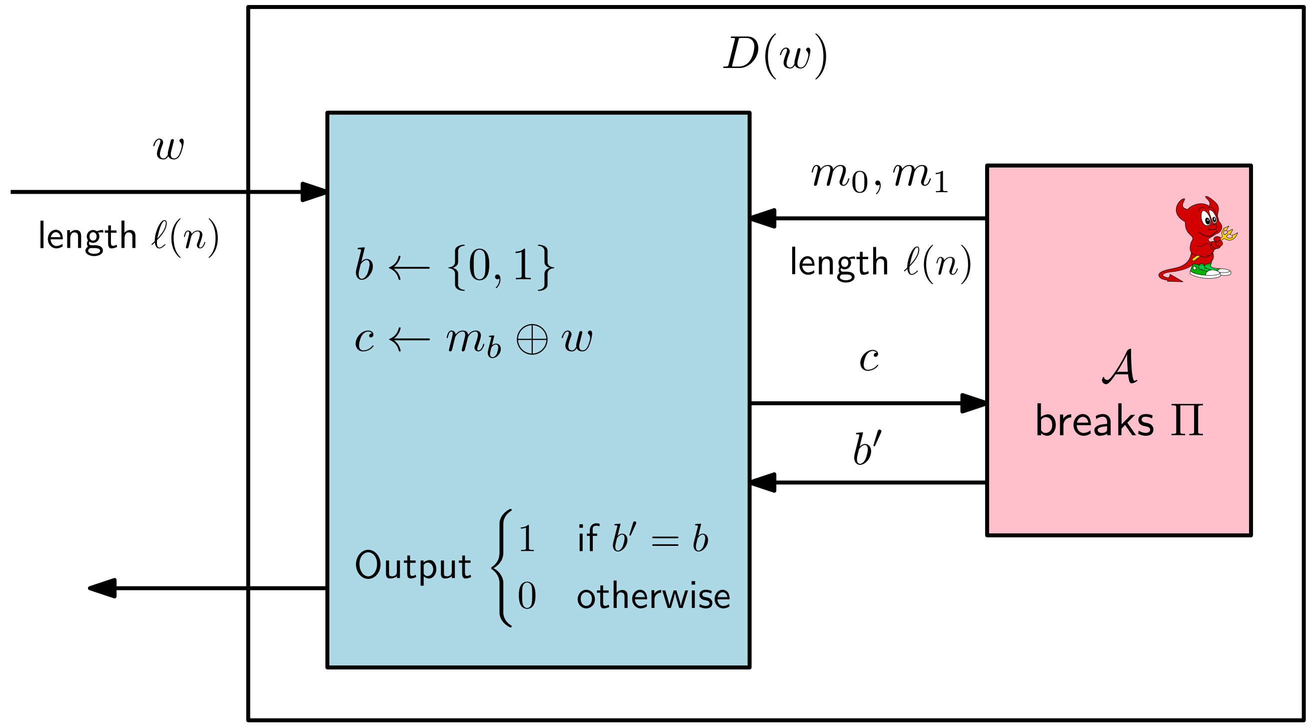

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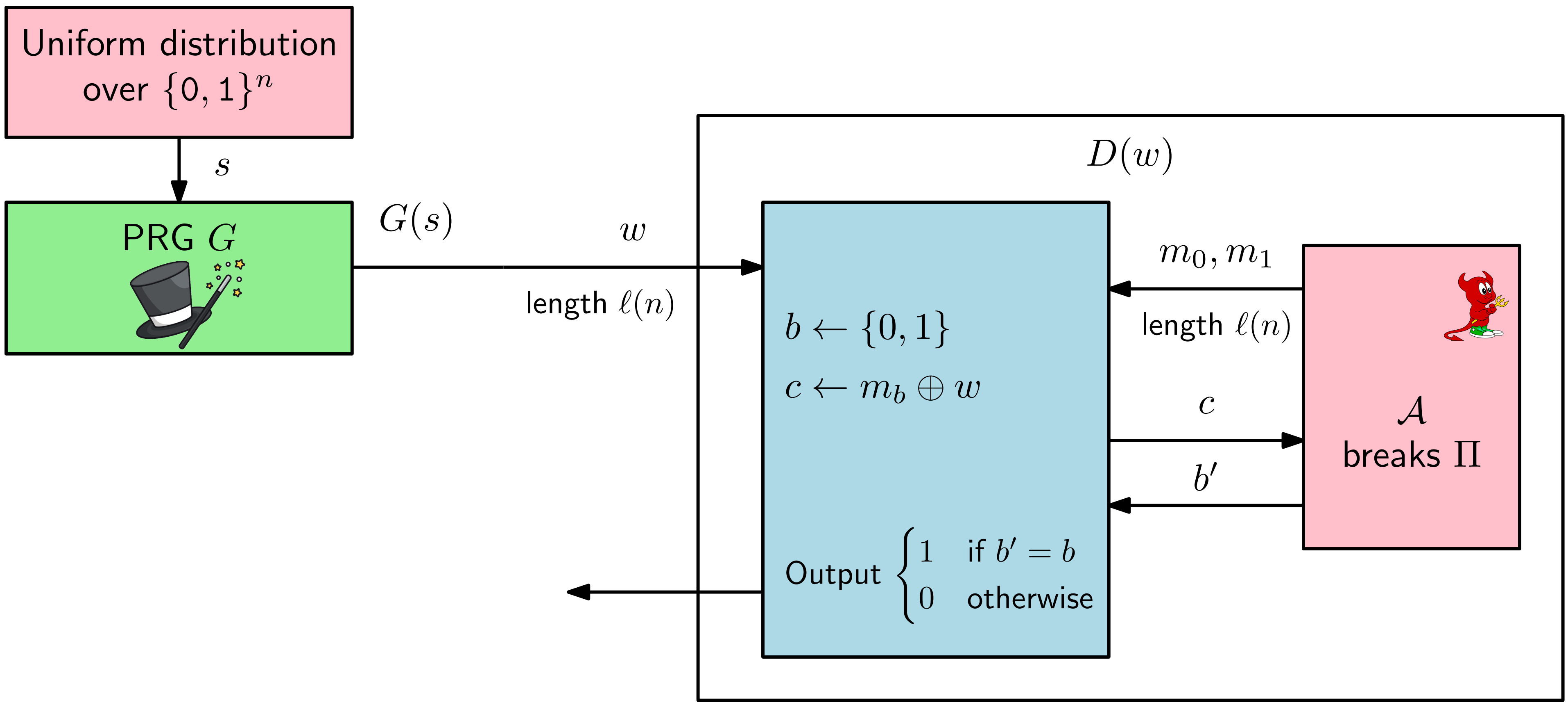

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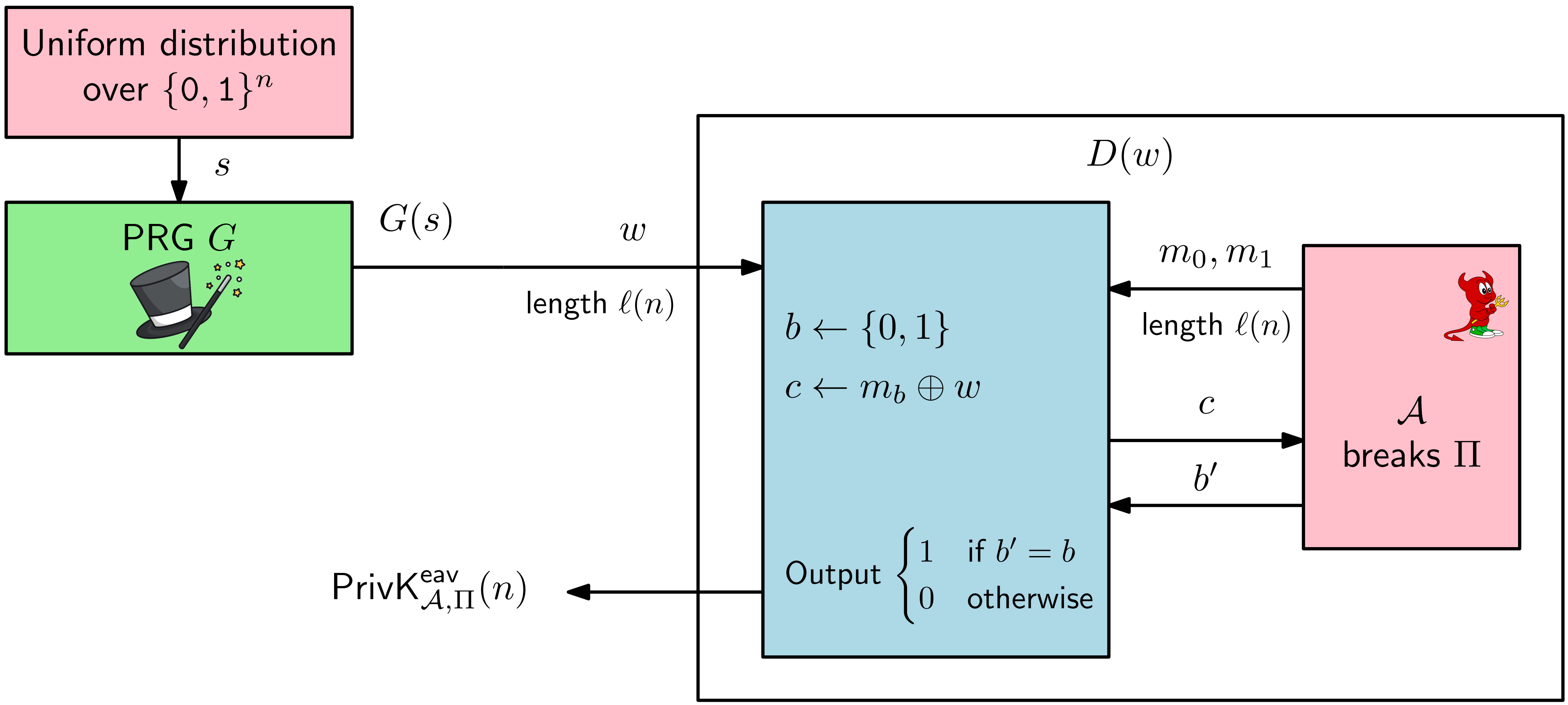

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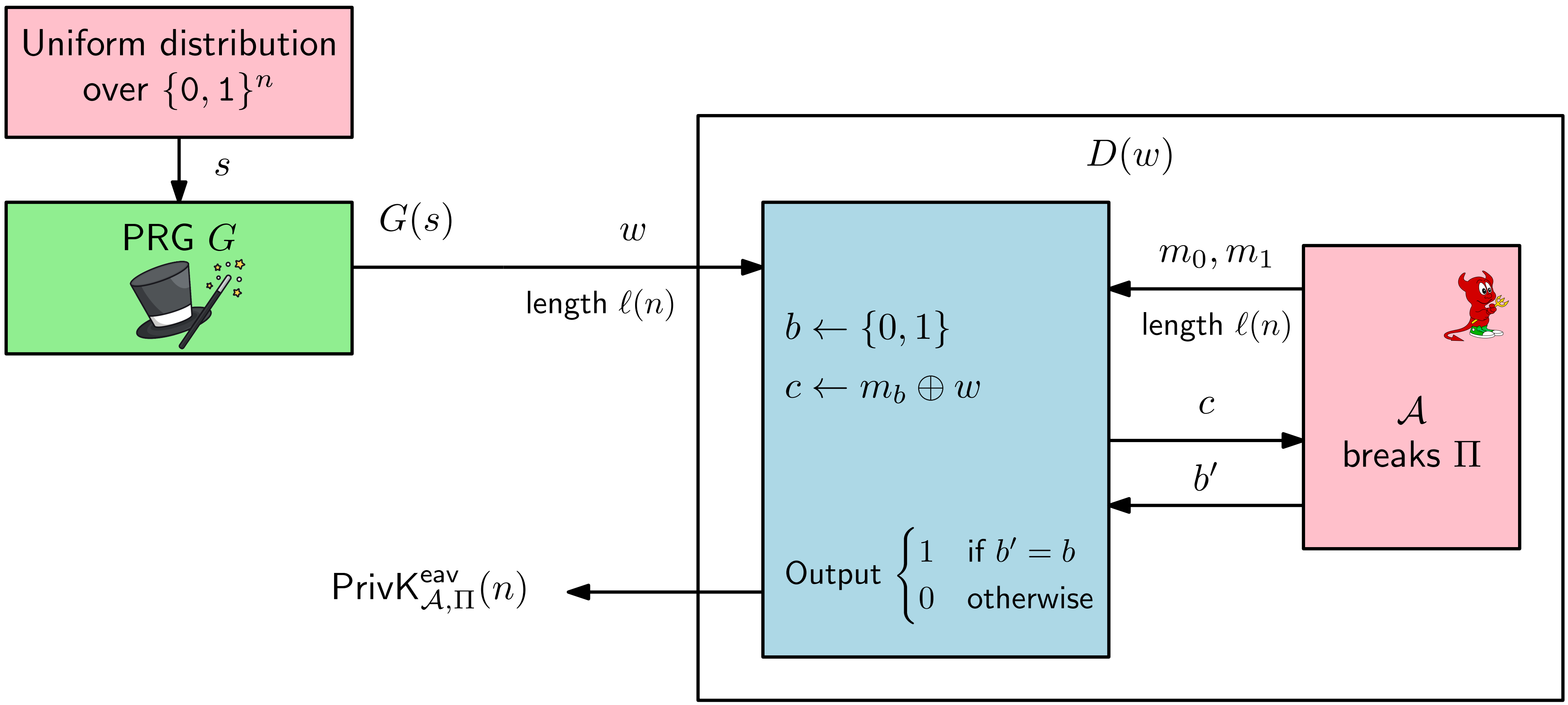

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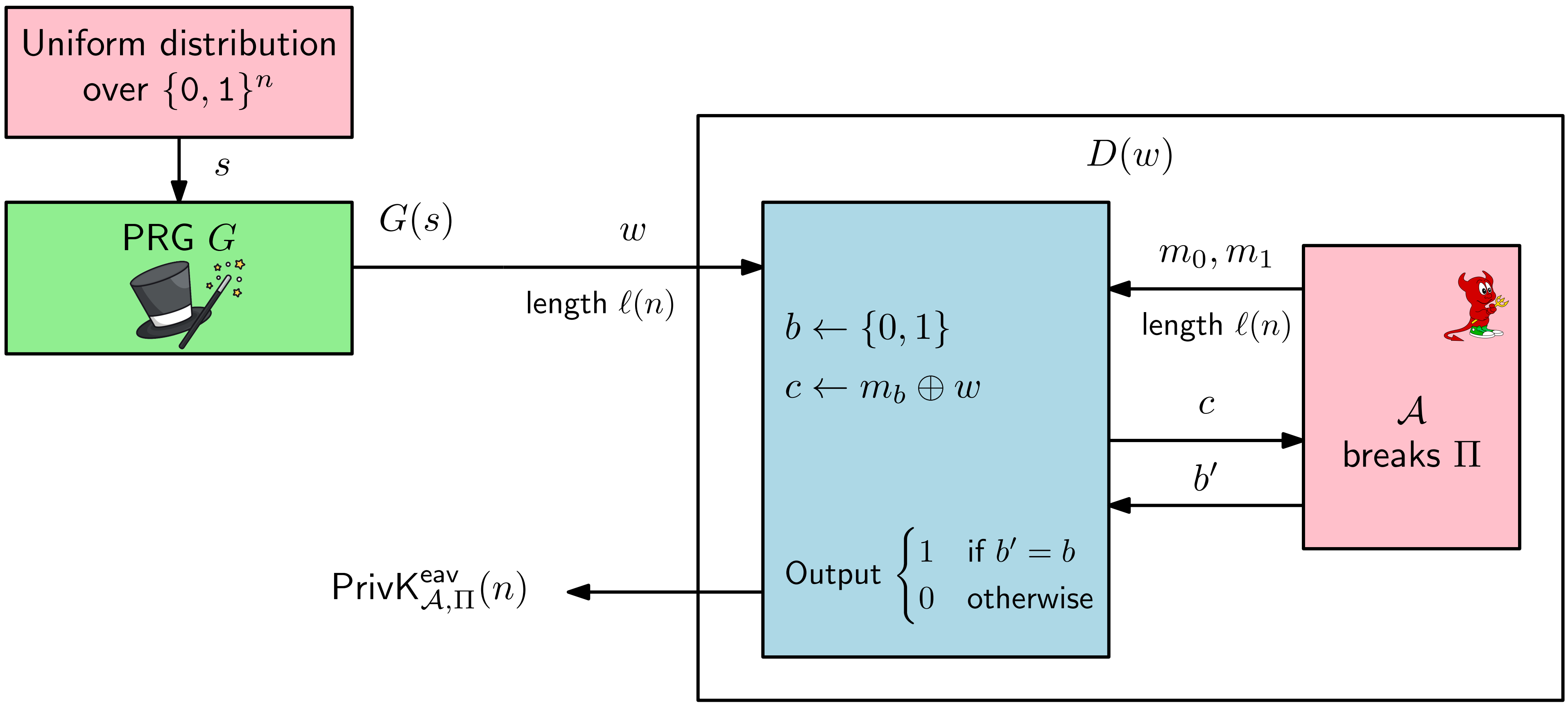

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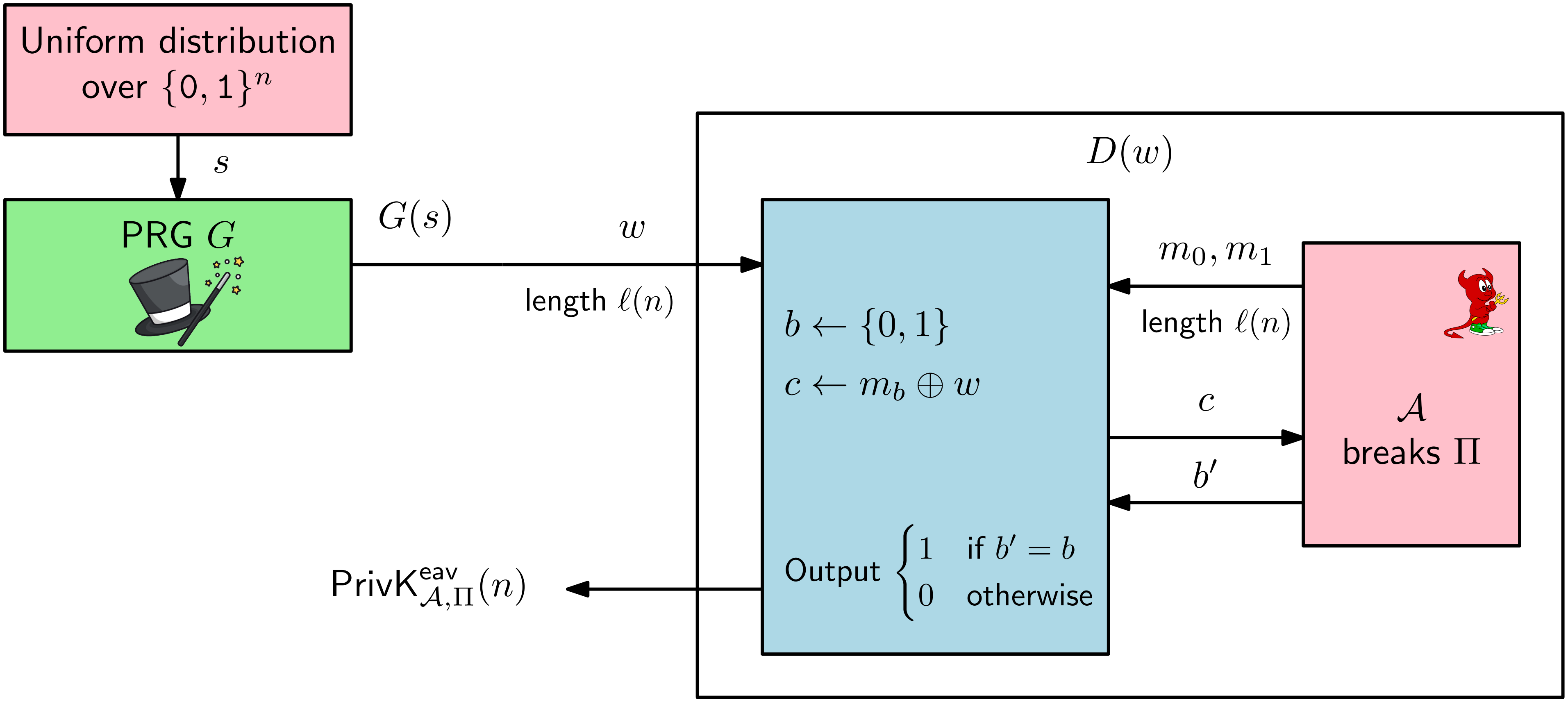

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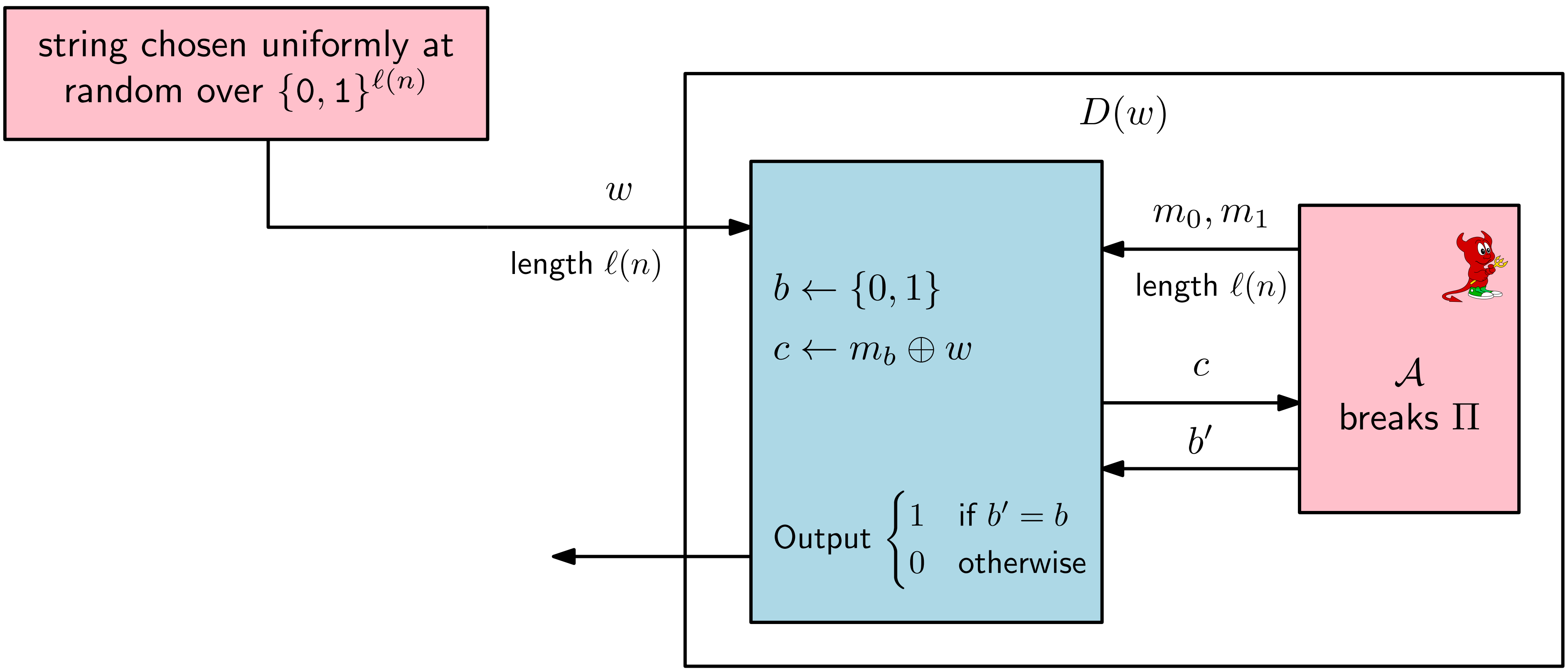

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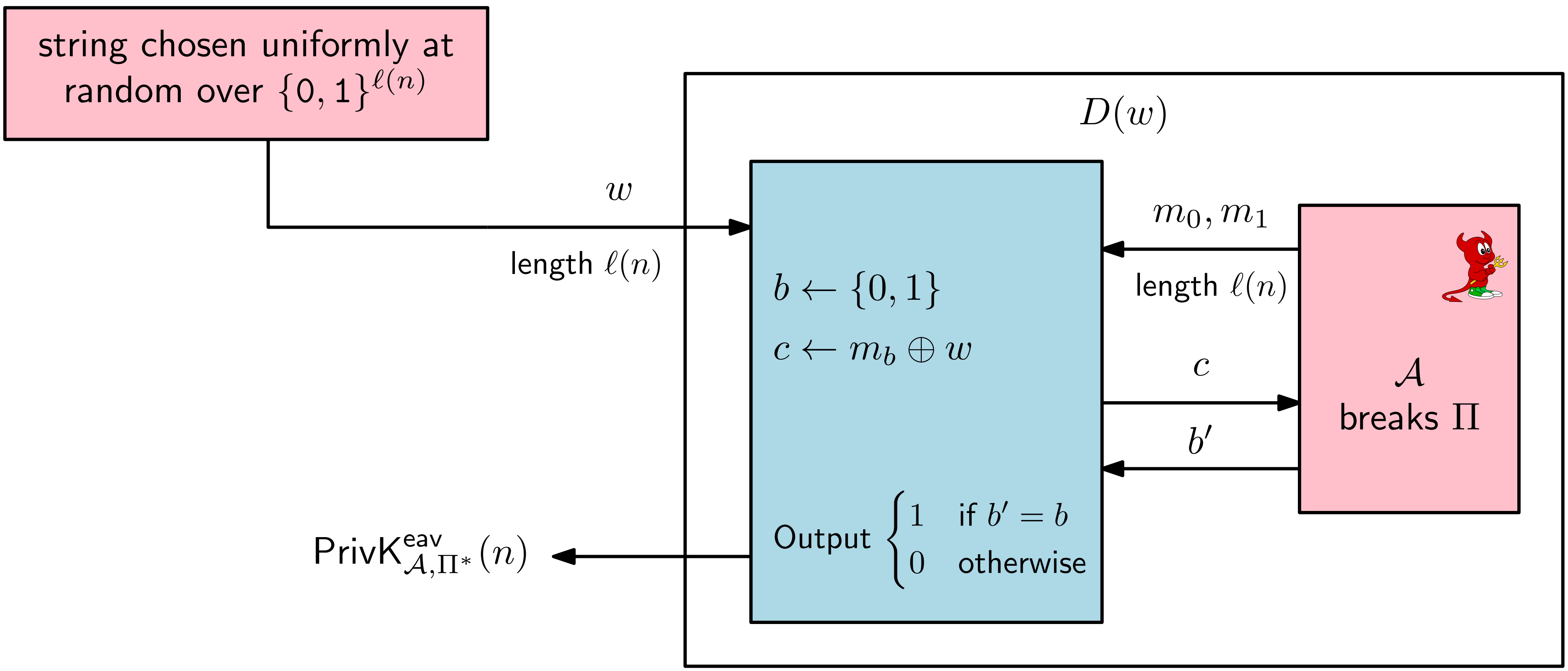

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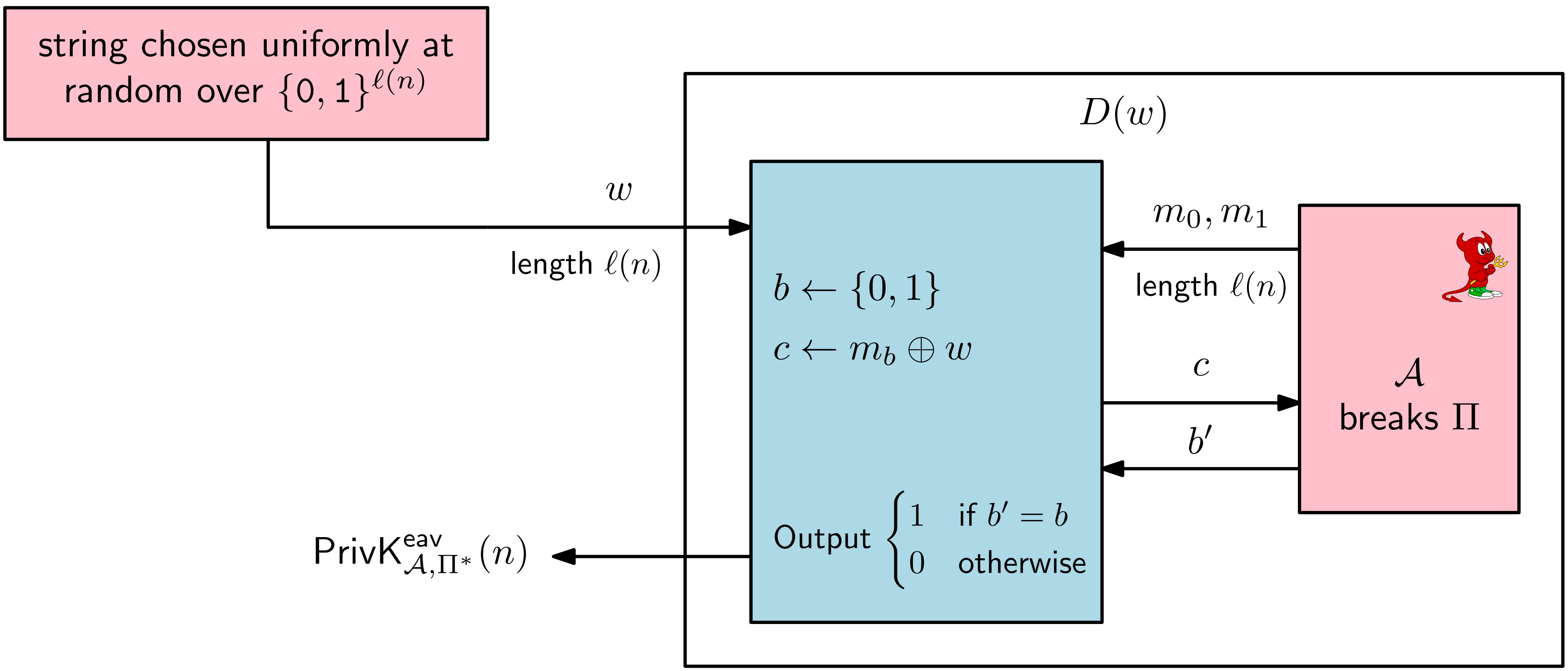

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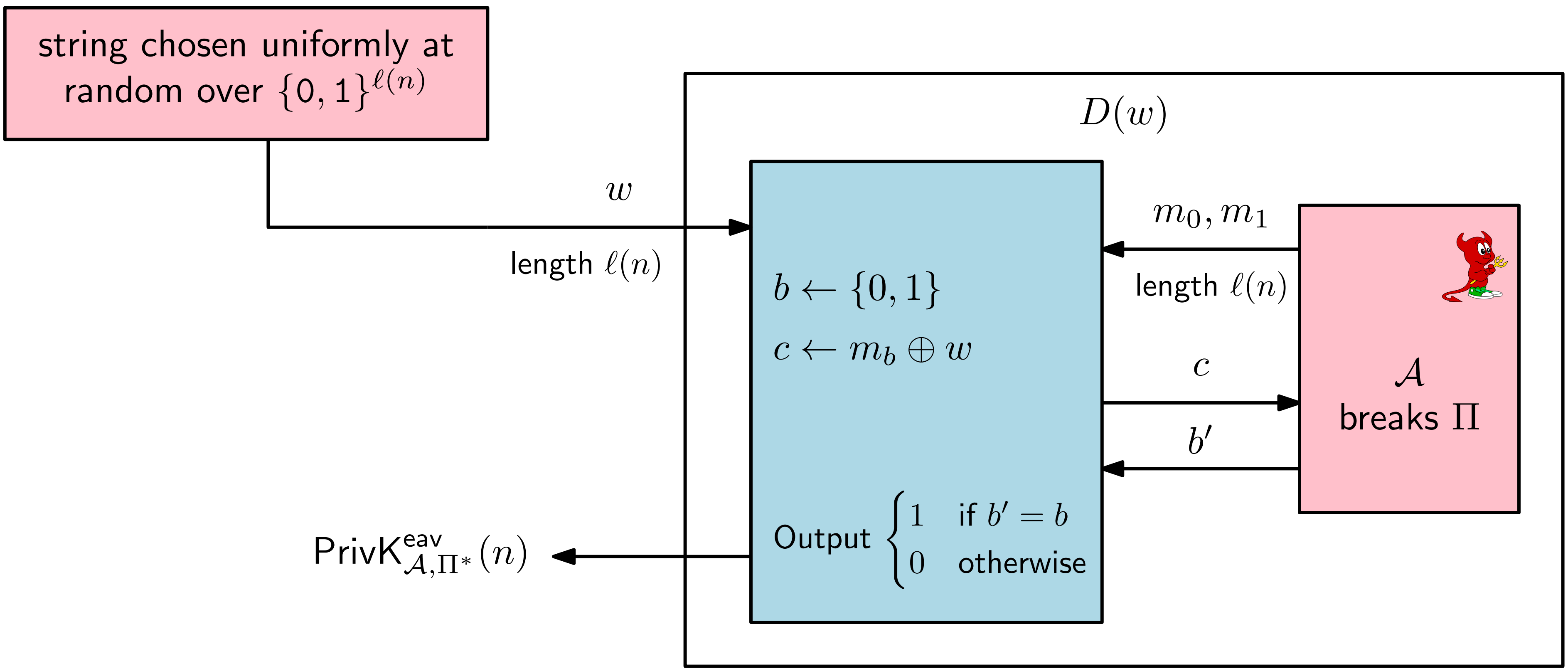

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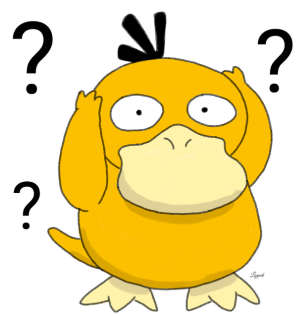

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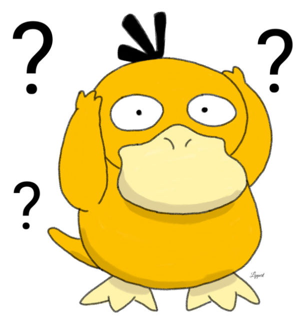

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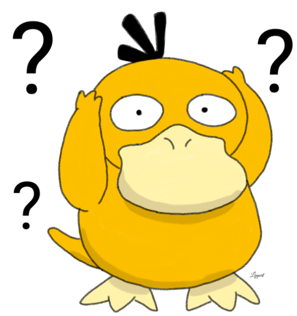

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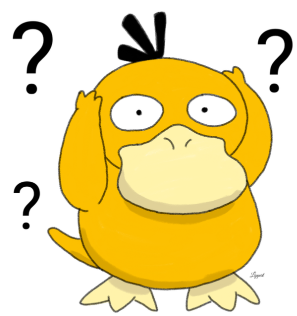

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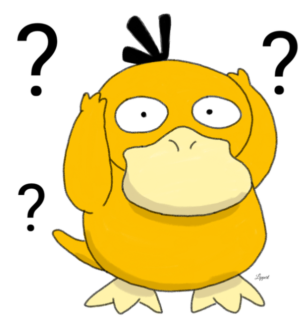

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- The adversary needs to provide a guess b' for the value of b

$$\mathsf{PrivK}^{\mathsf{mult}}_{\mathcal{A},\Pi}(n) = \begin{cases} 1 & \text{if } b' = b \\ 0 & \text{otherwise} \end{cases}$$

Multiple messages: security definition

Definition: *A private key encryption scheme* $\Pi = (Gen, Enc, Dec)$ *has* **indistinguishable multiple encryptions** *in the presence of an eavesdropper if, for every probabilistic polynomial-time adversary* $\mathcal{A}$*, there is a negligible function* ε *such that:*

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If a scheme has **indistinguishable multiple encryptions** in the presence of an eavesdropper then it is also **EAV-secure**

Multiple message (in)security of OTP

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We are exploiting the fact that, in OTP (and in pseudo OTP), the function Enc_k is **deterministic**!

Multiple message security and deterministic schemes

Observation: The previous adversary works against all schemes with a deterministic encryption function

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- Randomized encryption functions: multiple encryptions of the same message result in different ciphertexts
- Stateful schemes: Enc stores some additional information that is preserved between calls and it is used to produce different ciphertexts even when the same message is encrypted twice

An even stronger threat model

We will **not** focus on designing schemes with indistinguishable multiple encryptions

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We adopt an even stronger threat model instead!

security against chosen-plaintext attacks (CPA)

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All modern encryption schemes should be **at least** CPA-secure

Chosen-plaintext attack

The adversary learns the ciphertexts corresponding to one or more plaintexts <u>of its choice</u>.

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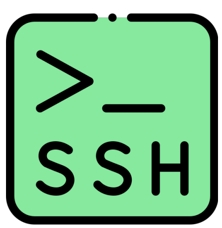

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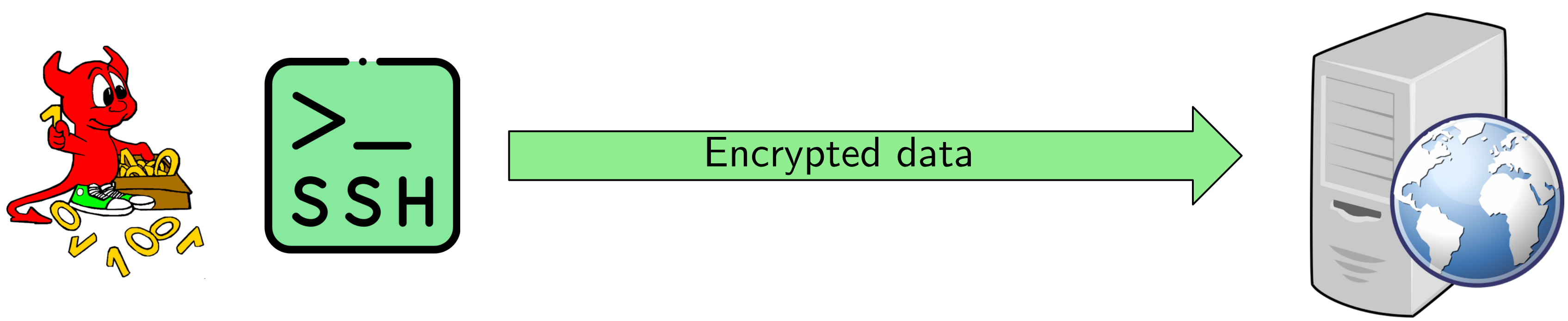

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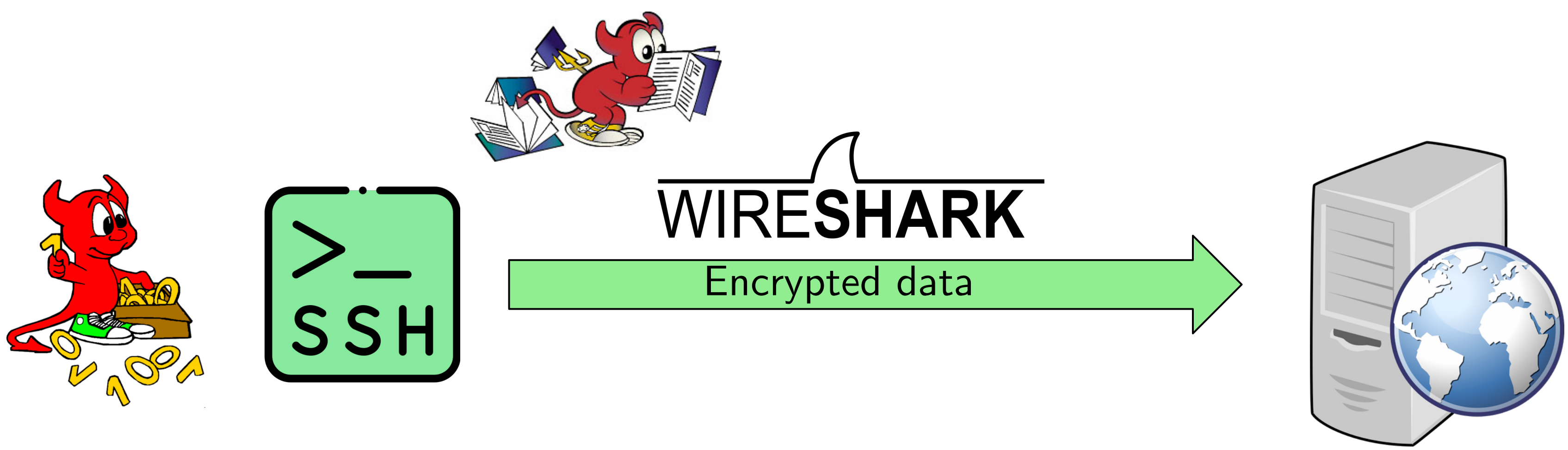

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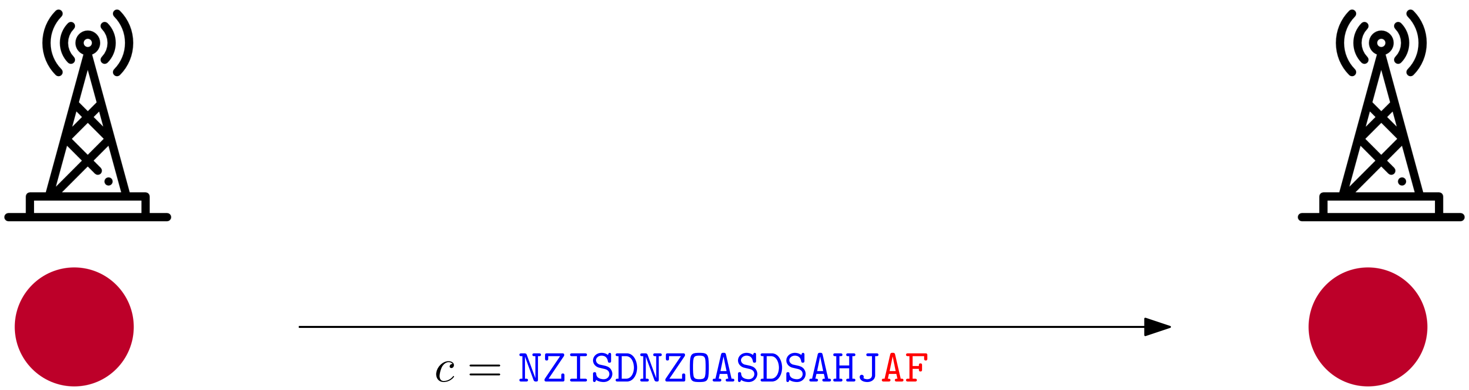

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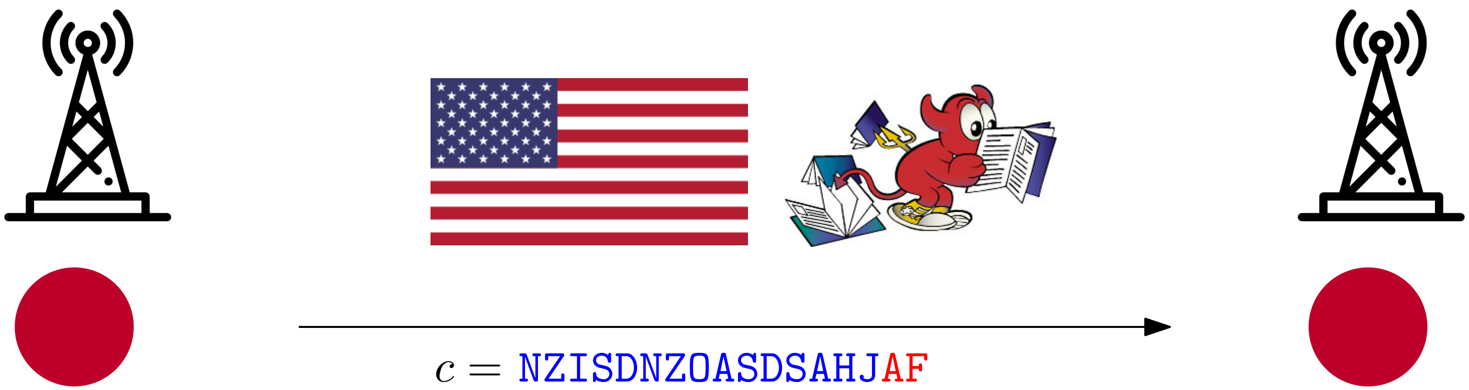

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$m =$ `We are planning to attack AF`

$c =$ `NZISDNZOASDSAHJAF`

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The U.S. cryptanalysts believed that AF meant Midway Island, but they were not 100% sure

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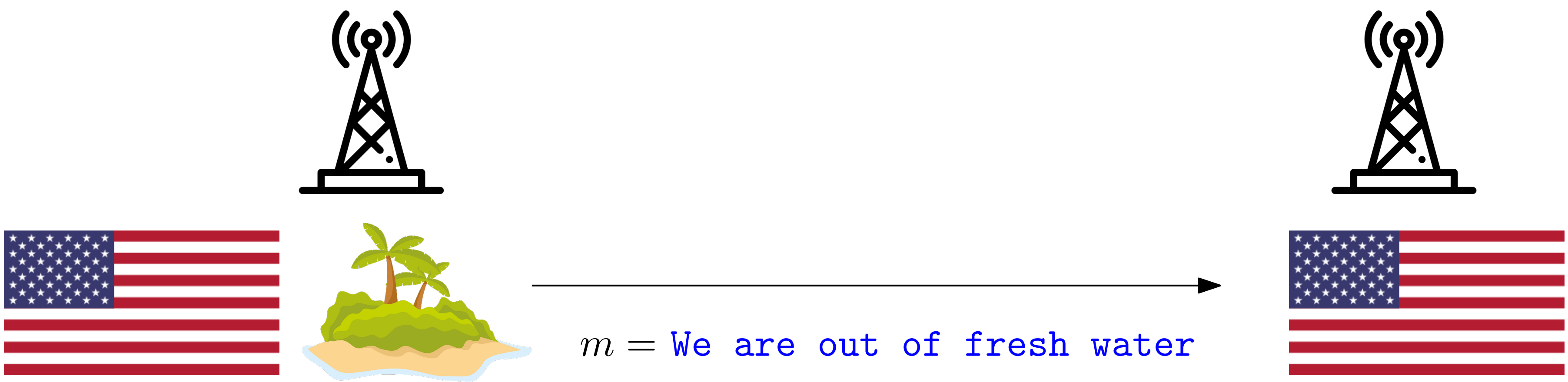

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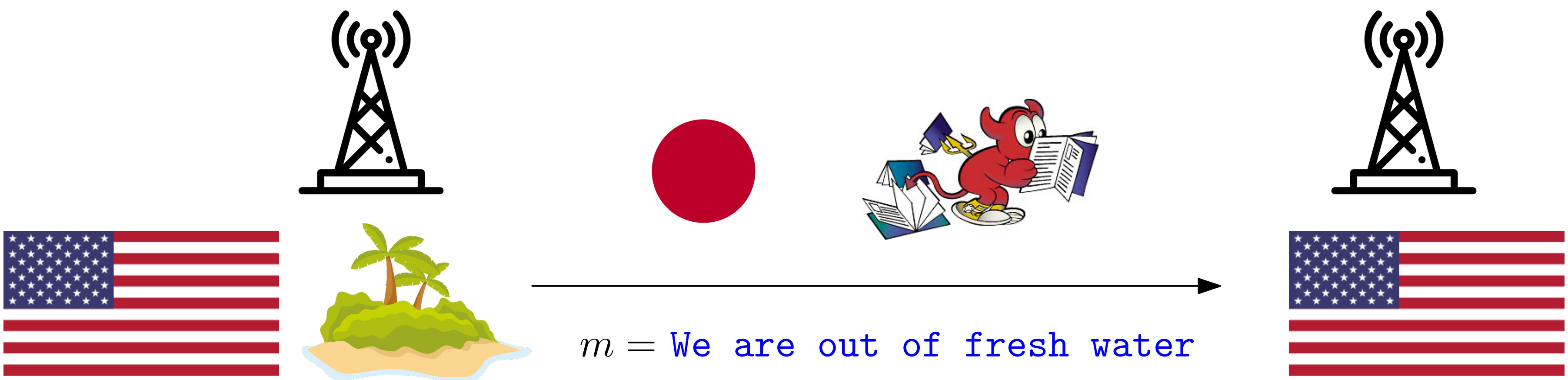

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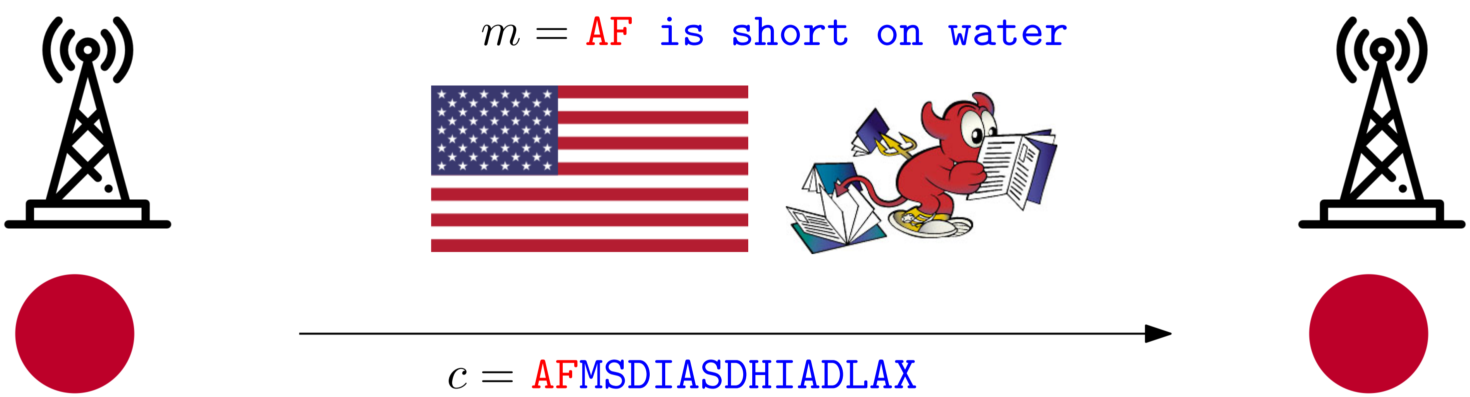
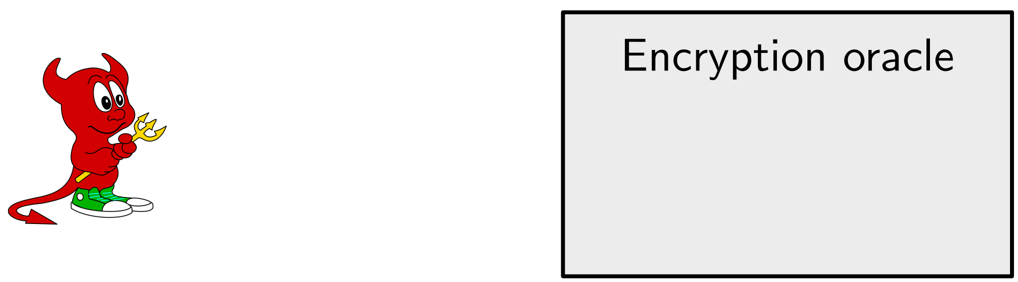


Modeling CPA security

A key $k \leftarrow \mathsf{Gen}(1^n)$ is generated and the adversary $\mathcal{A}$ is given access to an **encryption oracle**



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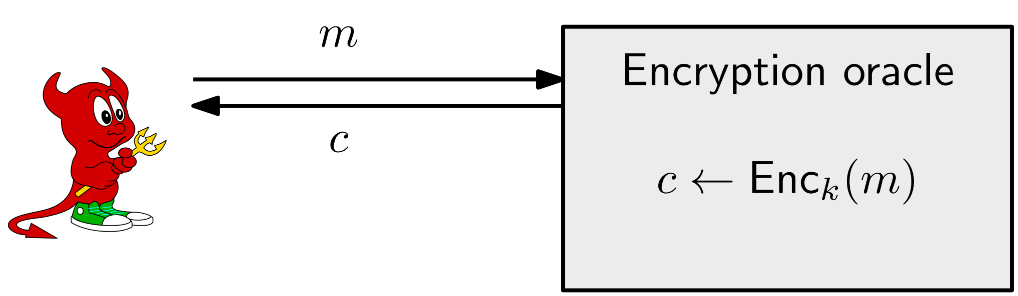

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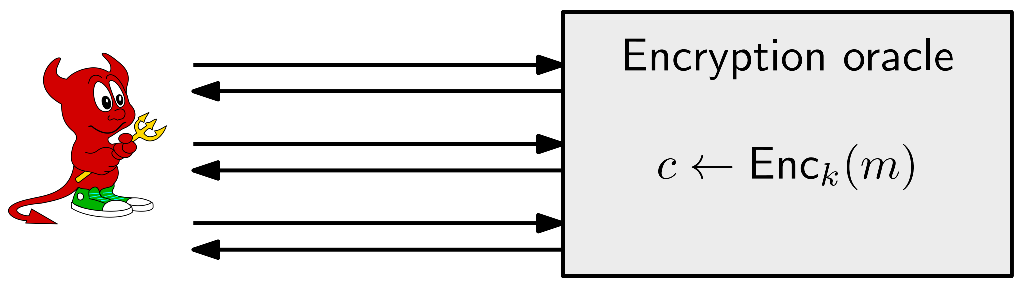

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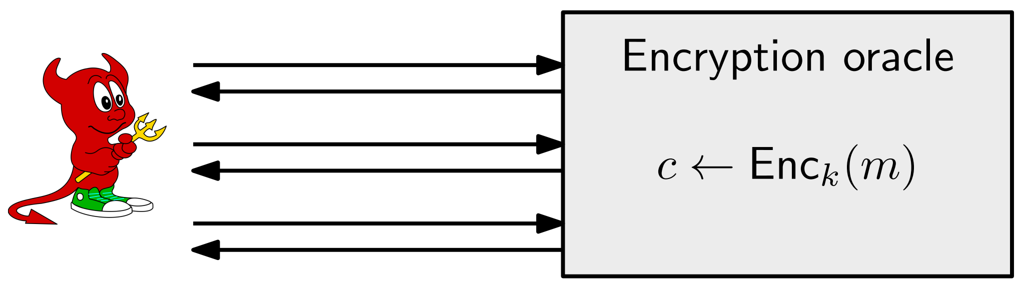

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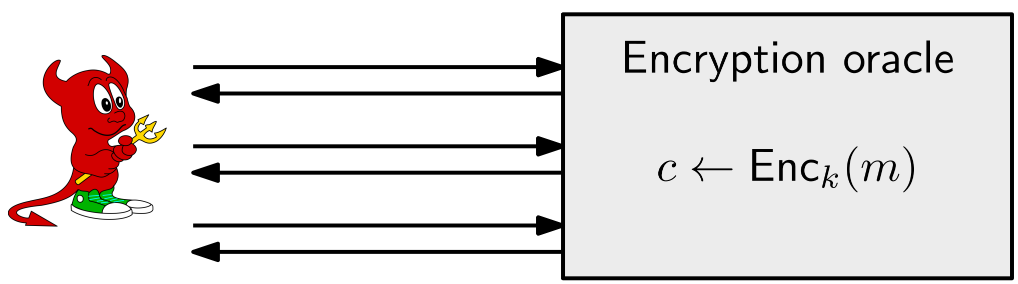

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A key $k \leftarrow \mathsf{Gen}(1^n)$ is generated

Encryption oracle

key k

Adversary $\mathcal{A}$

Verifier

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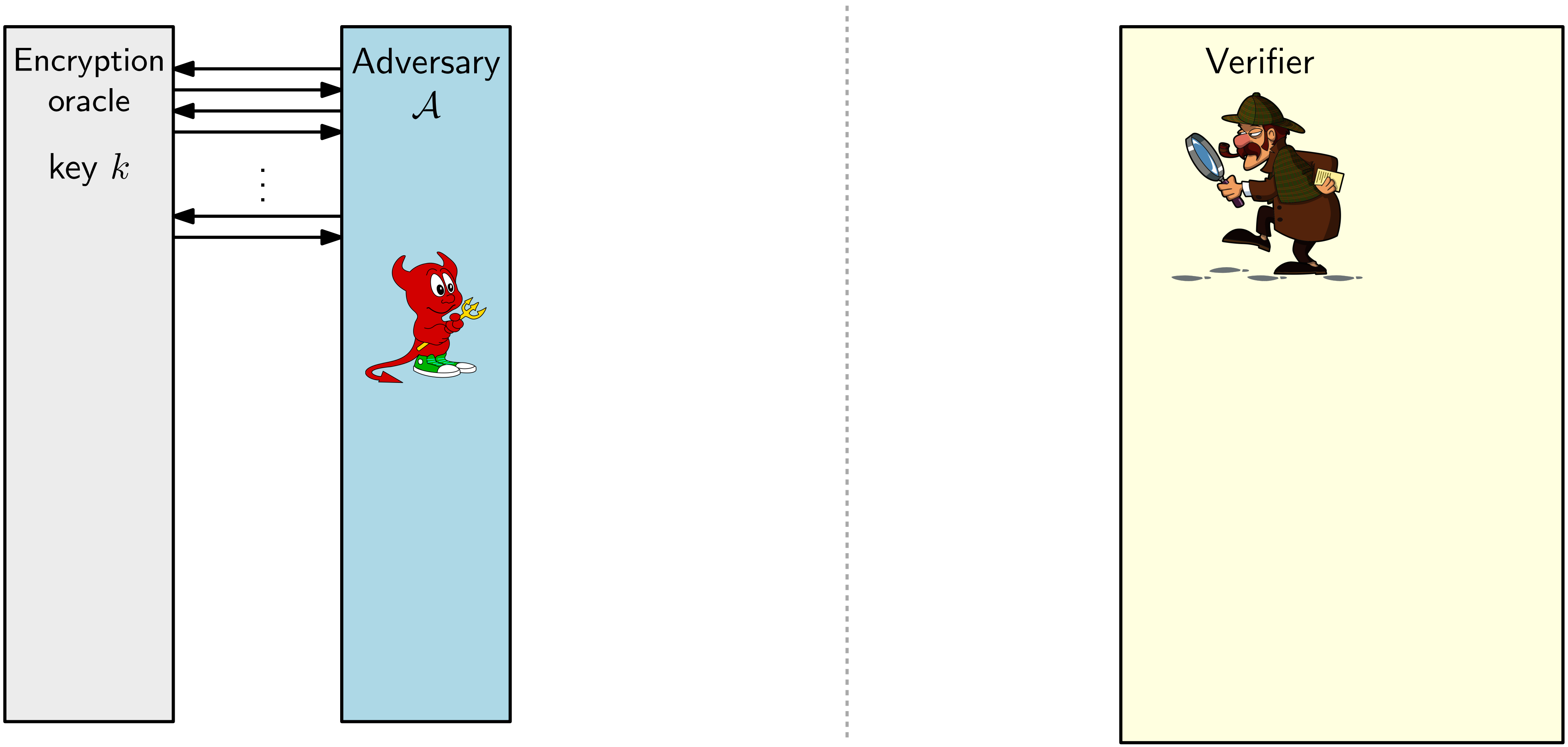

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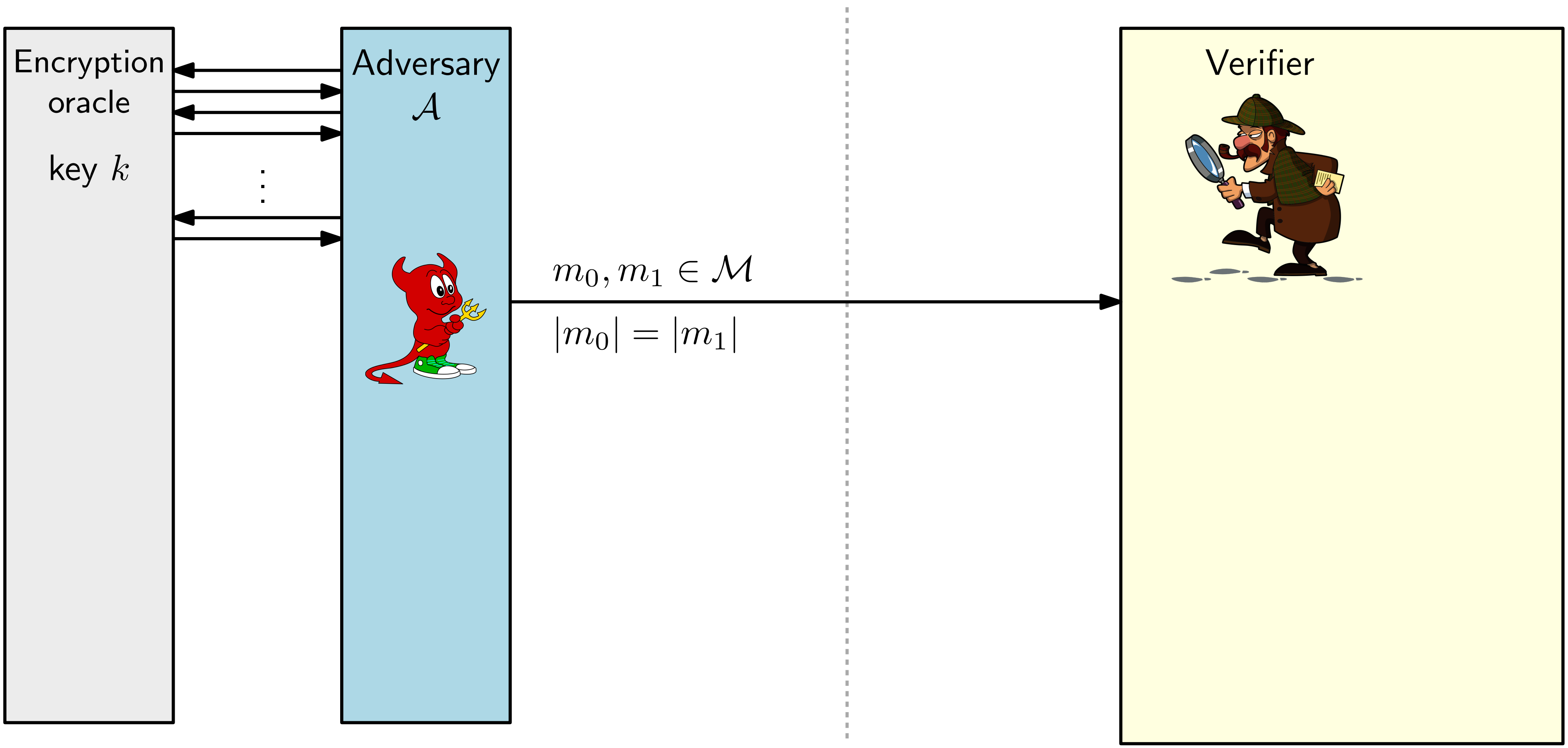

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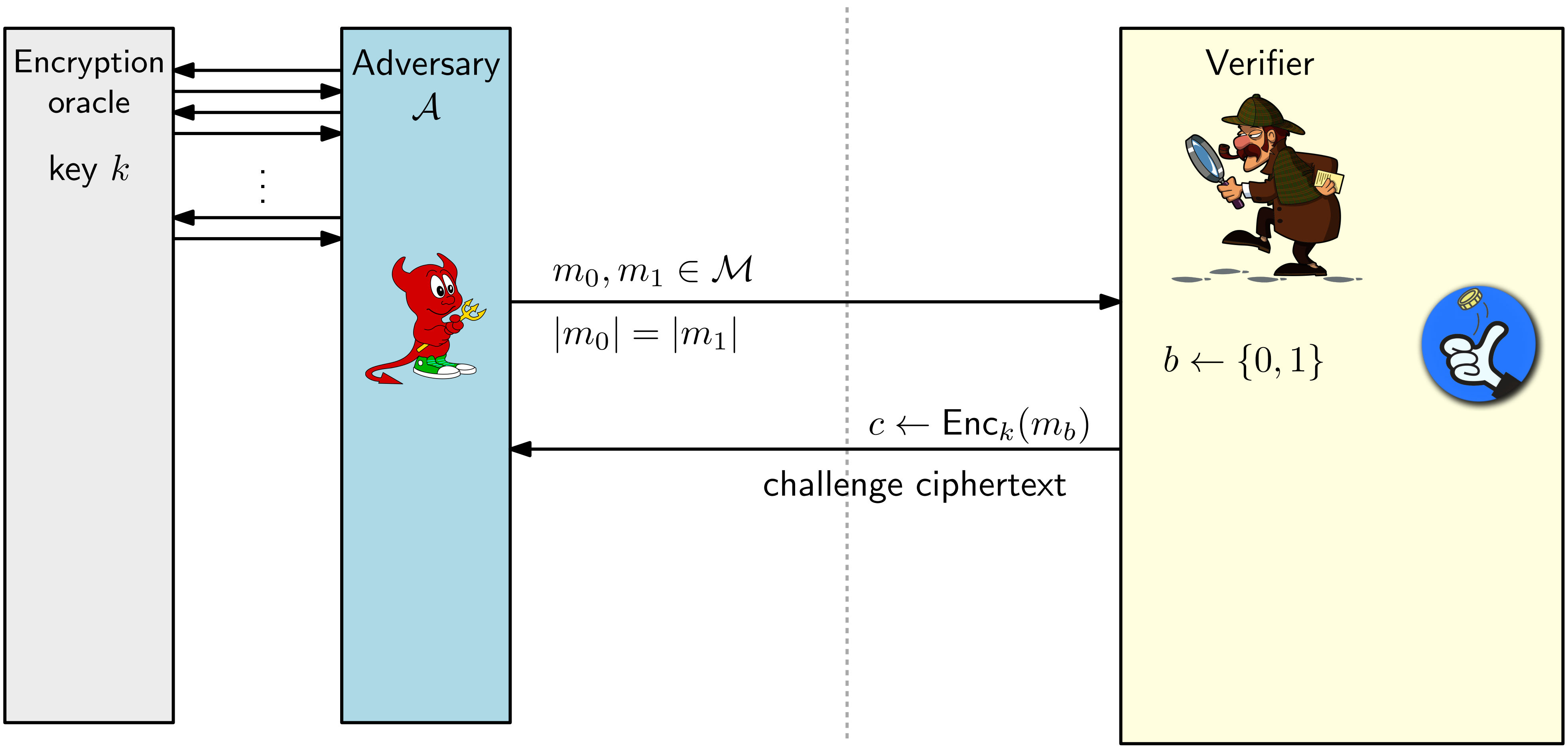

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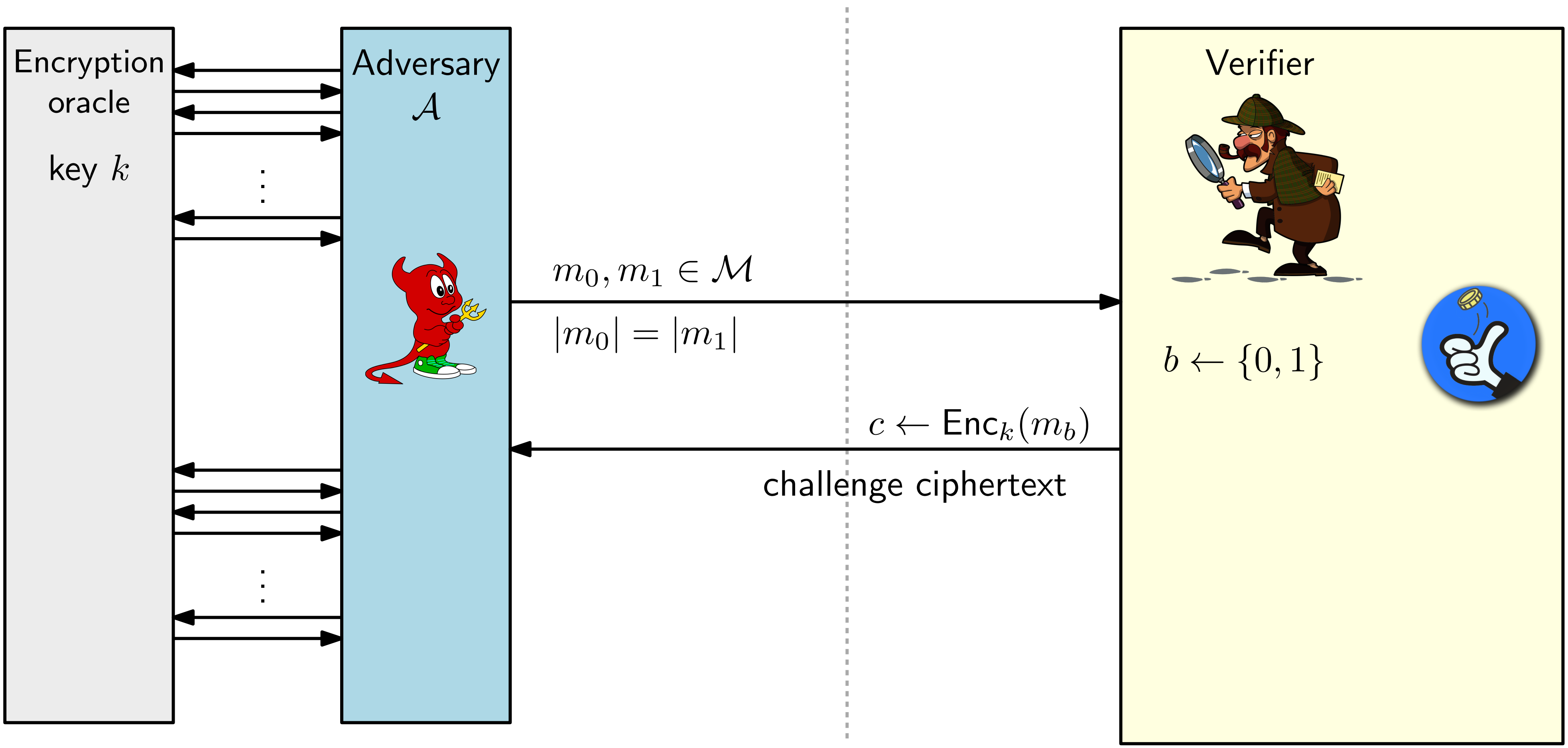

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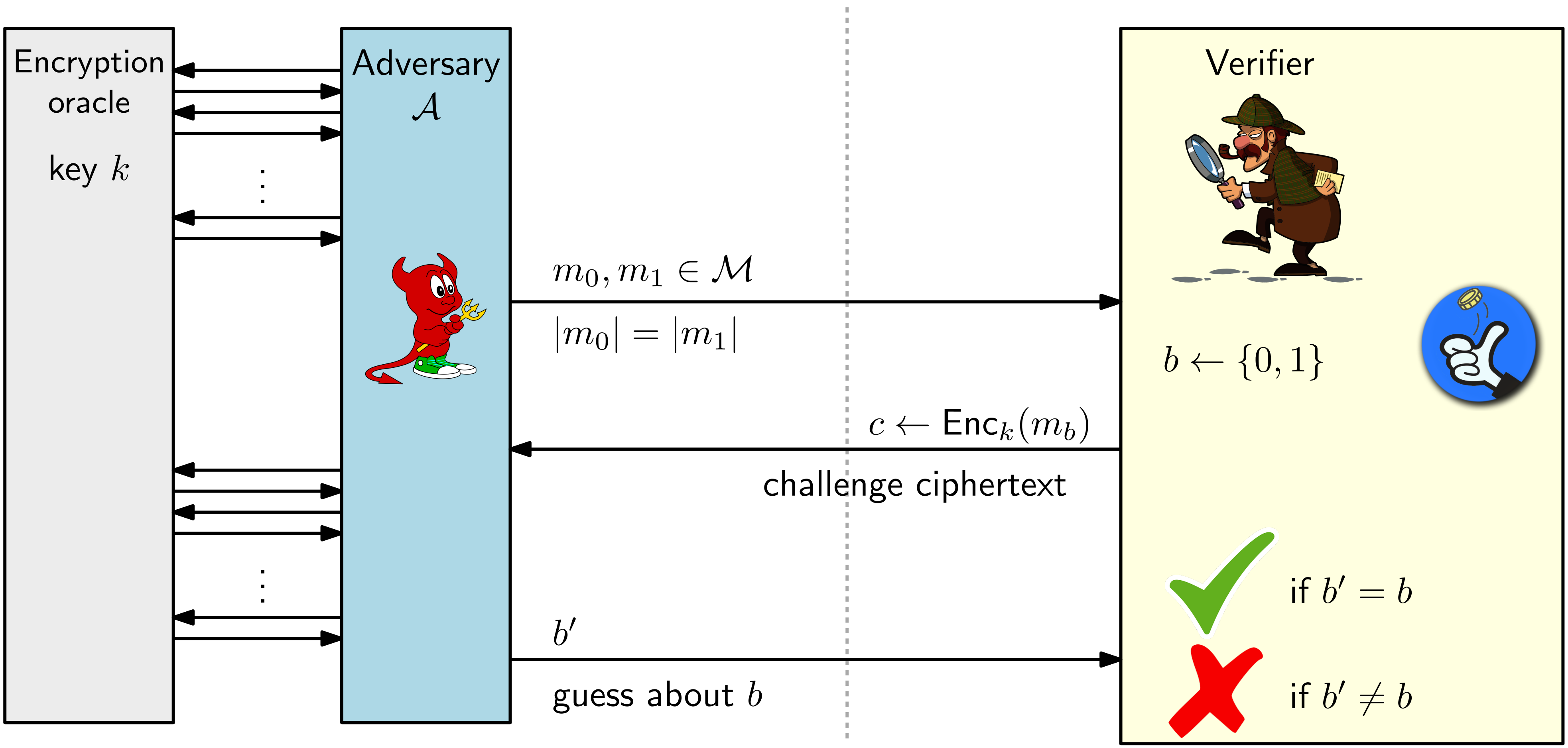

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Definition of CPA-security

Definition: *A private-key encryption scheme* Π *has indistinguishable encryptions under a chosen-plaintext attack (is* **CPA-secure***) if, for every probabilistic polynomial-time adversary* $\mathcal{A}$*, there is a negligible function* ε *such that:*

$$\Pr\left[\mathit{PrivK}^{\mathit{cpa}}_{\mathcal{A},\Pi}(n) = 1\right] \leq \frac{1}{2} + \varepsilon(n)$$

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Any private-key encryption scheme that is CPA-secure is also CPA-secure for multiple encryptions

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