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- **Known plaintext attack:** The adversary knows x and $F_k(x)$, where x is not chosen by the attacker
- **Chosen plaintext attack:** The attacker can query F_k (with values of its choice)
- **Chosen ciphertext attack:** The attacker can query both F_k and F_k^{-1} (with values of its choice)

Designing Block Ciphers

- To design a block cipher, we want the computed function to be “indistinguishable” from a uniform permutation over $\{0, 1\}^\ell$
- If x and x' differ, even just by one bit, the outputs of $F_k(x)$ and $F_k(x')$ should look unrelated (except for $F_k(x) \neq F_k(x')$)

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Substitution Permutation Networks (SPNs)

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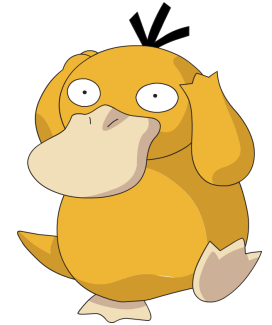
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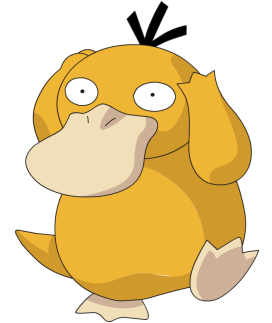
- **Confusion:** A small change in the input produces a small “random” change in the output
- **Diffusion:** The bits in the input are mixed so that a local change is spread throughout the block



Confusion

There are **many** random permutations

- Recall that $|\text{Perm}_\ell| = (2^\ell)!$
- How many bits are needed to identify one of these permutations?

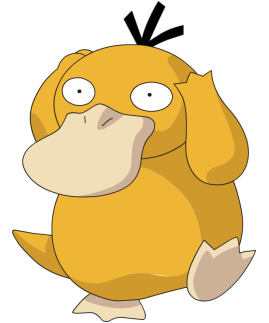


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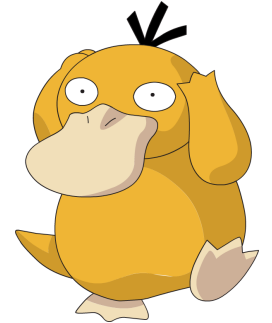
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Idea: Build a “random” permutation on **long** inputs by using many “random” permutations on **short** inputs

Example: To store 8 permutations over $\{0, 1\}^8$ we need less than $8 \cdot (8 \cdot 2^8) \text{ b} = 2 \text{ KB}$



Confusion

Consider a keyed PRP F_k with a block length 64 bits defined as follows: (the length is just an example)

$$F_k(x) = f_{k_1}(x_1) \parallel f_{k_2}(x_2) \parallel f_{k_3}(x_3) \parallel \dots \parallel f_{k_8}(x_8)$$

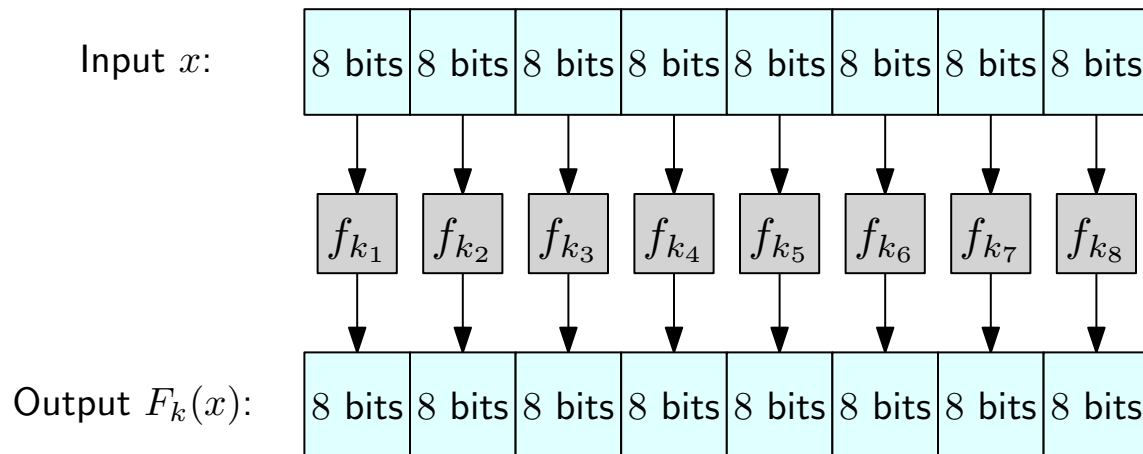
where $x = x_1 \parallel x_2 \parallel x_3 \parallel \dots \parallel x_8$, $k = k_1 \parallel k_2 \parallel k_3 \parallel \dots \parallel k_8$, all x_i are 8-bit long, and all f_{k_i} are permutations

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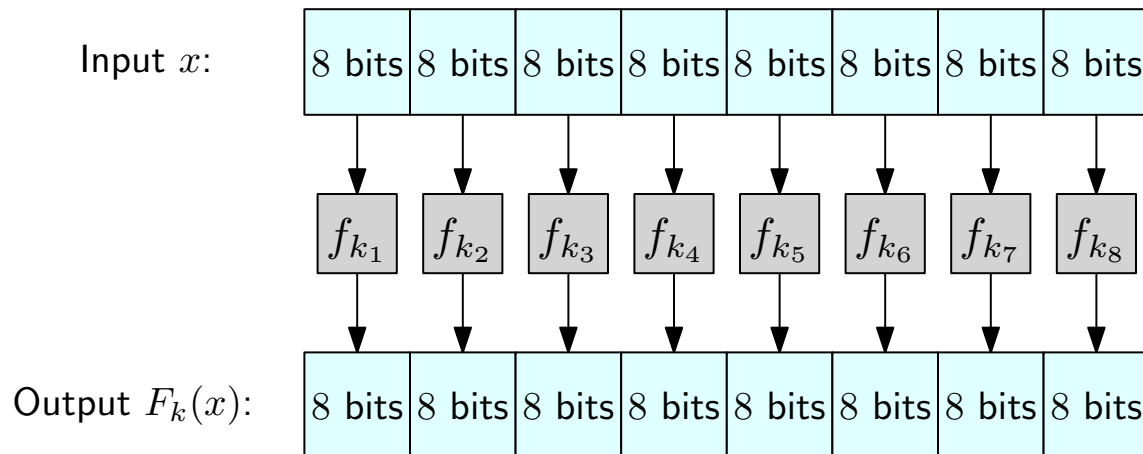


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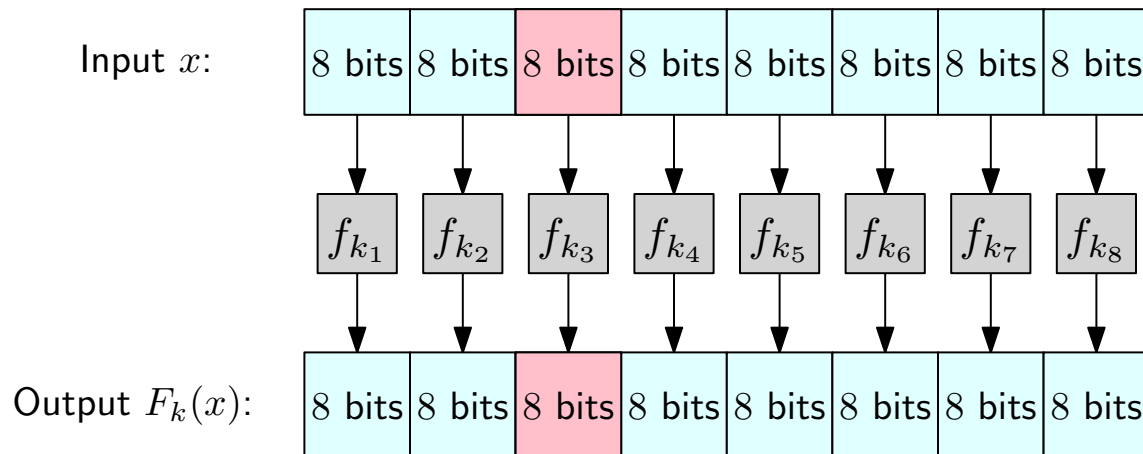
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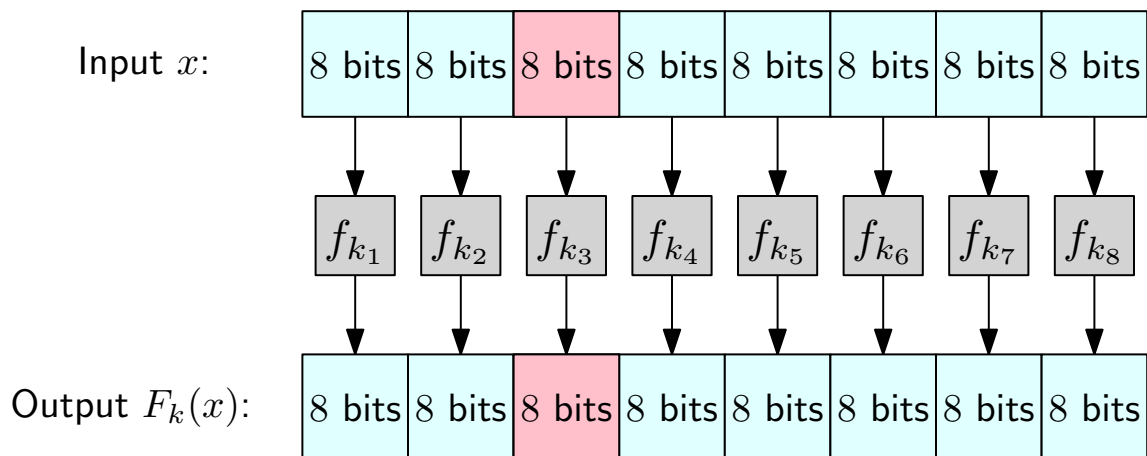
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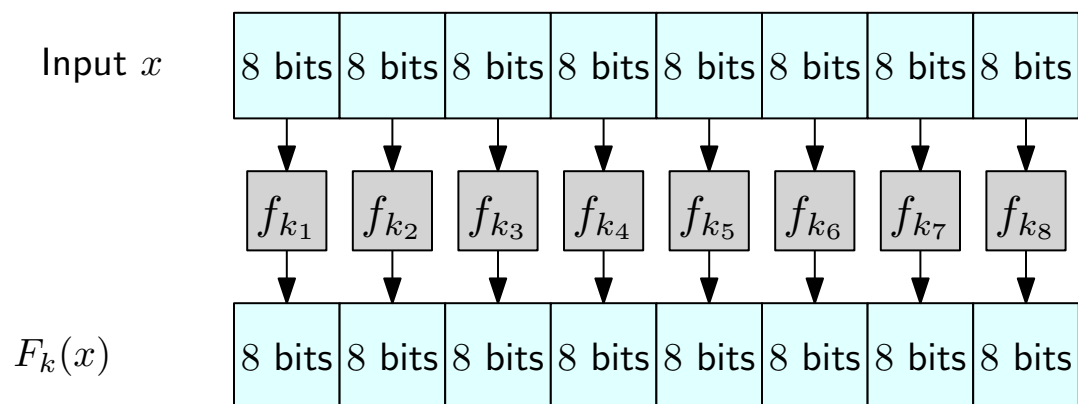
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Confusion but no diffusion

Adding diffusion

We use a **mixing permutation** π to add diffusion

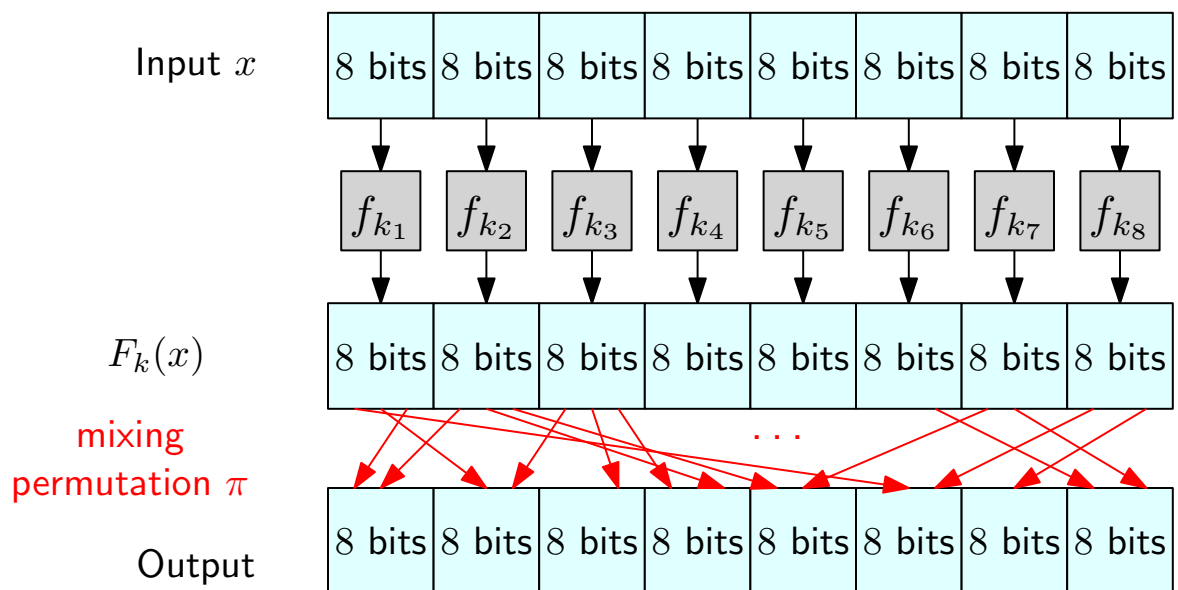
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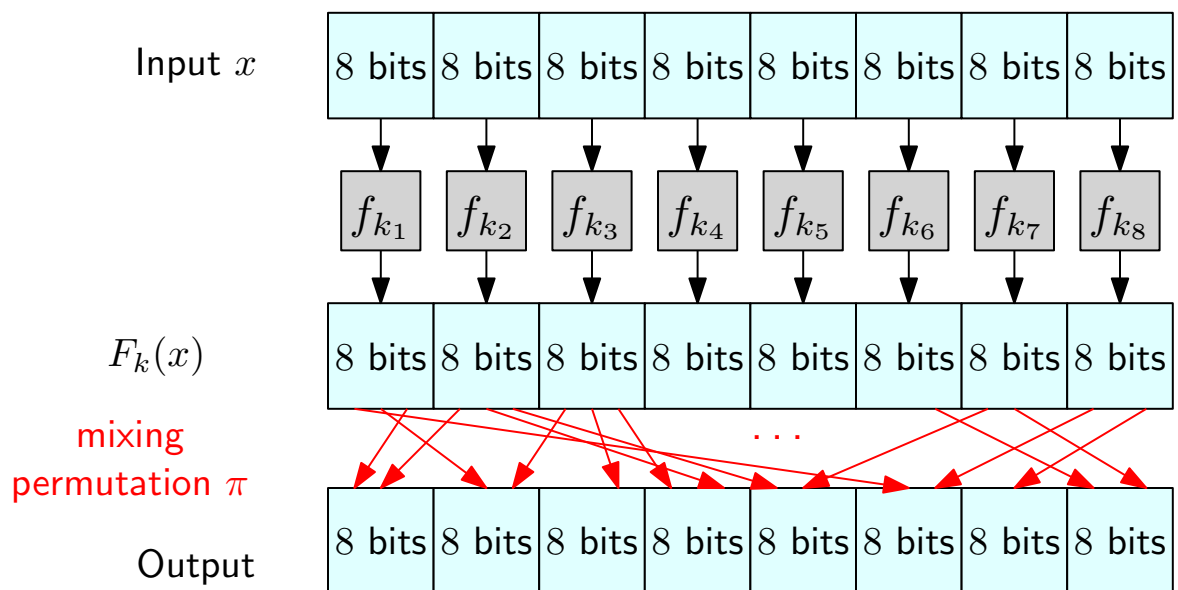
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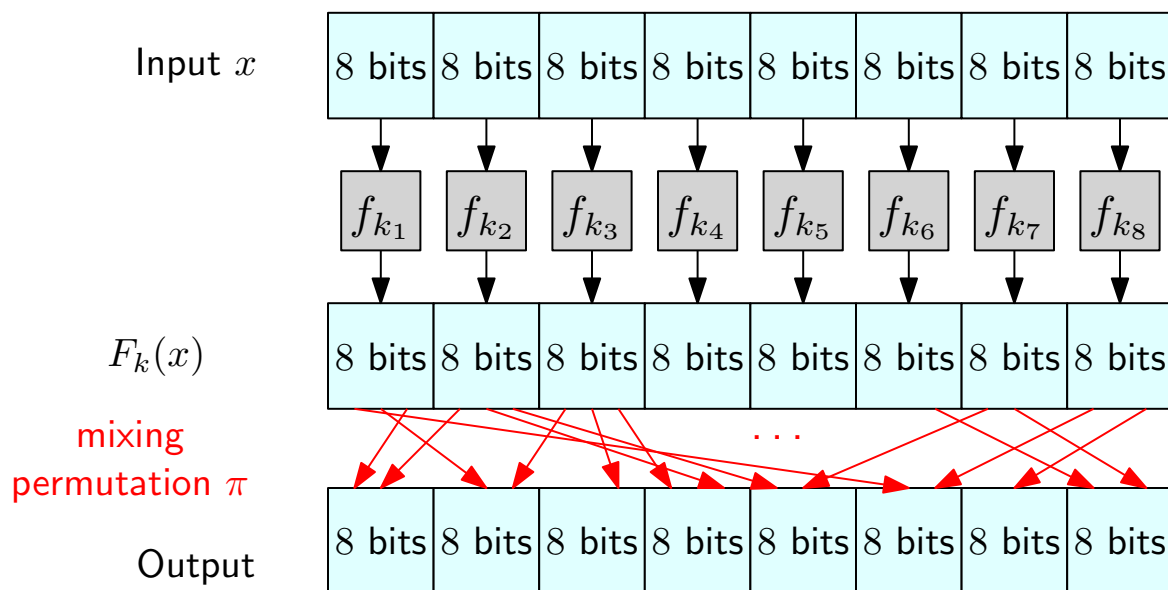
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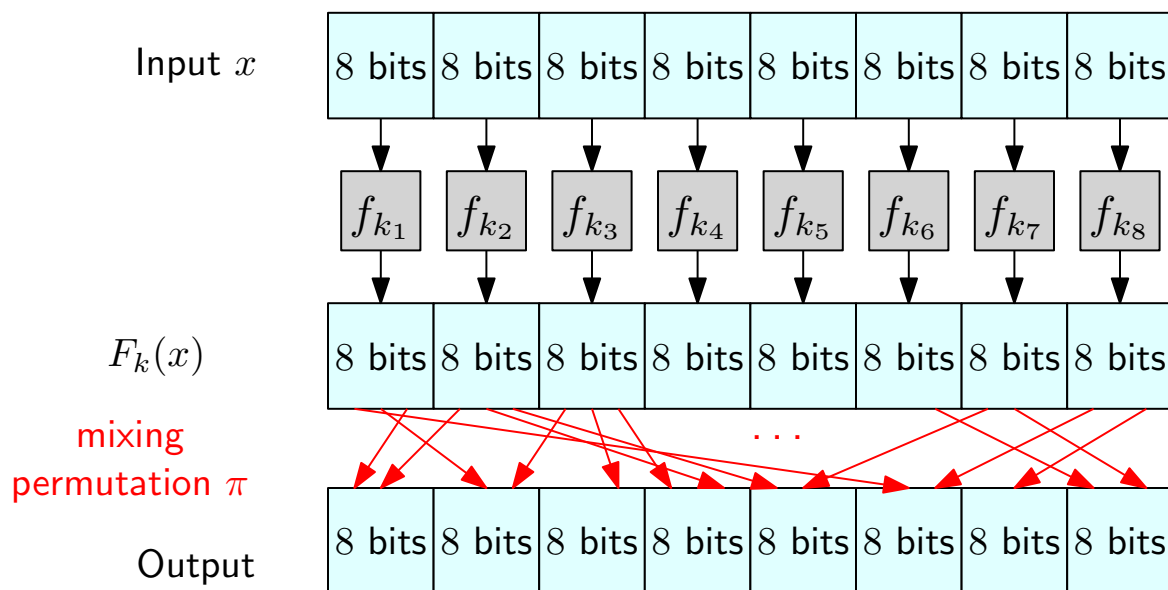
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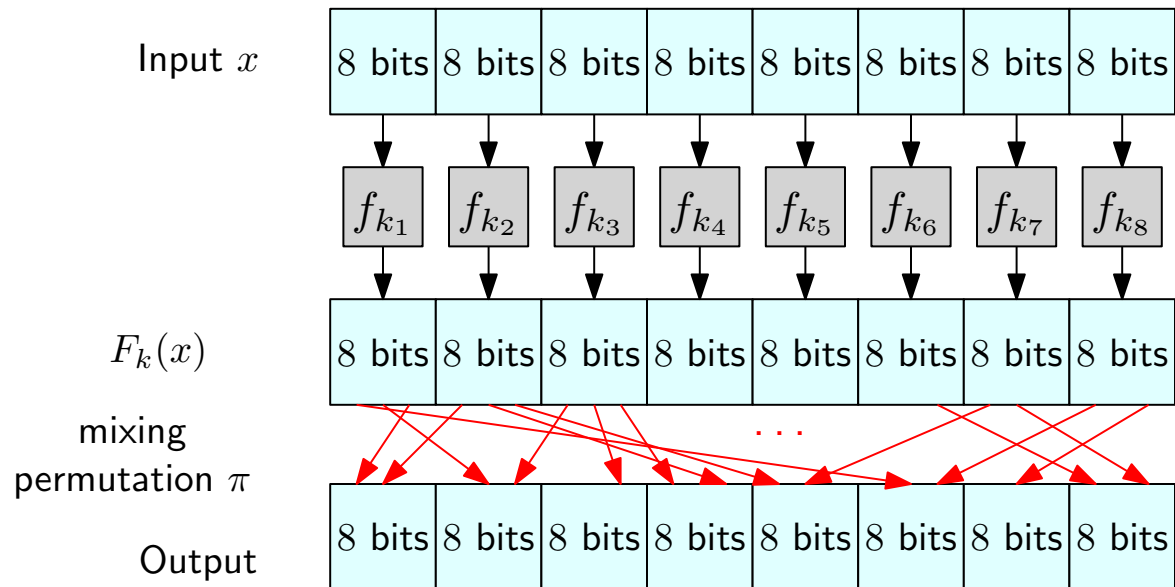
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In practice the mixing permutation does not depend on the key and is carefully designed and **fixed**

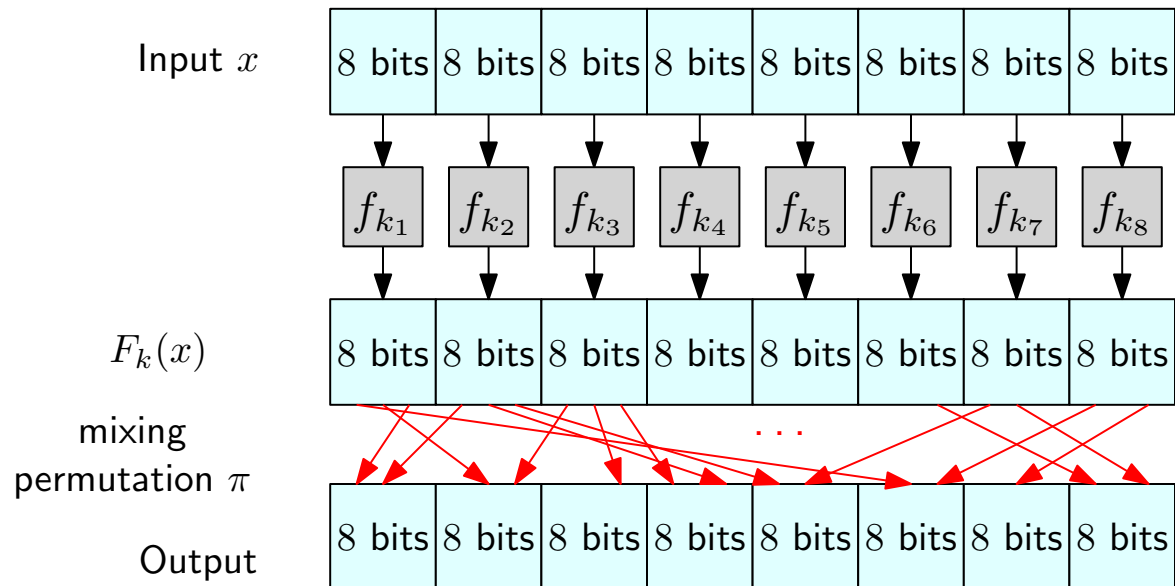


We have a Substitution Permutation Network (SPN)



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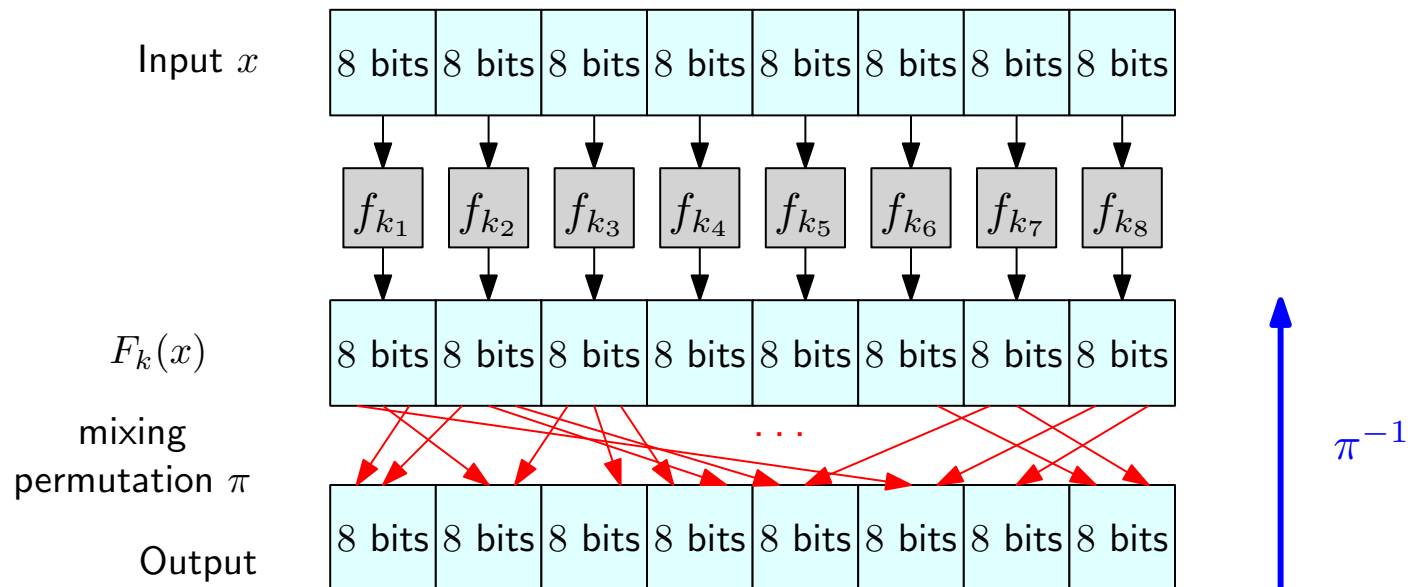
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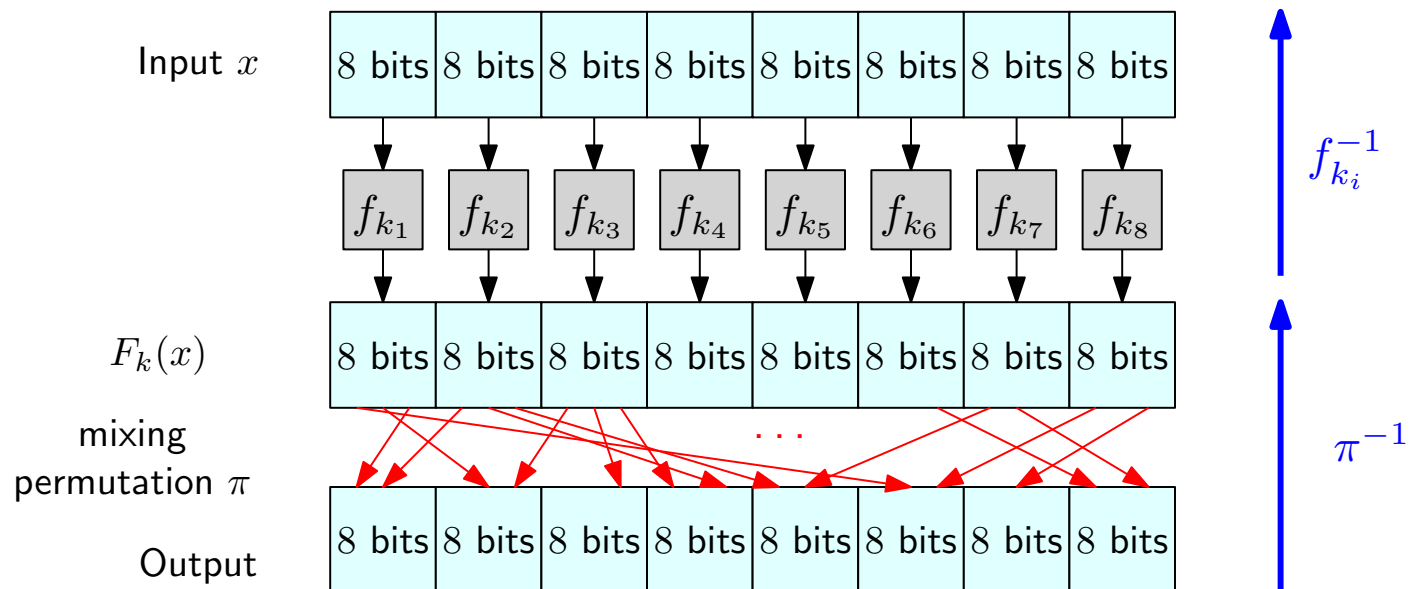


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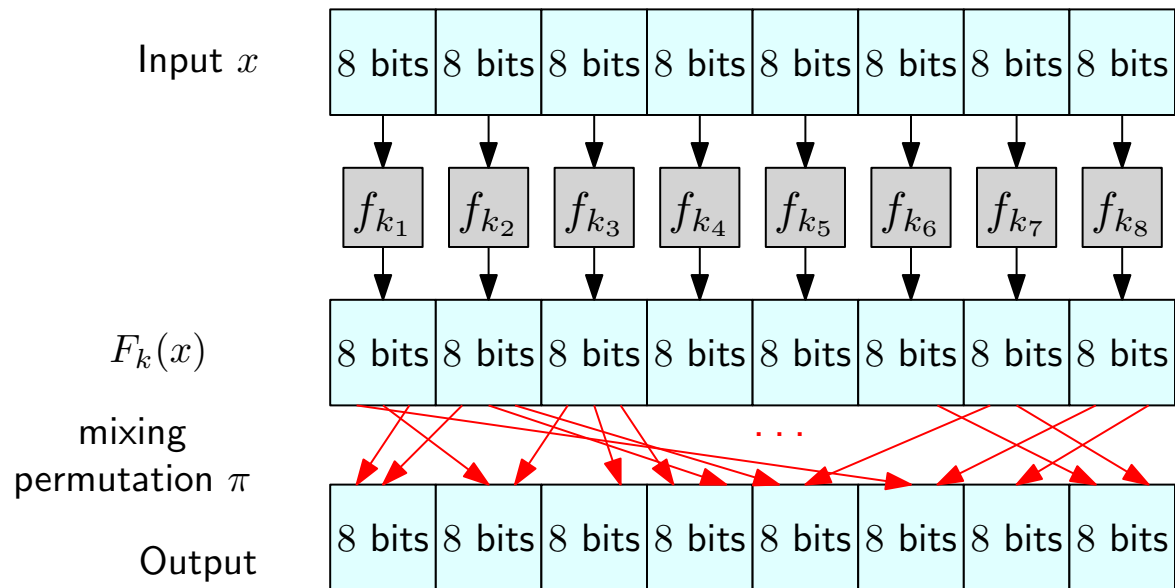


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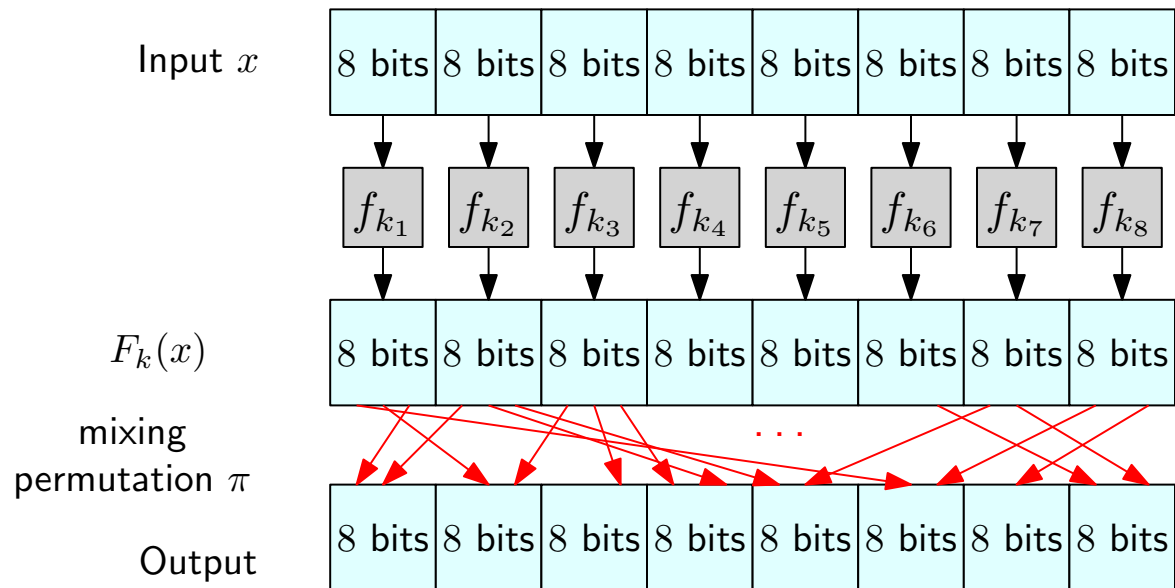
- The mixing permutation is... a permutation, and hence invertible
- Each function f_{k_i} is also a permutation, and hence invertible

Substitution Permutation Networks



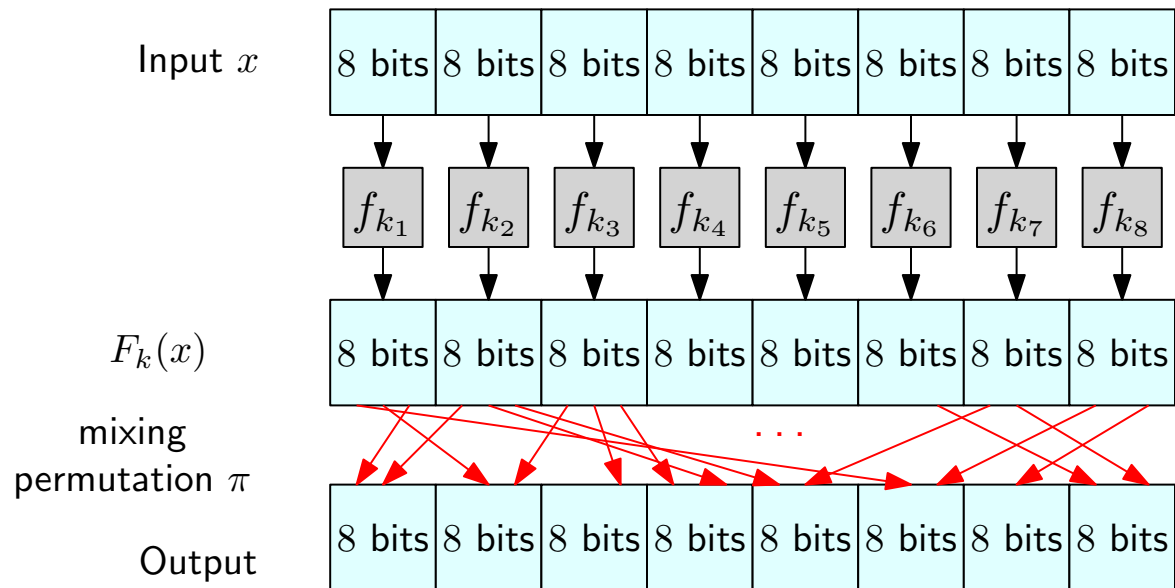
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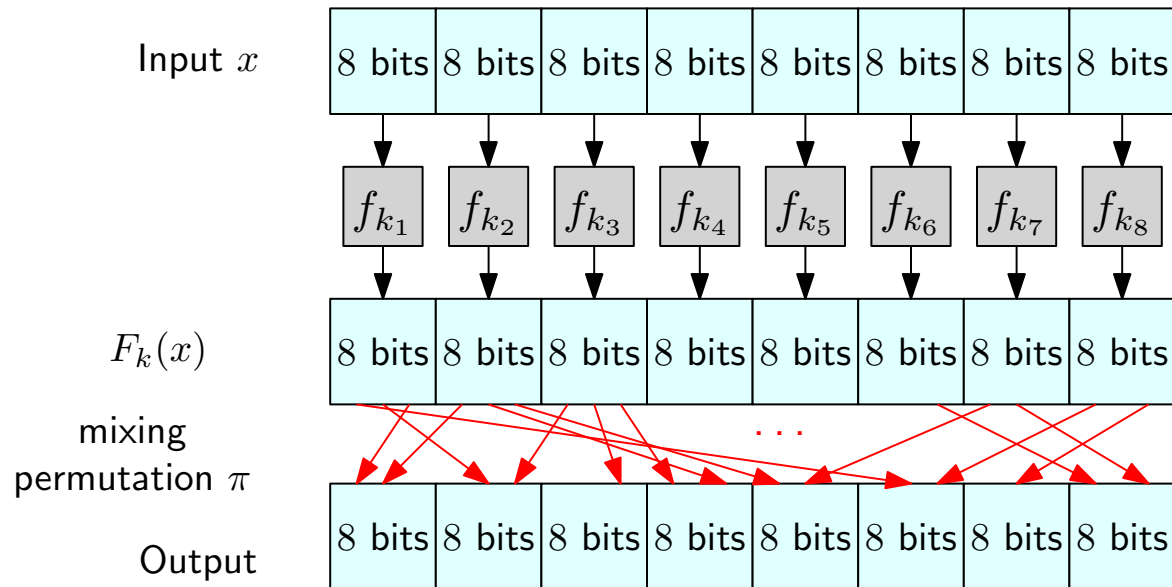
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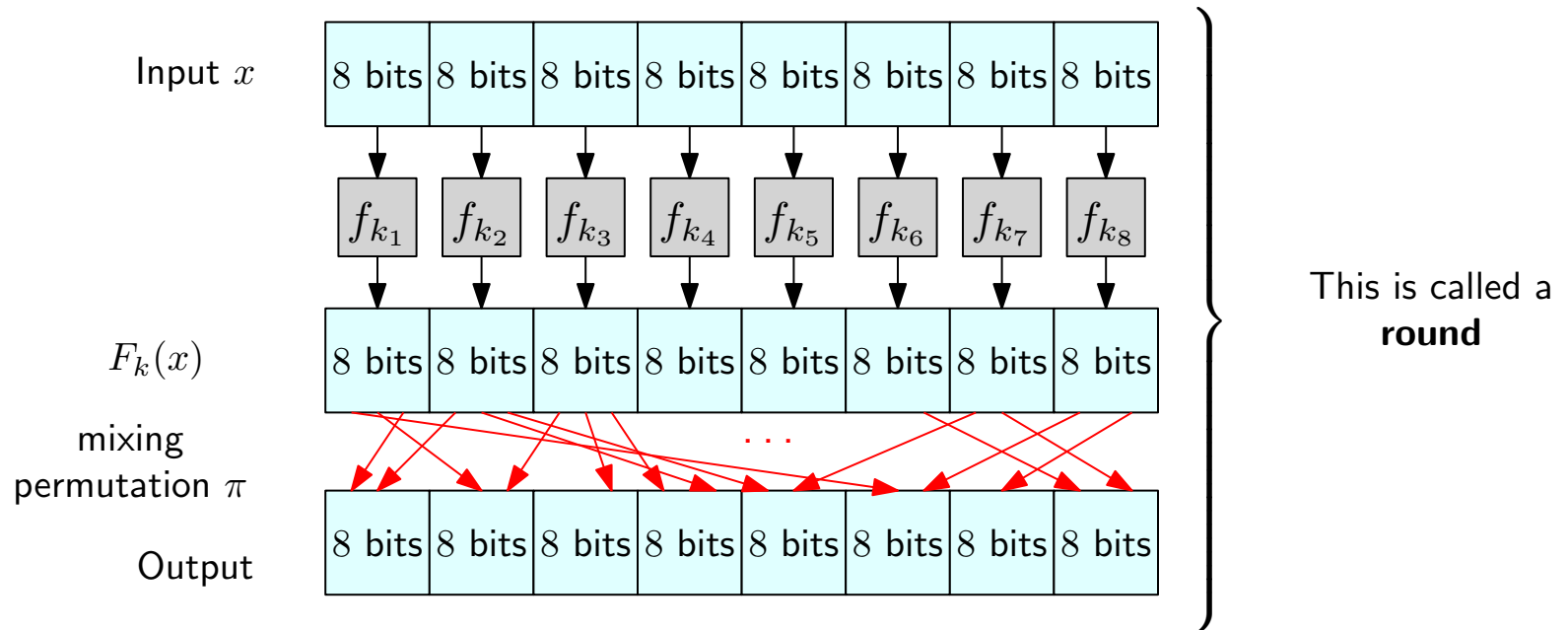
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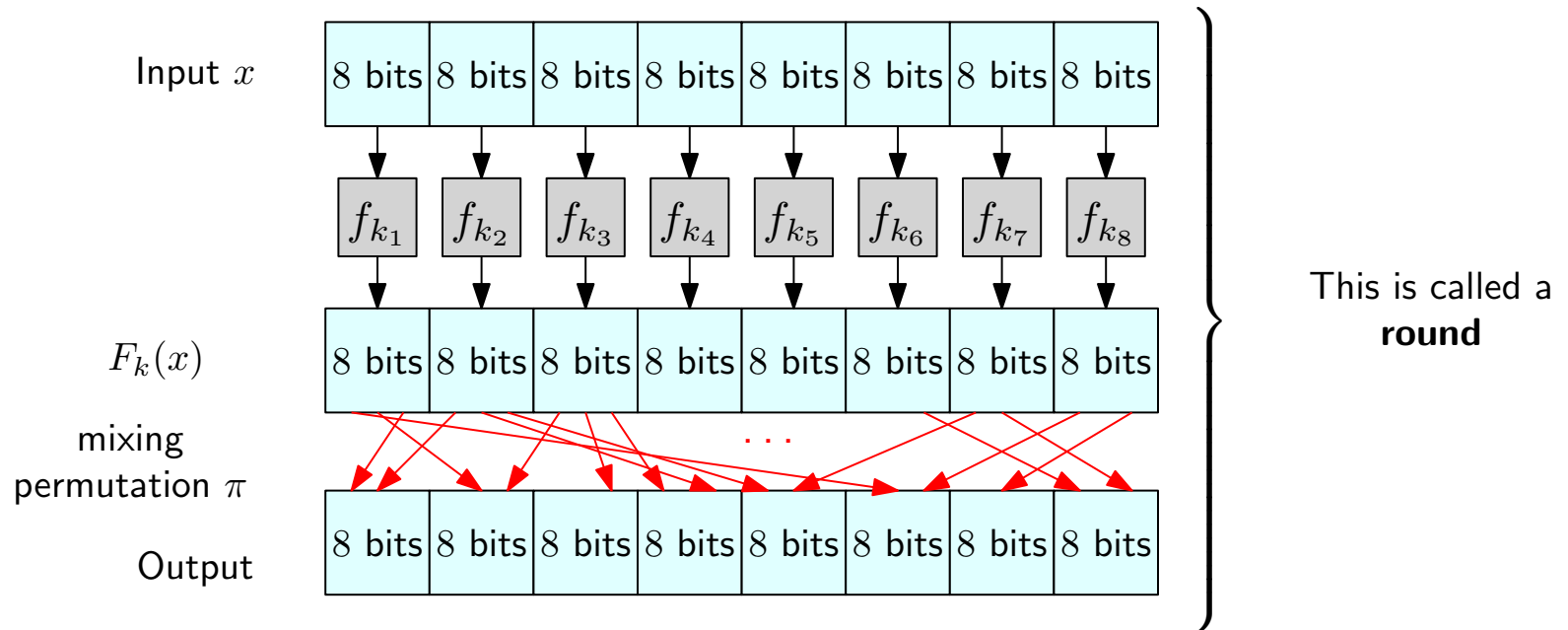
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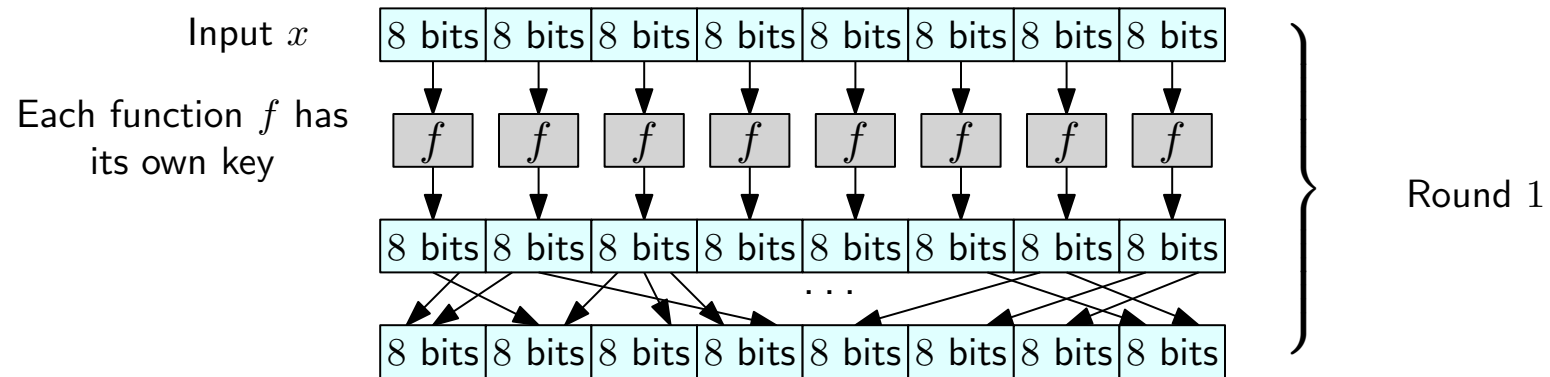


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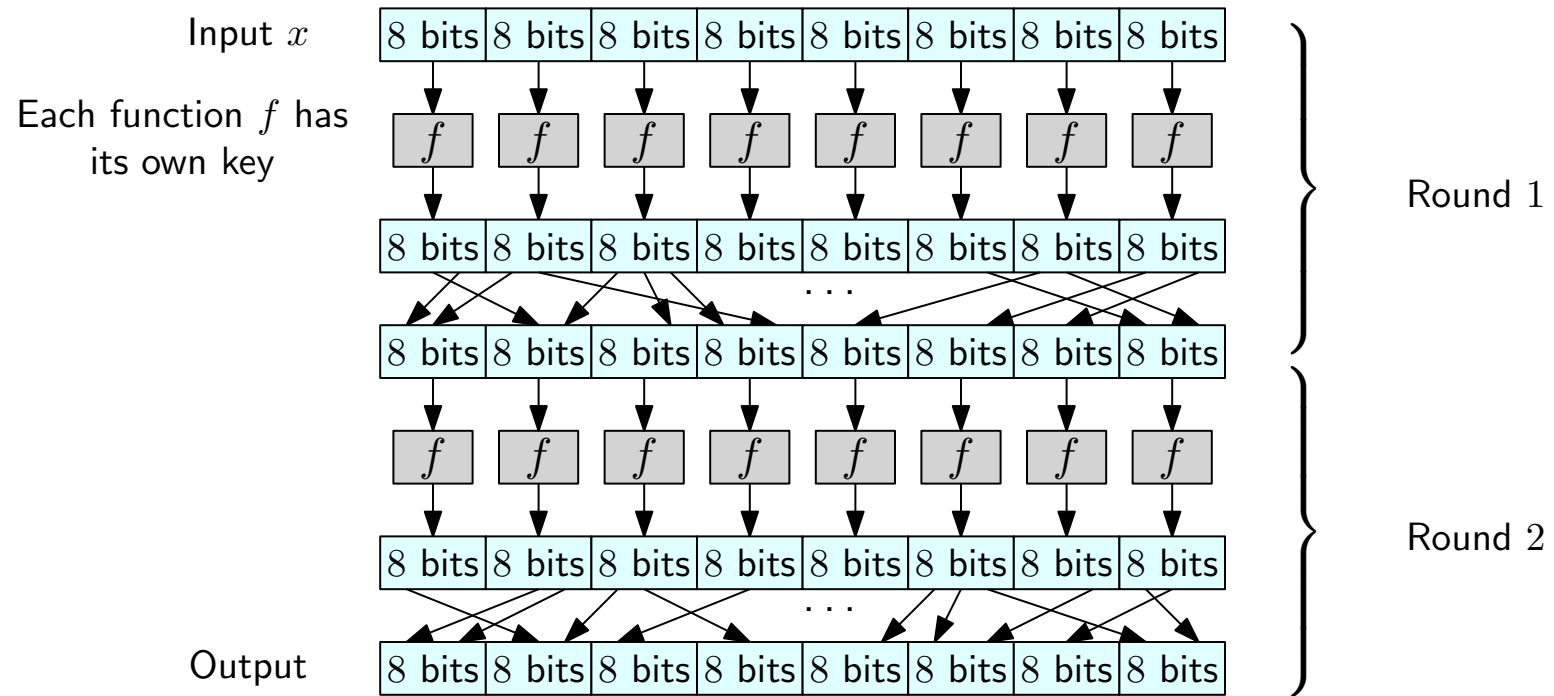
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What if we do another round with fresh functions f_{k_i} ?

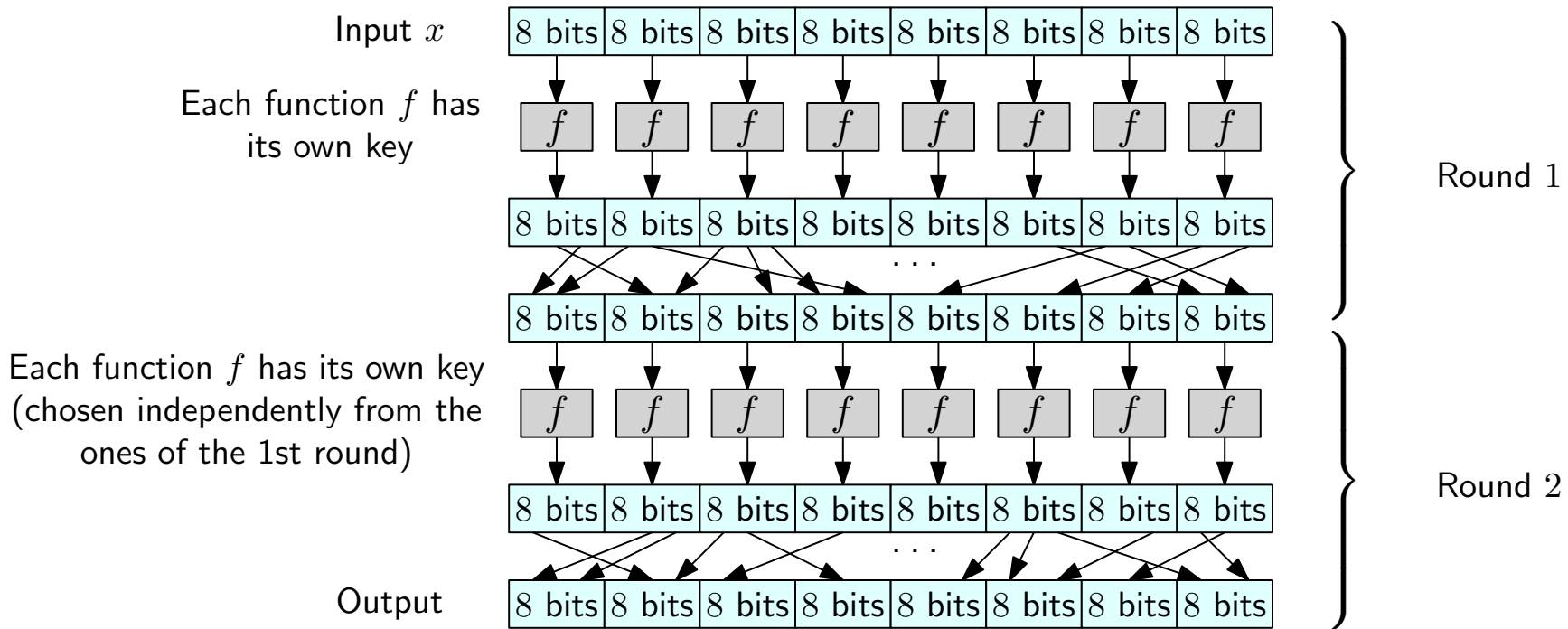
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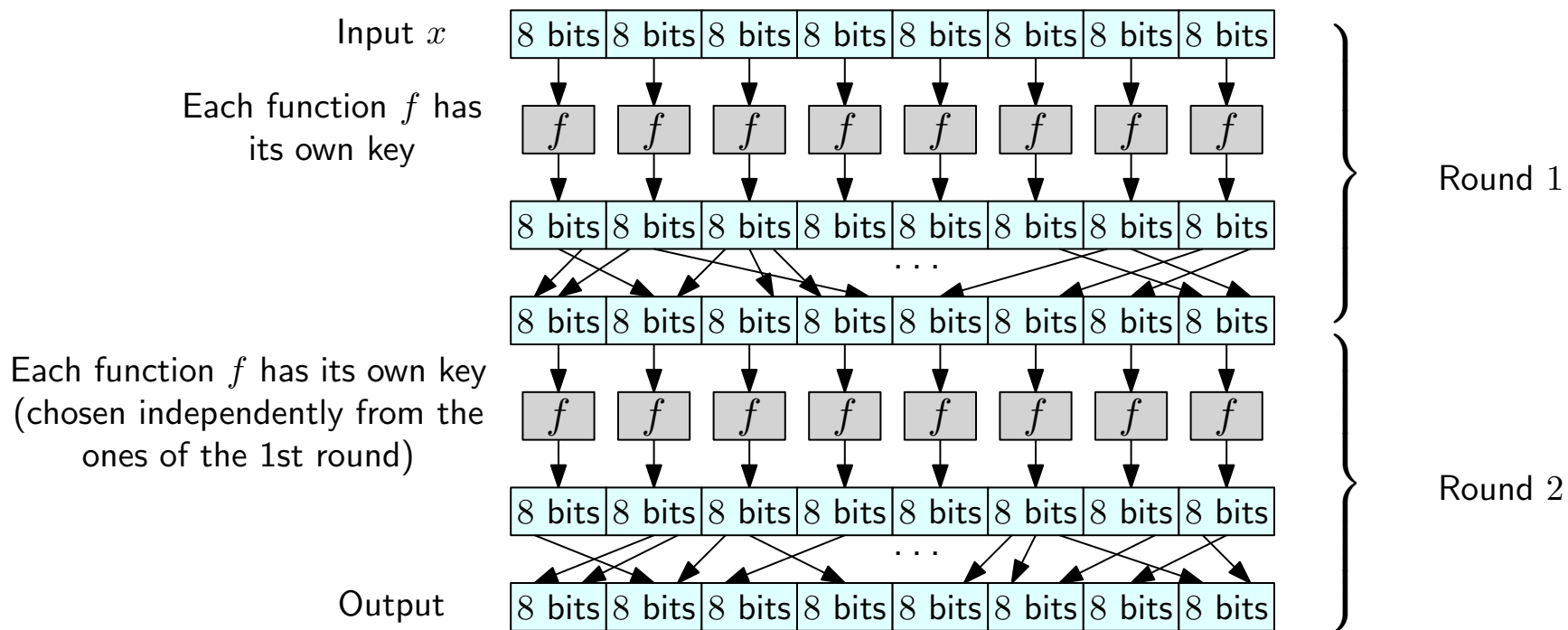
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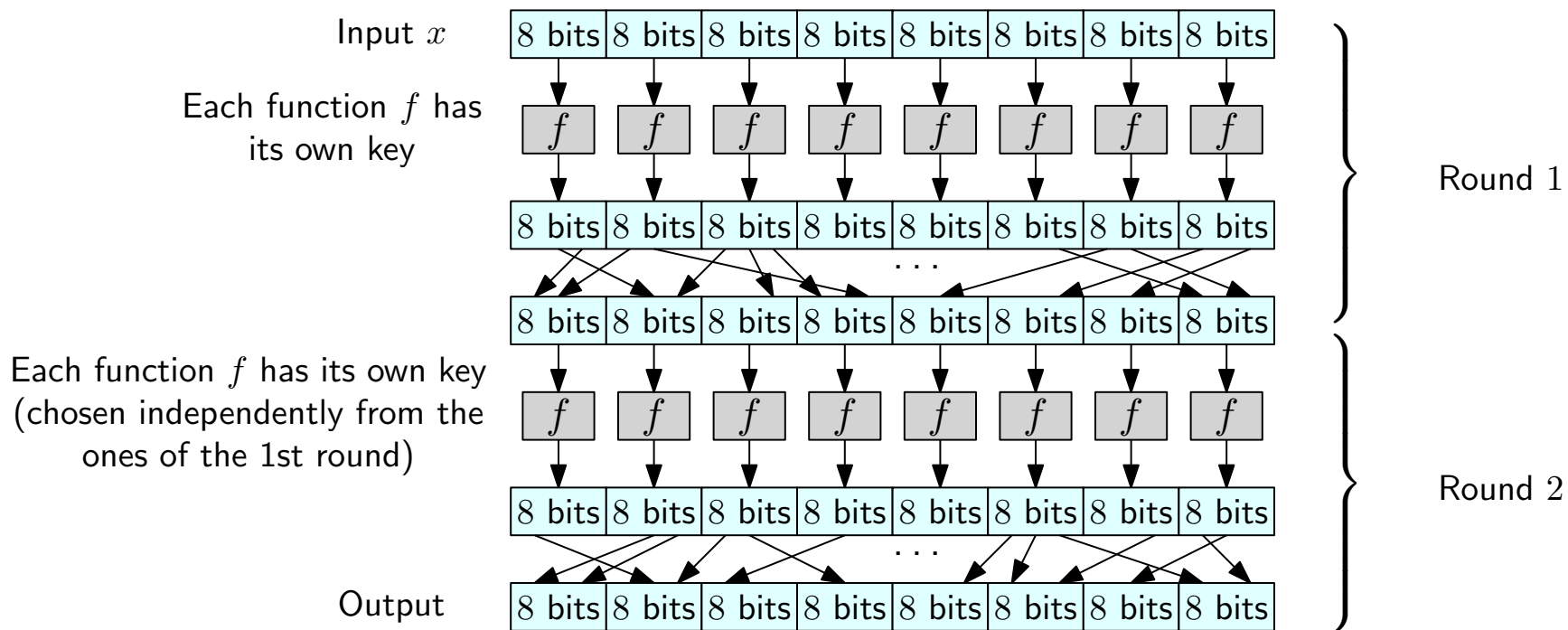


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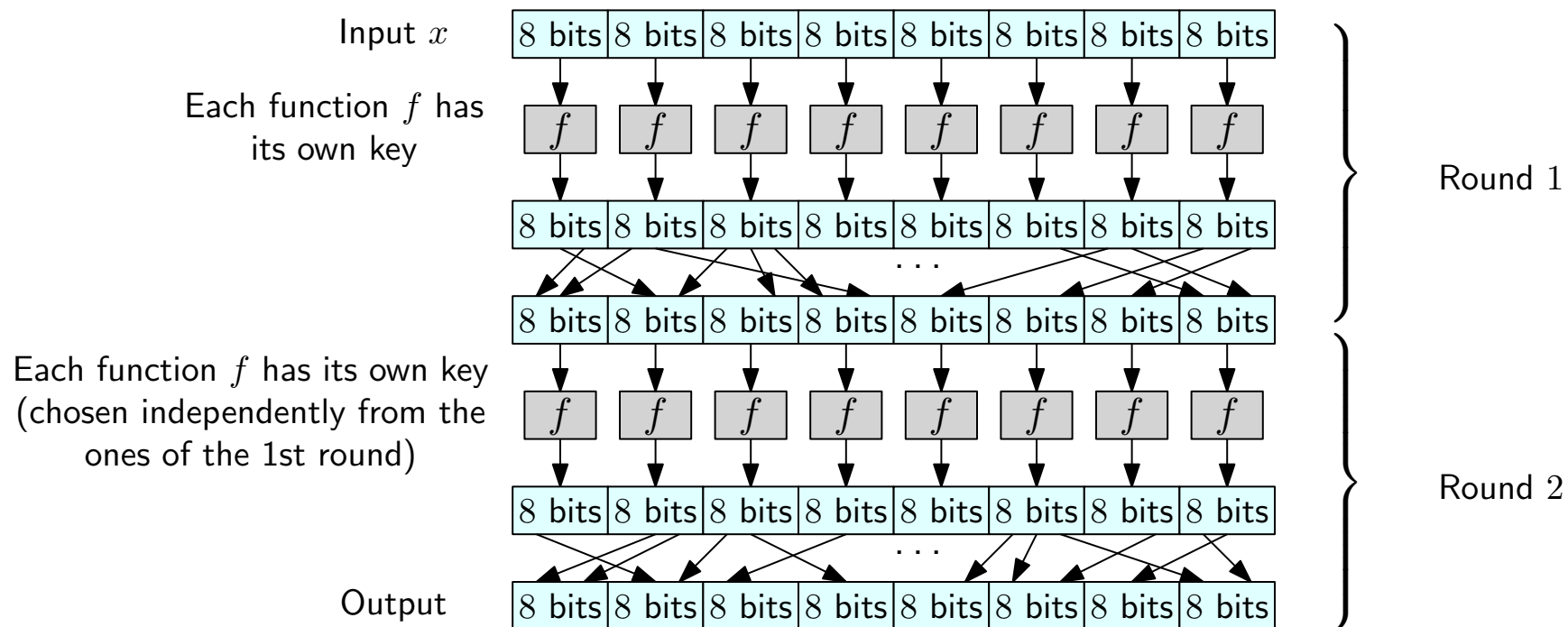
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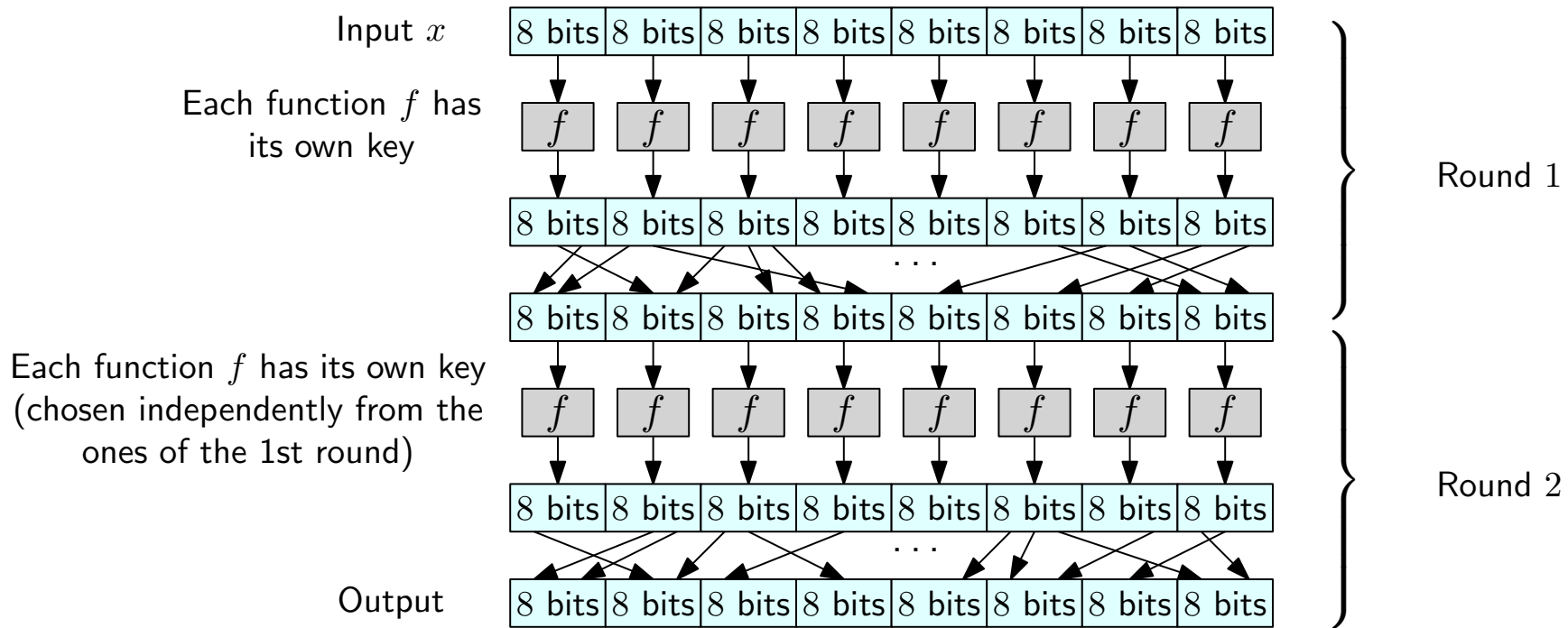


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Observation: the overall permutation remains invertible regardless of the number of rounds

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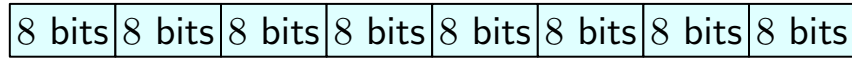
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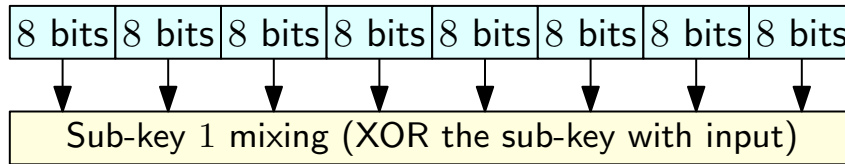
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- The round keys are derived from the master key according to a **key schedule**

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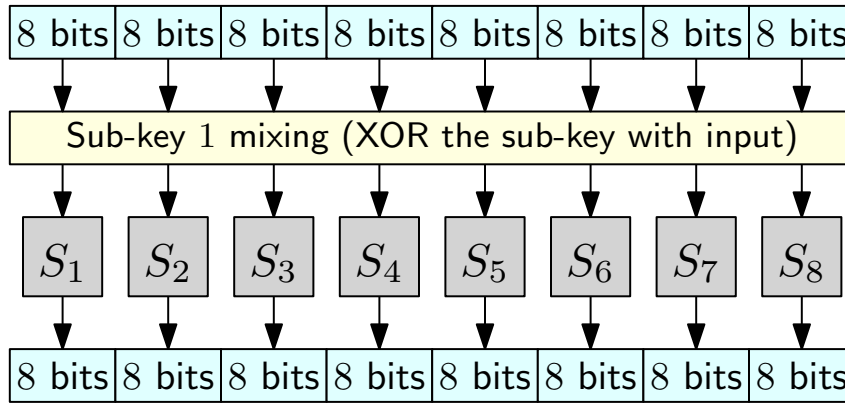
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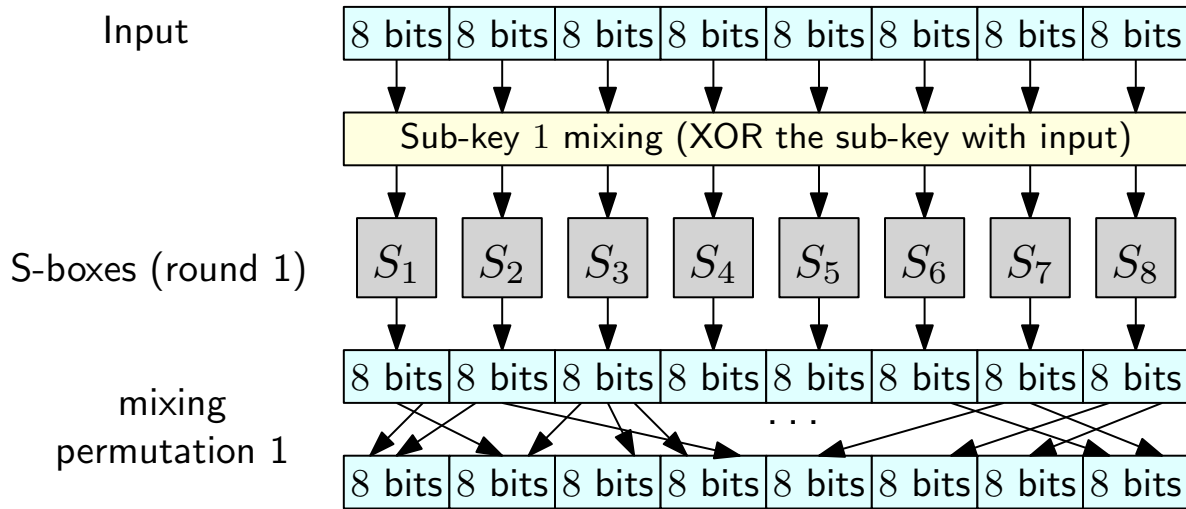
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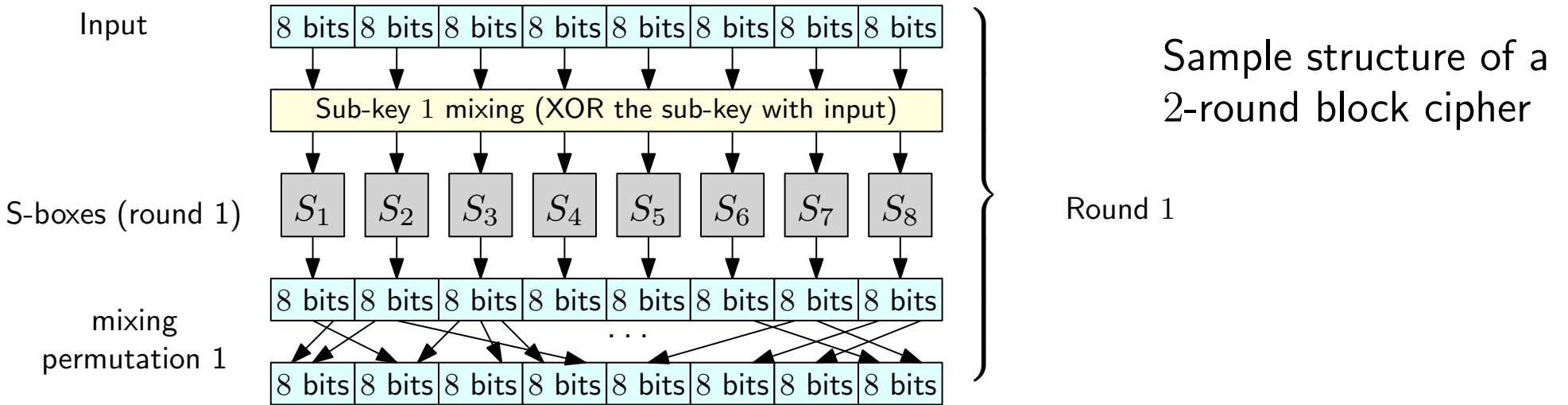


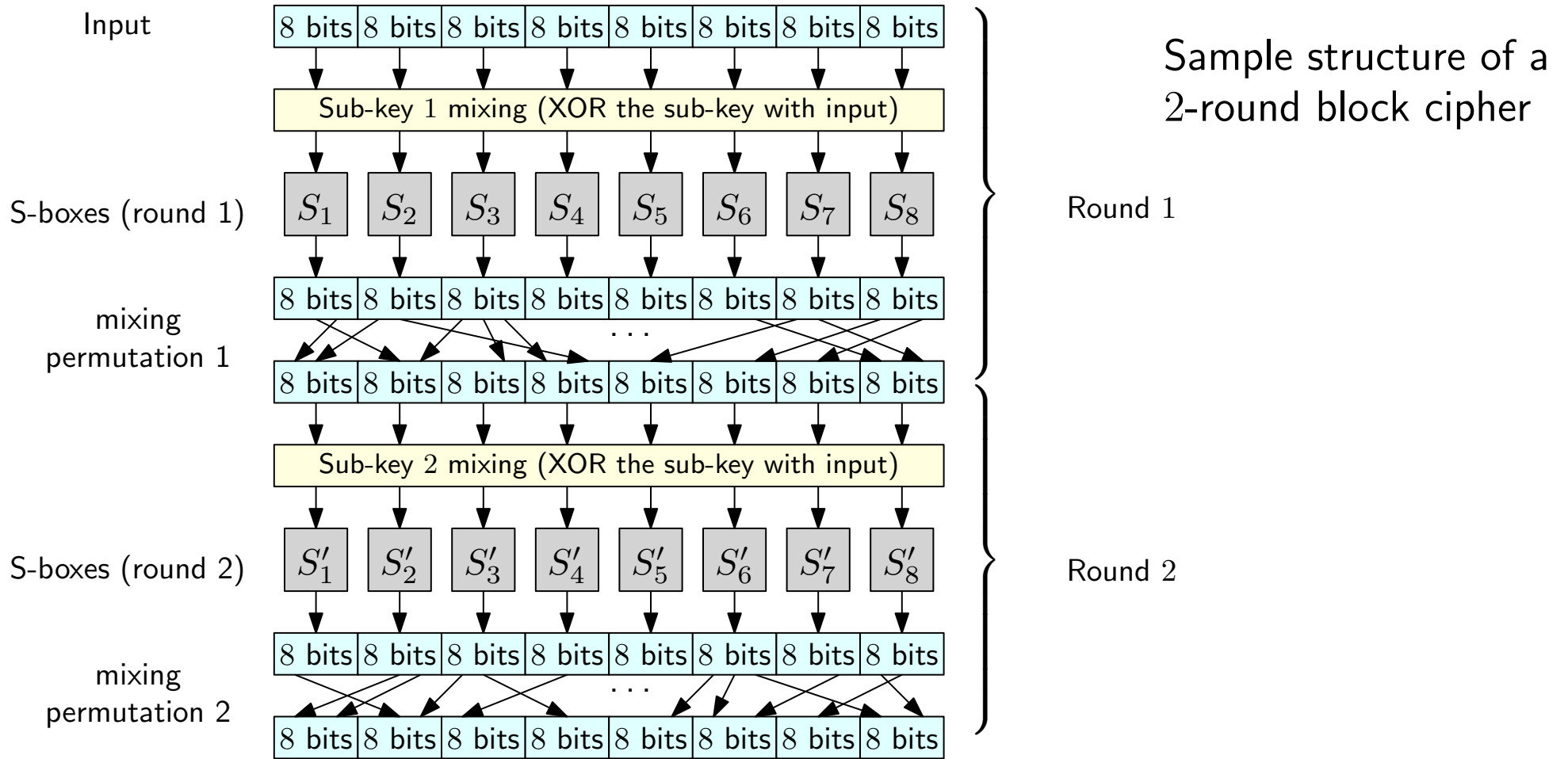
S-boxes (round 1)

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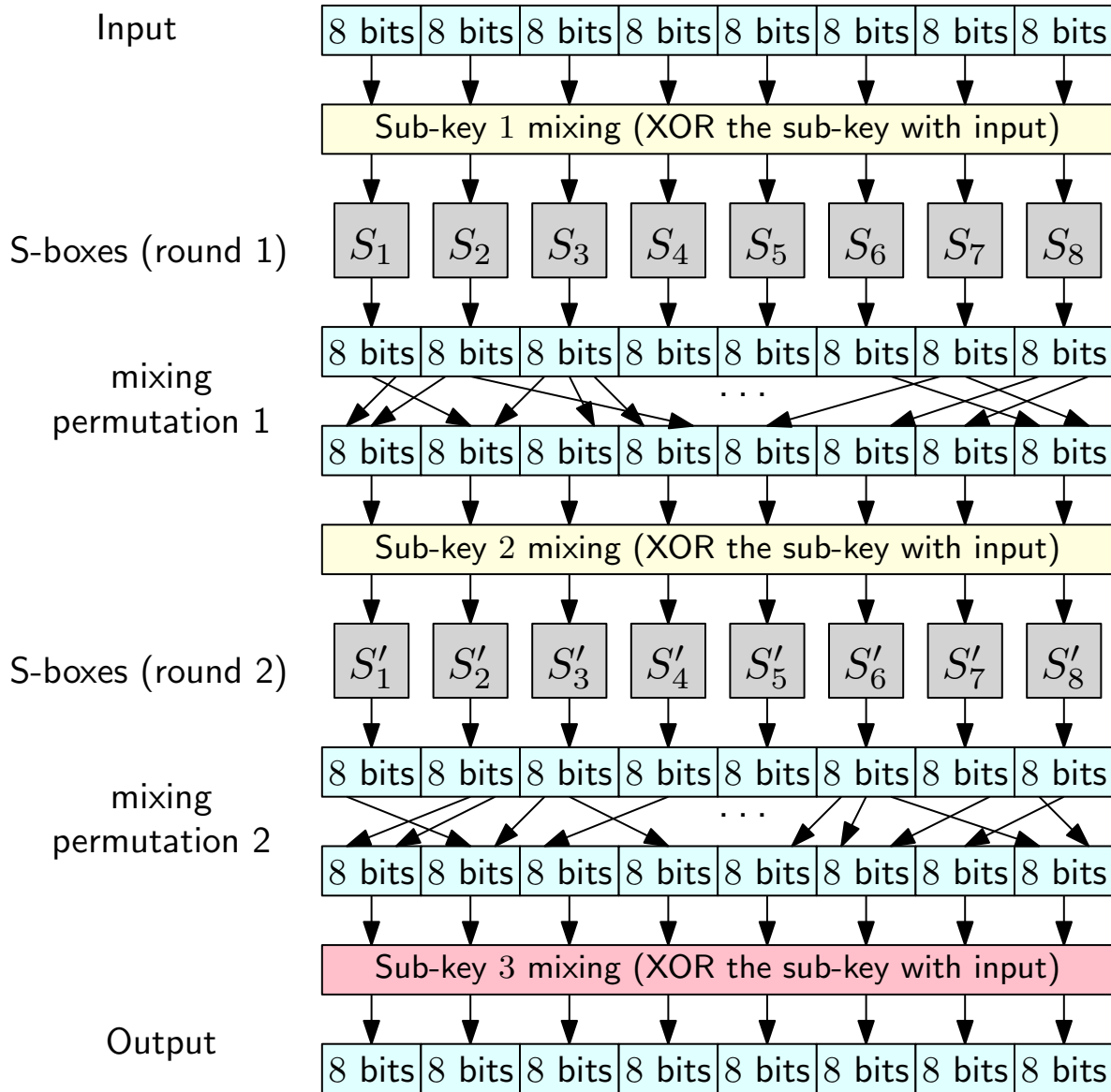


Sample structure of a 2-round block cipher





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Round 1

Round 2

After the last round, we perform one final key mixing step (recall that it is useless to apply a mixing permutation as the last step)

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We want to design the S-boxes and the mixing permutation to achieve the **avalanche effect**

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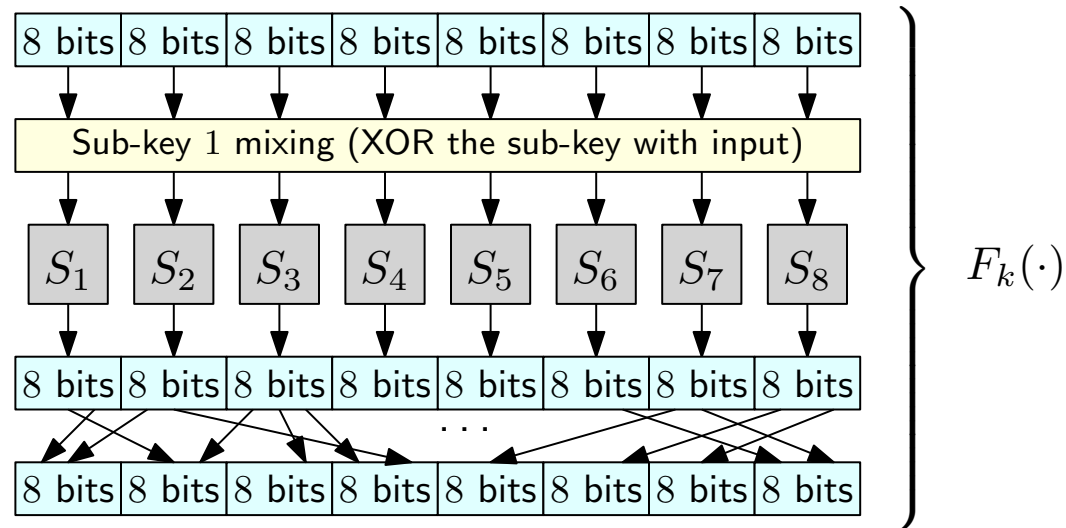
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- This adds diffusion



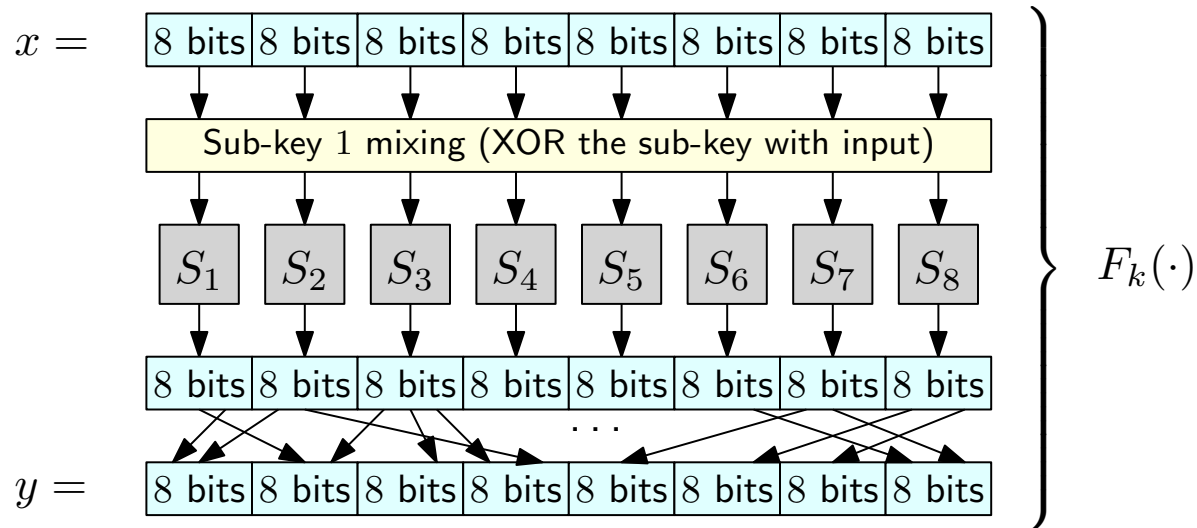
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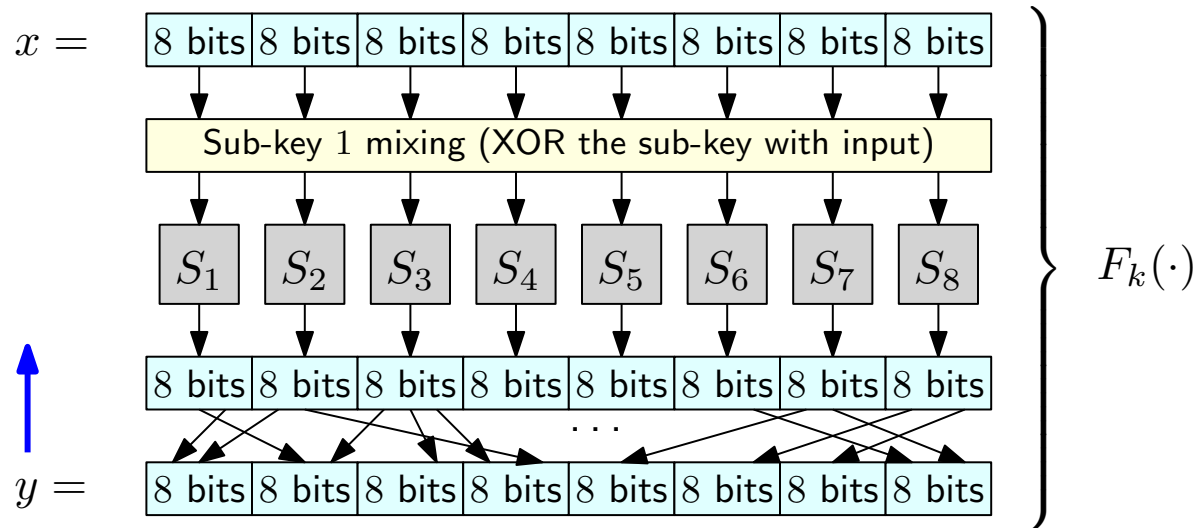


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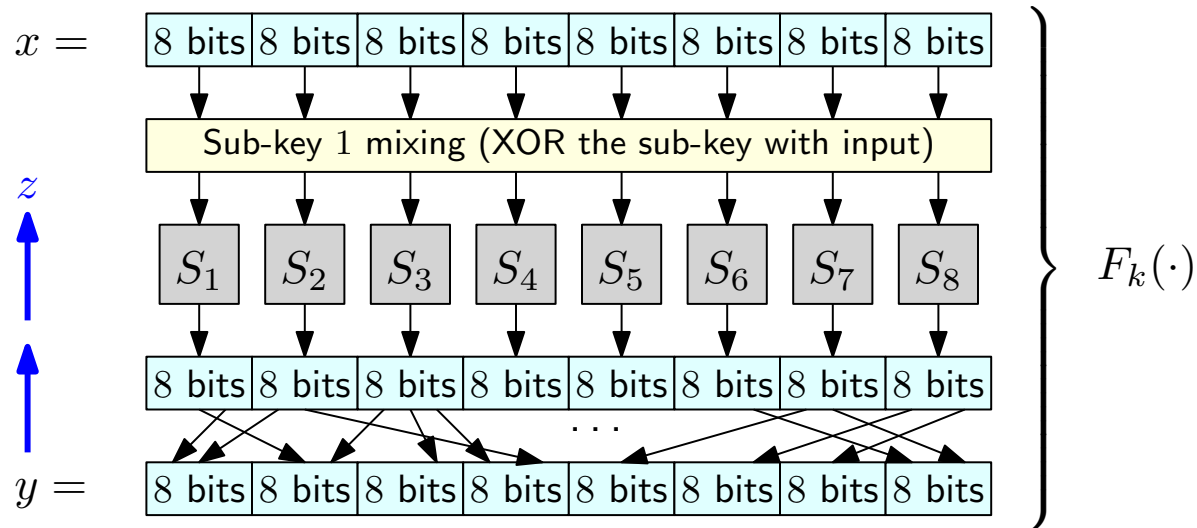
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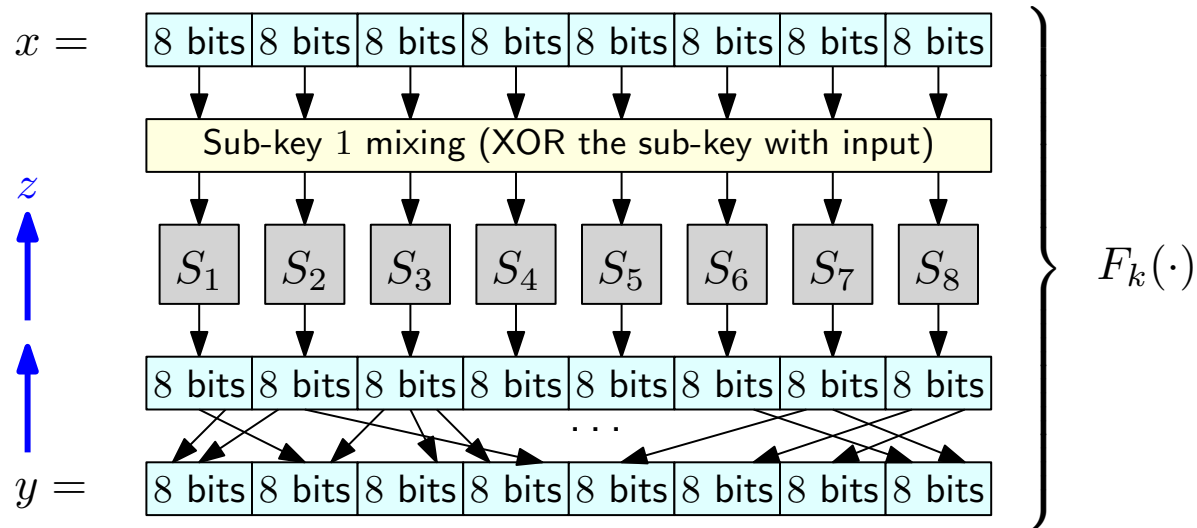
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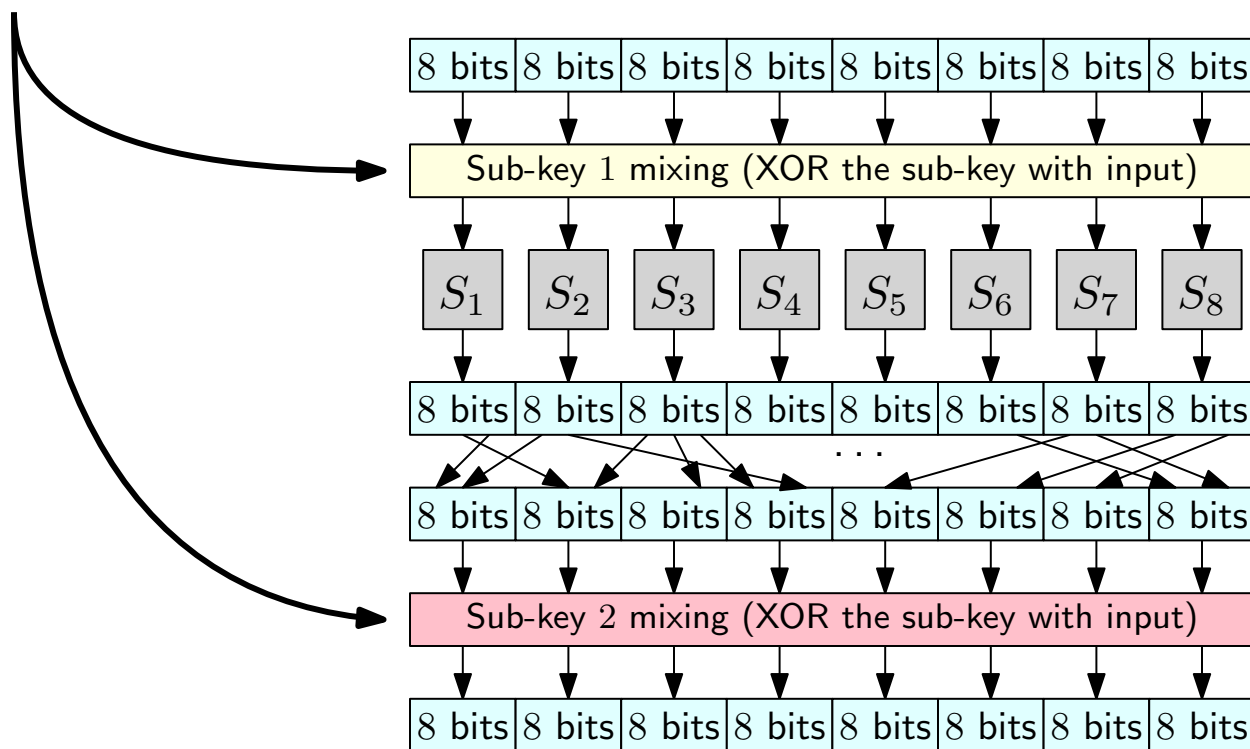
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- The (round and master) key is $k = z \oplus x = (x \oplus k) \oplus x$

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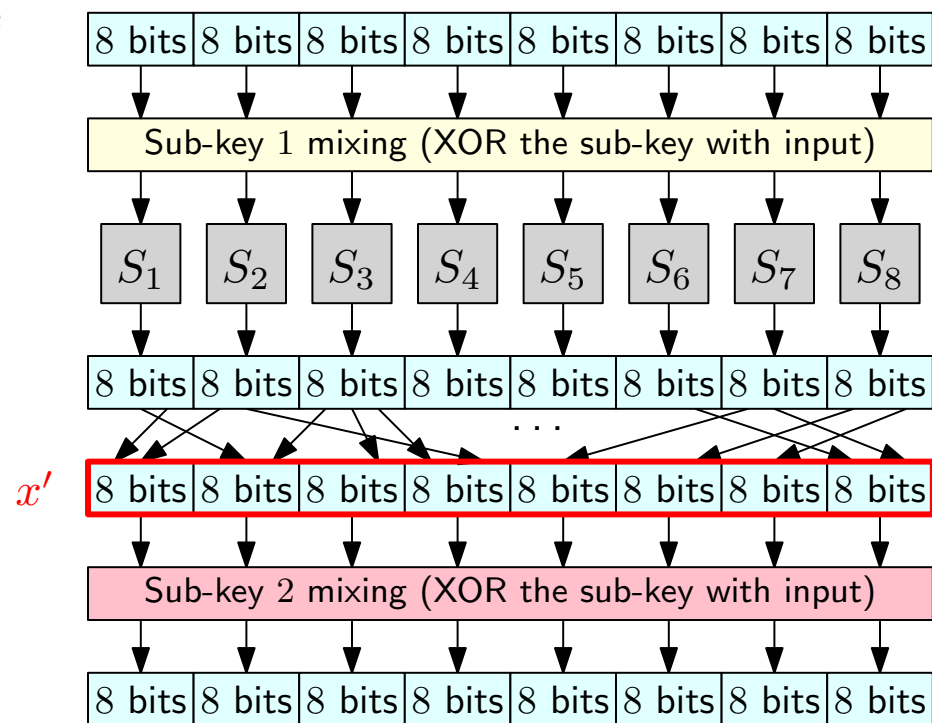
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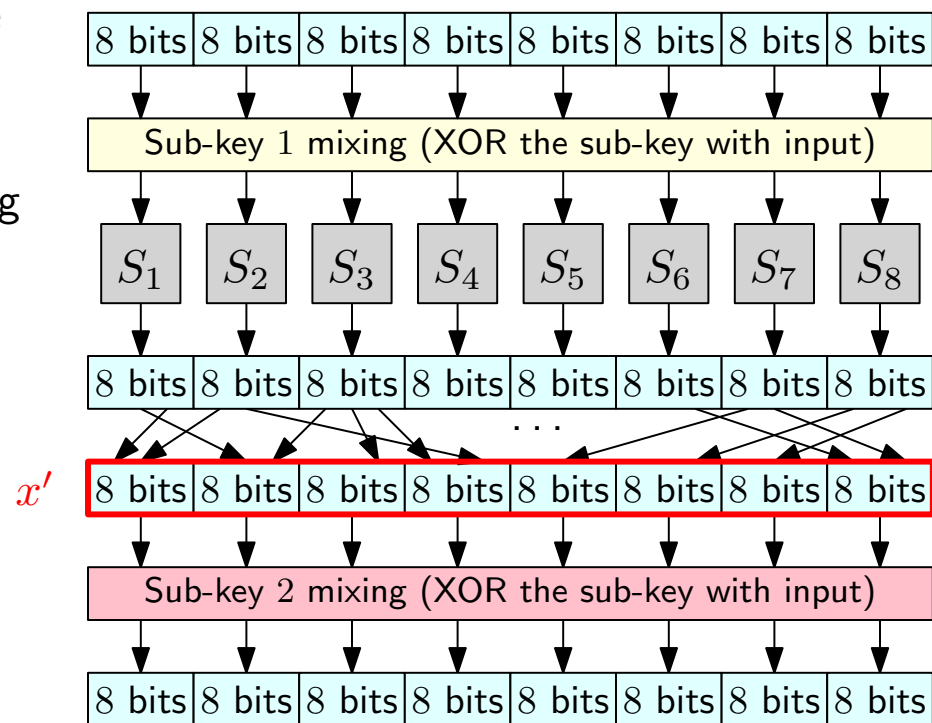
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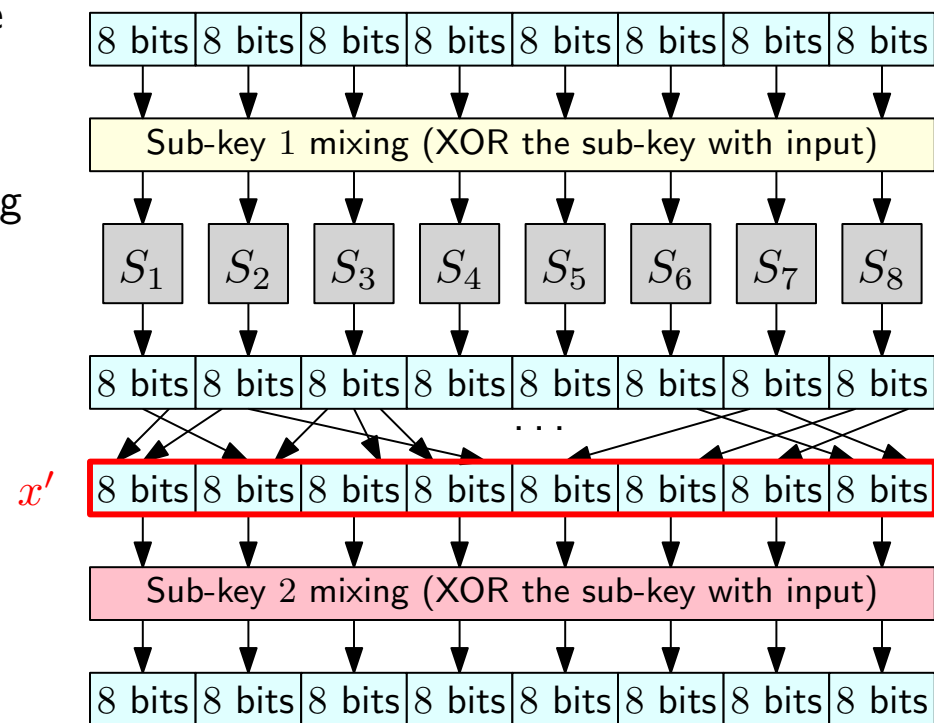
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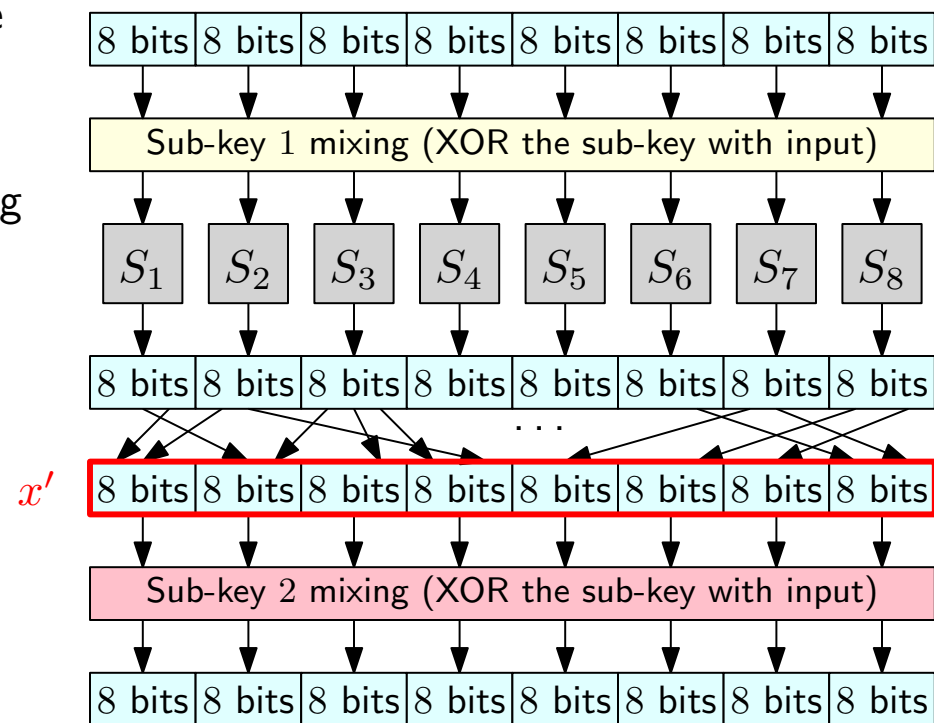
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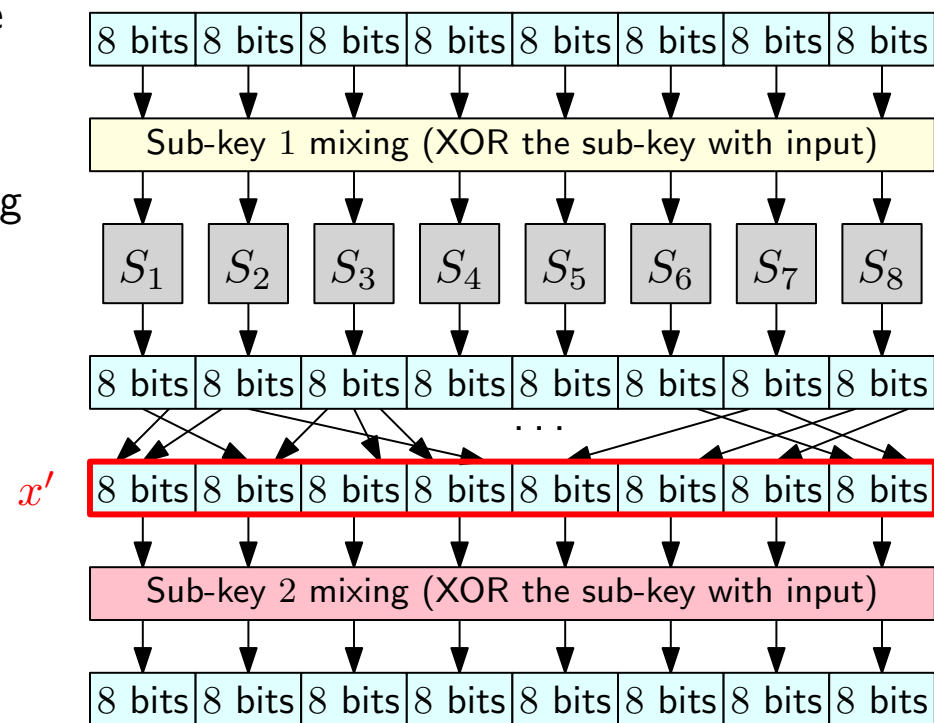
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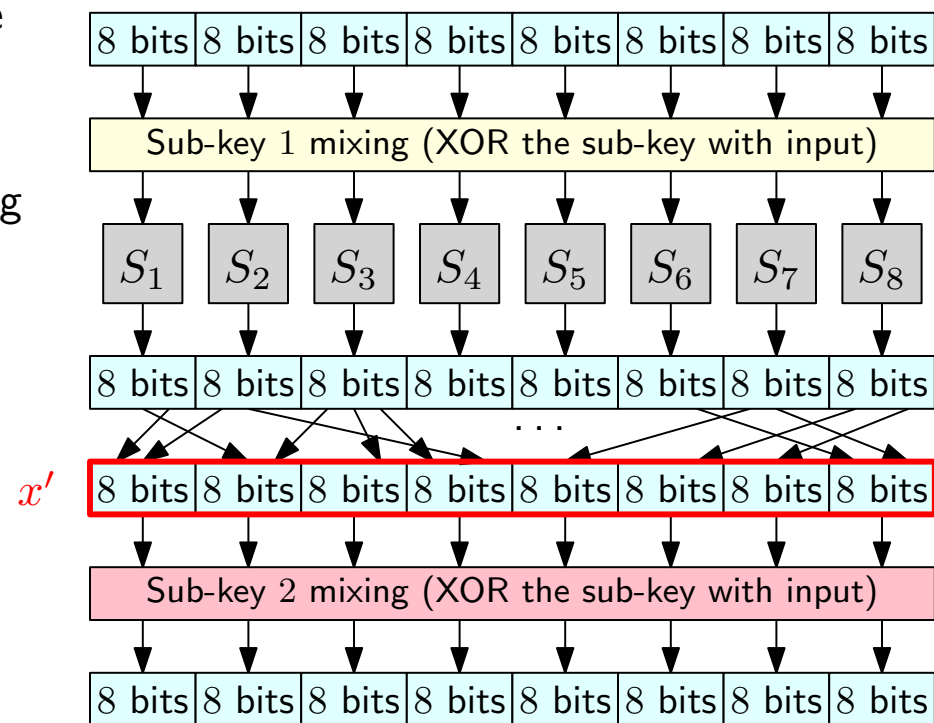
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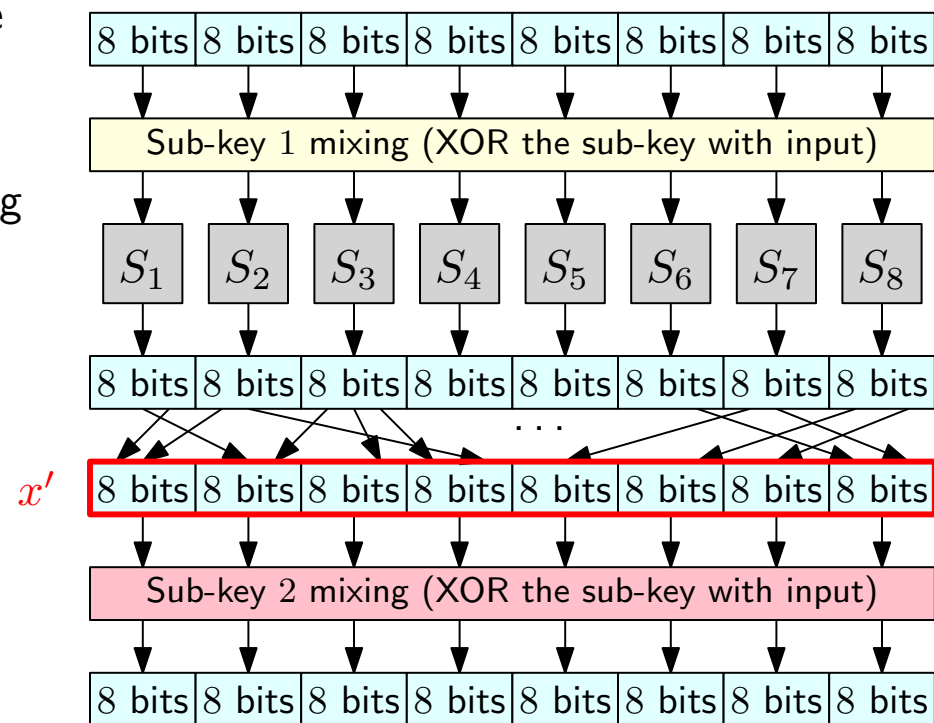
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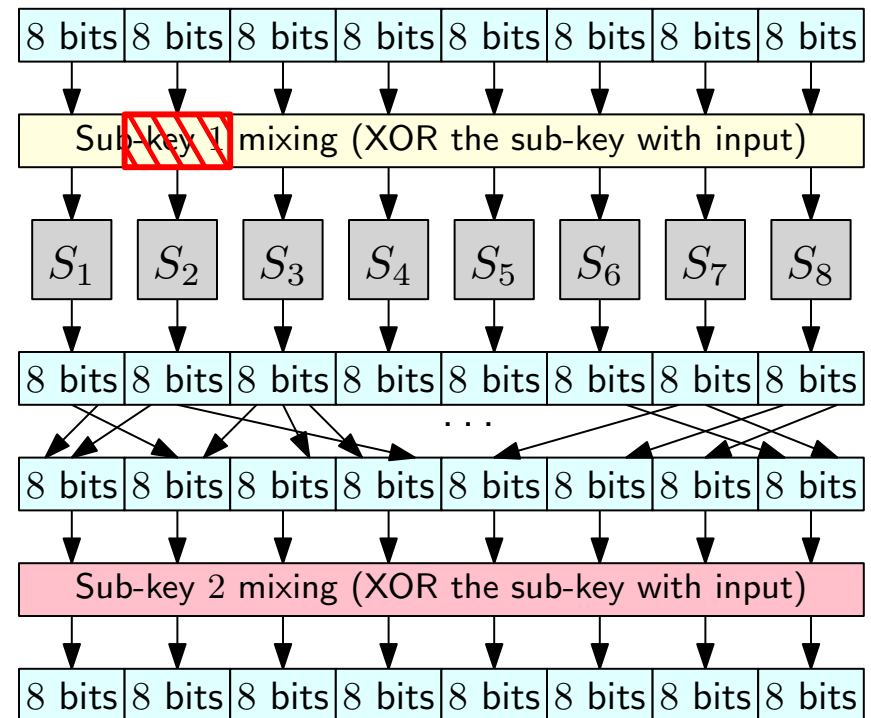
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Indeed... we can design a better attack!

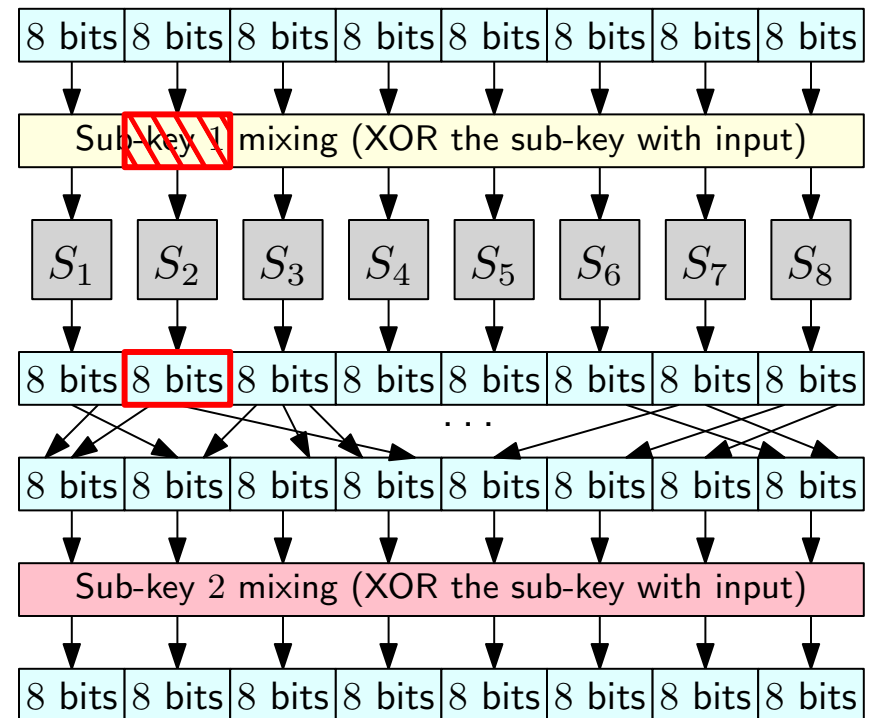
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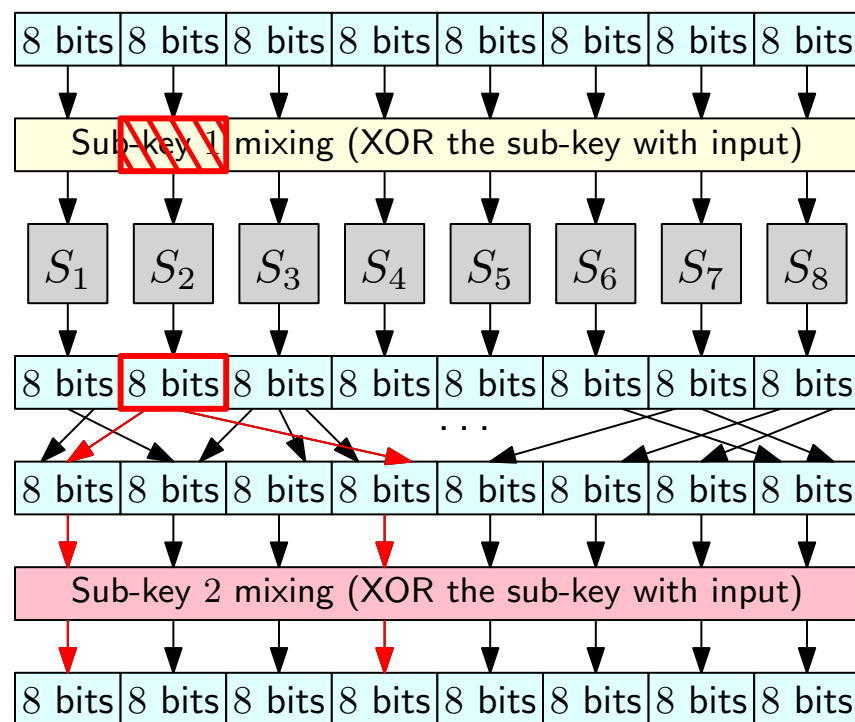
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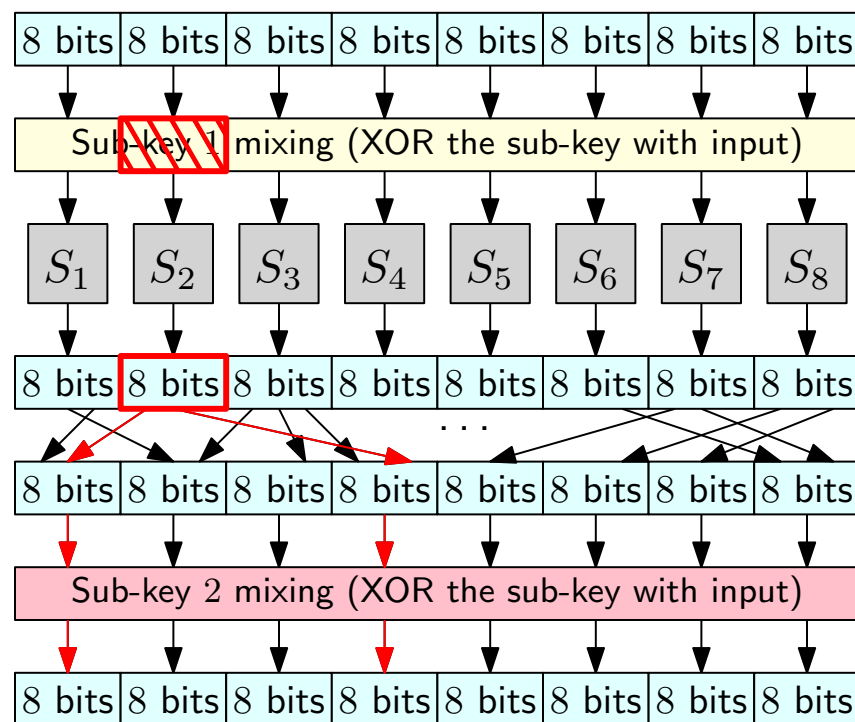
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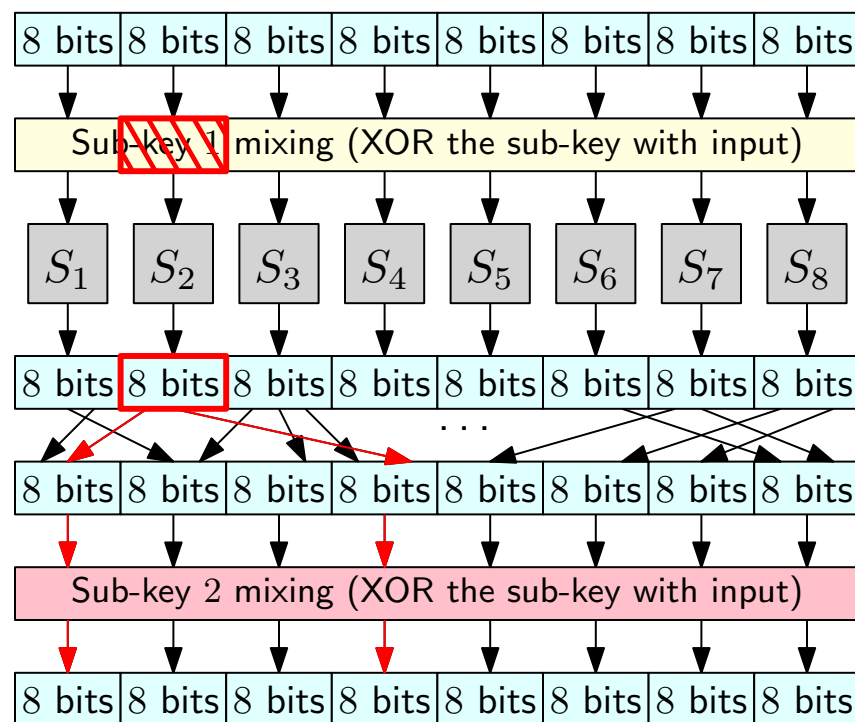
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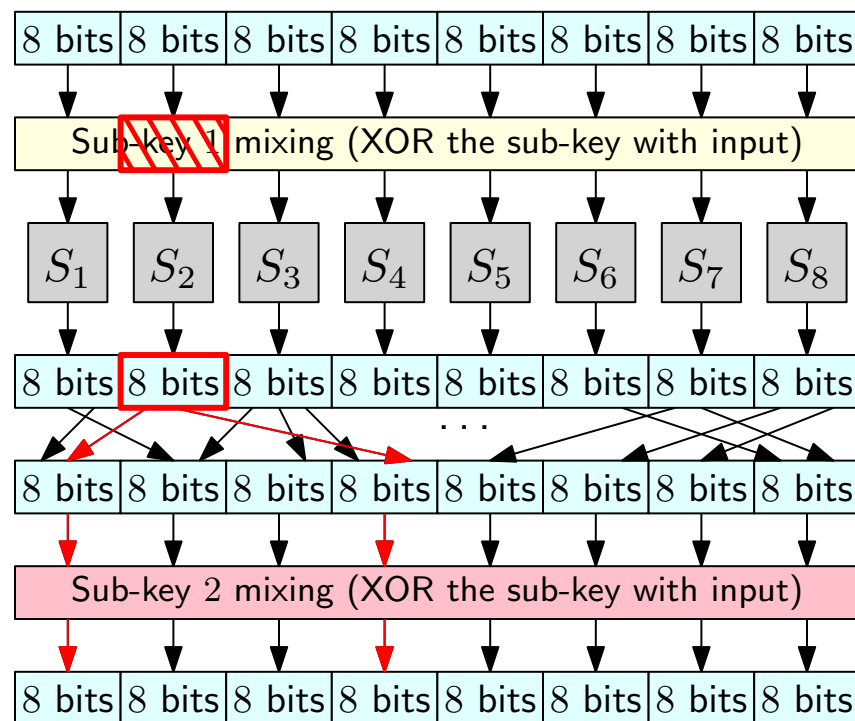
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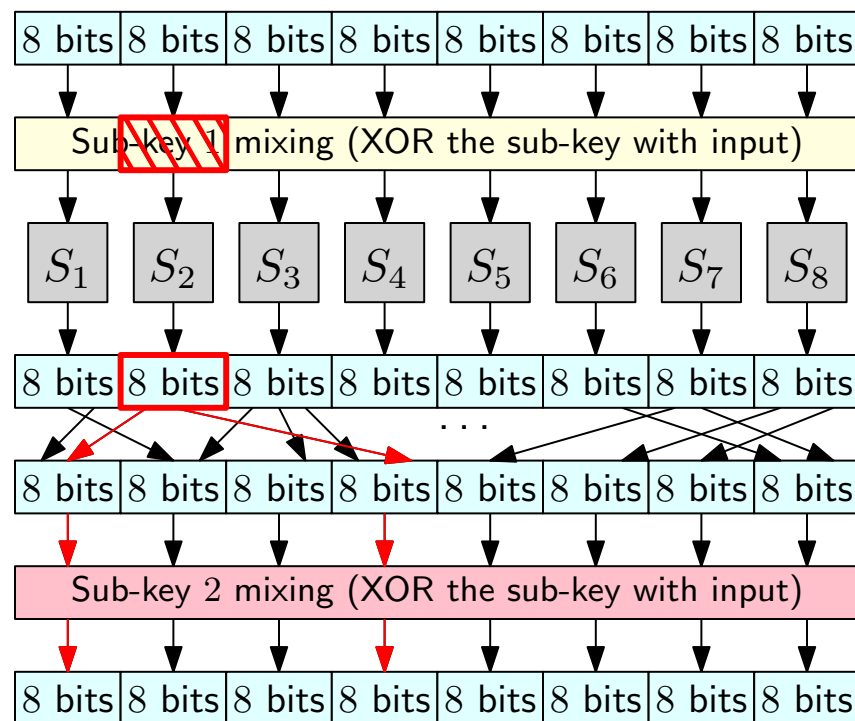
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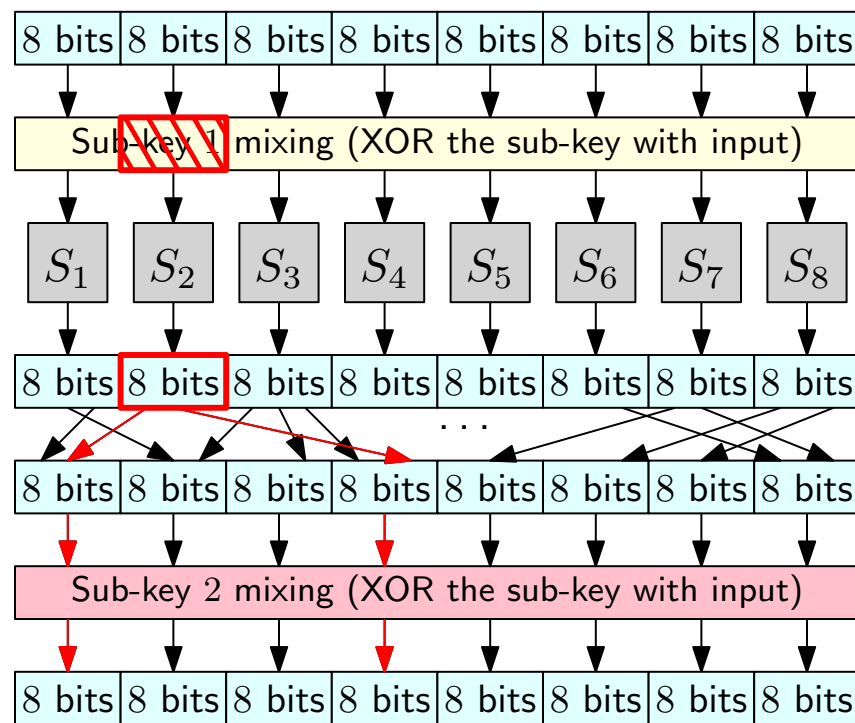
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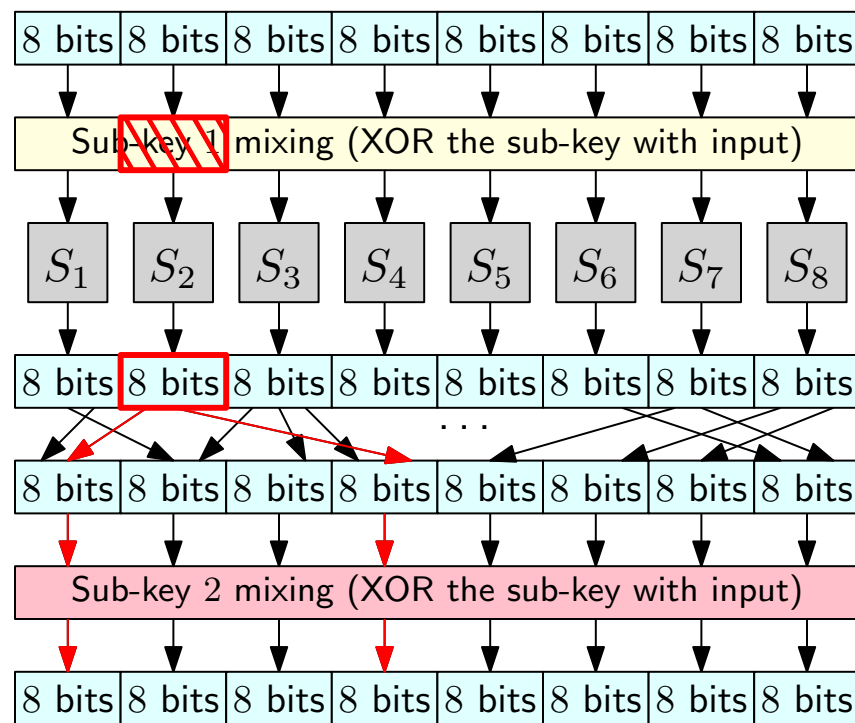


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$$\text{In the example: } \approx 8 \cdot 2^8 = 2^{11}$$

(instead of 2^{64} of the previous attack or 2^{128} of a naive bruteforce)

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It's common to see results of the form:

“A reduced version of [block cipher] using X instead of Y rounds has been broken”

Designing Block Ciphers

- To design a block cipher, we want the computed function to be “indistinguishable” from a uniform permutation over $\{0, 1\}^\ell$
- If x and x' differ, even just by one bit, the outputs of $F_k(x)$ and $F_k(x')$ should look unrelated (except for $F_k(x) \neq F_k(x')$)
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- To keep notation simple, define $f_i : \{0, 1\}^{\ell/2} \rightarrow \{0, 1\}^{\ell/2}$ as $f_i(x) = \hat{f}(k_i, x)$, where k_i is the i -th sub-key

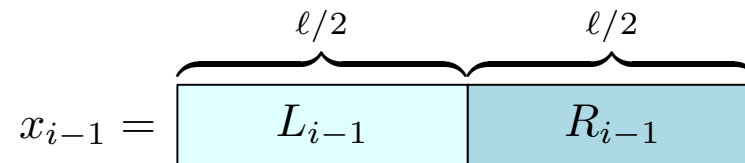
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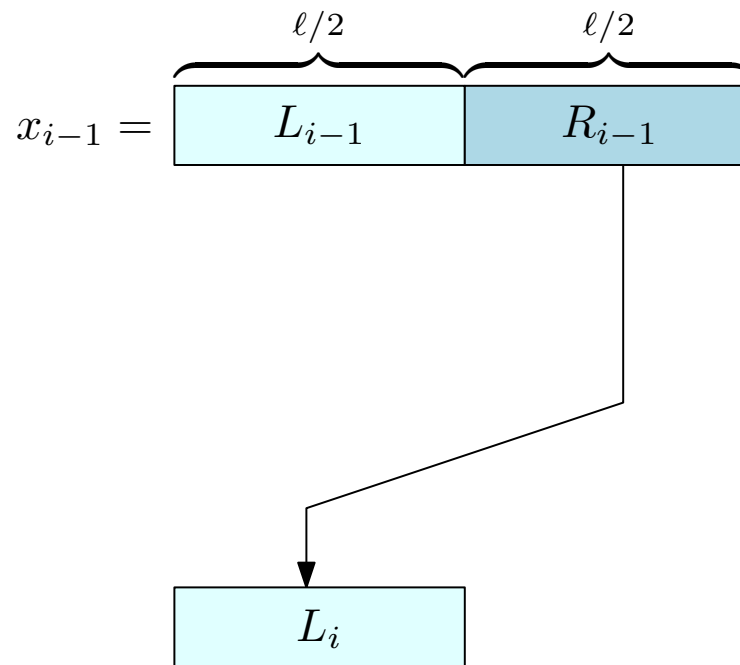


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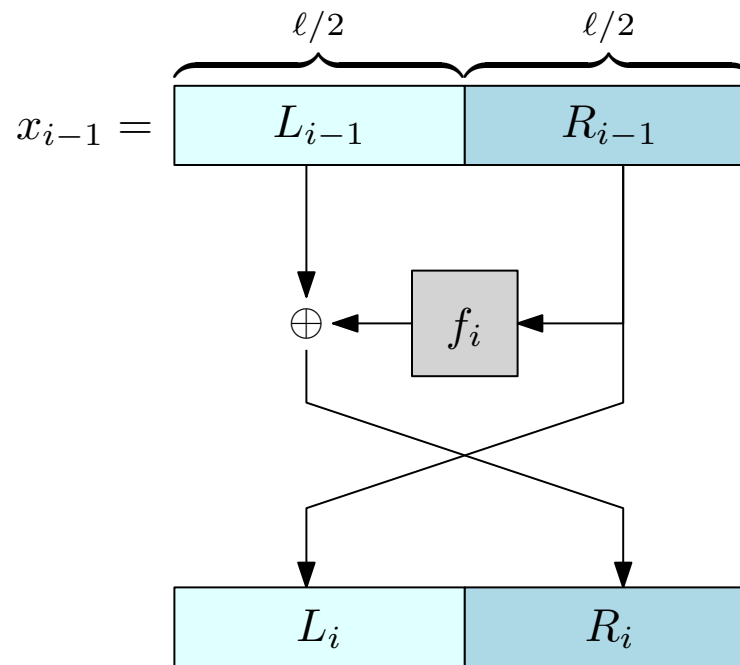
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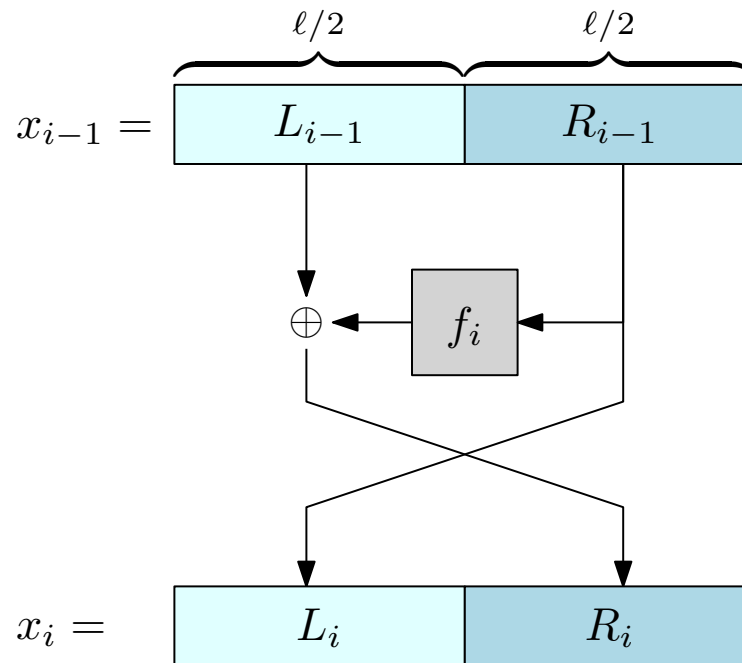
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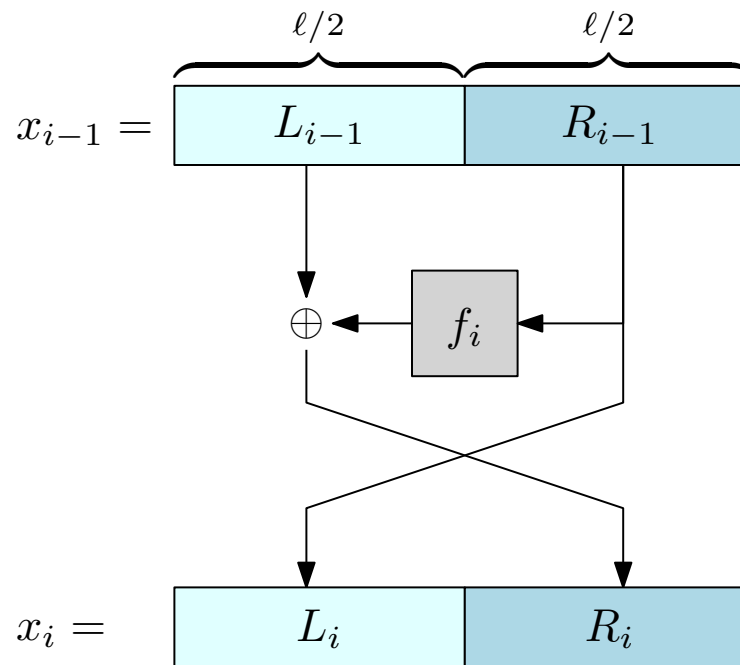
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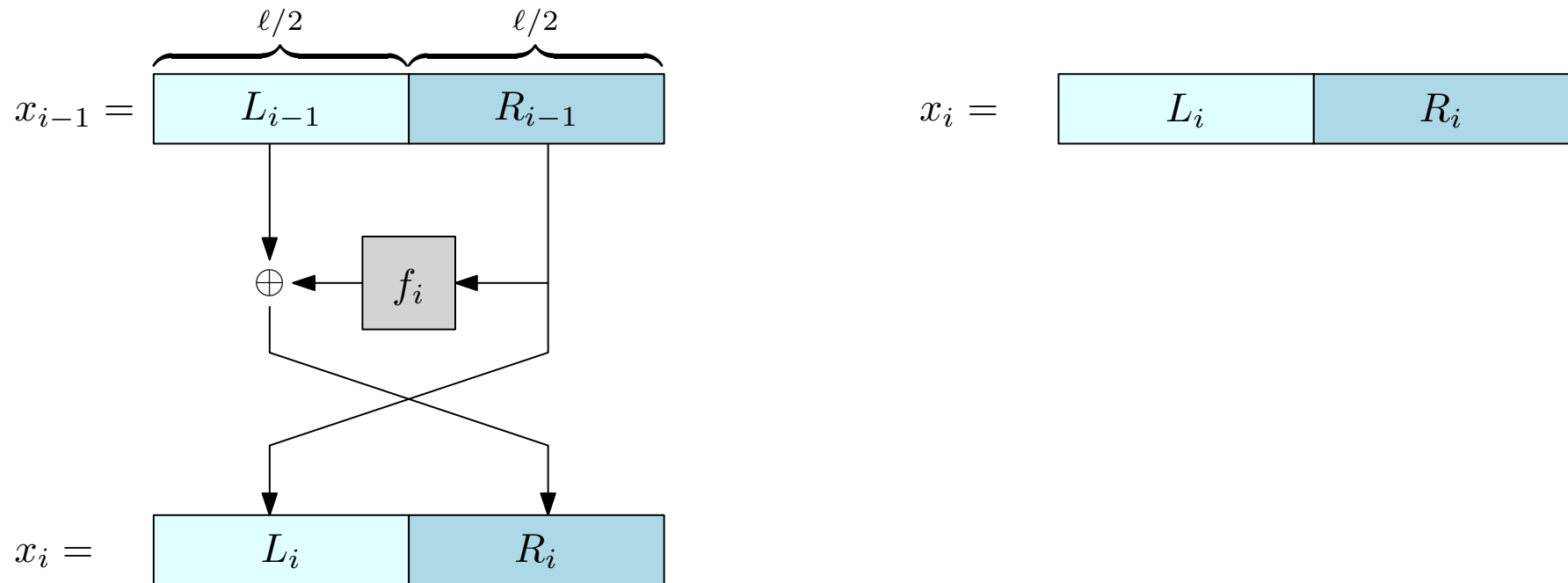
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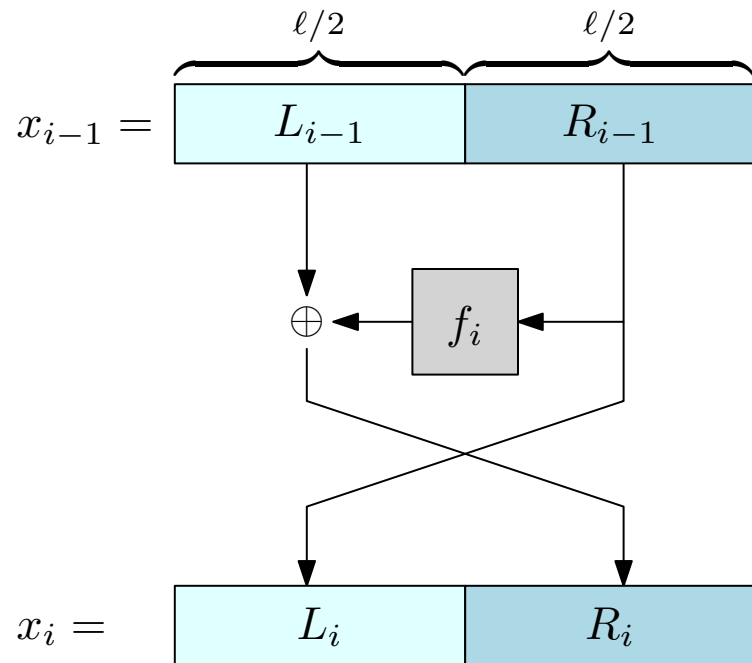
Is a Feistel Network round invertible? (How?)

Inverting a Round of Feistel Network

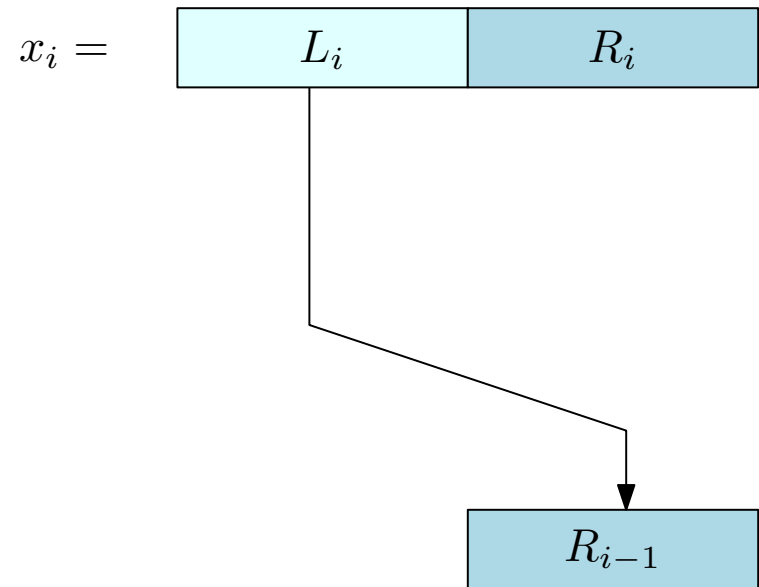


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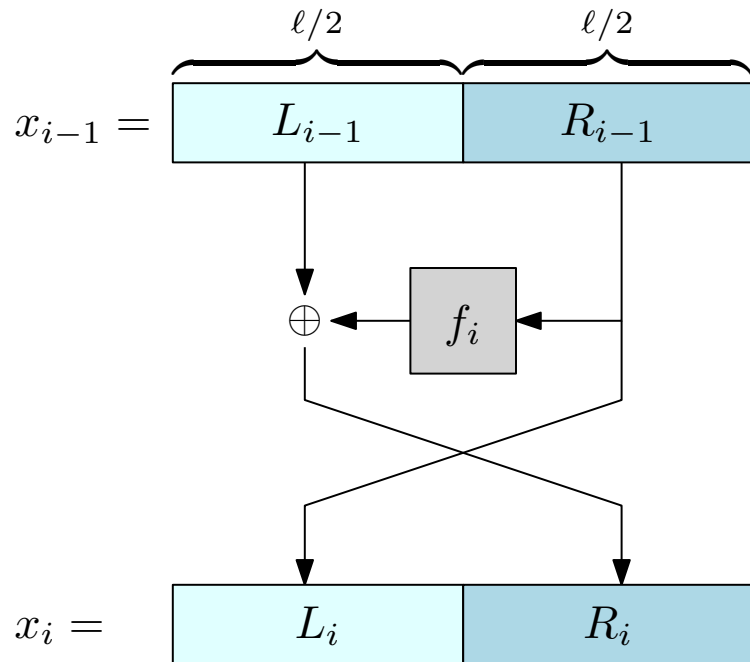


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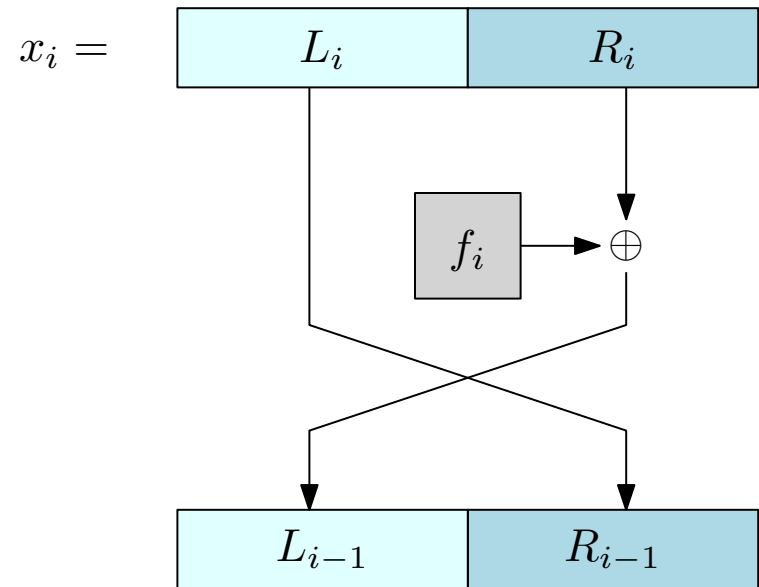


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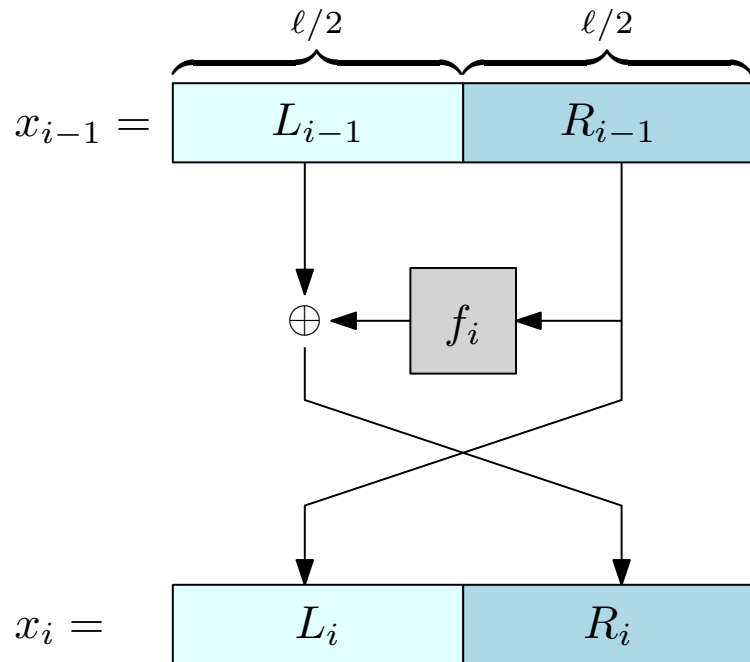


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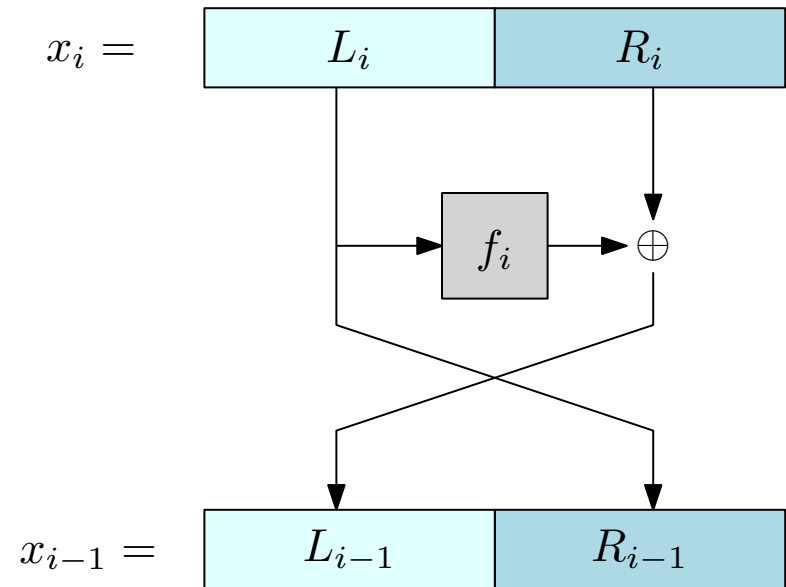


- $R_{i-1} = L_i$
- $L_{i-1} = R_i \oplus f_i(R_{i-1})$

Inverting a Round of Feistel Network

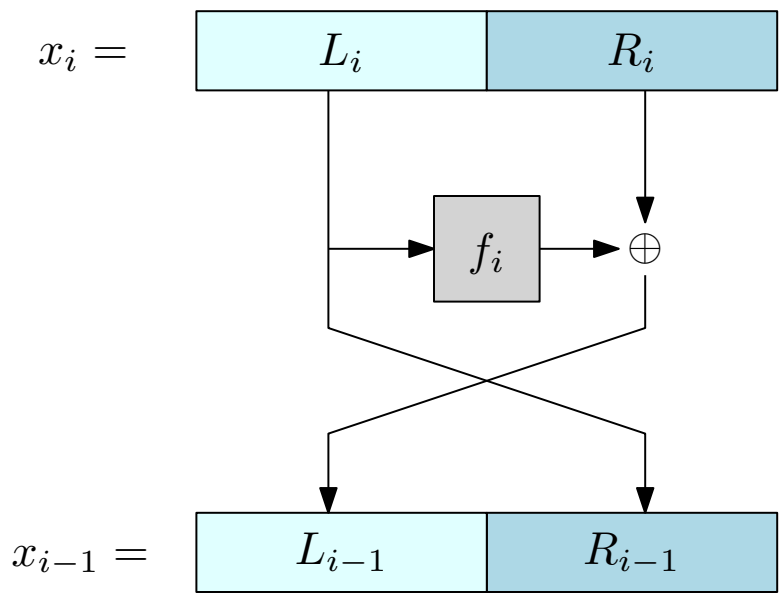
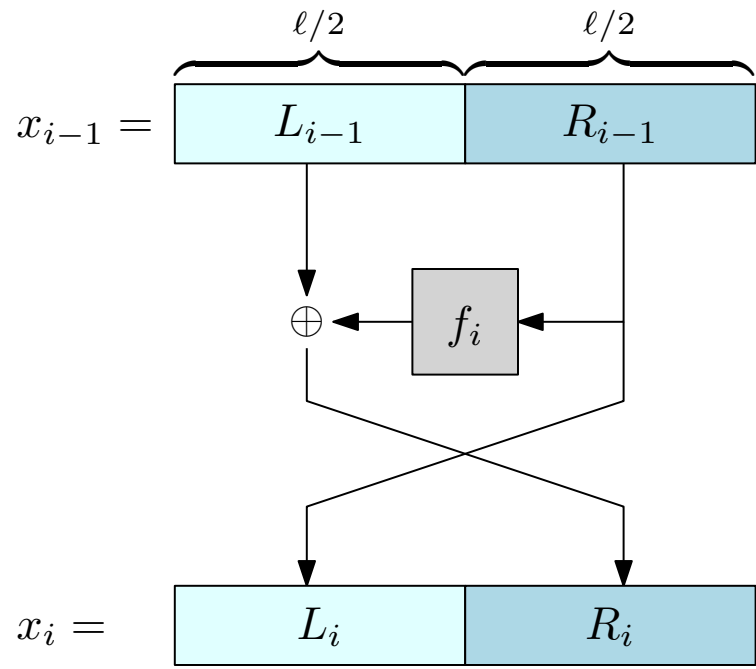


- $L_i = R_{i-1}$
- $R_i = L_{i-1} \oplus f_i(R_{i-1})$



- $R_{i-1} = L_i$
- $L_{i-1} = R_i \oplus f_i(R_{i-1}) = R_i \oplus f_i(L_i)$

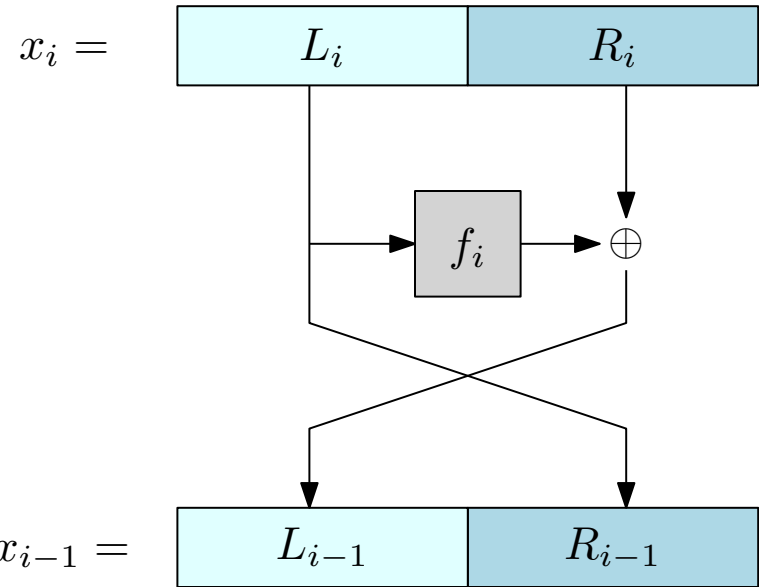
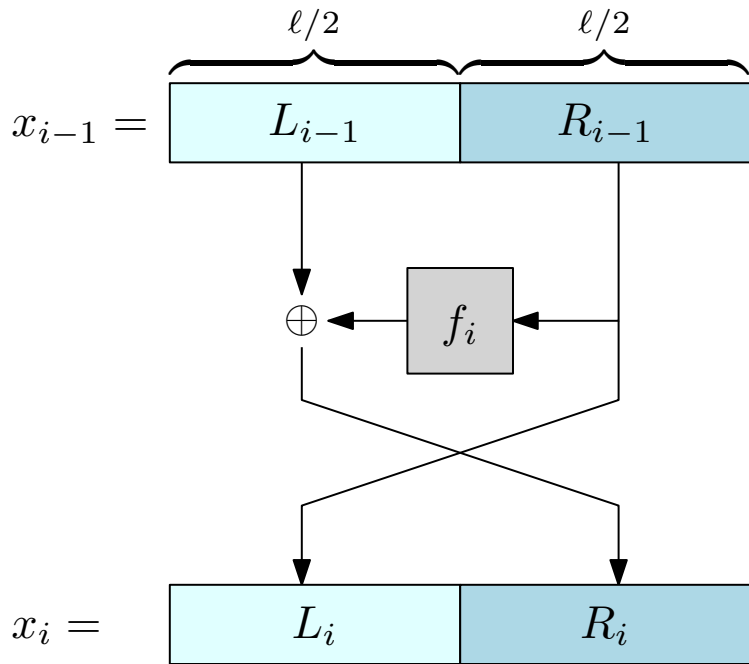
Inverting a Round of Feistel Network



Let F be a keyed function defined by a Feistel network. Then regardless of the key schedule, the round functions \hat{f}_i , and the number of rounds, F_k is a permutation for any k .

Inverting a Round of Feistel Network

F^{-1} is the same as F once the “left” and “right” sides are swapped!
How to invert multiple rounds?



Let F be a keyed function defined by a Feistel network. Then regardless of the key schedule, the round functions \hat{f}_i , and the number of rounds, F_k is a permutation for any k .

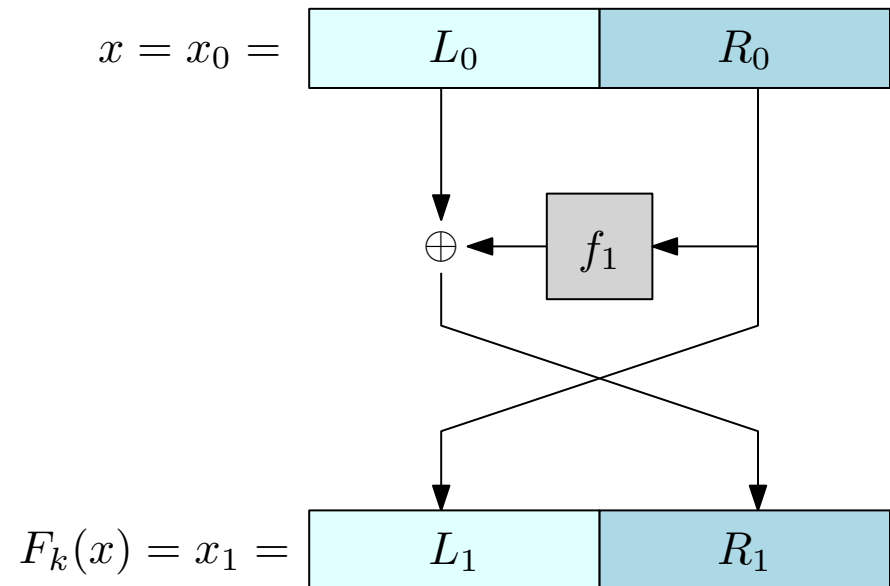
Security of 1-Round Feistel Networks

$$F_k(L_0 \parallel R_0) = L_1 \parallel R_1$$

$$L_1 = R_0$$

$$R_1 = L_0 \oplus f_1(R_0)$$

Is this a Pseudorandom permutation?



Security of 1-Round Feistel Networks

$$F_k(L_0 \parallel R_0) = L_1 \parallel R_1$$

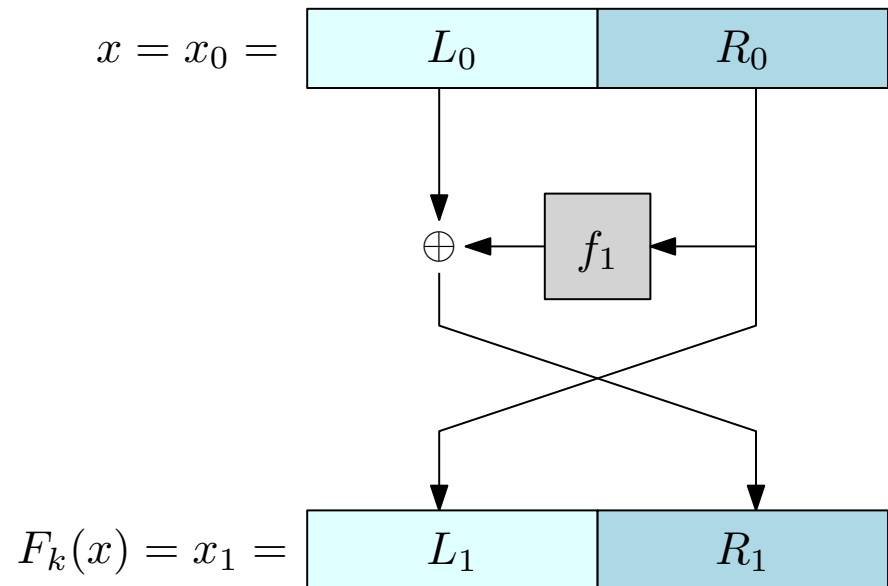
$$L_1 = R_0$$

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Is this a Pseudorandom permutation?

- No! $F_k(x)$ can be easily distinguished from a random permutation

How?



Security of 1-Round Feistel Networks

$$F_k(L_0 \parallel R_0) = L_1 \parallel R_1$$

$$L_1 = R_0$$

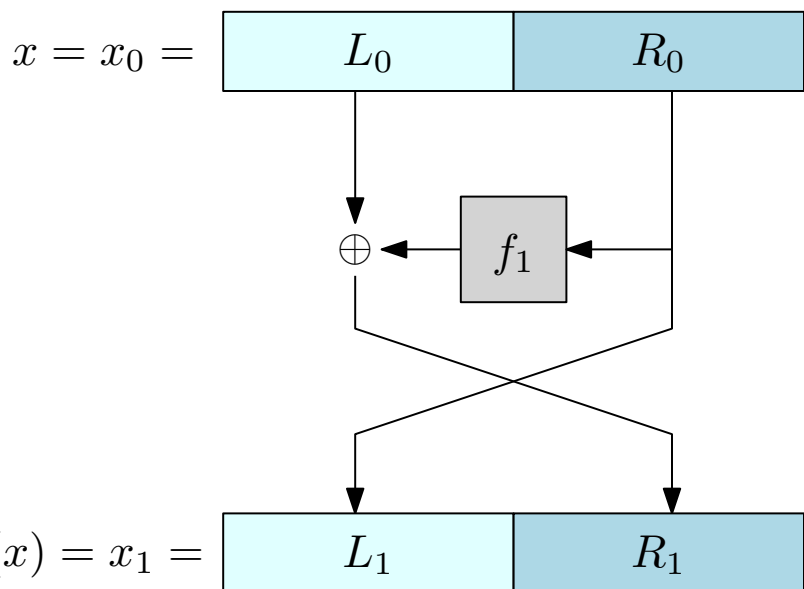
$$R_1 = L_0 \oplus f_1(R_0)$$

Is this a Pseudorandom permutation?

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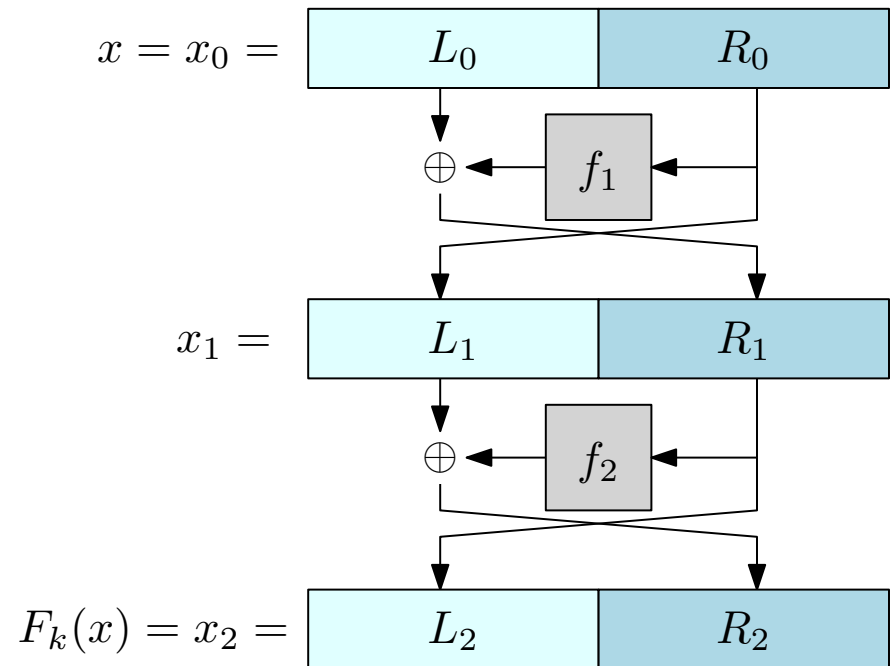
How?

- The adversary can simply query $x = 0^\ell$ and check whether the left $\ell/2$ bits of $F_k(x)$ are all 0 (or use any other string x and check whether the left half of $F_k(x)$ is equal to the right half of x)



Security of 2-Round Feistel Networks

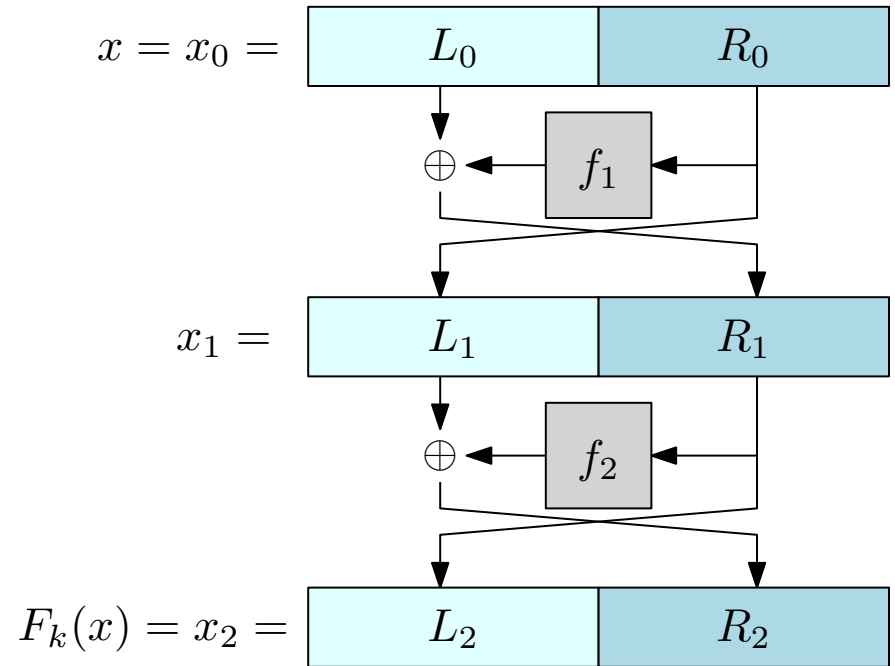
$$F_k(L_0 \parallel R_0) = L_2 \parallel R_2$$



Security of 2-Round Feistel Networks

$$F_k(L_0 \parallel R_0) = L_2 \parallel R_2$$

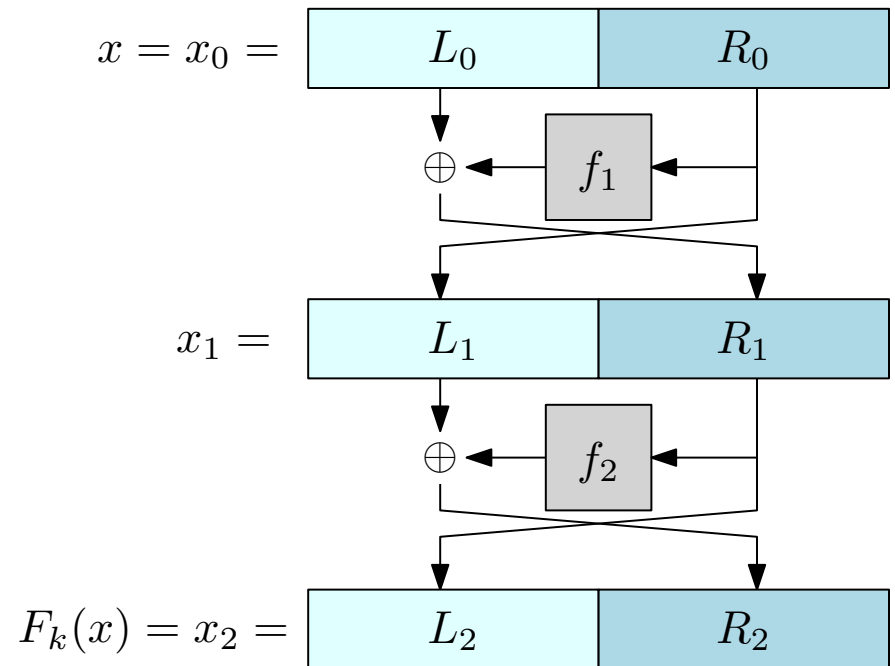
$$L_2 = R_1$$



Security of 2-Round Feistel Networks

$$F_k(L_0 \| R_0) = L_2 \| R_2$$

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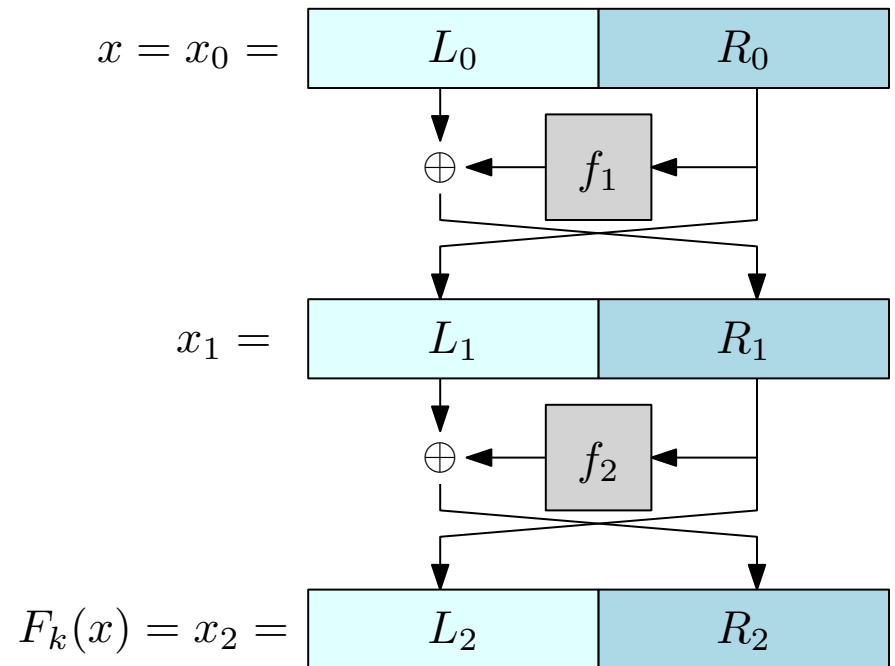


Security of 2-Round Feistel Networks

$$F_k(L_0 \| R_0) = L_2 \| R_2$$

$$L_2 = R_1 = L_0 \oplus f_1(R_0)$$

$$R_2 = L_1 \oplus f_2(R_1)$$

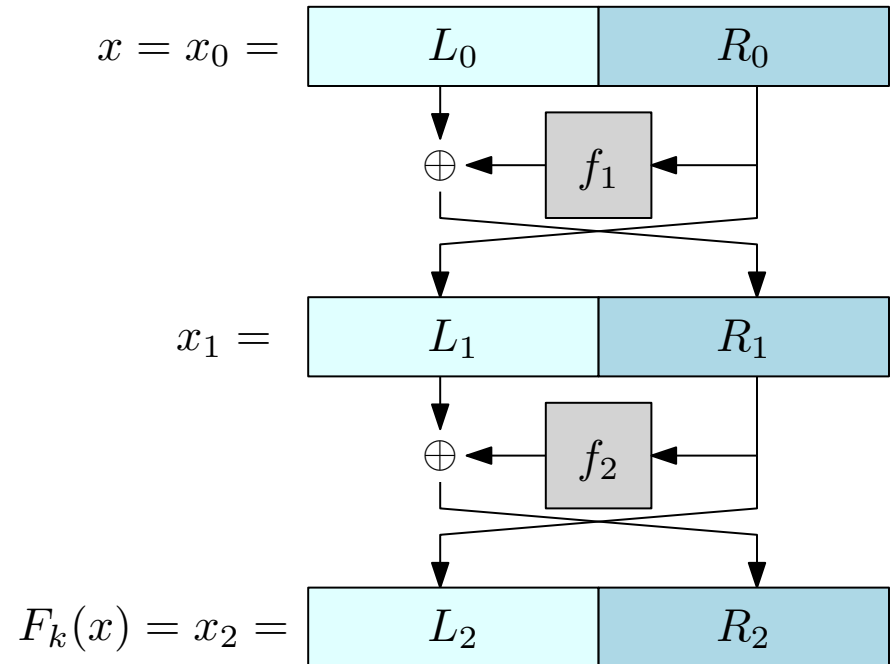


Security of 2-Round Feistel Networks

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$$L_2 = R_1 = L_0 \oplus f_1(R_0)$$

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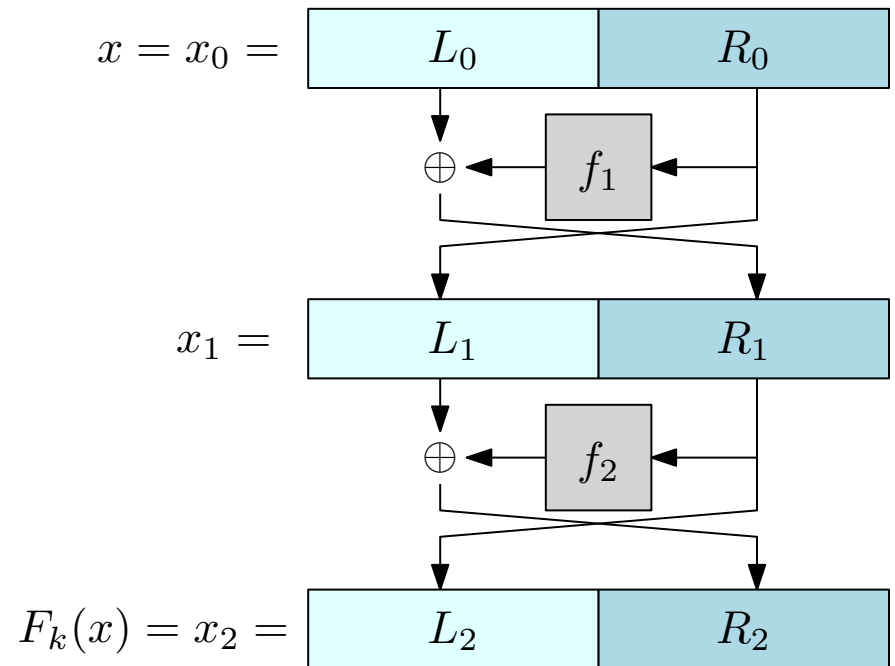


Security of 2-Round Feistel Networks

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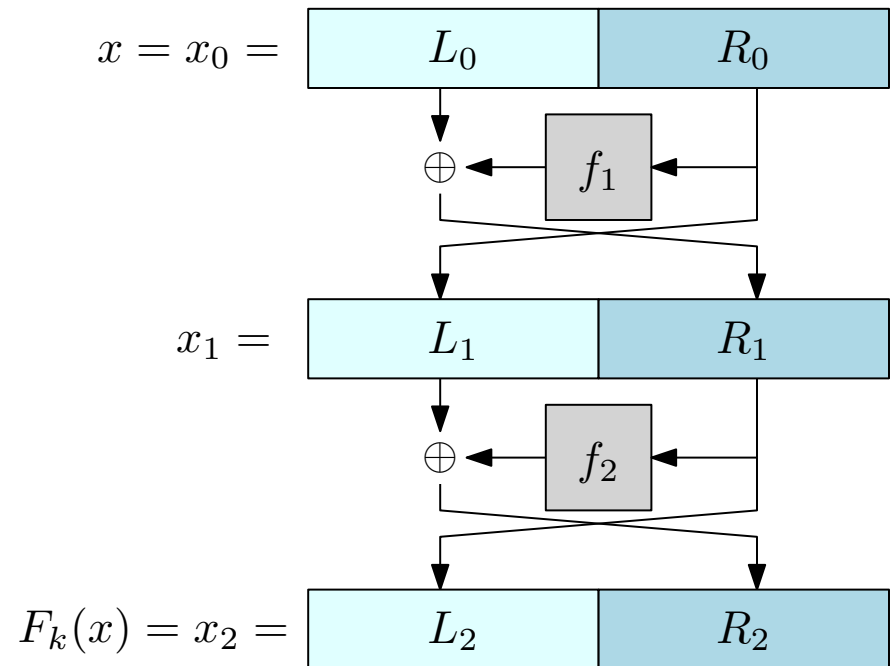
Security of 2-Round Feistel Networks

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Is this a Pseudorandom permutation?



Security of 2-Round Feistel Networks

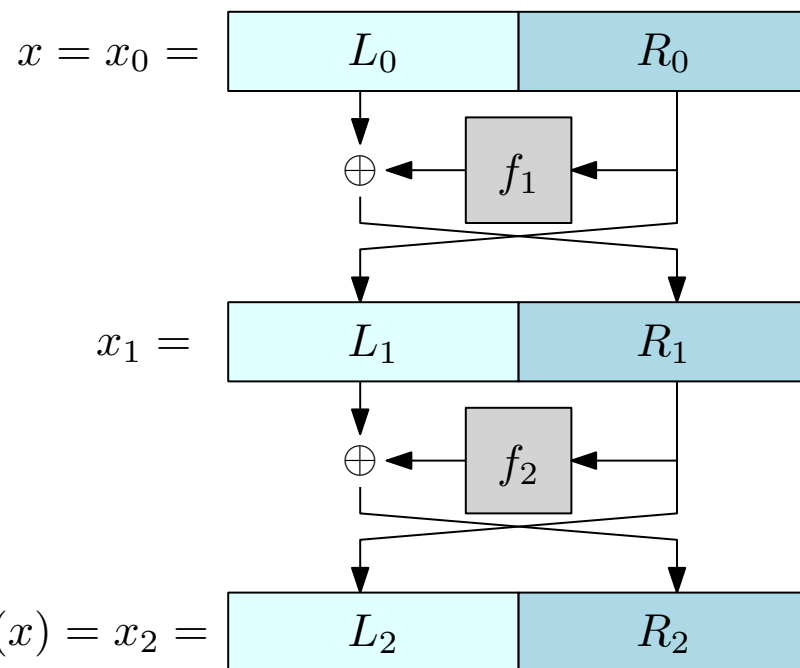
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Is this a Pseudorandom permutation?

No! Consider two different inputs
 $L_0 \| R_0$ and $L'_0 \| R'_0$



Security of 2-Round Feistel Networks

$$F_k(L_0 \| R_0) = L_2 \| R_2$$

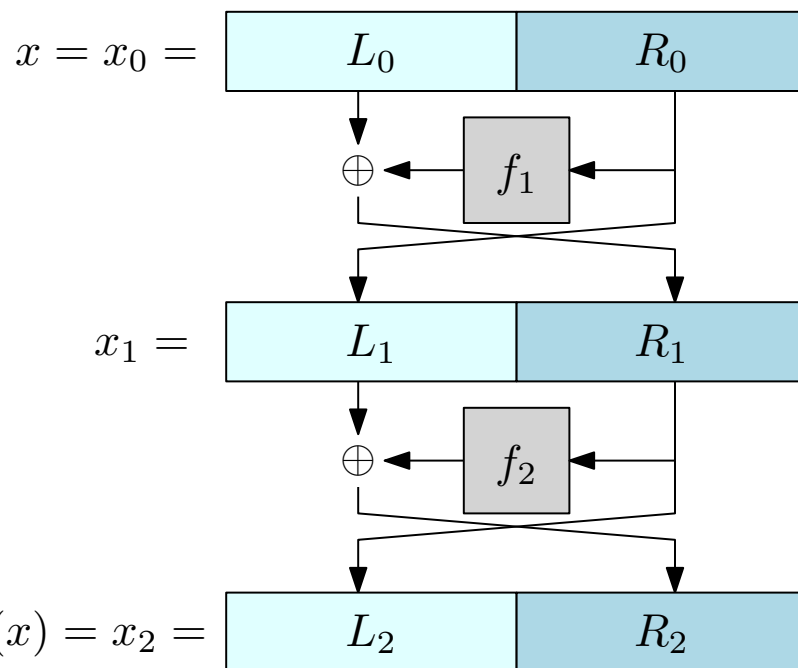
$$L_2 = R_1 = L_0 \oplus f_1(R_0)$$

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$$L_2 \oplus L'_2$$



Security of 2-Round Feistel Networks

$$F_k(L_0 \| R_0) = L_2 \| R_2$$

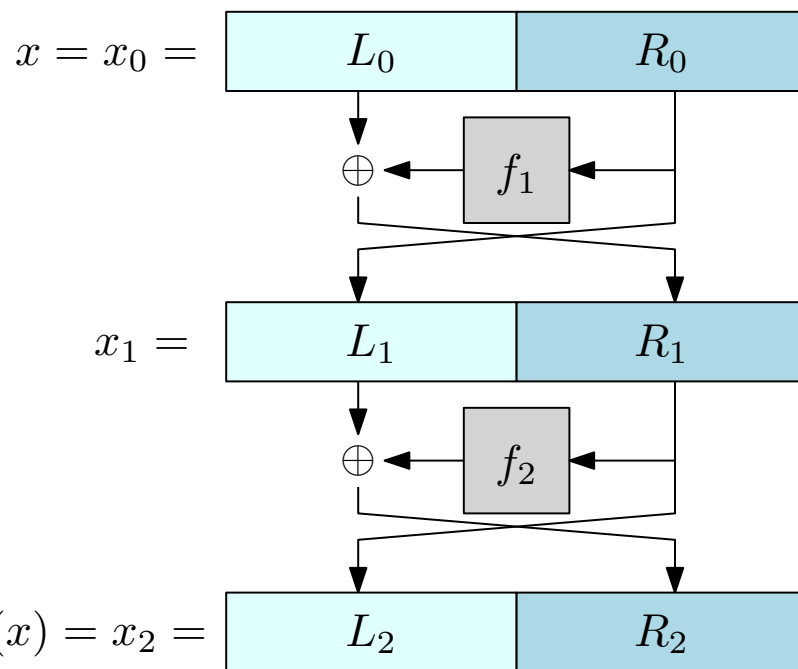
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No! Consider two different inputs
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$$L_2 \oplus L'_2 = L_0 \oplus f_1(R_0) \oplus L'_0 \oplus f_1(R'_0)$$



Security of 2-Round Feistel Networks

$$F_k(L_0 \| R_0) = L_2 \| R_2$$

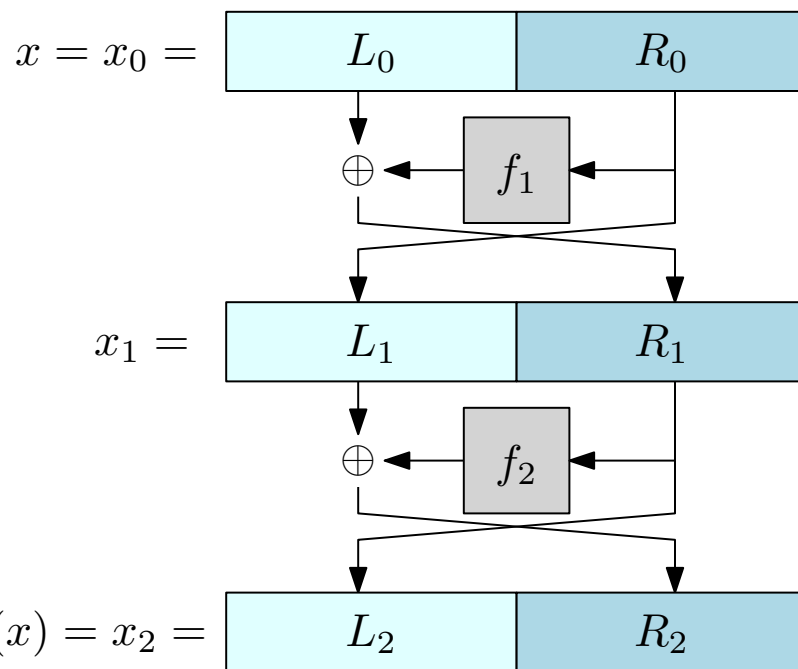
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How can we exploit this?

Security of 2-Round Feistel Networks

$$F_k(L_0 \| R_0) = L_2 \| R_2$$

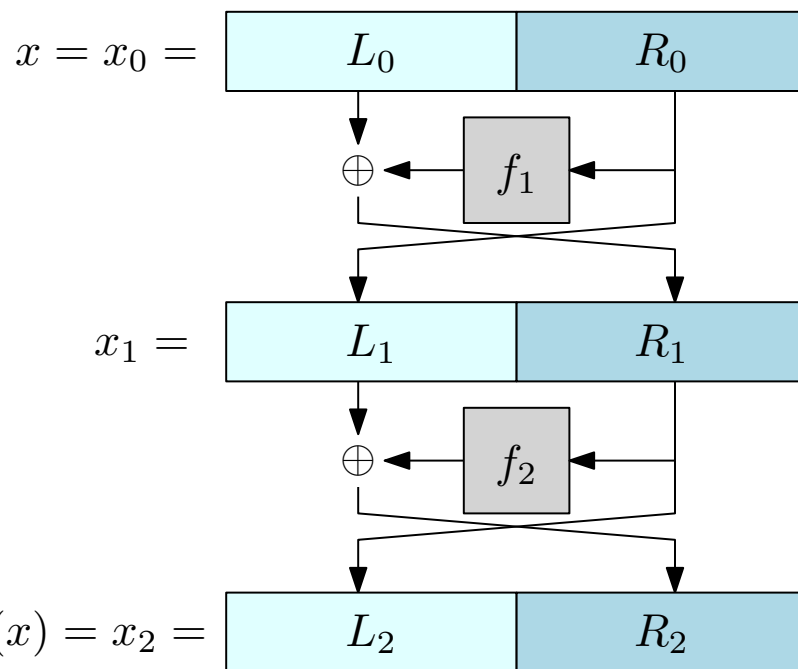
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How can we exploit this? Pick $R_0 = R'_0$

Security of 2-Round Feistel Networks

$$F_k(L_0 \| R_0) = L_2 \| R_2$$

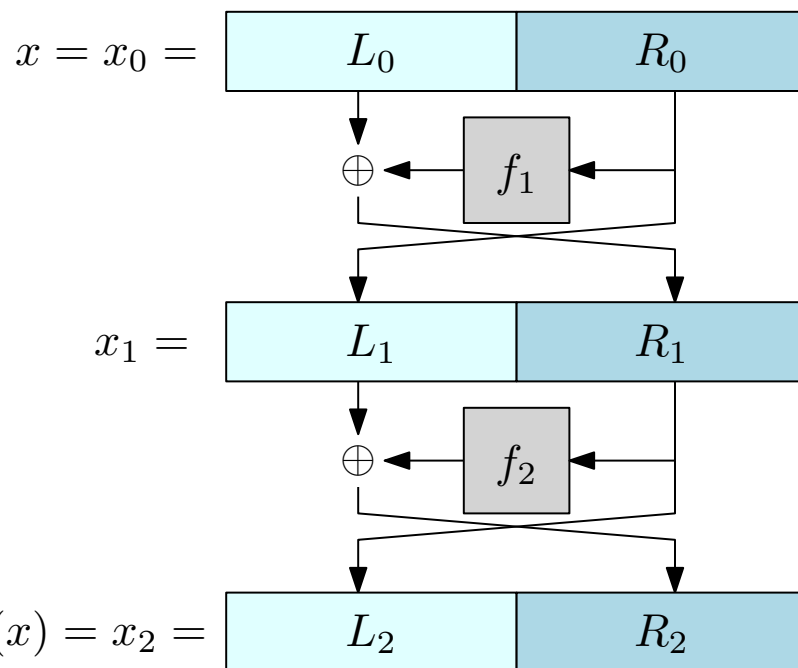
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Security of 2-Round Feistel Networks

$$F_k(L_0 \| R_0) = L_2 \| R_2$$

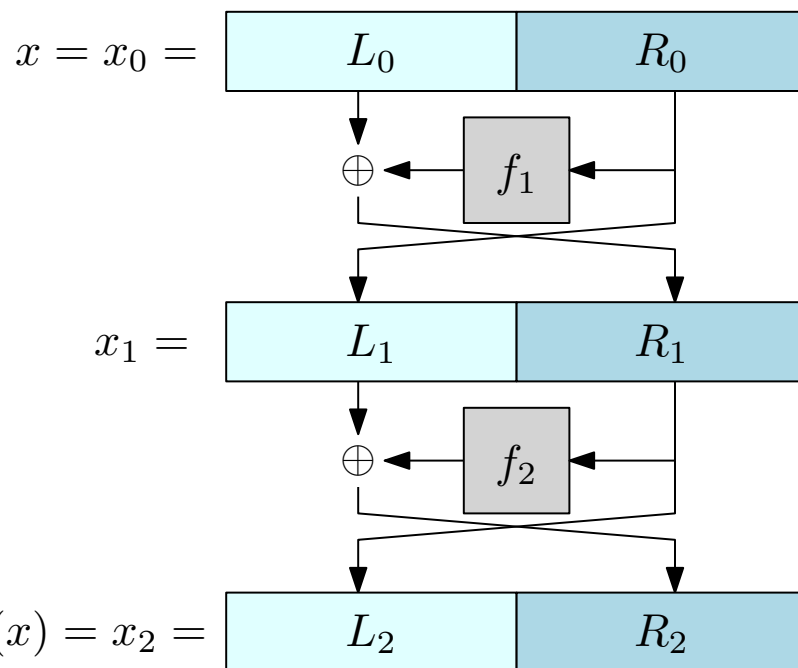
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How can we exploit this? Pick $R_0 = R'_0$

This is easy to distinguish from a random function. E.g., pick $L_0 = 0^{\ell/2}$ and $L'_0 = 1^{\ell/2}$

Security of 2-Round Feistel Networks

$$F_k(L_0 \| R_0) = L_2 \| R_2$$

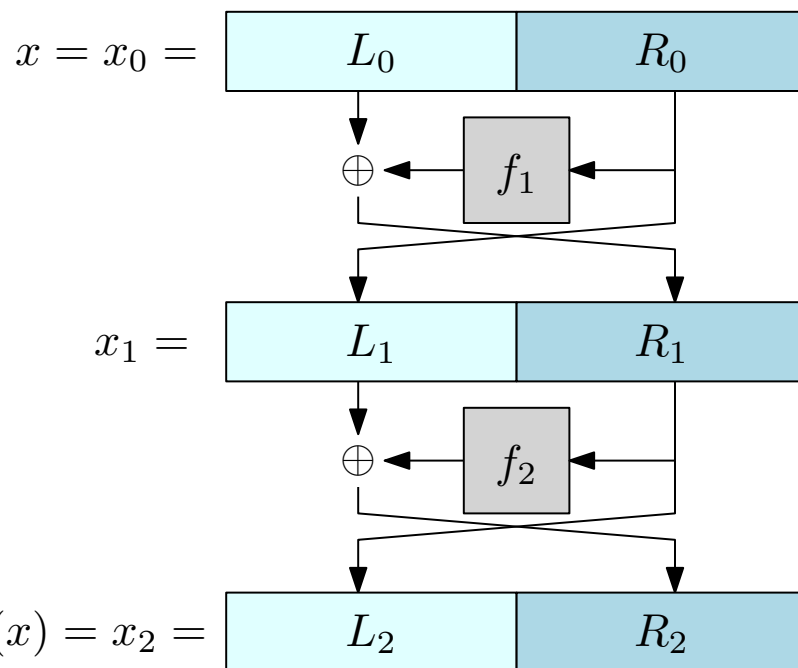
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Security of 2-Round Feistel Networks

$$F_k(L_0 \| R_0) = L_2 \| R_2$$

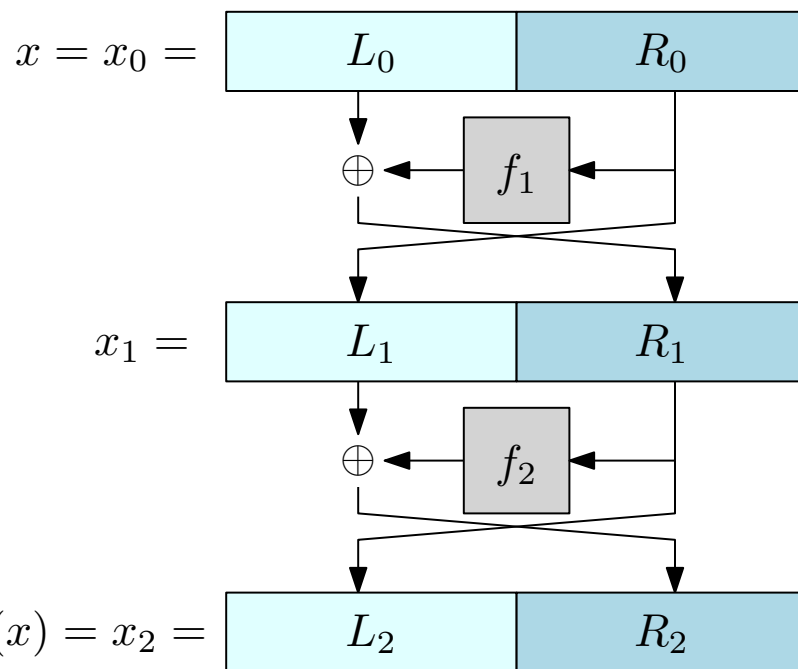
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Is this a Pseudorandom permutation?

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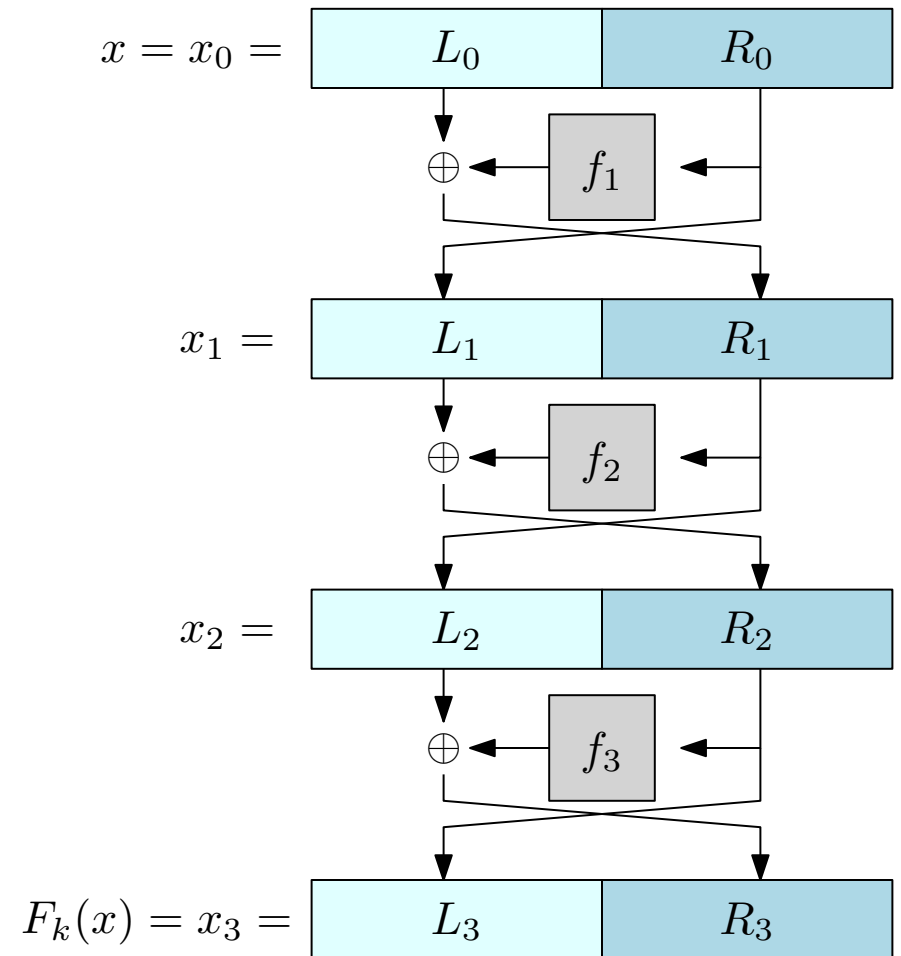


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Security of 3-Round Feistel Networks

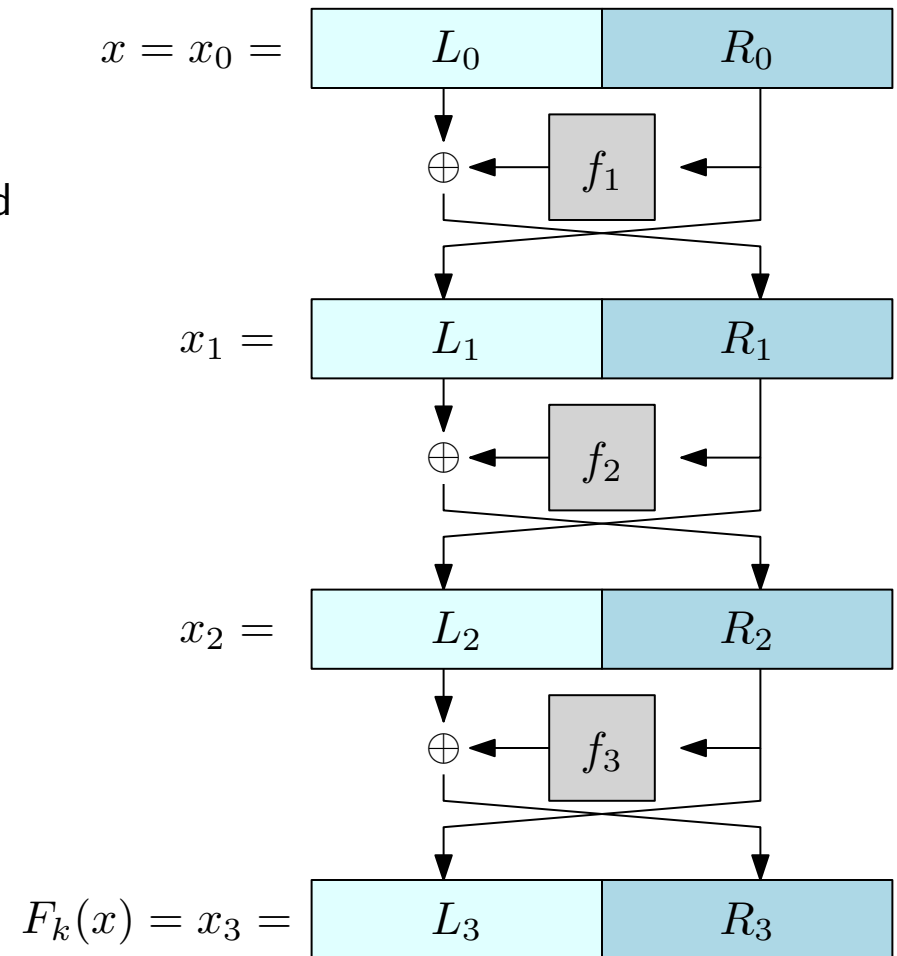
Is this a pseudorandom permutation?



Security of 3-Round Feistel Networks

Is this a pseudorandom permutation?

- Yes!
(If $f_i = F_{k_i}$ for some pseudorandom function F and the keys k_i are chosen independently and u.a.r.)

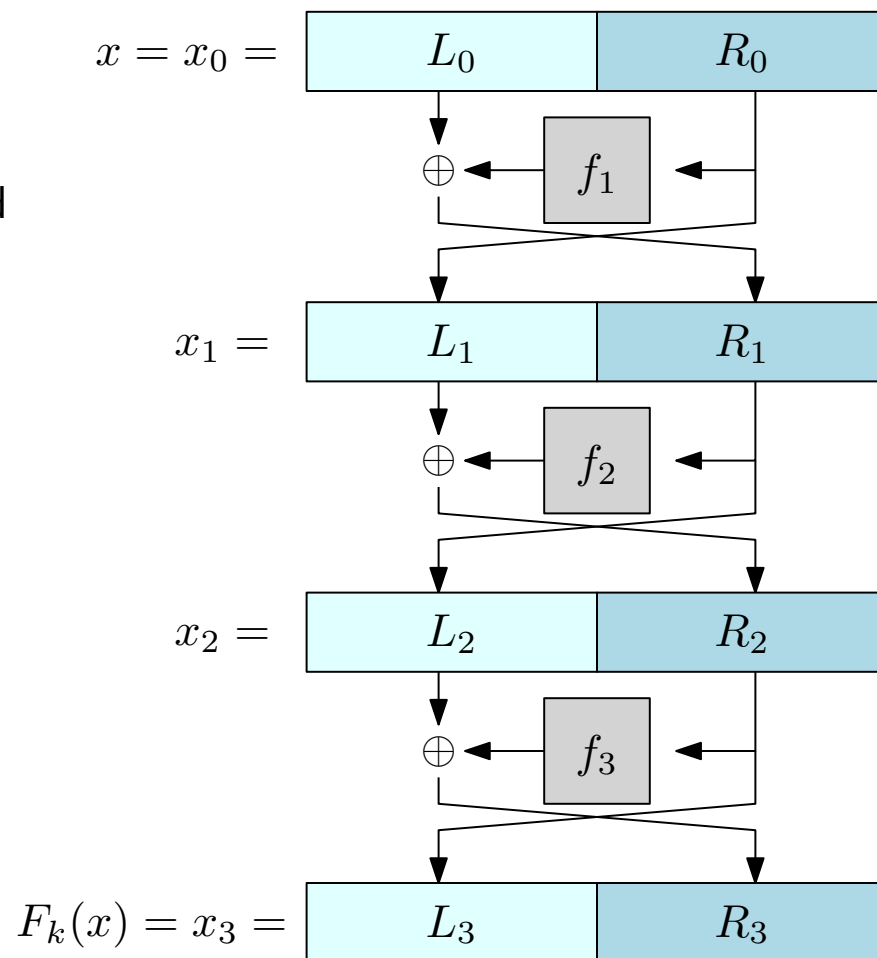


Security of 3-Round Feistel Networks

Is this a pseudorandom permutation?

- Yes!
(If $f_i = F_{k_i}$ for some pseudorandom function F and the keys k_i are chosen independently and u.a.r.)

Is this a **strong** pseudorandom permutation?



Security of 3-Round Feistel Networks

Is this a pseudorandom permutation?

- Yes!
(If $f_i = F_{k_i}$ for some pseudorandom function F and the keys k_i are chosen independently and u.a.r.)

Is this a **strong** pseudorandom permutation?

- No
- But 4-round Feistel networks are!

